

Risk Management and Decision Analysis

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Recall

Structuring Decisions

- Element of Decision Problems
 - Decision to made
 - Risk: Uncertainty
 - Outcomes & Consequences
 - Values & Objectives
- Environment of Decision Problems
 - Scope / Objectives / Attributes / Goals / Alternatives
- Influence Diagram vs. Decision Tree
- Decision Making Rules

Research Paper Review #1: Structuring Decisions

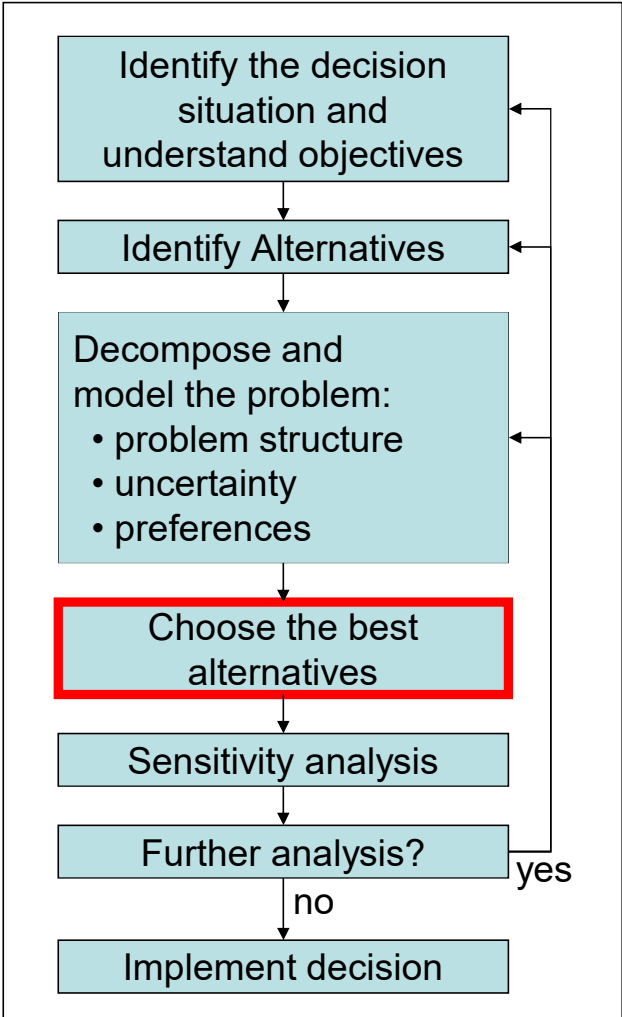
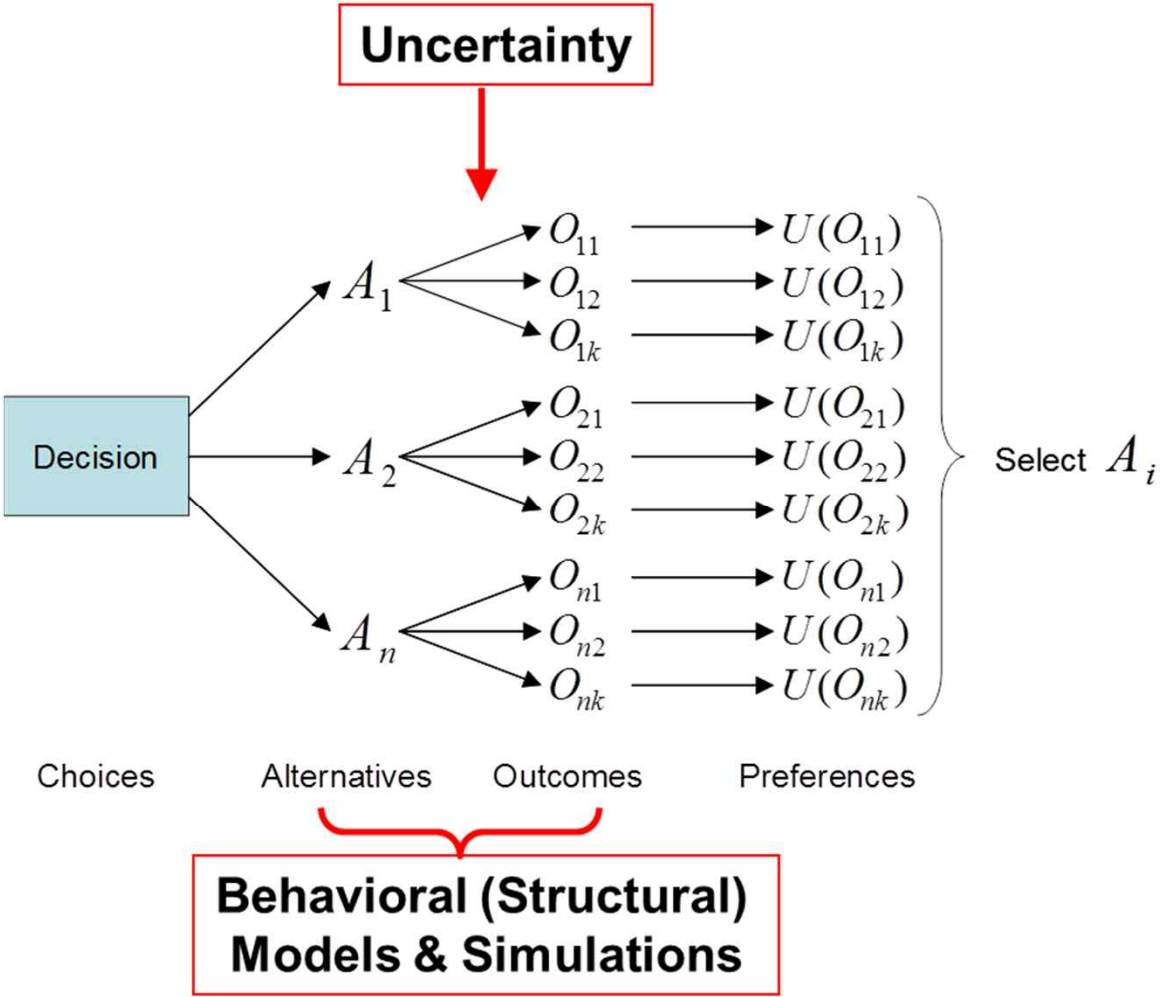
Group Presentation: 24th March 2021

1. Moussa, M., Ruwanpura, J., and Jergeas, G. (2006). "Decision Tree Modeling Using Integrated Multilevel Stochastic Networks." *Journal of Construction Engineering and Management* 132(12): 1254-1266.
2. Yet, B., Neil, M., Fenton, N., Constantinou, A., and Dementiev, E. (2018) "An Improved Method for Solving Hybrid Influence Diagrams." *International Journal of Approximate Reasoning*, Volume 95, 2018, Pages 93-112, ISSN 0888-613X, <https://doi.org/10.1016/j.ijar.2018.01.006>.
3. Abbas, A. E. (2006). "Entropy Methods for Joint Distributions in Decision Analysis." *IEEE Transactions on Engineering Management* 53(1): 146-159.
4. Medda, F. (2007). "A Game Theory Approach for the Allocation of Risks in Transport Public Private Partnerships." *International Journal of Project Management* 25(3): 213-218.

PART I

Decision Making Rules

Decision Analysis Process



Evaluation of Outcomes

- Profit – Measured in # Dollars
- Casualties – Measured in # Deaths
- Environmental Damage – Measured in # Polluted Soil
- Health Risk – Measured in # Infected People

Trade off between has to be made in almost any decision problem

van Dorp & Mazzuchi 2006

- Profit, Casualties, Environmental Damage, Health Risk – Single measure modeling trade-off needs to be developed: Measured in # Utilities.

Example: Condo Complex Size

The statement of a developer's decision problem

- To select the size of the new luxury condominium project that will lead to the largest profit
 - Given the uncertainty concerning the demand for the condominium
- Three decision alternatives
 - D1 = a small complex with 30 condos
 - D2 = a medium complex with 60 condos
 - D3 = a large complex with 90 condos
- Two states of nature concerning the chance event of the demand for the condominium
 - S1 = strong demand for the condos
 - S2 = weak demand for the condos

Example: Condo Complex Size

Payoff Table for the Condo Complex Size

- The consequence resulting from a specific combination of a decision alternative and a state of nature

Decision Alternative	State of Nature	
	Strong Demand (S1)	Weak Demand (S2)
Small complex (D1)	\$8 M	\$7 M
Medium complex (D2)	\$14 M	\$5 M
Large complex (D3)	\$20 M	- \$9 M

Example: Condo Complex Size

Decision-Making under **Certainty**

- If the state of nature is Strong Demand (S1)
 - The best complex size is Large complex (D3) & payoff is $V_{31} = \$20 \text{ M}$
- If the state of nature is Weak Demand (S2)
 - The best complex size is Small complex (D1) & payoff is $V_{12} = \$7 \text{ M}$

Decision Alternative	State of Nature	
	Strong Demand (S1)	Weak Demand (S2)
⇒ Small complex (D1)	\$8 M	\$7 M
<u>Dominated Alternative</u> Medium complex (D2)	\$14 M	\$5 M
⇒ Large complex (D3)	\$20 M	-\$9 M

Exercise #1. DM under Certainty

Consider the following problem with three decision alternatives and three states of nature with the following payoff table representing profits:

- If the state of nature is S1 → D1
- If the state of nature is S2 → D3
- If the state of nature is S3 → D2

Decision Alternative		State of Nature		
		S1	S2	S3
→	D1	4	4	-2
→	D2	0	3	-1
→	D3	1	5	-3

Decision-Making under **Uncertainty**

Decision-making under **uncertainty** without probabilities

- Use when the decision maker has little confidence in his ability to assess the probabilities
- Simple best-case or worst-case analysis is desirable
- Select the most appropriate approach according to the decision maker's judgment

Decision Making Approaches

- Optimistic Approach
- Conservative Approach
- Minimax Regret Approach

Decision-Making under **Uncertainty**

Decision-making under **uncertainty** without probabilities

1. Optimistic Approach

- Evaluate each decision alternative in terms of the best payoff that can occur
- Maximax Approach

Decision Alternative	State of Nature		Maximum Payoff
	Strong Demand (S1)	Weak Demand (S2)	
Small complex (D1)	\$8 M	\$7 M	\$8 M
Medium complex (D2)	\$14 M	\$5 M	\$14 M
→ Large complex (D3)	\$20 M	-\$9 M	\$20 M

Maximum of the maximum payoff values

Exercise #2. Optimistic Approach

Consider the following problem with three decision alternatives and three states of nature with the following payoff table representing profits:

- An optimistic decision maker would use the optimistic (maximax) approach. We choose the decision that has the largest single value in the payoff table.

Decision Alternative	State of Nature			Maximum Payoff
	S1	S2	S3	
D1	4	4	-2	4
D2	0	3	-1	3
D3	1	5	-5	5

Maximum of the maximum payoff values

Decision-Making under **Uncertainty**

Decision-making under **uncertainty** without probabilities

2. Conservative Approach

- Evaluate each decision alternative in terms of the worst payoff that can occur
- Maximin Approach

Decision Alternative	State of Nature		minimum Payoff
	Strong Demand (S1)	Weak Demand (S2)	
Small complex (D1)	\$8 M	\$7 M	\$7 M
Medium complex (D2)	\$14 M	\$5 M	\$5 M
Large complex (D3)	\$20 M	-\$9 M	-\$9 M


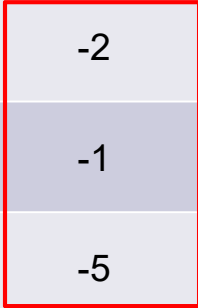

Maximum of the minimum payoff value

Exercise #3. Conservative Approach

Consider the following problem with three decision alternatives and three states of nature with the following payoff table representing profits:

- A conservative decision maker would use the conservative (maximin) approach. List the minimum payoff for each decision. Choose the decision with the maximum of these minimum payoffs.

Decision Alternative	State of Nature			minimum Payoff
	S1	S2	S3	
D1	4	4	-2	-2
D2	0	3	-1	-1
D3	1	5	-5	-5

   Maximum of the minimum payoff value

Decision-Making under **Uncertainty**

Decision-making under **uncertainty** without probabilities

3. Minimax Regret Approach

- Good for a decision maker that is neither purely optimistic nor purely conservative
- Opportunity Loss or Regret associated with decision alternative (D_i) when state of nature (S_j) has occurred: $R_{ij} = |V_j^* - V_{ij}|$
 V_j^* = the payoff value corresponding to the best decision for the state of nature S_j

Decision Alternative	State of Nature		Opportunity Loss or Regret		Maximum Regret
	Strong Demand (S1)	Weak Demand (S2)	Strong Demand (S1)	Weak Demand (S2)	
Small complex (D1)	\$8 M	\$7 M	$ 8 - 20 =$ \$12 M	$ 7 - 7 =$ \$0	\$12 M
→ Medium complex (D2)	\$14 M	\$5 M	$ 14 - 20 =$ \$6 M	$ 5 - 7 =$ \$2 M	\$6 M
Large complex (D3)	\$20 M	-\$9 M	$ 20 - 20 =$ \$0	$ -9 - 7 =$ \$16 M	\$16 M

Minimum of the maximum regret

Exercise #3. Minimax Regret Approach

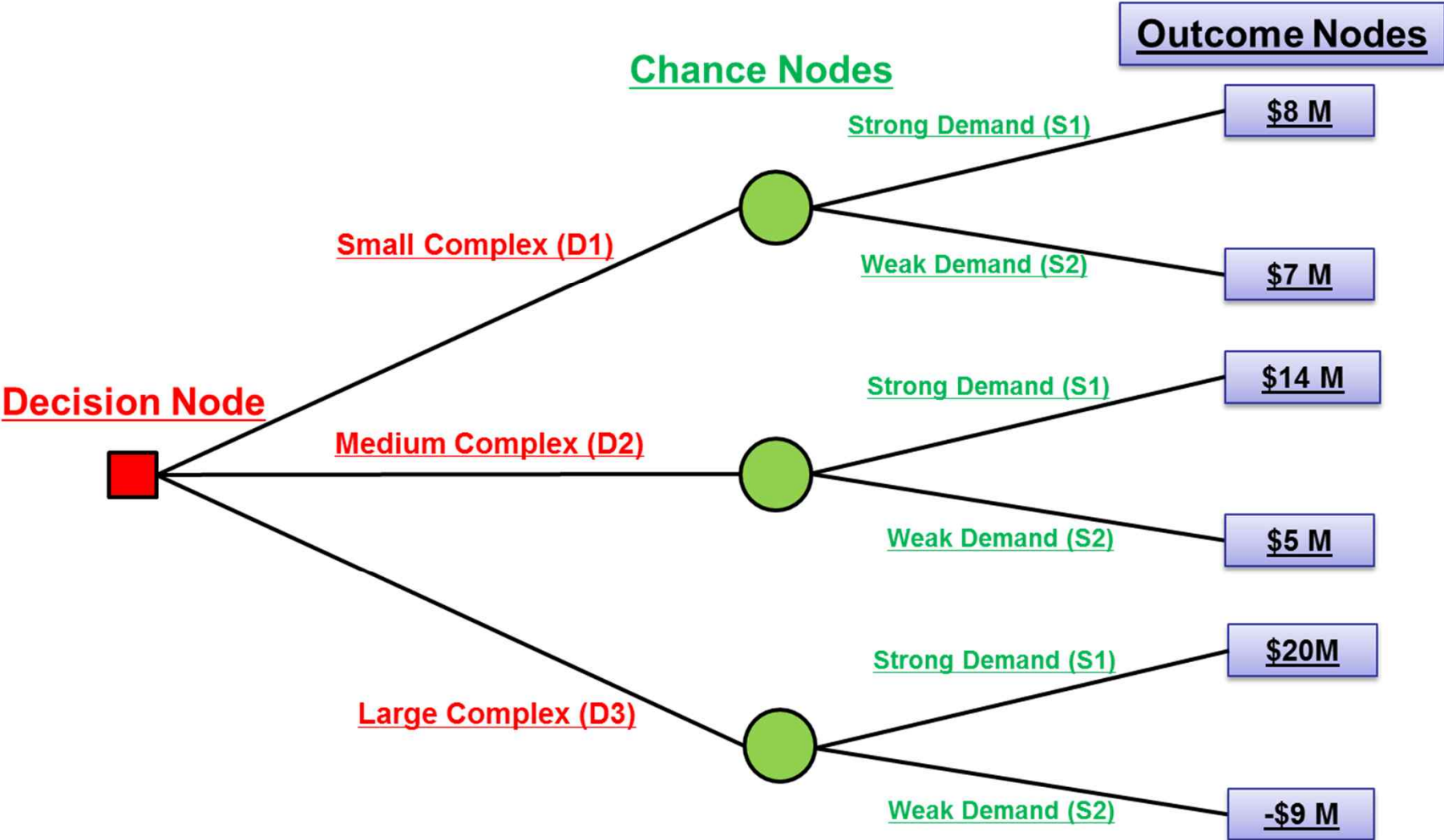
Consider the following problem with three decision alternatives and three states of nature with the following payoff table representing profits:

- For each decision list the maximum regret. Choose the decision with the minimum of these values.

Decision Alternative	State of Nature			Opportunity Loss or Regret			Maximum Regret
	S1	S2	S3	S1	S2	S3	
D1	4	4	-2	$ 4 - 4 = 0$	$ 4 - 5 = 1$	$ -2 + 1 = 1$	1
D2	0	3	-1	$ 0 - 4 = 4$	$ 3 - 5 = 2$	$ -1 + 1 = 0$	4
D3	1	5	-3	$ 1 - 4 = 3$	$ 5 - 5 = 0$	$ -3 + 1 = 2$	3

Minimum of the maximum regret

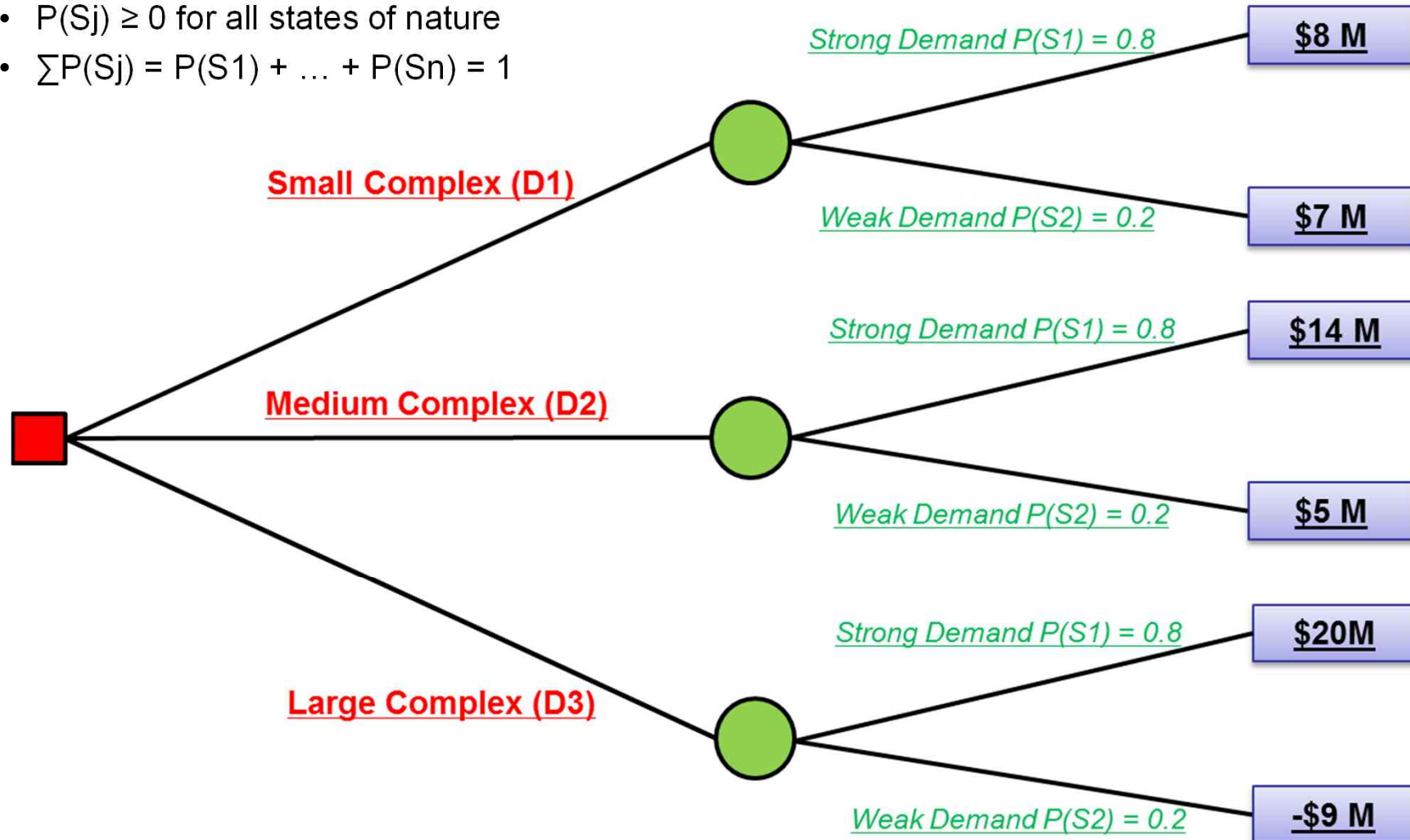
Decision Tree for the Condo Complex Size



Decision-making under **uncertainty** with probabilities

Assign Probabilities to Chance Nodes

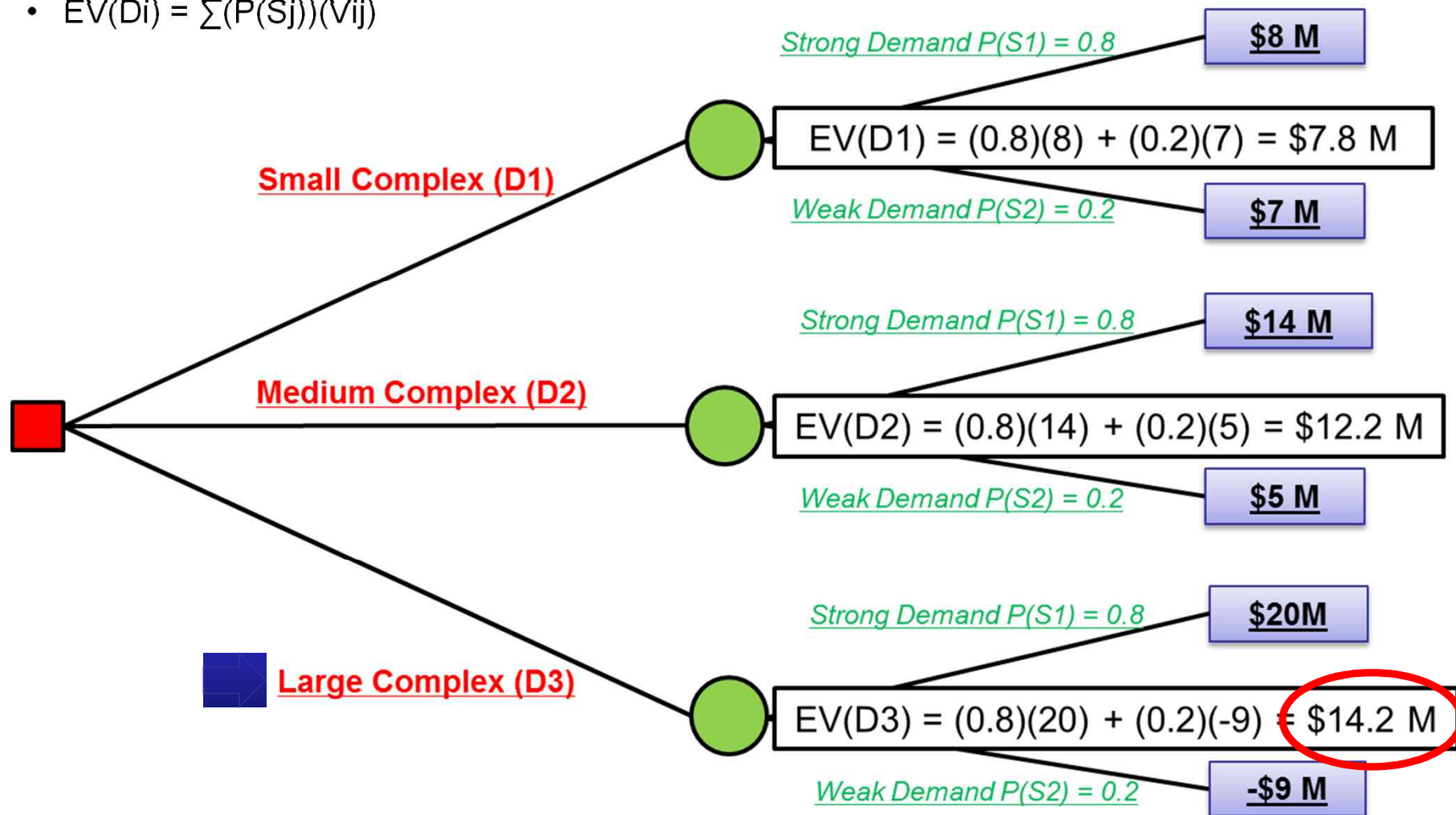
- $P(S_j) \geq 0$ for all states of nature
- $\sum P(S_j) = P(S_1) + \dots + P(S_n) = 1$



Decision-making under **uncertainty** with probabilities

- Expected Value (EV) Calculation & Decision Making

- $EV(D_i) = \sum(P(S_j))(V_{ij})$



Decision Tree Analysis Procedure

A decision tree is a graphical representation of the expected value calculations and consists of

- Decision Nodes: represented by squares, are variables or actions that the decision maker controls
- Chance Event Nodes: represented by circles, are variables or events that cannot be controlled by the decision maker
- Outcome (Terminal or End) Nodes: represented by unconnected branches, are endpoints where outcome values are attached

By convention, the tree is drawn from left to right & Calculations are done from right to left, as follows

- Chance node: calculate EV
- Replace a decision node with the value of its best alternative
- If a cost value lies along a branch, recognize it as the cost of passing from right to left

Example: Decision to Crash a Project

A contractor working on an outdoor construction project in a coastal area is reviewing progress on July 1

- If **normal** progress is maintained and **no time is lost** due to hurricanes, the job will be completed on July 31
- However, due to the poor weather conditions in the area after July 16, there will be only a **40 percent chance of finishing on time**
- It is estimated that there is a **50 percent chance of a minor hurricane**, which will cause a **delay of 5 days**
- There is a **10 percent chance of a major hurricane**, which will cause a **delay of 10 days**

Example: Decision to Crash a Project

Immediate Decision

It must be decided now whether to **start a crash program** (the 1st decision alternative) on July 2

- An additional cost of \$75 per day should be taken into account to finish the project on July 16

The 2nd decision alternative is to maintain the normal schedule and review the progress on July 31

Secondary Decisions

one has occurred and the project is delayed

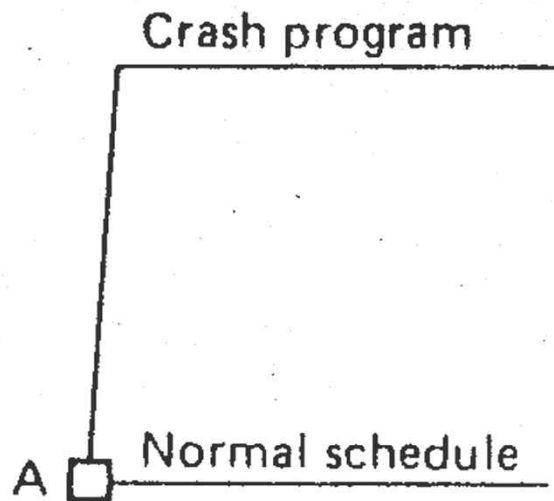
- There will be a choice of **accepting the delay at a certain penalty cost** or **trying to crash the program**
 - The **penalty cost** for delay of completion will be \$400 per day for the first 5 days and \$600 per day for the second 5 days
 - The **additional cost of a crash program** after the hurricane will be \$200 per day

Outcome: the **total additional cost** is computed as the sum of delay penalty cost and crash cost

Example: Decision to Crash a Project

A **decision Node** (□) is drawn to represent the most immediate decision that the contractor must make

- A branch is drawn from the node to represent each alternative that is available to the decision-maker at this decision point

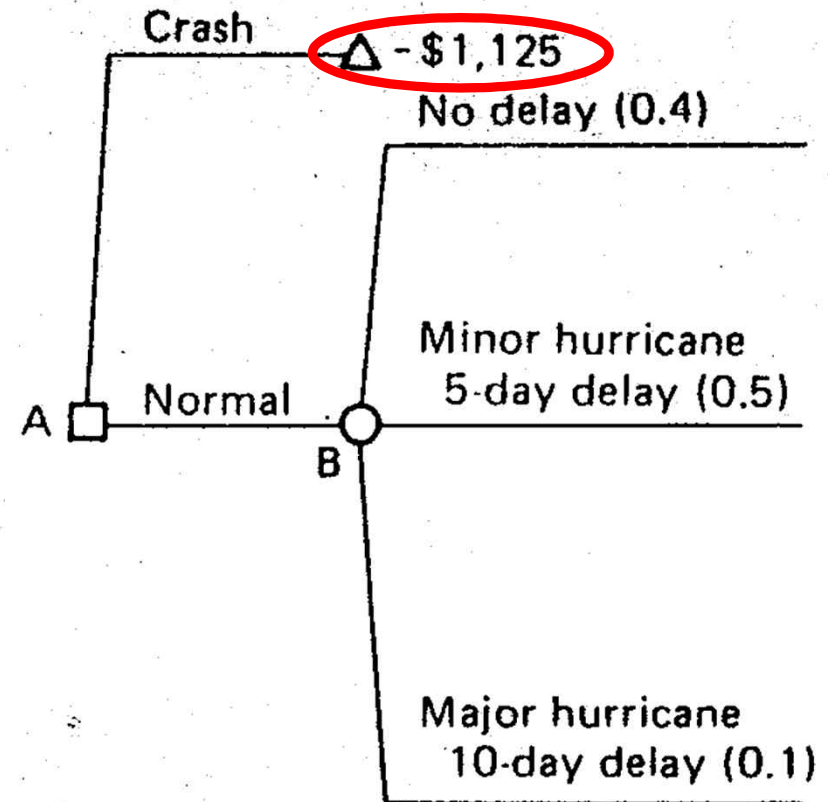


(a) Most immediate decision

Example: Decision to Crash a Project

The potential results of each of the decision alternatives are then modeled

- The outcome of immediately launching a **crash program** is that the project will be completed at an added cost of \$75 per day for 15 days, or a total of \$1125
- This is represented by assigning a terminal node with $-\$1125$

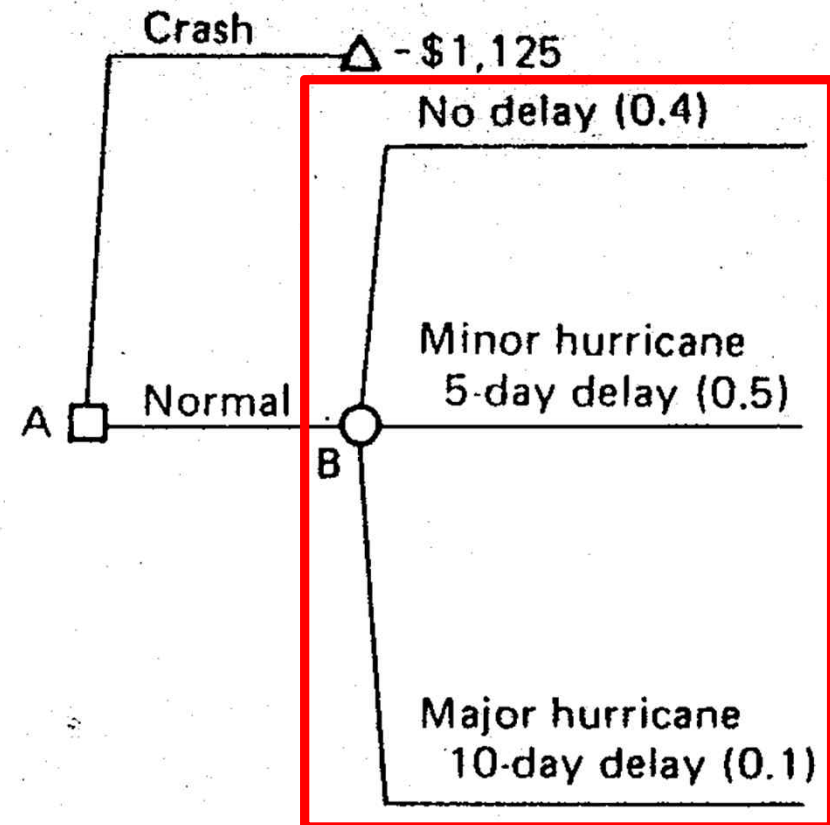


(b) Possible outcomes from first decision

Example: Decision to Crash a Project

The second alternative (**normal**) has three possible outcomes

- No delay with a probability of 0.4
- 5-day delay with a probability of 0.5
- 10-day delay with a probability of 0.1
- This is represented as a **chance Node** (○), where each possible result is represented by a branch and the probability of occurrence is entered along the branch



(b) Possible outcomes from first decision

Example: Decision to Crash a Project

The probabilities can be determined in a number of ways

- **Educated guess by the contractor**, whose judgment is based on present weather conditions and on recollection or weather conditions at the same period in previous years
- **Historical records** or local weather conditions
 - Suppose that the past 50 years of weather records for the area are available to the contractor
 - The number of years in which there was either a minor or major hurricane during the period August 16-31 can be counted

$$P(\text{no delay}) = \frac{\text{number of years with no hurricane}}{\text{total number of years counted}} = \frac{20}{50} = 0.4$$

$$P(\text{minor hurricane}) = \frac{\text{number of years with a minor hurricane}}{\text{total number of years counted}} = \frac{25}{50} \\ = 0.5$$

$$P(\text{major hurricane}) = \frac{\text{number of years with a major hurricane}}{\text{total number of years counted}} = \frac{5}{50} \\ = 0.1$$

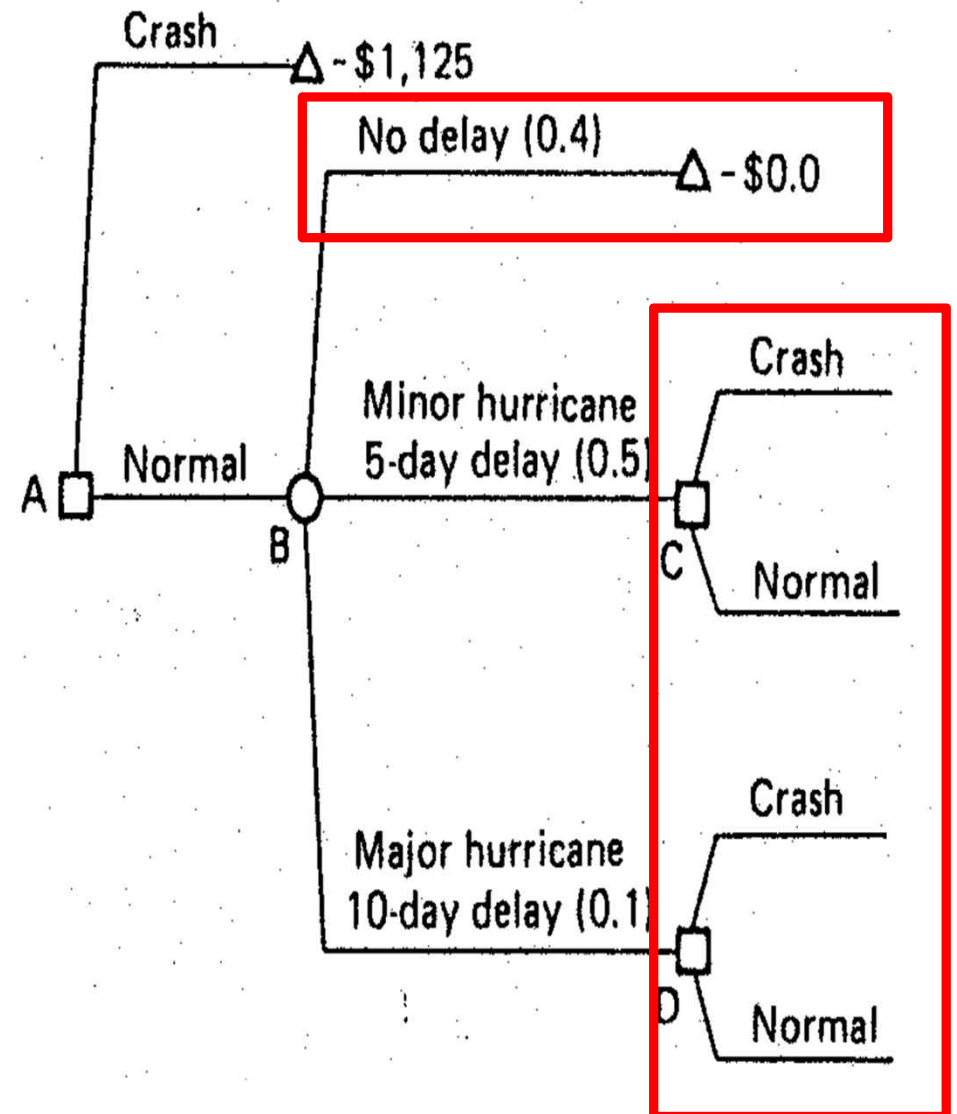
Example: Decision to Crash a Project

The only possible result of no delay on July 31 is that the project is completed on time with no delay cost

- Represented by a terminal node with a value of -\$0.0

For the other two outcomes

- The contractor must decide next **whether to launch a crash program or to maintain a normal pace** (Secondary Decisions)



Example: Decision to Crash a Project

The possible results (outcomes) of a **crash program** after a **minor hurricane** causes a 5-day delay

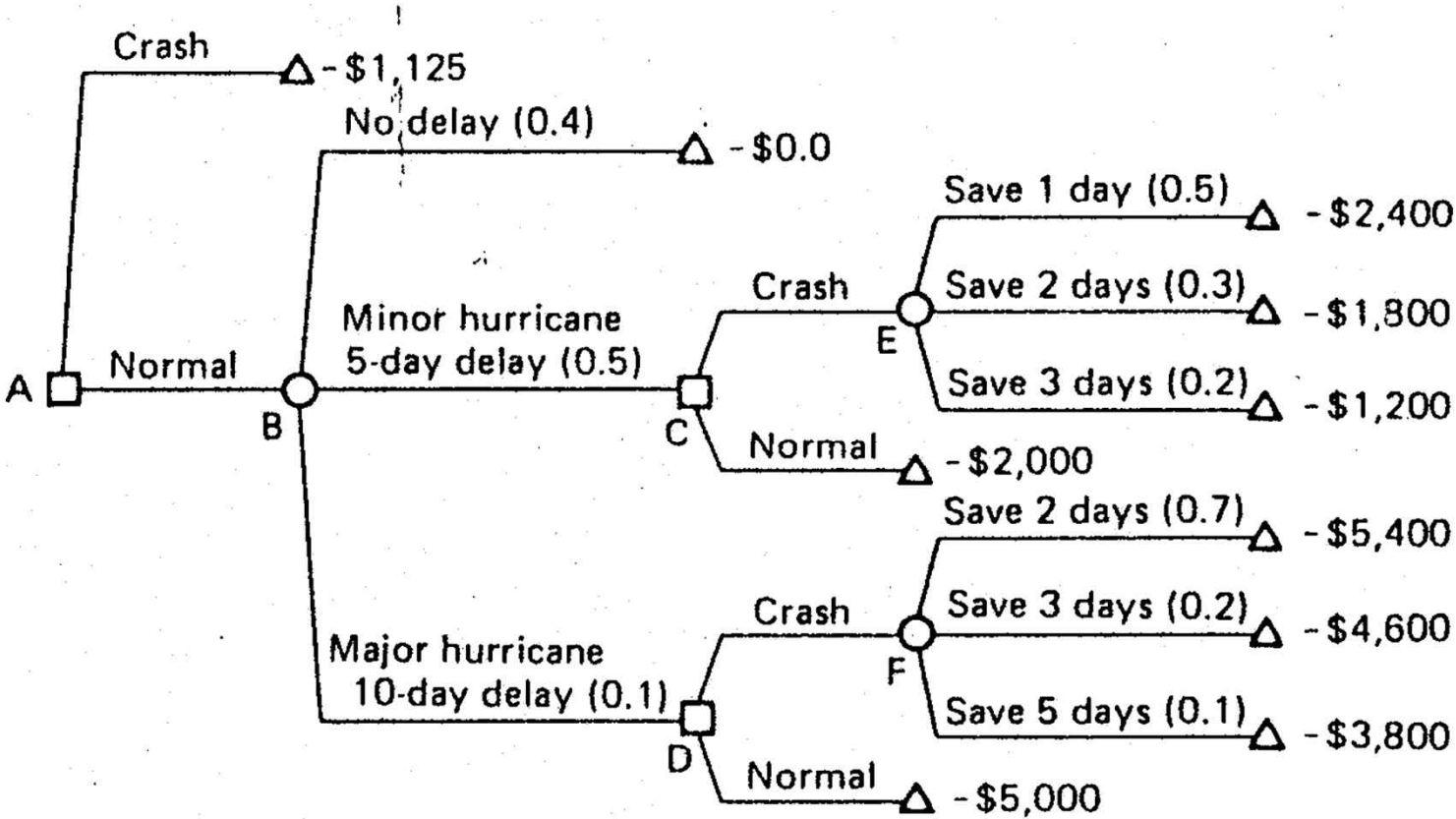
Crash Program Result	Probability	Total Additional Cost = Penalty Cost + Crash Cost
Save 1 day	0.5	$\$1600 + 800 = \2400
Save 2 days	0.3	$\$1200 + 600 = \1800
Save 3 days	0.2	$\$800 + 400 = \1200

The possible results of a **crash program** after a **major hurricane** causes a 10-day delay

Crash Program Result	Probability	Total Additional Cost
Save 2 days	0.7	$\$(2000 + 1800) + 1600 = \5400
Save 3 days	0.2	$\$(2000 + 1200) + 1400 = \4600
Save 4 days	0.1	$\$(2000 + 600) + 1200 = \3800

Example: Decision to Crash a Project

The decision tree is finally completed by drawing branches to represent *possible outcomes* of *secondary decisions* & *their costs* at terminal nodes

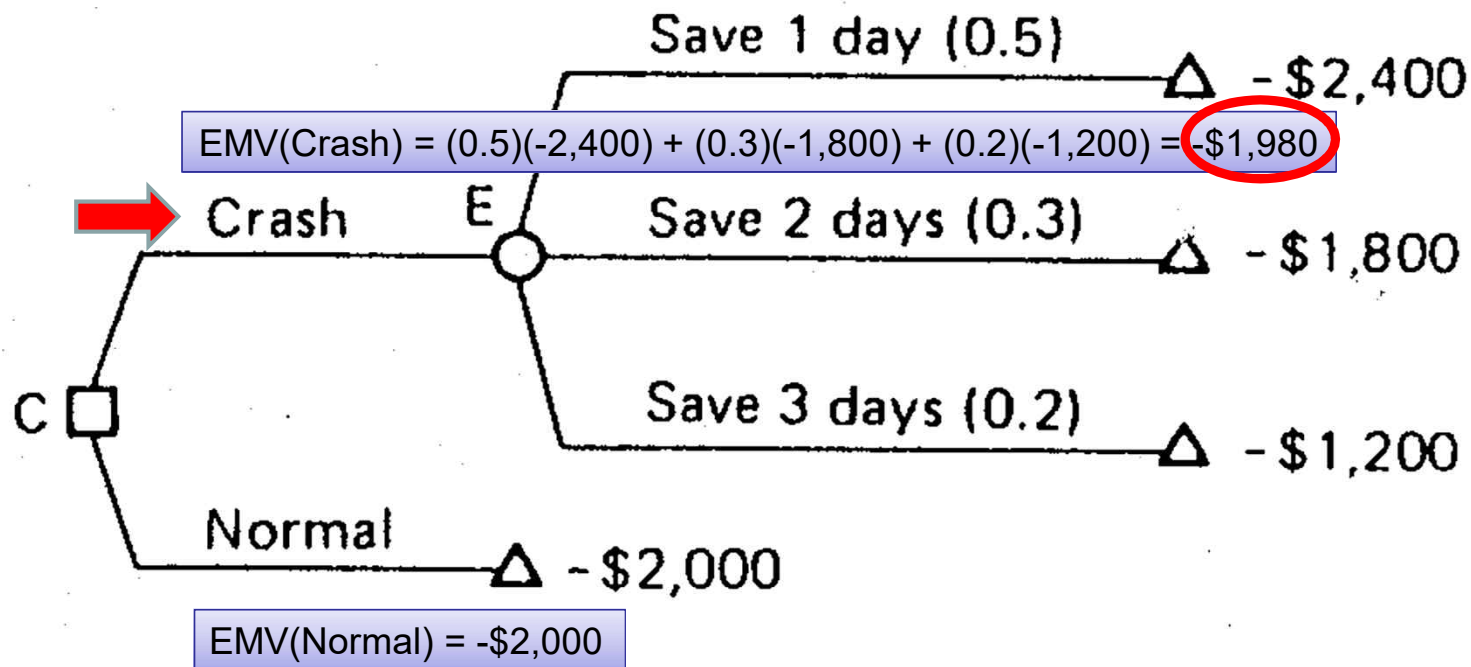


(d) Complete decision tree

Decision Analysis based on EMV

Expected Monetary Value (EMV)

- Comparing alternatives for the *minor hurricane secondary decision*

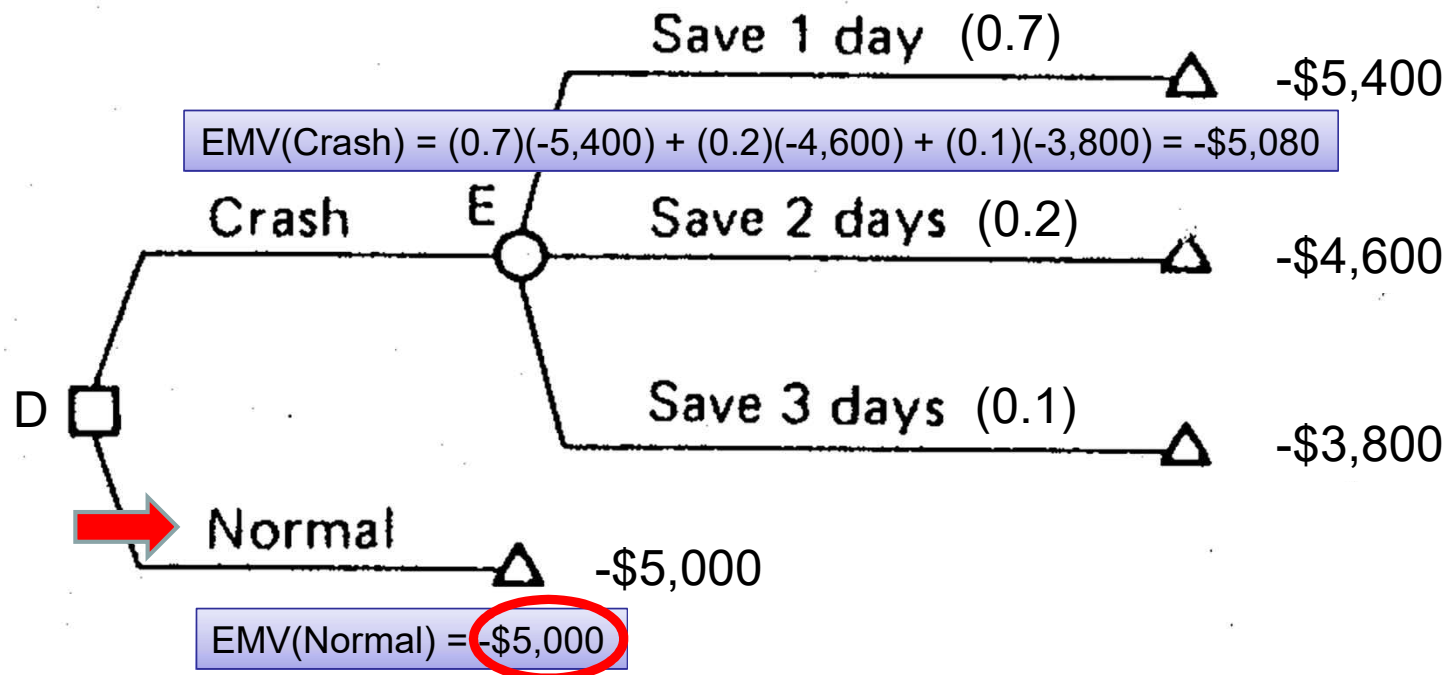


- If the decision maker is a **believer** in EMV, a **logical** choice is to start a crash program (**Decision Rule**)
 - The outcome at Decision Node C is $-\$1,980$

Decision Analysis based on EMV

Expected Monetary Value (EMV)

- Comparing alternatives according to Expected Monetary Value (EMV) for the *major hurricane*

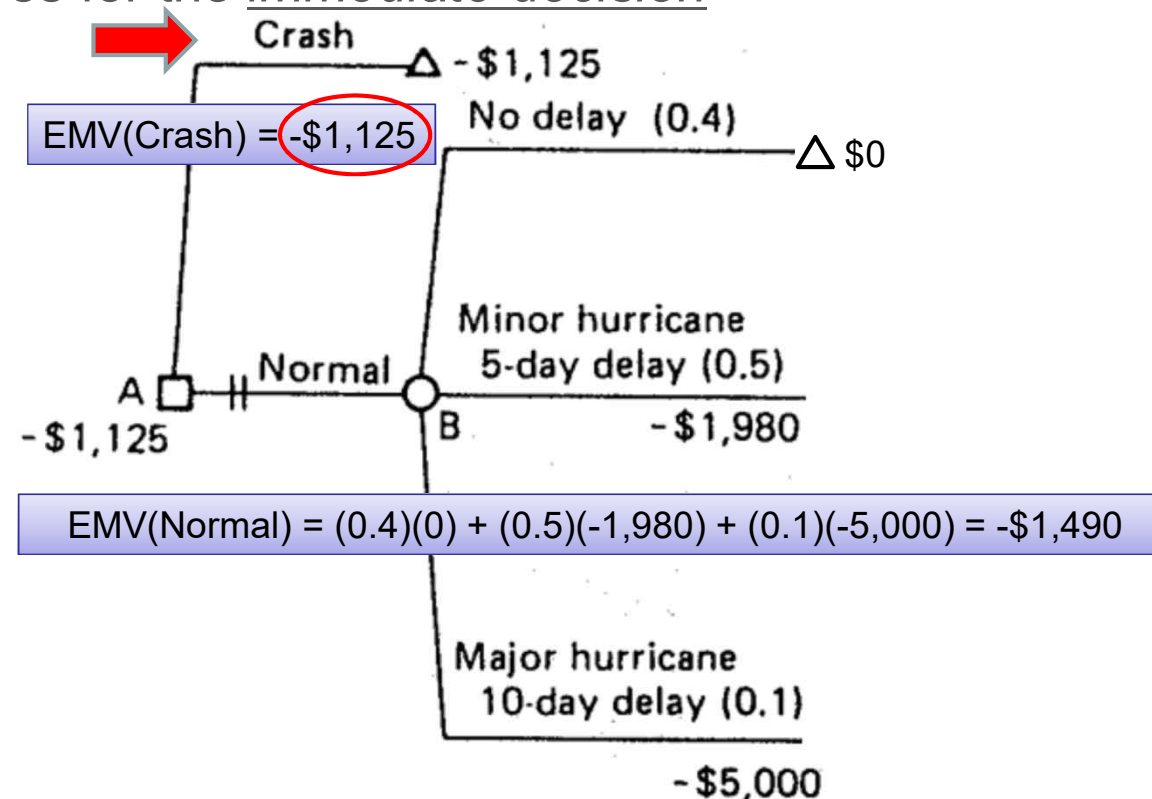


- If the decision maker is a **believer** in EMV, a **logical** choice is to start a *normal* program (**Decision Rule**)
 - The outcome at Decision Node D is -\$5,000

Decision Analysis based on EMV

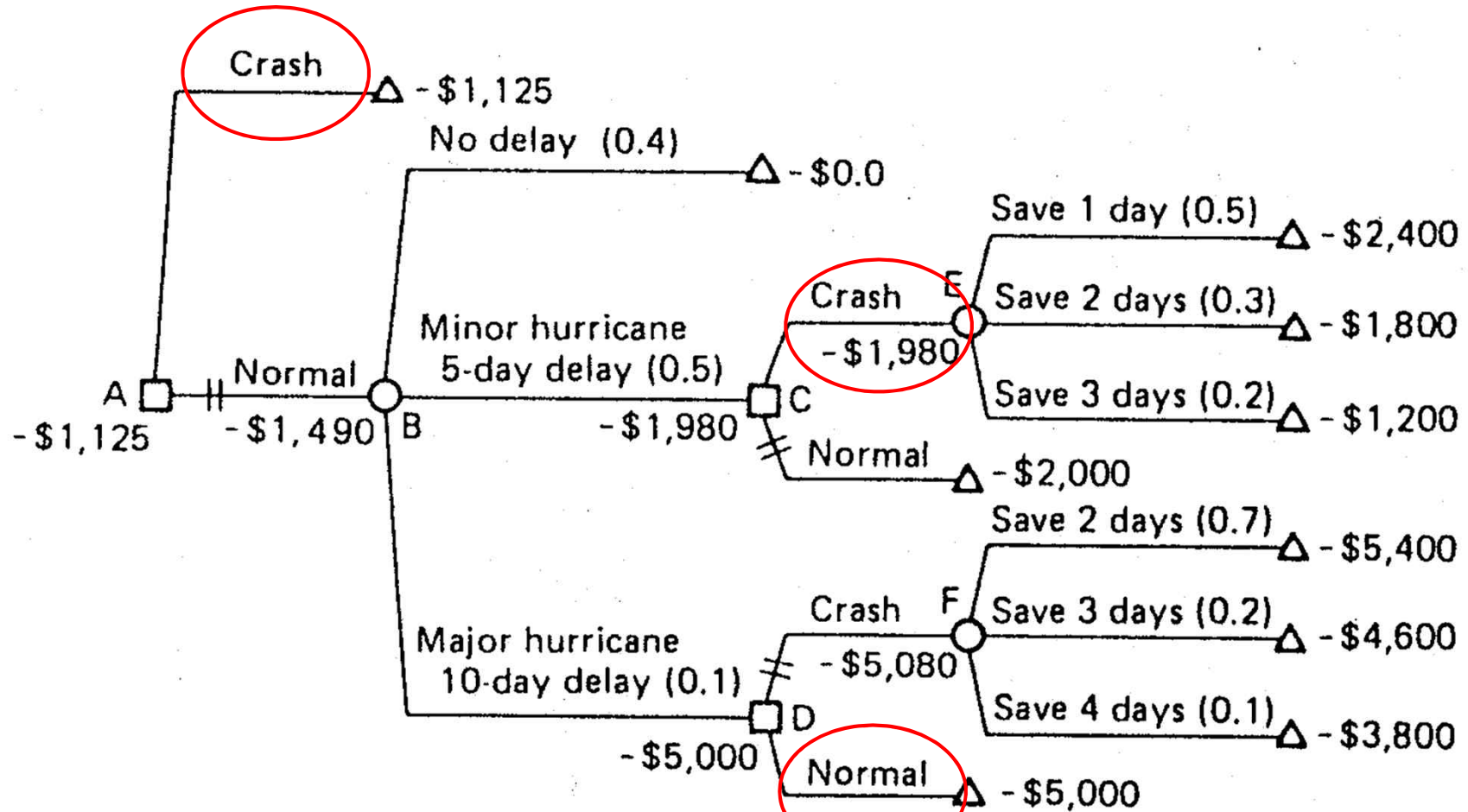
Expected Monetary Value (EMV)

- Comparing alternatives for the *immediate decision*



- If the decision maker is a **believer** in EMV, a **logical** choice is to start a crash program (**Decision Rule**)
 - The outcome at Decision Node A is -\$1,125

Decision Analysis based on EMV



Decision analysis based on EMV.

Decision Analysis based on EMV

In general, the **decision analysis procedure** based on **EMV** consists of the following steps (***backward calculation***)

- Starting from the *tips* of the tree, compute the EMV at the chance nodes closest to the tips
- At each decision node, compare the EMVs of the alternatives and choose the alternative with highest profit or minimum loss
 - Assign the EMV of the chosen alternative to that decision node
- Proceed node by node toward the root of the tree
 - The EMV for the decision problem is obtained at the root of the tree
- Trace back through the tree to determine the optimum decision as indicated by the branches that do not have hatch marks
 - This set of decisions constitutes the optimum strategy

Value of Information

- Frequently **information** is available which can improve **the probability estimates for the states of nature** (certainty).
- The **Expected Value of Perfect Information (EVPI)** is the increase in the expected profit that would result if one knew with certainty which state of nature would occur.
- The EVPI provides an **upper bound on the expected value of any sample or survey information.**

Value of Information

EVPI Calculation

- Step 1
 - Determine the **optimal return** corresponding to each state of nature.
- Step 2
 - Compute **the expected value of these optimal returns**.
- Step 3
 - Subtract the **EV of the optimal decision** from the amount determined in step (2).

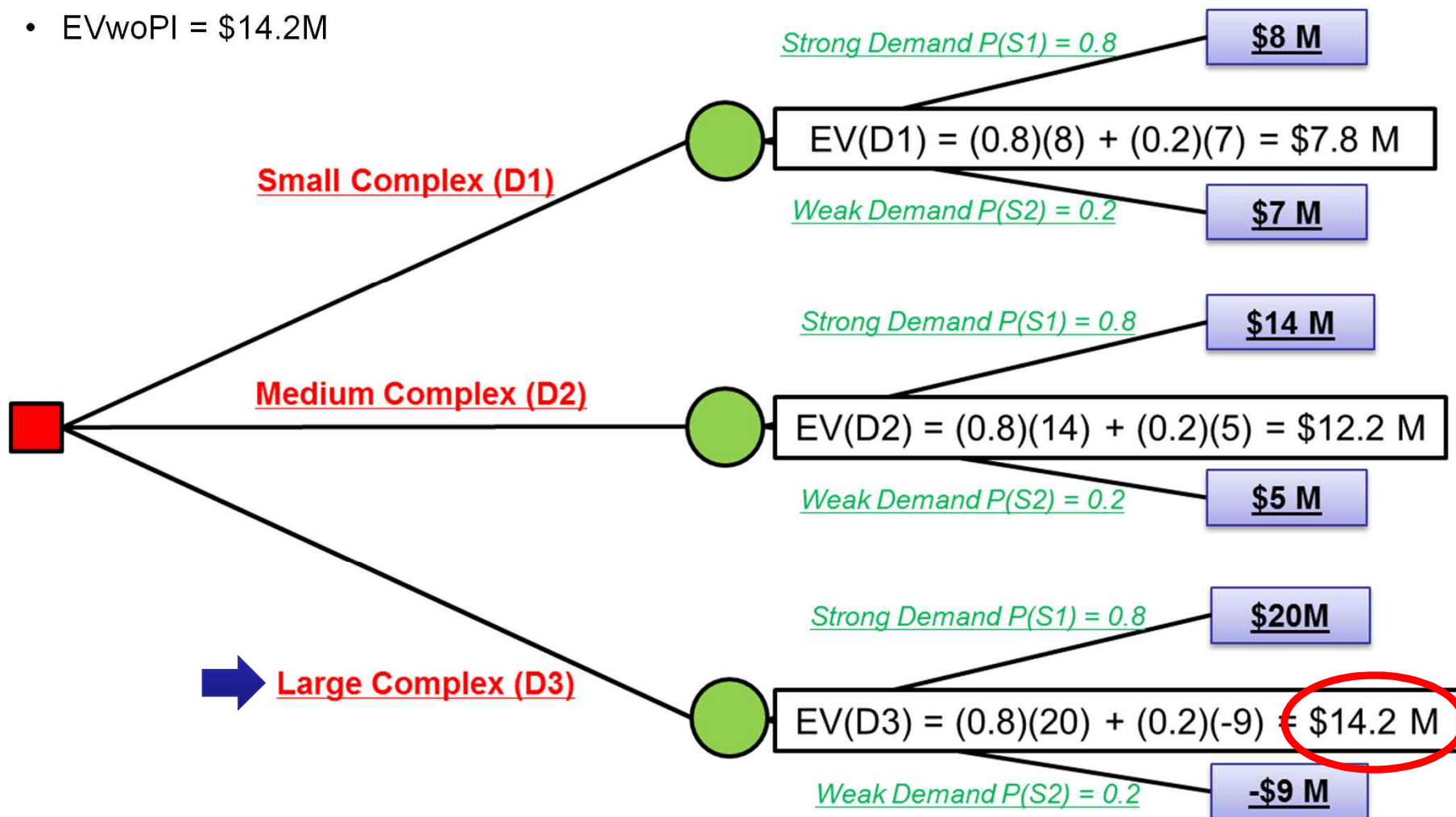
Value of Information

Example: Condo Complex Size

Decision-Making without Perfect Information

- Expected Value without Perfect Information (EVwoPI)

- EVwoPI = \$14.2M



Value of Information

Example: Condo Complex Size

Decision-Making with Perfect Information

- If we know that the state of nature is (S1)
 - The best complex size is Large complex (D3) & payoff is $V_{31} = \$20 \text{ M}$
- If we know that the state of nature is (S2)
 - The best complex size is Small complex (D1) & payoff is $V_{12} = \$7 \text{ M}$

	State of Nature	
Decision Alternative	Strong Demand (S1)	Weak Demand (S2)
Small complex (D1)	\$8 M	\$7 M
Medium complex (D2)	\$14 M	\$5 M
Large complex (D3)	\$20 M	-\$9 M

- Consider probabilities of strong & weak demand
 - ♦ $P(S1) = 0.8$ & $P(S2) = 0.2$
 - ♦ **Expected Value with Perfect Information (EVwPI)** = $(0.8)(20) + (0.2)(7) = \$17.4\text{M}$

Value of Information

- Expected Value of Perfect Information (EVPI)
- Expected Value **with** Perfect Information about the states of nature (EVwPI)
- Expected Value **without** Perfect Information about the states of nature (EVwoPI)
 - $EVPI = |EVwPI - EVwoPI|$
 - In our condo size complex decision
 - ♦ $EVPI = |17.4 - 14.2| = \$3.2M$

Expected Opportunity Loss (EOL)


- Minimax Regret Approach

Decision Alternative	State of Nature		Opportunity Loss or Regret		Maximum Regret
	Strong Demand (S1)	Weak Demand (S2)	Strong Demand (S1)	Weak Demand (S2)	
Small complex (D1)	\$8 M	\$7 M	$ 8 - 20 =$ \$12 M	$ 7 - 7 =$ \$0	\$12 M
Medium complex (D2)	\$14 M	\$5 M	$ 14 - 20 =$ \$6 M	$ 5 - 7 =$ \$2 M	\$6 M
Large complex (D3)	\$20 M	-\$9 M	$ 20 - 20 =$ \$0	$ -9 - 7 =$ \$16 M	\$16 M

Minimum of the maximum regret

Expected Opportunity Loss (EOL)

- Expected Opportunity Loss (EOL) for each decision alternative
 - Minimum EOL always provide the best decision alternative under this decision rule
 - Minimum Expected Opportunity Loss = Expected Value of Perfect Information (Minimum EOL = EVPI = \$3.2M)

	Opportunity Loss or Regret		
Decision Alternative	Strong Demand P(S1) = 0.8	Weak Demand P(S2) = 0.2	Expected Opportunity Loss (EOS)
Small complex (D1)	\$12 M	\$0	$(0.8)(12)+(0.2)(0) = \$9.6M$
Medium complex (D2)	\$6 M	\$2 M	$(0.8)(6)+(0.2)(2) = \$5.2M$
 Large complex (D3)	\$0	\$16 M	$(0.8)(0)+(0.2)(16) = \$3.2M$

Implications of the EMV Criterion

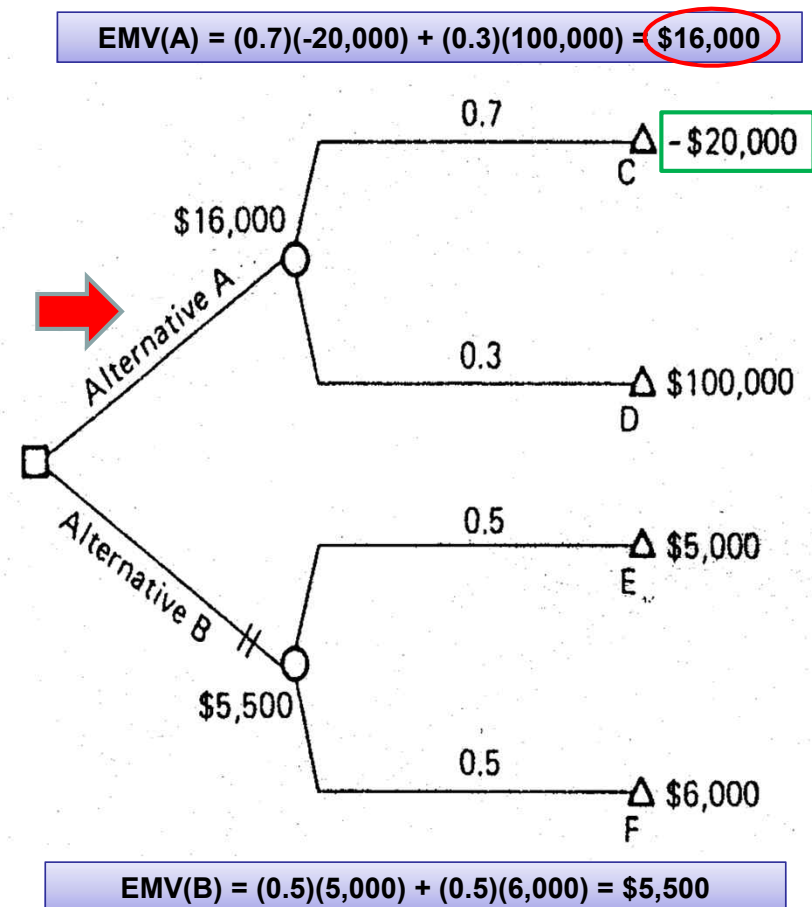
In using the EMV as a '**decision criterion**,' a decision maker must always keep in mind that this approach carries the following two implications:

- The decision maker is betting on the law of averages, since the EMV of an alternative means that if he or she chooses this alternative many times under similar conditions he or she would receive this much in return on the average
 - This may be quite different from the amount that is actually obtained if the choice is only made once because only one final outcome results
- The EMV is a completely objective measure of the value of money and implies that *every dollar within a sum of money provides the same amount of satisfaction*
 - It does not consider personal differences about the value of money

Implications of the EMV Criterion

Consider the decision problem:

- **Alternative A** has a 30 percent chance of making \$100,000 profit and a 70 percent chance of losing \$20,000
- **Alternative B** has a 50 percent chance of making a \$5000 profit and an equal chance of gaining \$6,000
- According to a strict EMV analysis, alternative A has an EMV of \$16,000 and alternative B has an EMV of only \$5,500; hence, alternative A is the better choice



Implications of the EMV Criterion

On the other hand, it is obvious that in practice many people would be quite happy to accept alternative B but absolutely refuse to accept alternative A

- However, alternative A may be the preferred choice of one who draws little satisfaction from the meager sum of \$20,000
- In reality, a decision maker often bases the financial decisions on the **available capital** and on **the willingness to take risk**
- Consider an aggressive investor who has a capital of \$50,000
 - Confronted with the decision problem above, the decision would probably be strongly influenced by the **opportunity of gaining** \$100,000 and thus **tripling the capital**
 - The prospect of losing \$20,000 would not seriously deter the selection of alternative A, because even there is a loss of \$20,000, there is still \$30,000 remaining as capital

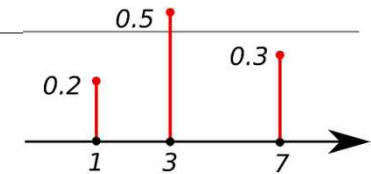
Implications of the EMV Criterion

Suppose next that the decision maker is a **conservative investor** who has a capital of only \$10,000

- Although there would undoubtedly be interest in gaining \$100,000, there would also be a strong deterrence by the prospect of losing \$20,000
 - It would not only wipe out the capital, but would result in a \$10,000 debt
- Since alternative B means a certain gain of either \$5000 or \$6000, which is more than 50 percent of present capital, there would be a strong inclination to choose alternative B
 - To this investor, the computed EMVs fail to measure truly the relative values of the two alternatives
- The **EMV criterion failed** in this example because the monetary values assigned to the various outcomes at the terminal nodes C, D, E, and F do not **truly reflect the values of these outcomes to the decision maker**

Risk Profiles & Cumulative Risk Profiles

Risk Profile



- Graph that shows probabilities for each of the possible outcomes given a particular decision strategy
- Risk Profile is a **probability mass function** for the **discrete** random variable (Y) representing the outcomes for the given decision strategy

Cumulative Risk Profile

- Graph that shows cumulative probabilities associated with a risk profile
- Cumulative risk profile is a cumulative distribution function for the discrete random variable (Y) representing the outcomes for the given decision strategy

Decision Path and Strategy

- **Decision Path**

- A path starting at the left most node up to the values at the end of a branch by selecting one alternative from decision nodes or by following one outcome from uncertainty nodes
- Represents a possible future scenario

- **Decision Strategy**

- The collection of decision paths connected to one branch of the immediate decision by selecting one alternative from each decision node along these paths
- Represents specifying at every decision in the decision problem what we would do, if we get to that decision
- We may not get there due to outcome of previous uncertainty nodes

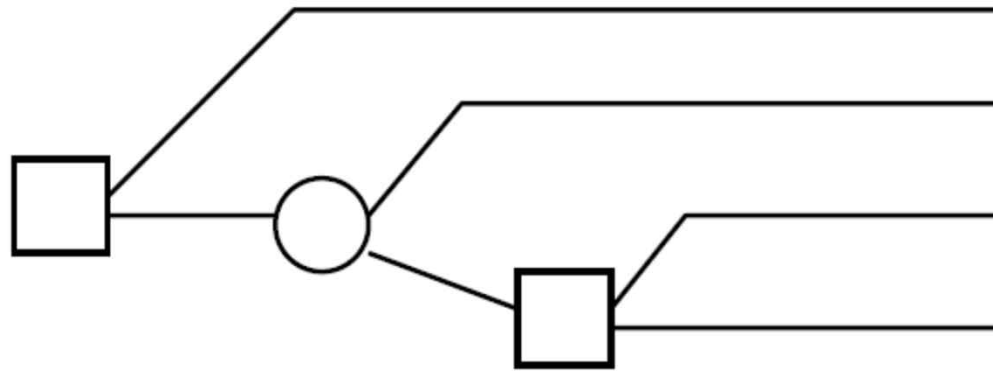
- **Optimal decision strategy**

- That decision strategy which results in the highest EMV if we maximize profit and the lowest EMV if we minimize cost

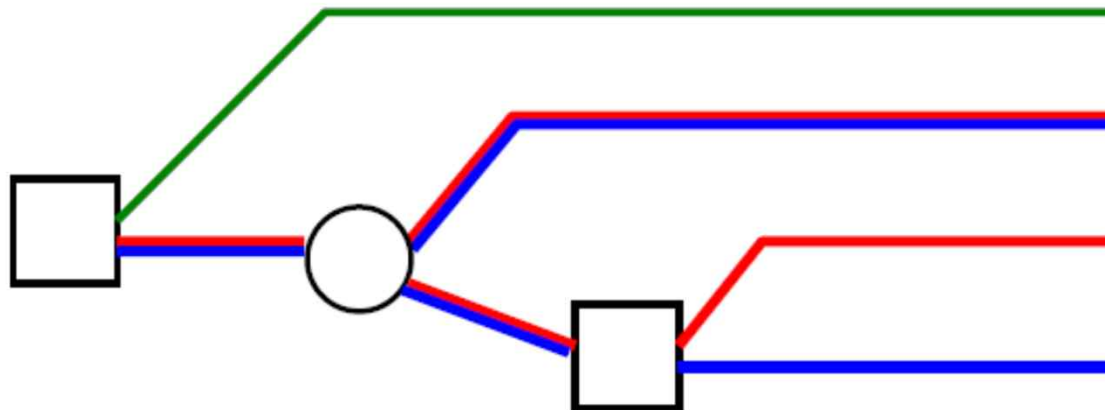
Counting Decision Strategies

How many decision strategies in Example 1?

Example 1



Example 1



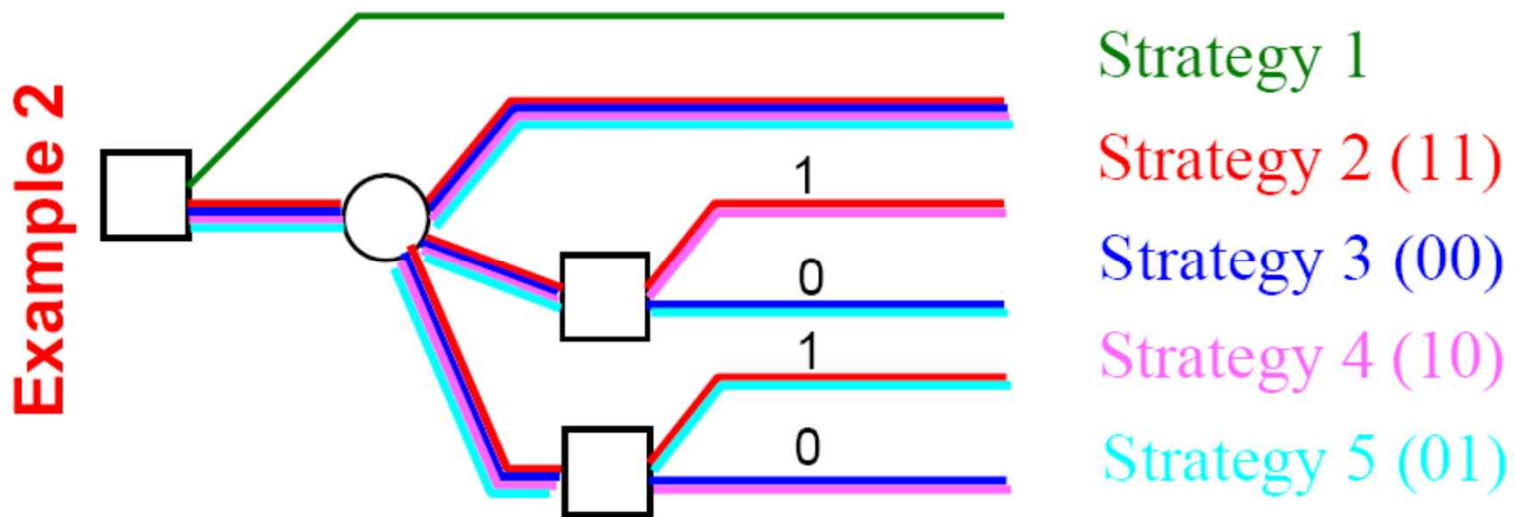
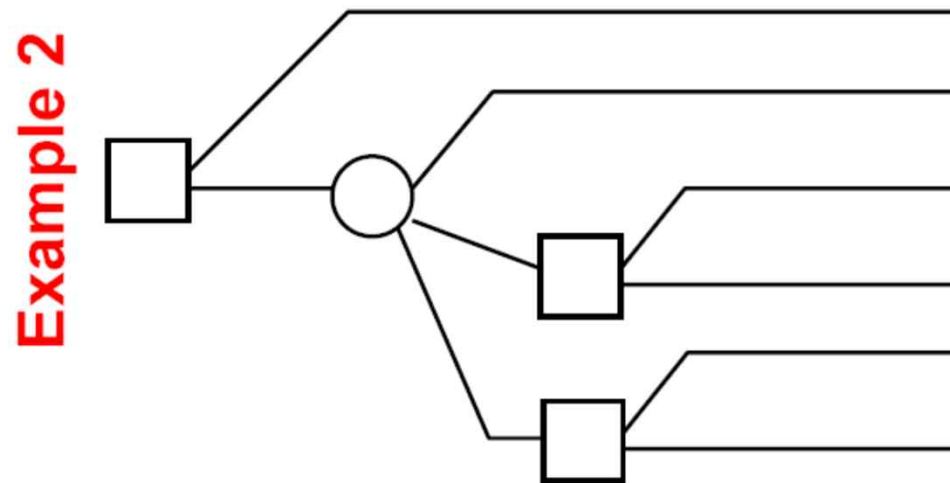
Strategy 1

Strategy 2

Strategy 3

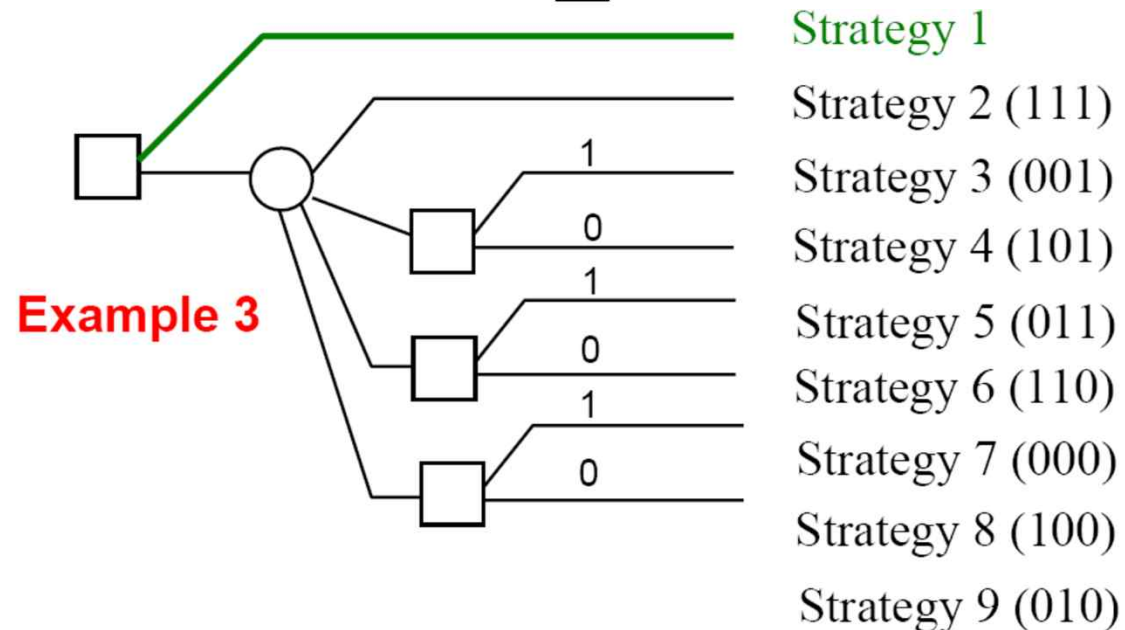
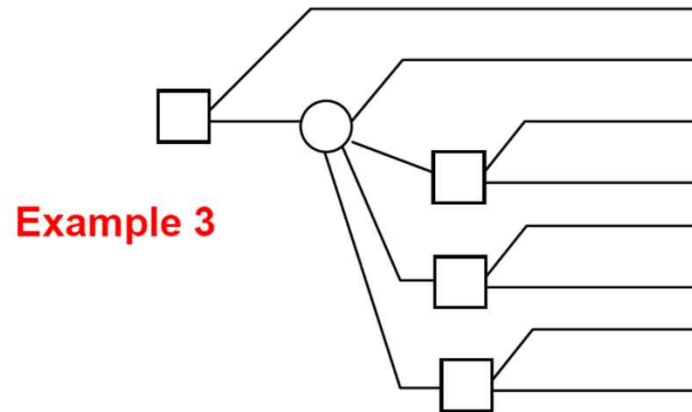
Counting Decision Strategies (2)

How many decision strategies in Example 2?



Counting Decision Strategies (3)

How many decision strategies in Example 3?



Deterministic Dominance

Deterministic Dominance

- If the worst outcome of Alternative B is at least as good as that of the best outcome of Alternative A, then Alternative B deterministically dominates Alternative A

Deterministic dominance may also be concluded by drawing cumulative risk profiles

Range of a Cumulative Risk Profile = [L, U]

- where, L= Smallest 0% point in Cumulative Risk Profile and
- U= Largest 100% point in Cumulative Risk Profile

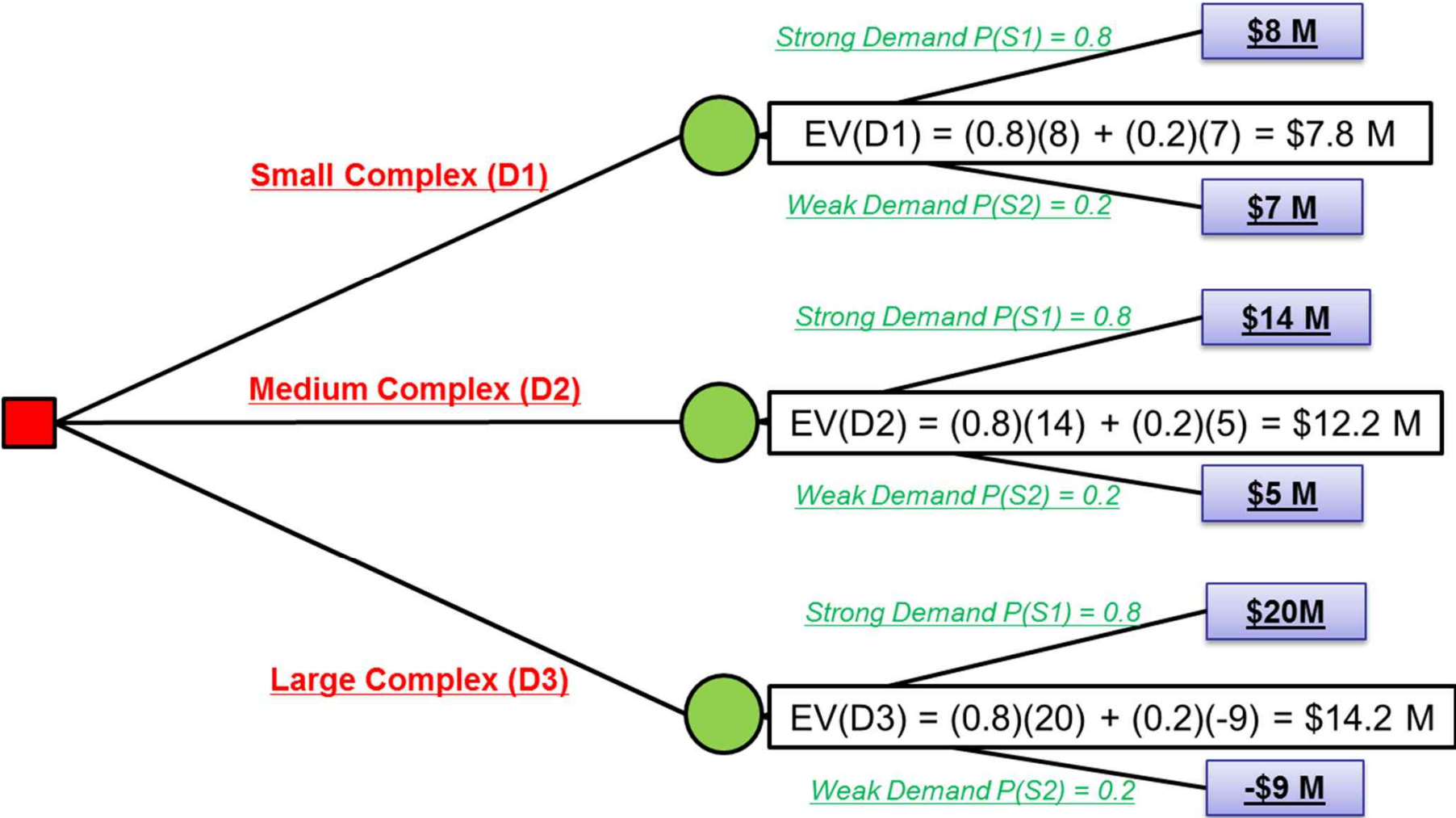
Stochastic Dominance

Conditions of Cumulative Risk Profile plots for **Stochastic Dominance**

- When the objective is to Maximize EMV
 1. Cumulative risk profiles in both plots do not cross
 2. The CRP that is toward the “right and below” stochastically dominates

Risk Profile

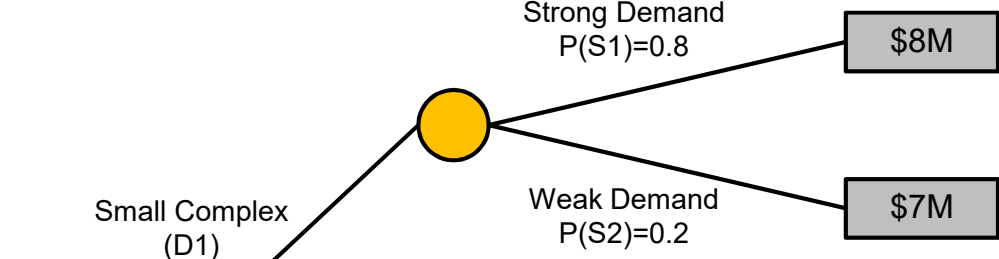
Example: Condo Complex Size



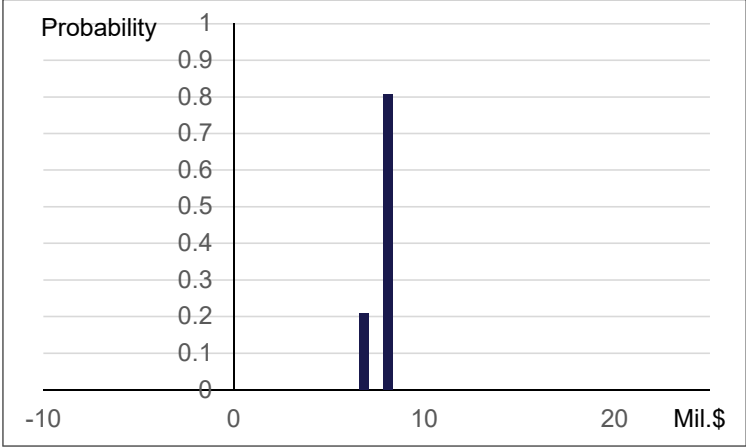
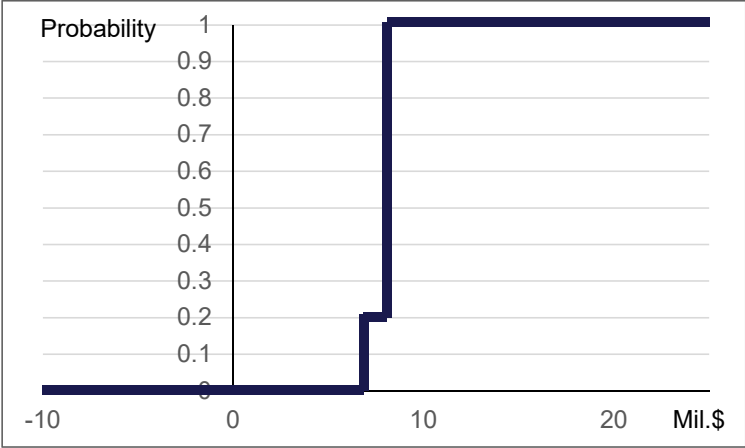
Risk Profile

Example: Condo Complex Size

- The first strategy: Small Complex



Outcome	Probability
\$7M	0.2
\$8M	0.8



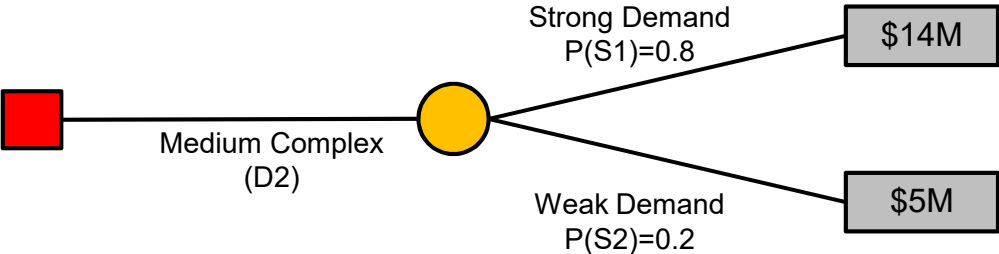
Cumulative Risk Profile

Risk Profile

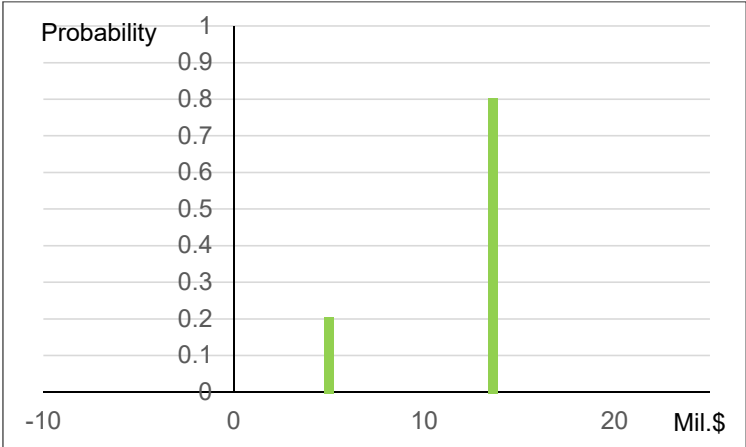
Risk Profile

Example: Condo Complex Size

- The second strategy: Medium Complex

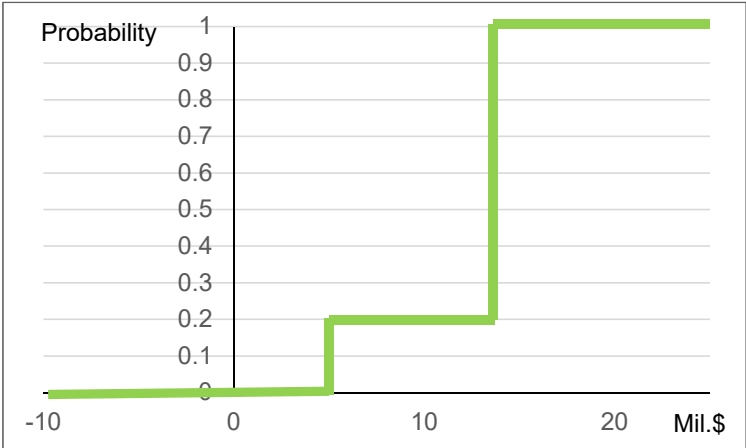


Risk Profile



Outcome	Probability
\$5M	0.2
\$14M	0.8

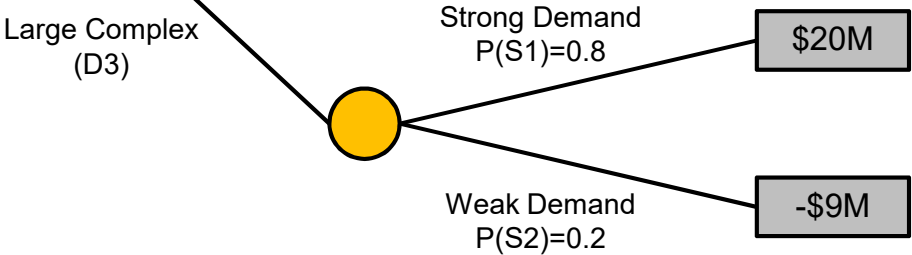
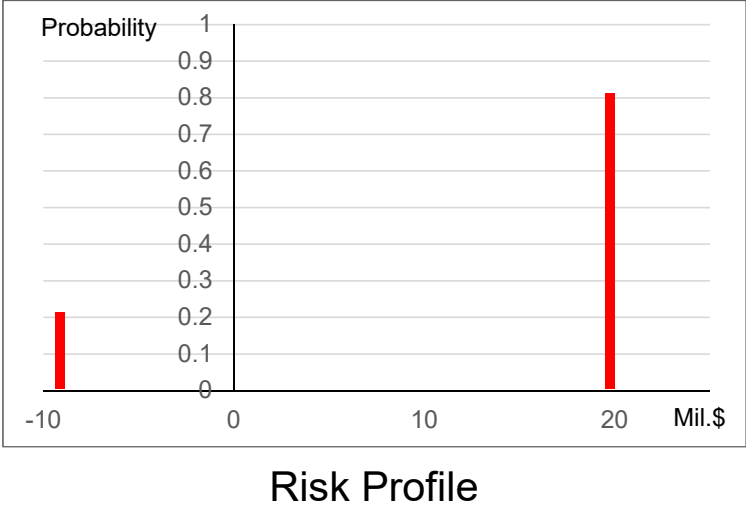
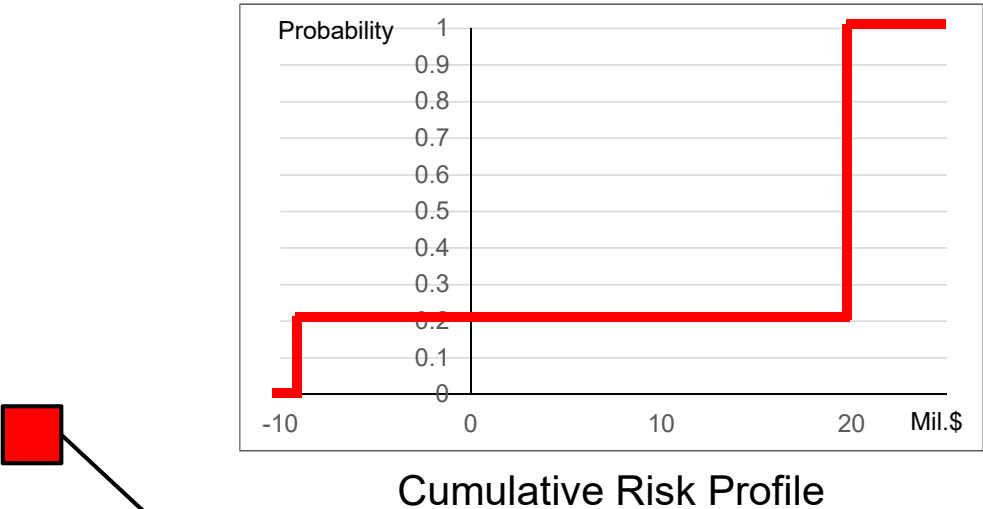
Cumulative Risk Profile



Risk Profile

Example: Condo Complex Size

- The third strategy: Large Complex

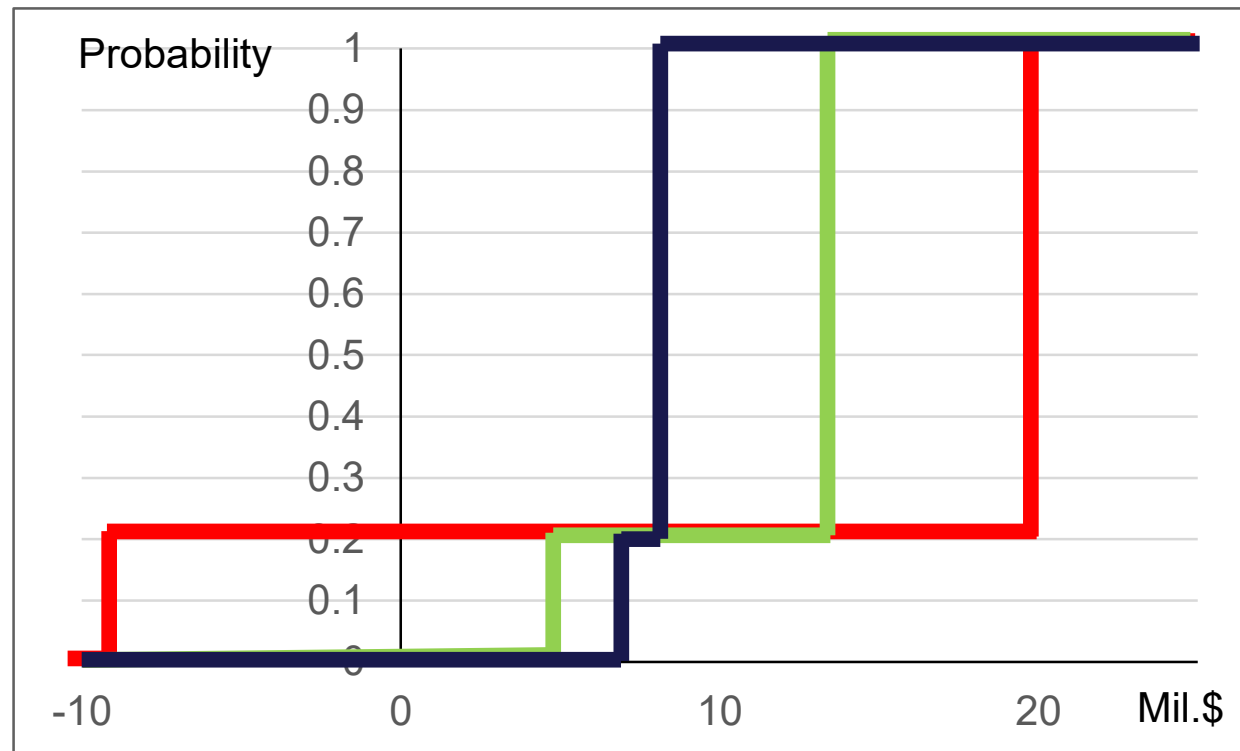


Outcome	Probability
-\$9M	0.2
\$20M	0.8

Risk Profile

Example: Condo Complex Size

- Cumulative Risk Profile



Any Deterministic or Stochastic Dominance?

Case Study: Texaco versus Pennzoil

In early 1984, Pennzoil and Getty Oil agreed to the terms of a merger

- But before any formal documents could be signed, Texaco offered Getty a substantially better price
 - Gordon Getty, who controlled most of the Getty Stock, reneged on the Pennzoil deal and *sold to Texaco*
 - Naturally, Pennzoil felt as if it had been dealt with **unfairly** and **immediately** files a lawsuit against Texaco alleging that Texaco had interfered illegally in the Pennzoil-Getty negotiations
- Pennzoil **won** the case: in late 1985, it was awarded \$11.1 billion, the largest judgment ever in the United States
 - A Texas appeal court **reduced** the judgment to **\$2 billion**, but interest and penalties drove the total back up to \$10.3 billion
 - James Kinnear, Texaco's Chief executive officer, had said that Texaco would file for **bankruptcy** if Pennzoil obtained court permission to secure the judgment by filing liens(유치권) against Texaco's

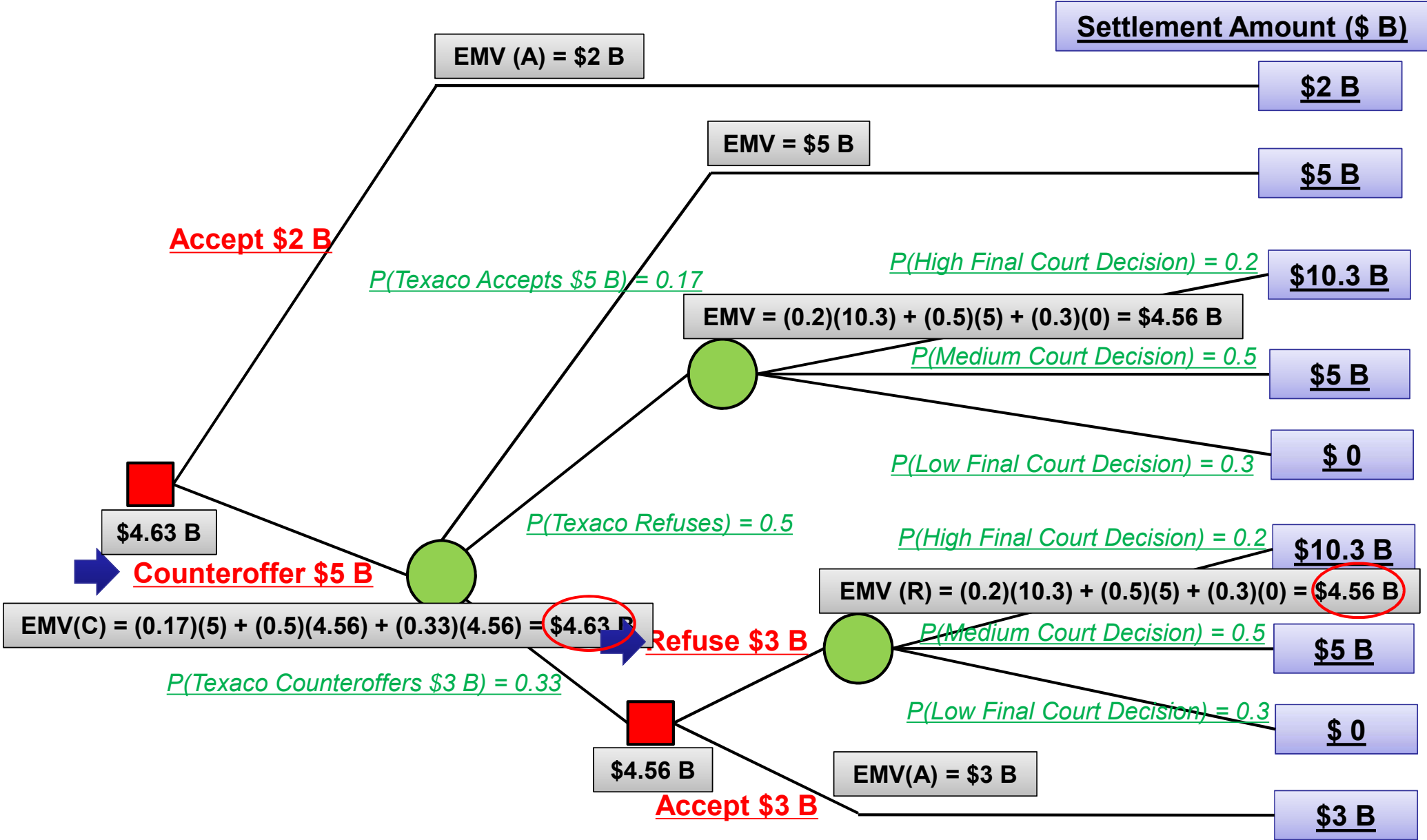
Case Study: Texaco versus Pennzoil (Cont'd)

- Furthermore, Kinnear had promised to fight the case all the way to the U.S. Supreme Court if necessary, arguing in part that Pennzoil had not followed Security and Exchange Commission regulations in its negotiations with Getty
- In April 1987, just before Pennzoil began to file liens, Texaco **offered** to Pennzoil **\$2 billion dollars** to settle the entire case
 - Hugh Liedtke, chairman of Pennzoil, indicated that his advisors were telling him that *a settlement between \$3 billion and \$5 billion* would be fair
- What should Hugh Liedtke do?
 1. Accept \$2 Billion
 2. Refuse \$2 Billion and counter offer \$5 Billion

Case Study: Texaco versus Pennzoil (Cont'd)

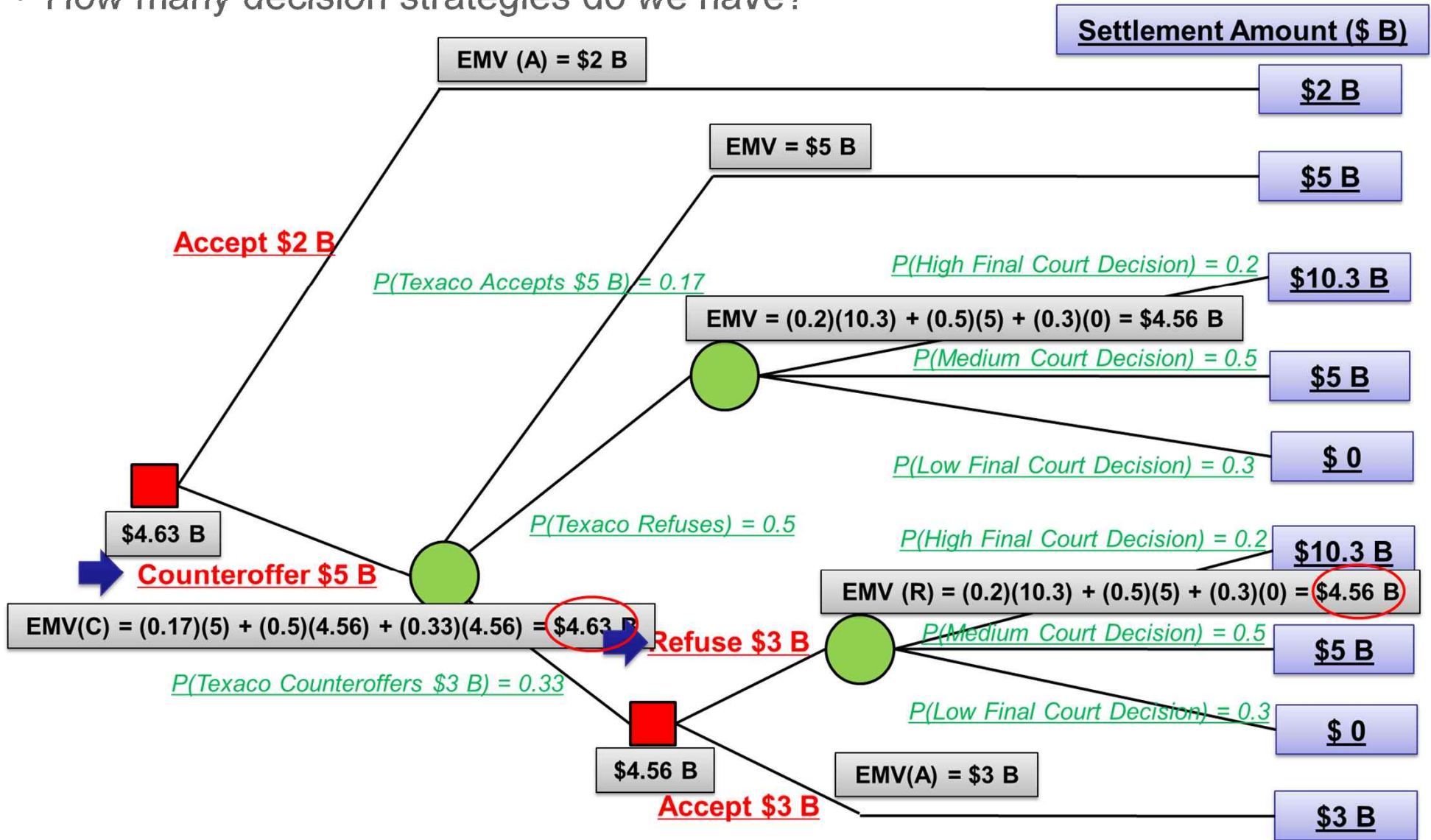
- Given tough negotiation positions of the two executives, it could be an even chance (50%) that Texaco will **refuse** to negotiate further
- Liedtke and advisor figure that it is twice as likely that Texaco would counter offer \$3 billion than accepting the \$5 billion
 - Hence, because there is a 50% of refusal, there must be a 33% chance of a Texaco counter offer, & a 17% chance of Texaco accepting \$5 B
- What are the probabilities of the final court decision?
 - Liedtke admitted that Pennzoil could lose the case
 - ✓ There is a significant possibility (30%) *that the outcome would be zero*
 - Given the strength of the Pennzoil case it is also possible that the court will upheld the judgment as it stands
 - ✓ This probability is assessed at 20%
 - Finally, the possibility exists that the judgment could be reduced somewhat to \$5 billion
 - ✓ Thus there must be a chance of 50% of this happening

Decision Tree Analysis: Texaco vs. Pennzoil



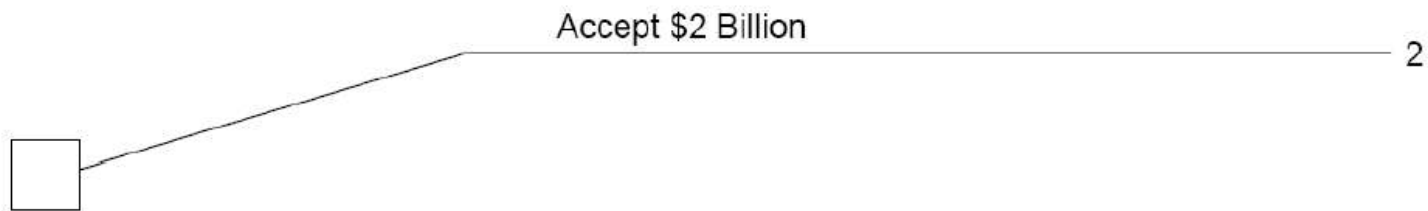
Decision Strategies: Texaco-Pennzoil Case

- How many decision strategies do we have?



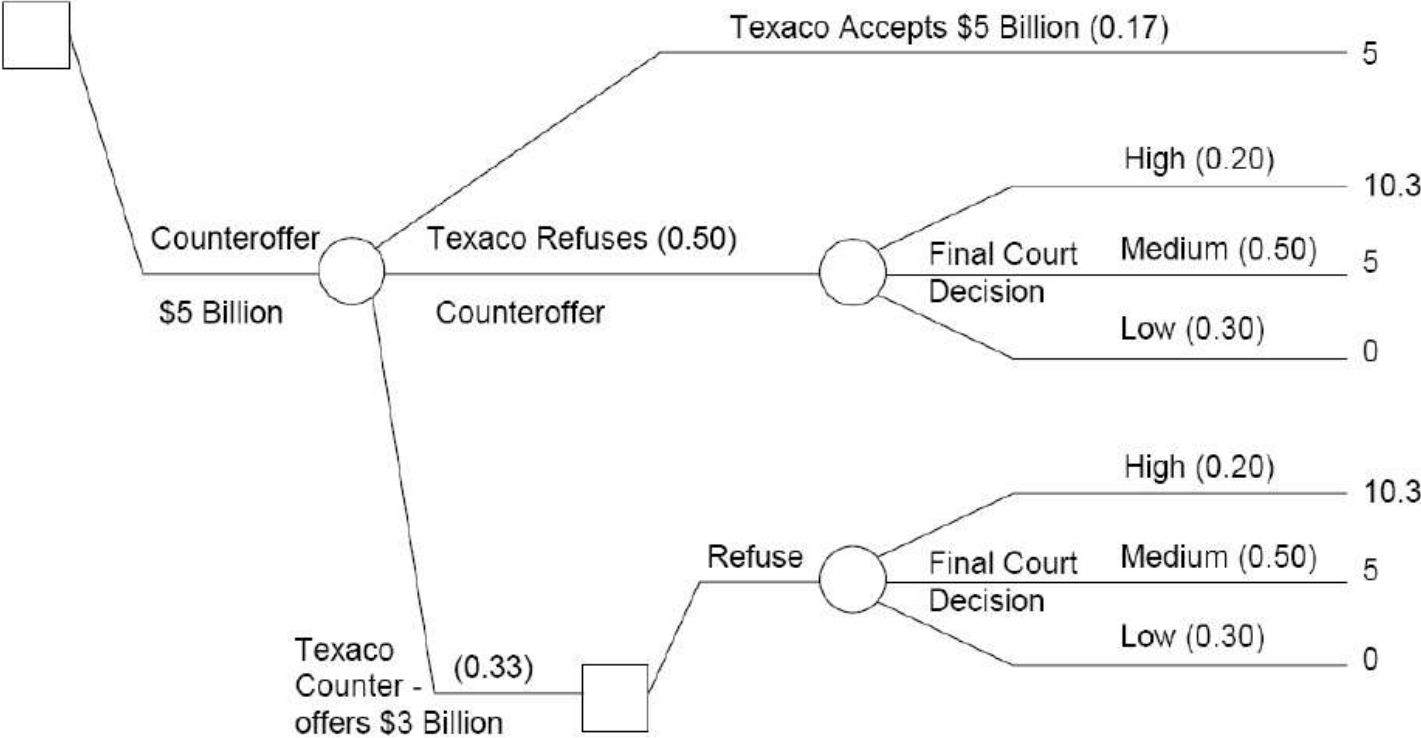
Decision Strategies: Texaco-Pennzoil Case

First strategy: "Accept \$2 billion"



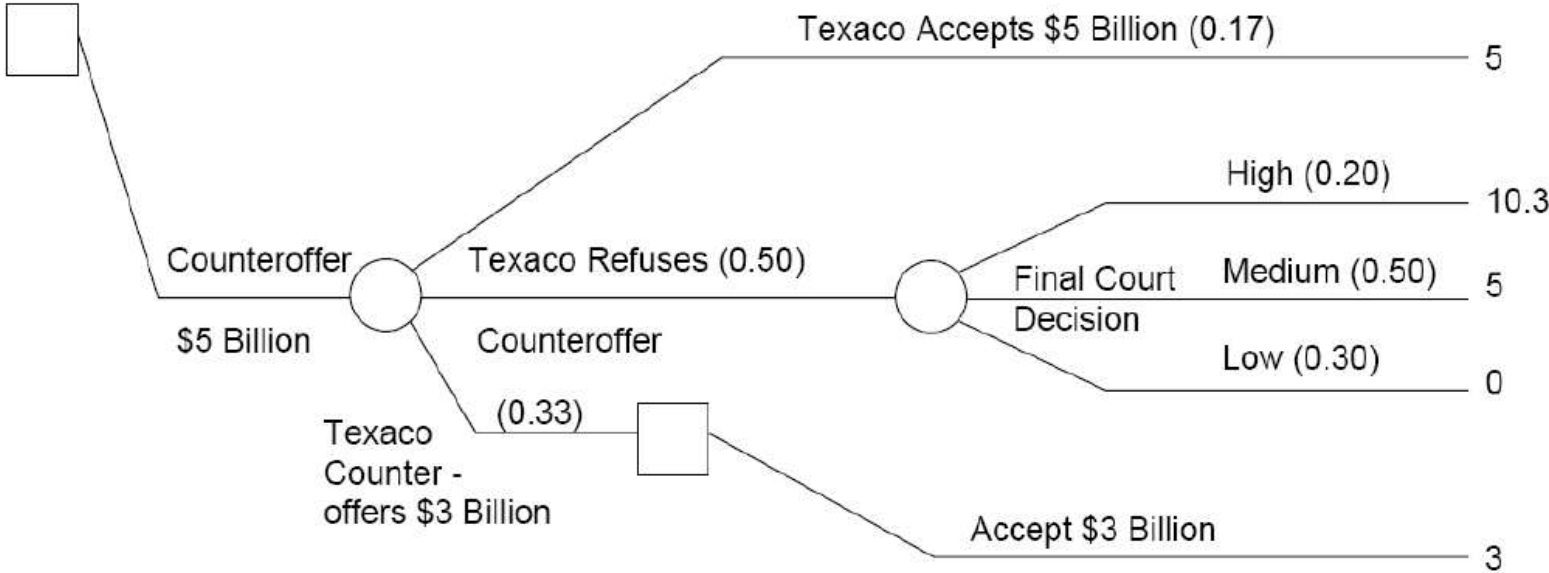
Decision Strategies: Texaco-Pennzoil Case

Second strategy: “Counter \$5 billion and if Texaco counter offers \$3 billion refuse this counteroffer of \$3 Billion”



Decision Strategies: Texaco-Pennzoil Case

Third strategy: “Counter \$5 billion and if Texaco counter offers \$3 billion accept this counteroffer of \$3 Billion”

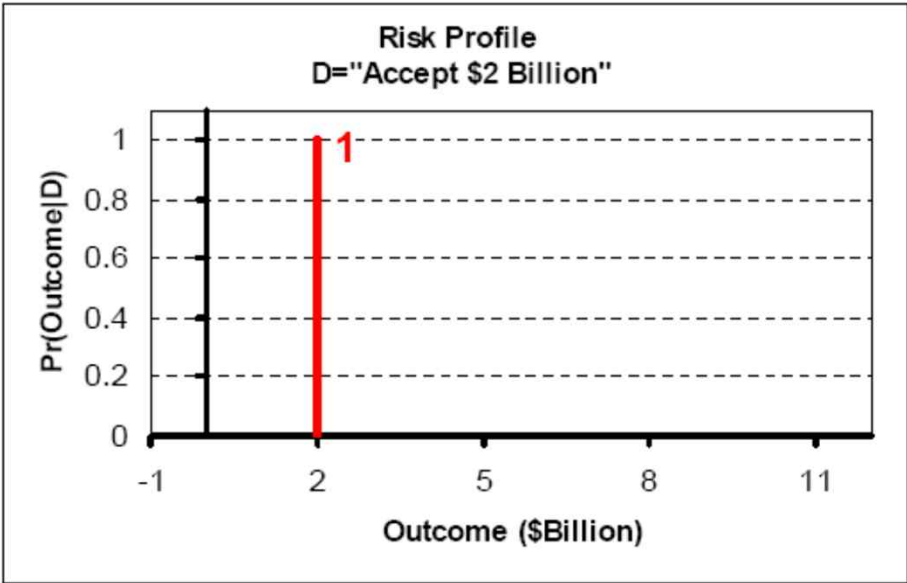


Risk Profiles: Texaco-Pennzoil Case

First strategy: "Accept \$2 billion"

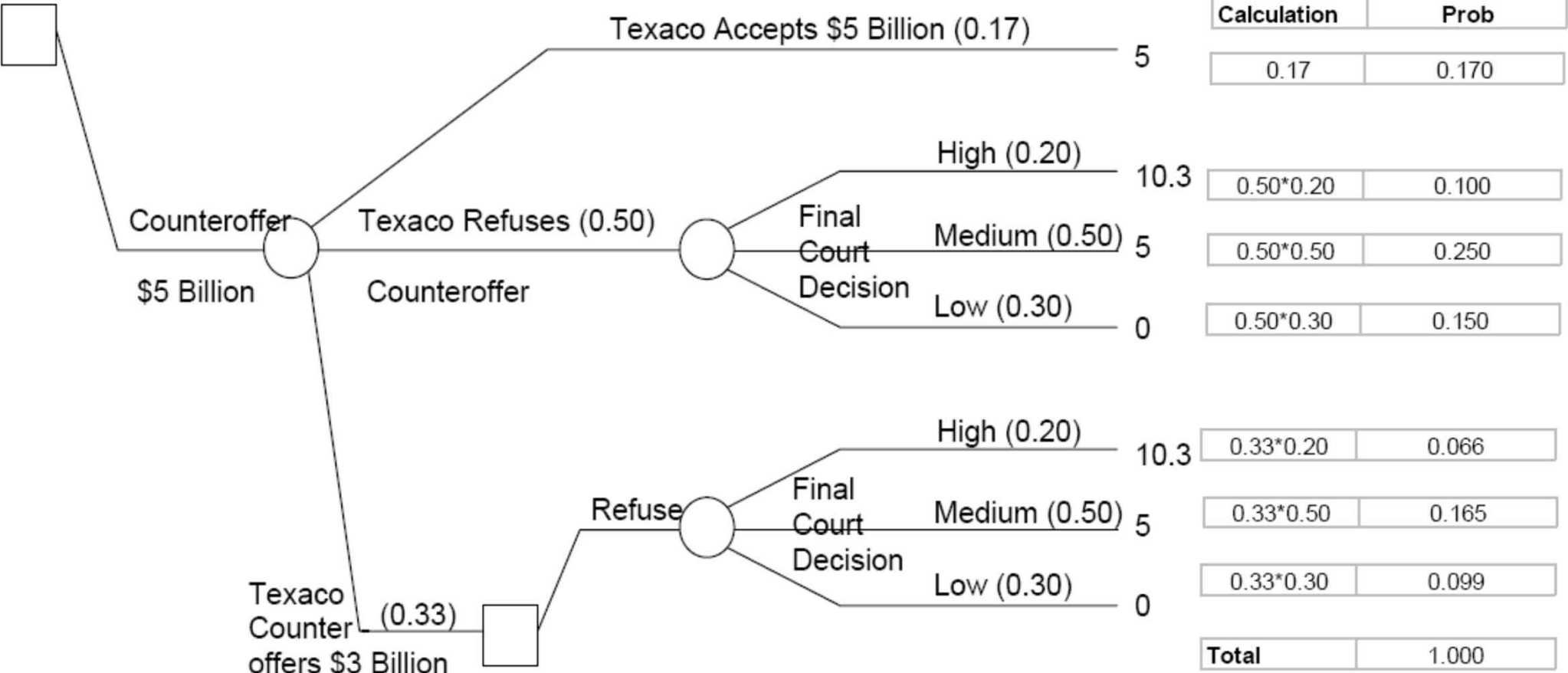


Outcome x (\$Billion)	Pr(Outcome D)
2	1



Risk Profiles: Texaco-Pennzoil Case (2)

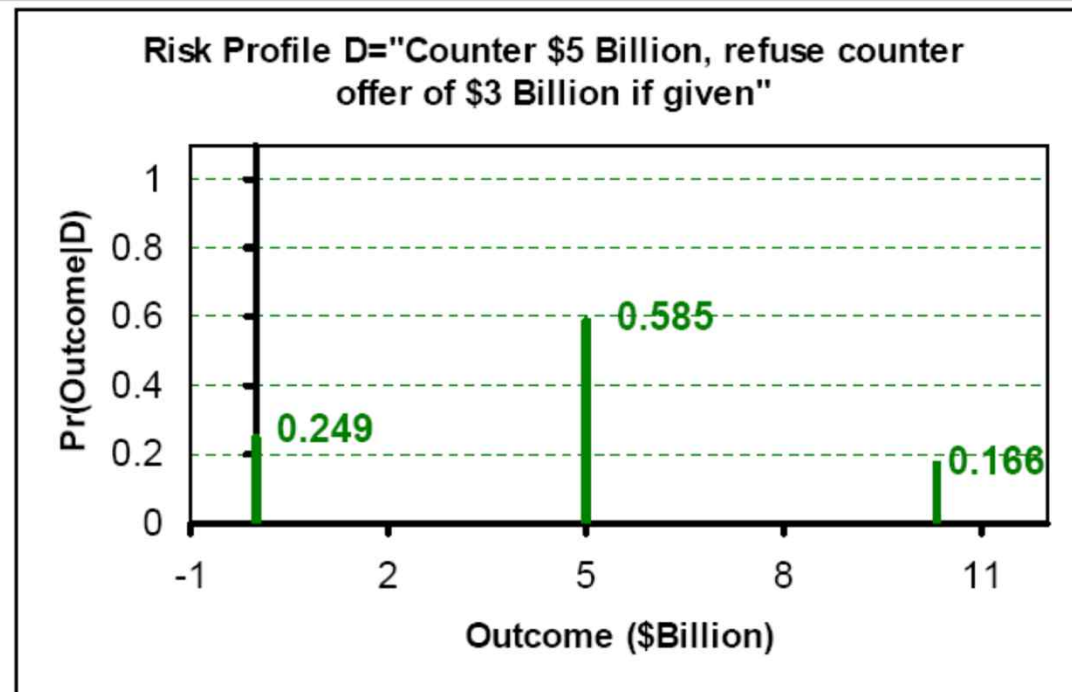
Second strategy: “Counter \$5 billion and if Texaco counter offers \$3 billion refuse this counteroffer of \$3 Billion”



Risk Profiles: Texaco-Pennzoil Case (3)

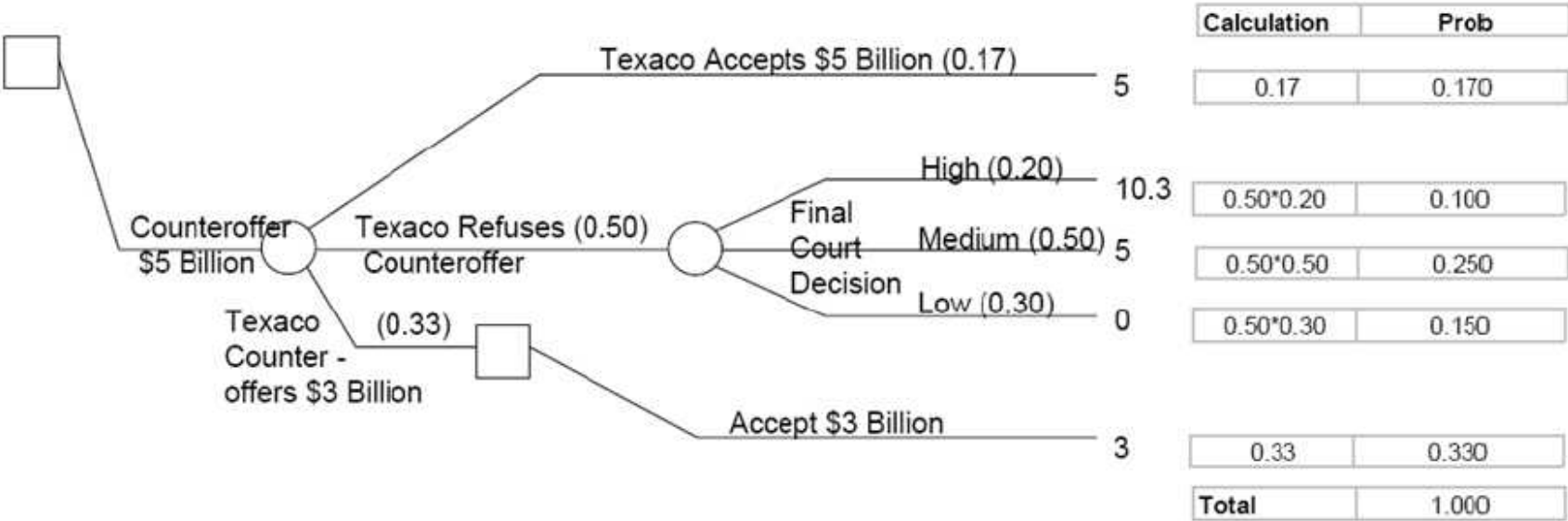
Second strategy: “Counter \$5 billion and if Texaco counter offers \$3 billion refuse this counteroffer of \$3 Billion”

Outcome x (\$Billion)	Calculation	Pr(Outcome D)
0	$0.150+0.099$	0.249
5	$0.170+0.250+0.165$	0.585
10.3	$0.100+0.066$	0.166
		1.000



Risk Profiles: Texaco-Pennzoil Case (4)

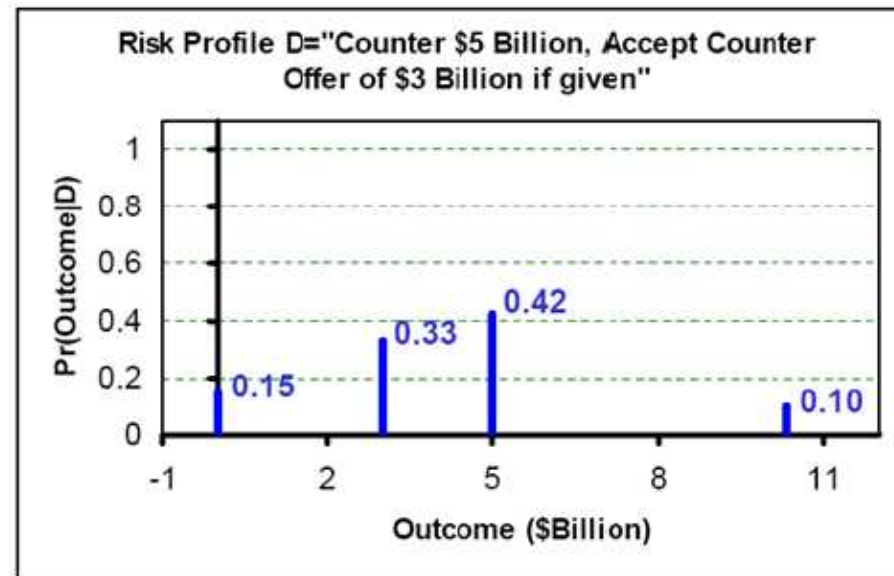
Third strategy: “Counter \$5 billion and if Texaco counter offers \$3 billion accept this counteroffer of \$3 Billion”



Risk Profiles: Texaco-Pennzoil Case (5)

Third strategy: “Counter \$5 billion and if Texaco counter offers \$3 billion accept this counteroffer of \$3 Billion”

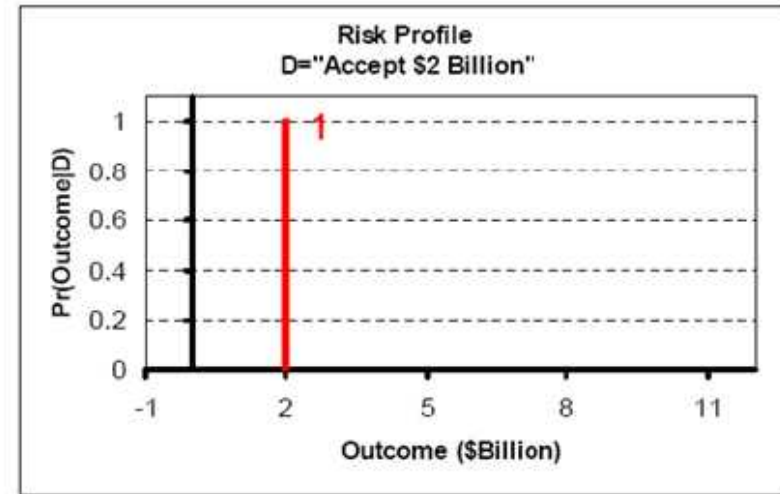
Outcome x (\$Billion)	Calculation	Pr(Outcome D)
0	0.15	0.15
3	0.33	0.33
5	0.170+0.250	0.42
10.3	0.1	0.1
		1.000



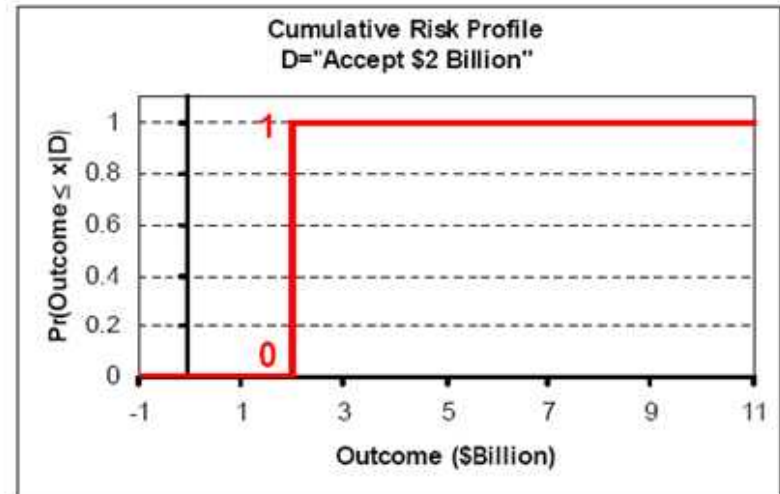
Cumulative Risk Profiles: Texaco-Pennzoil

First strategy: "Accept \$2 billion"

Outcome x (\$Billion)	Pr(Outcome D)
2	1



Outcome x (\$Billion)	Pr(Outcome ≤ x D)
2	1

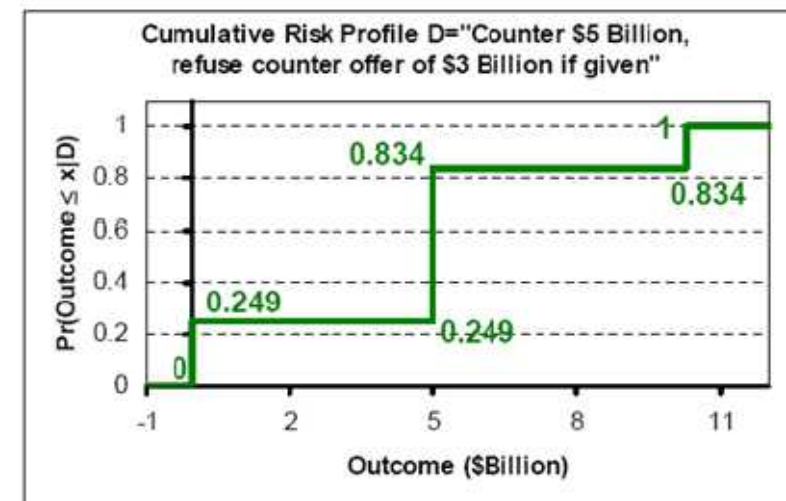
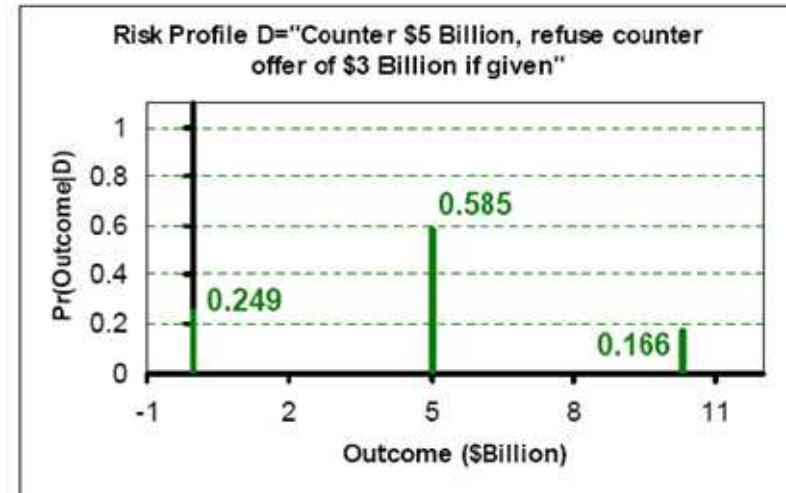


Cumulative Risk Profiles: Texaco-Pennzoil (2)

Second strategy: “Counter \$5 billion and if Texaco counter offers \$3 billion refuse this counteroffer of \$3 Billion”

Outcome x (\$Billion)	$\Pr(\text{Outcome} D)$
0	0.249
5	0.585
10.3	0.166

Outcome x (\$Billion)	$\Pr(\text{Outcome} \leq x D)$
0	0.249
5	$0.249 + 0.585 = 0.834$
10.3	$0.834 + 0.166 = 1$

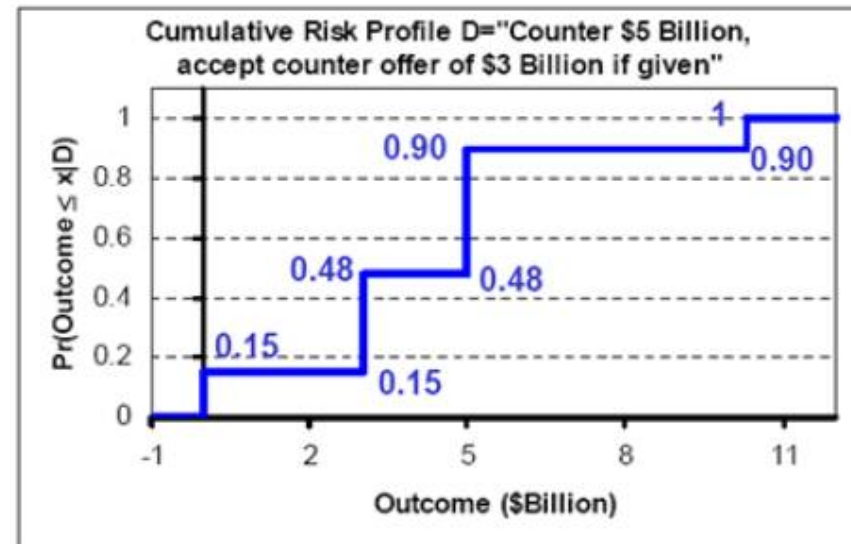
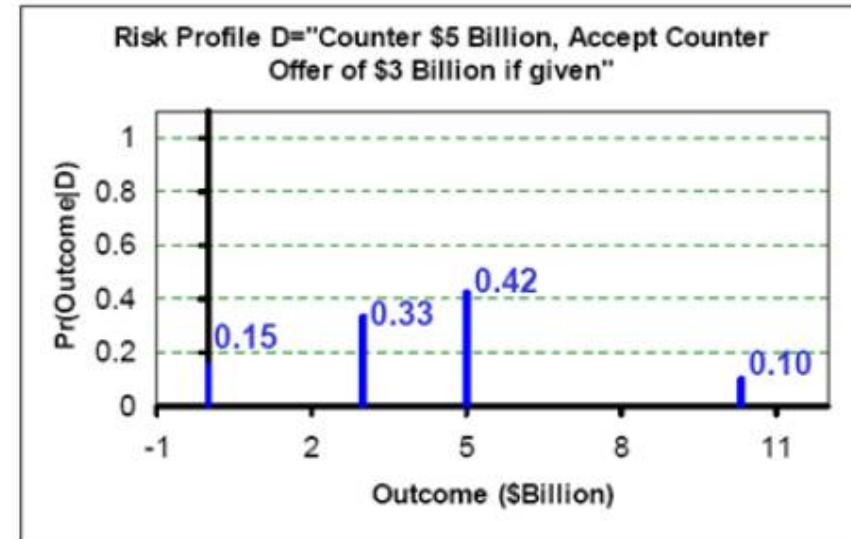


Cumulative Risk Profiles: Texaco-Pennzoil (3)

Third strategy: “Counter \$5 billion and if Texaco counter offers \$3 billion accept this counteroffer of \$3 Billion”

Outcome x (\$Billion)	$\Pr(\text{Outcome} D)$
0	0.15
3	0.33
5	0.42
10.3	0.1

Outcome x (\$Billion)	$\Pr(\text{Outcome} \leq x D)$
0	0.15
3	$0.15 + 0.33 = 0.48$
5	$0.48 + 0.42 = 0.90$
10.3	$0.90 + 0.10 = 1$



Deterministic Dominance

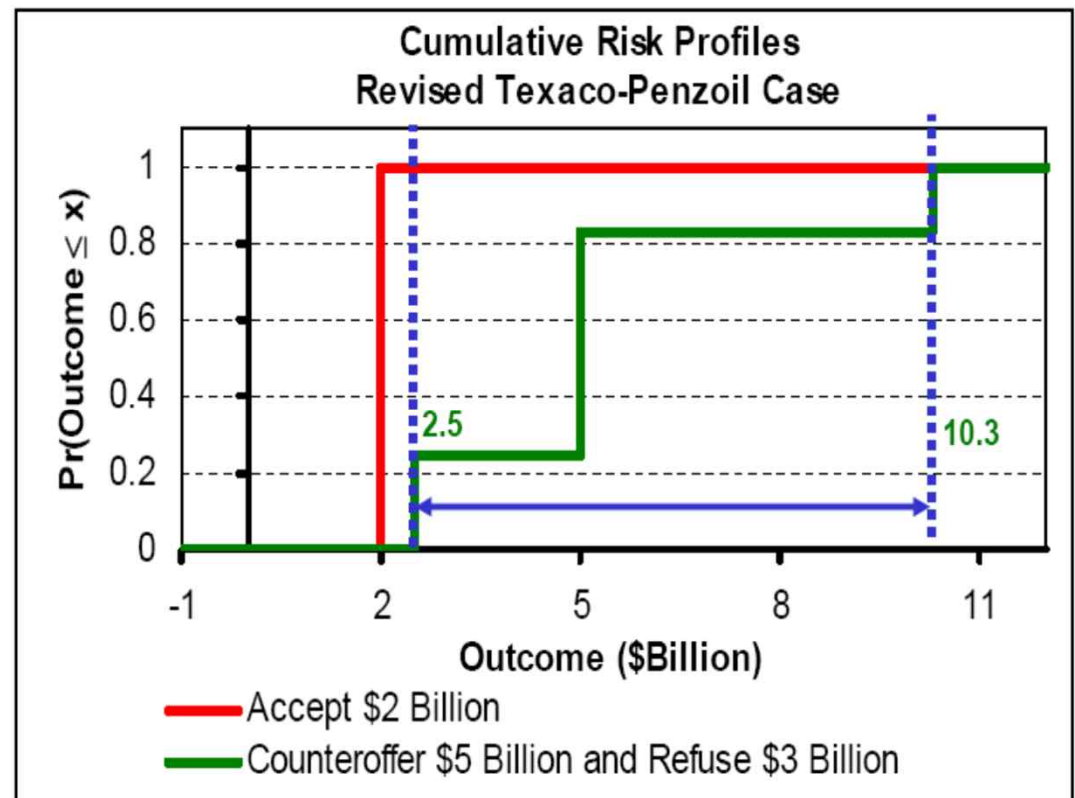
Deterministic dominance via cumulative risk profiles

1. Draw cumulative risk profiles in one graph
2. Determine range for each risk profile
3. If ranges are disjoint or their intersections contain a single point

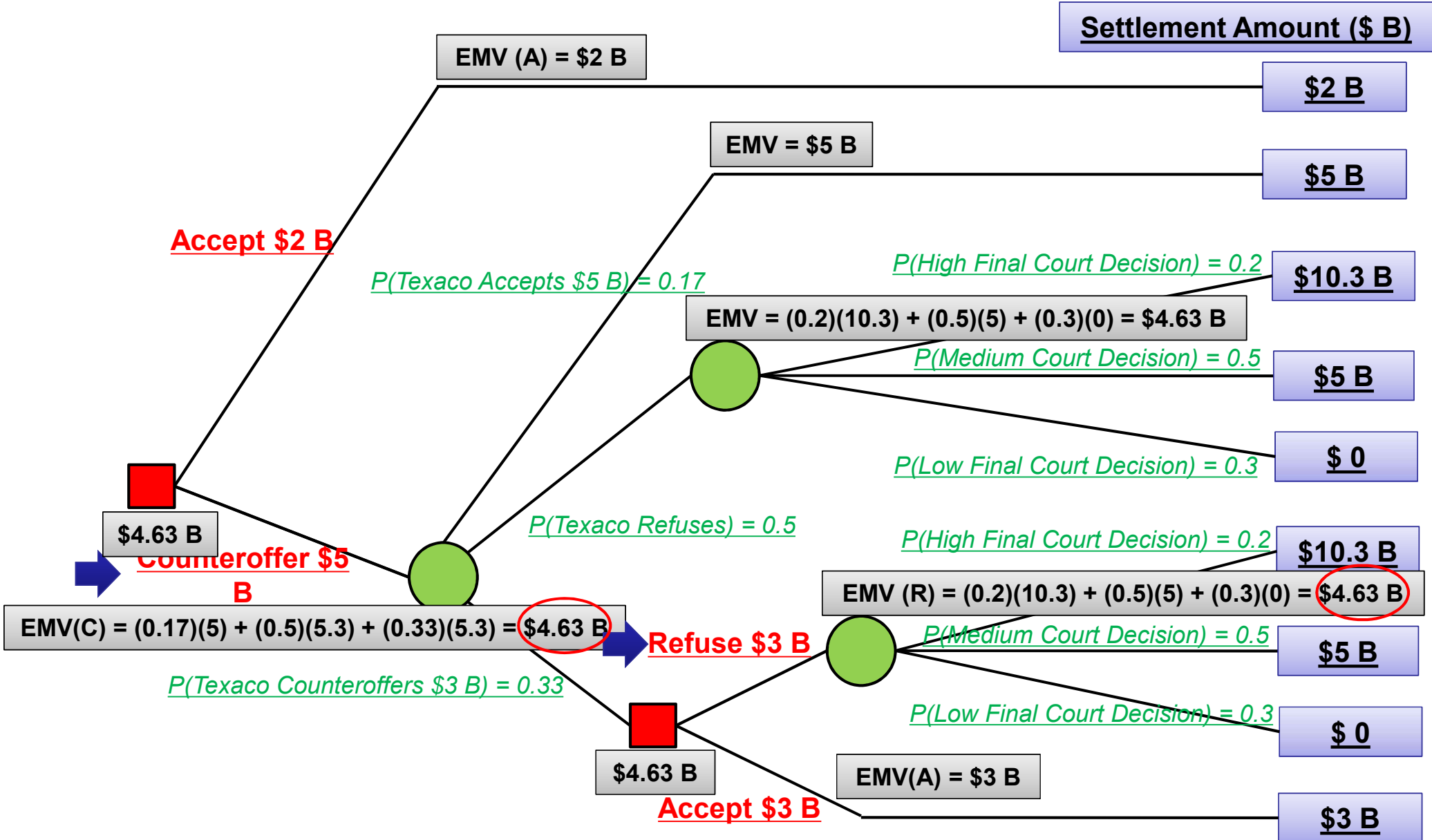
Range 1: {2}

Range 2: {2.5, 5, 10.3}

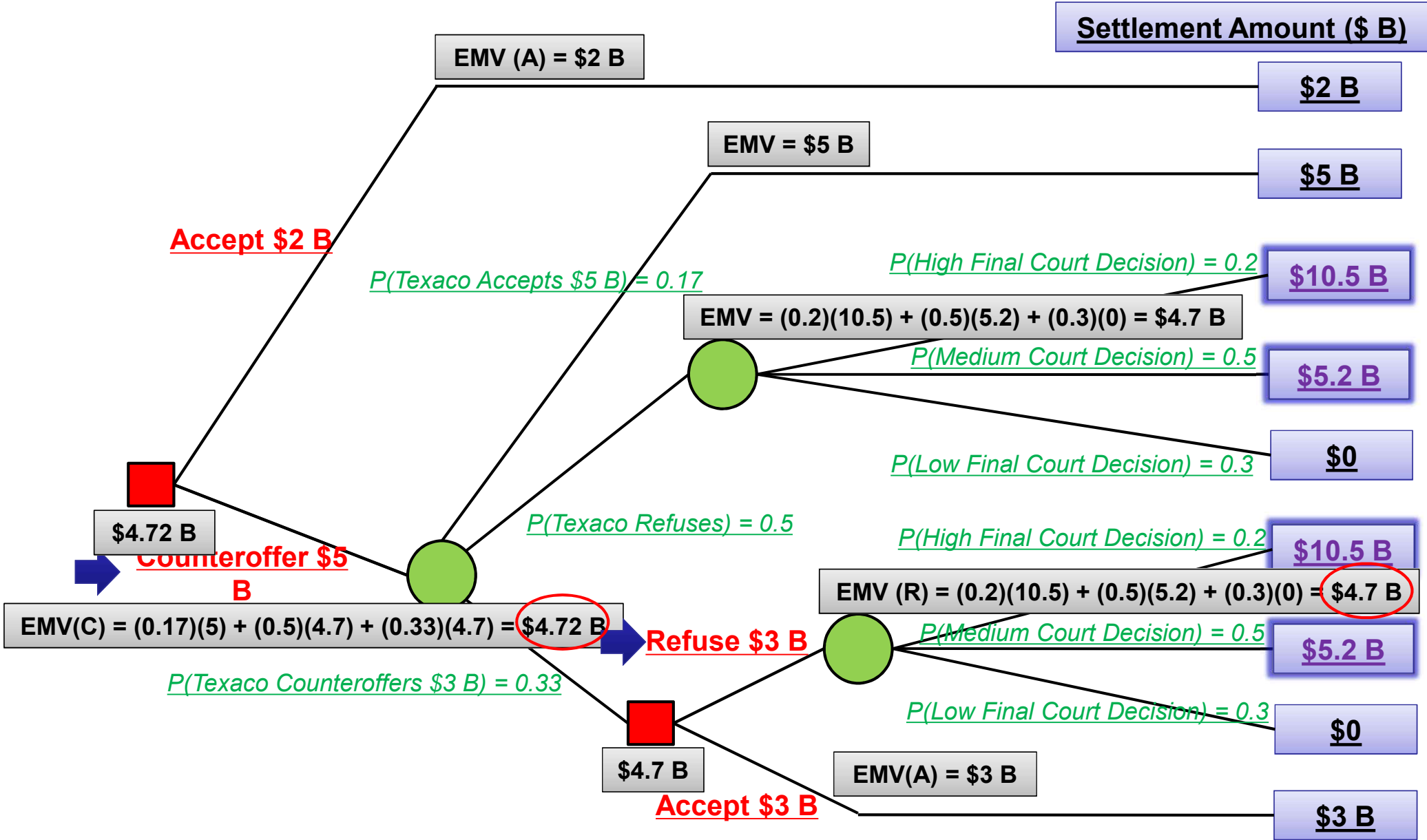
- Ranges 1 and 2 are disjoint
- The Objective is Max EMV
- Hence, **the Green CRP** *deterministically dominates* **the Red CRP**



Decision Tree Analysis: Law Firm A



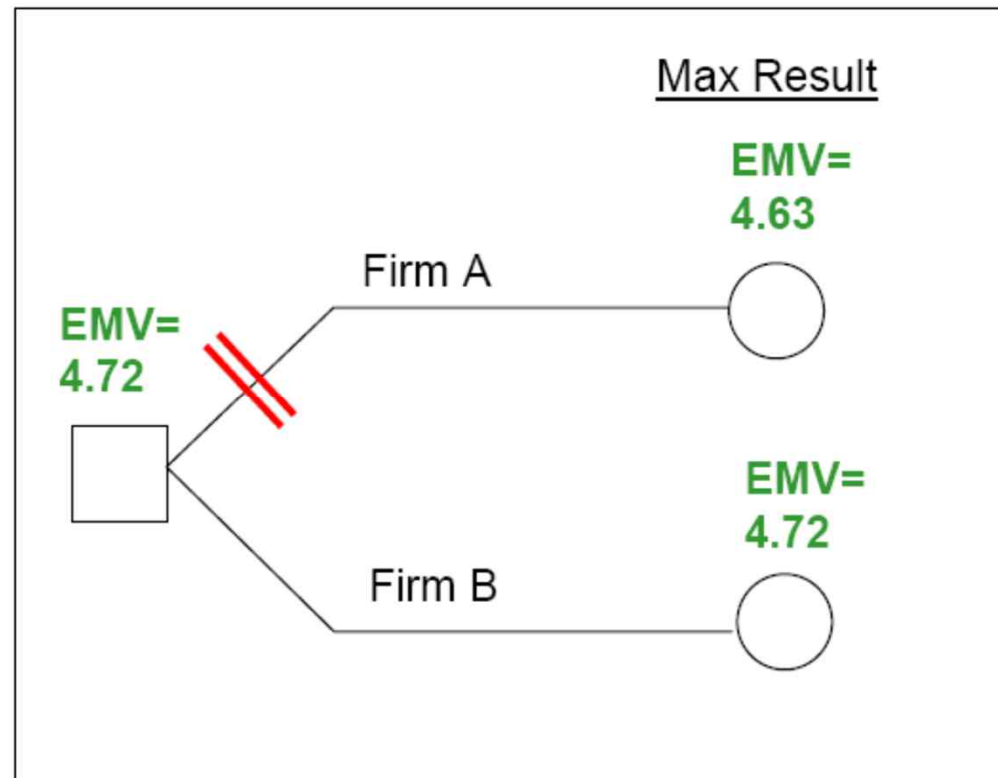
Decision Tree Analysis: Law Firm B



EMV: Law Firm A or Law Firm B

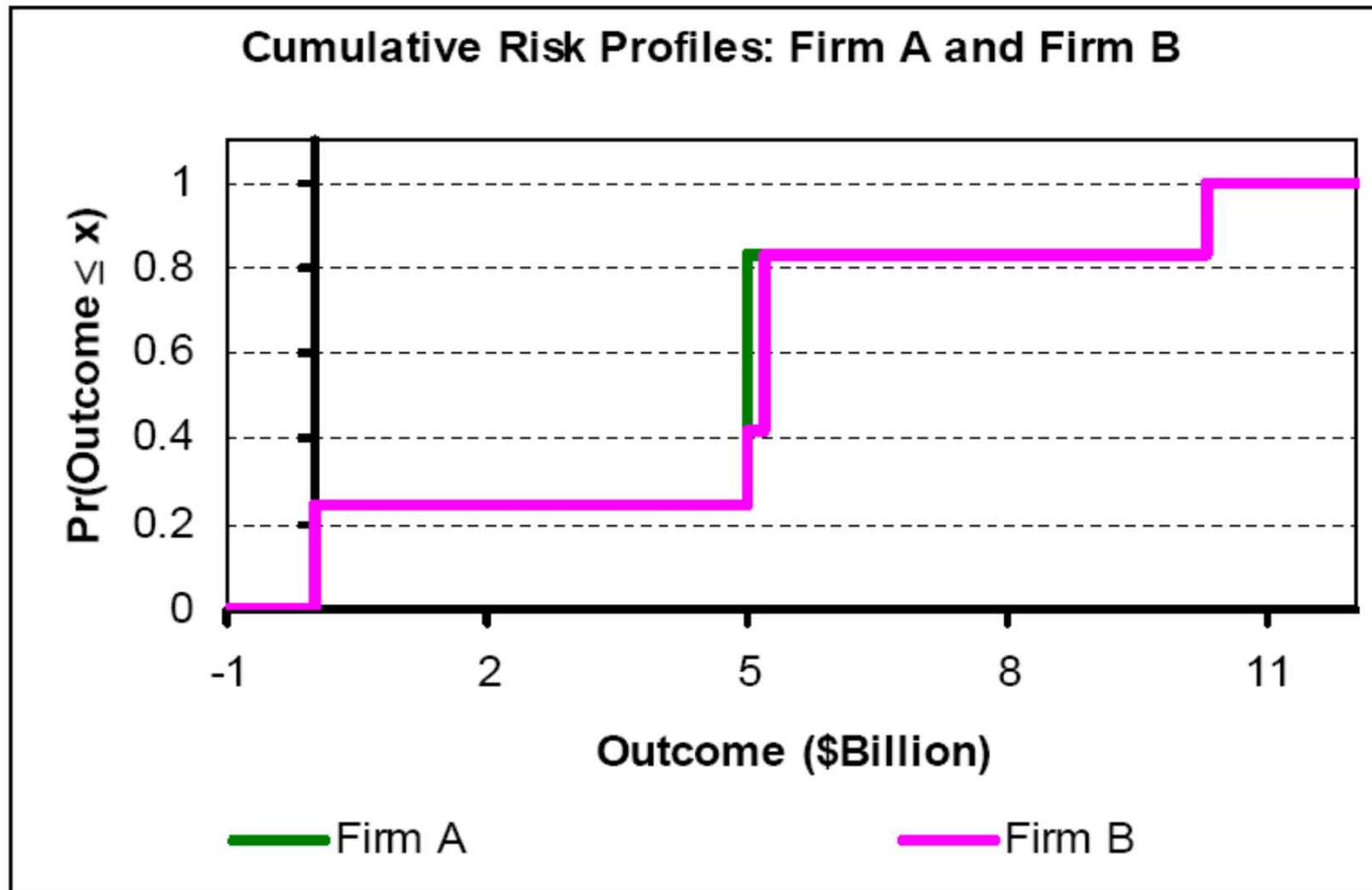
Which law firm to use to present the case?

Based on EMV analysis we choose “Law Firm B”



Stochastic Dominance

Optimal Cumulative risk profiles of “**Law Firm A**” Decision Tree & “**Law Firm B**” Decision Tree



Stochastic Dominance

For all possible values of x

- $\Pr(\text{Outcome} \leq x | \text{Law Firm B}) \leq \Pr(\text{Outcome} \leq x | \text{Law Firm A})$

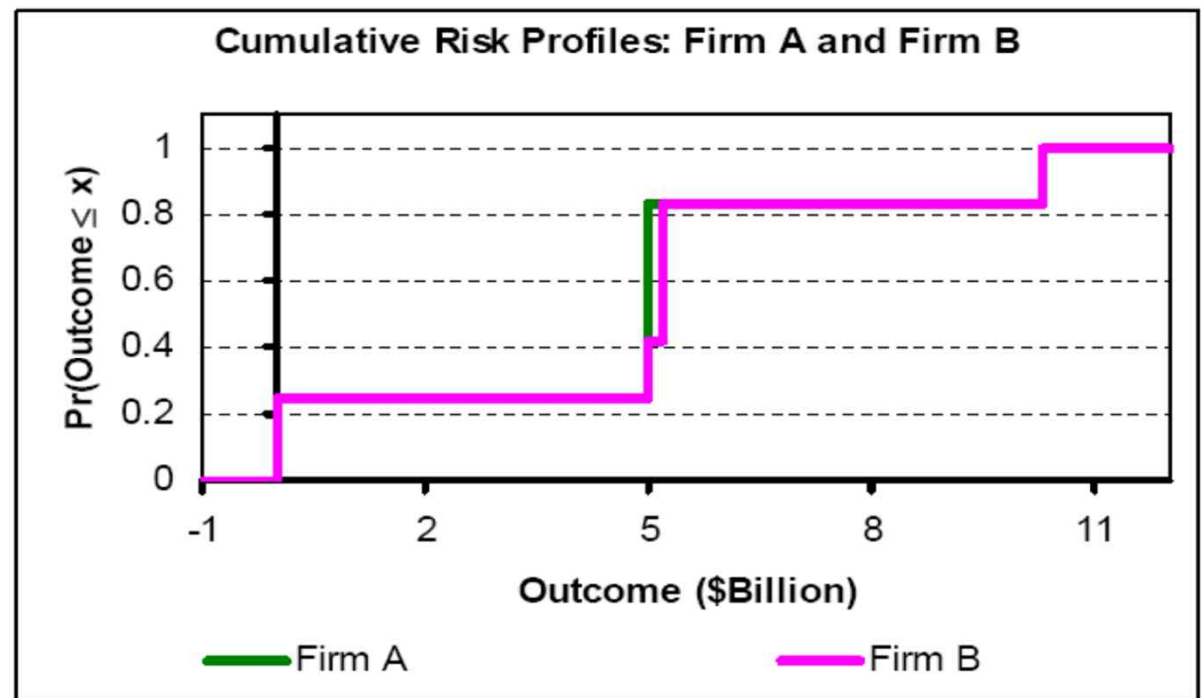
or equivalently,

- $\Pr(\text{Outcome} \geq x | \text{Law Firm B}) \geq \Pr(\text{Outcome} \geq x | \text{Law Firm A})$

- Hence the *chances of winning* with **Law Firm B** are *always better* than that of **Law Firm A**

- **Law Firm B alternative**
stochastically dominates

- **Law Firm A alternative**



Assignment #2-1: Dante Development Corporation

Dante Development Corporation is considering bidding on a contract for a new office building complex. First, the company should decide whether to bid for this project. The cost of preparing the bid is \$200,000. The company has a 0.8 probability of winning the contract if it submits a bid. If the company wins the bid it will have to pay \$2,000,000 to become a partner in the project. The company will then consider conducting a market research study to forecast demand for the office units prior to the construction phase. The cost of study is \$150,000. The company should decide whether to conduct the study or not. The possible outcomes of the market research study are 'Forecast High' with probability 0.6 and 'Forecast Moderate' with probability 0.4.

Finally, the company should decide whether to build the office complex or to sell the right to another developer. The decision to build the complex will result in an income of \$5,000,000 if demand is high and \$3,000,000 if demand is moderate. If Dante chooses to sell its rights in the project to another developer, income from the sale is estimated to be \$3,500,000. The probabilities of market demand for each possible outcome of the market research study are shown as following table.

Market Research Study		Actual Market Demand	
		High	Moderate
Forecast Results	High	0.85	0.15
	Moderate	0.225	0.775
No Market Research Study		0.6	0.4

Assignment #2-1: Dante Development Corporation

- a. Structure the decision tree for Dante's problem and solve it.
- b. What is the optimal decision strategy for Dante? What is the expected value of the Dante's profit in this project?
- c. What would the cost of the market research study have to be in order to motivate Dante to conduct it?
- d. Outline the entire decision strategies of this development company. How many decision strategies does the development company have? Draw a risk profile and a cumulative risk profile for each Dante's decision strategy.
- e. Compare each decision strategy with Dante's optimal decision strategy. Draw the cumulative risk profile of each decision strategy and the cumulative risk profile of the optimal decision strategy on the same graph for the purpose of comparison. Compare each strategy with the optimal decision strategy. Is there any deterministic or stochastic dominance for each comparison?

Assignment #2-2: Hemmingway, Inc.

Hemmingway, Inc. is considering a \$5 million research and development (R&D) project. Profit projections appear promising but Hemmingway President is concerned because the probability that the R&D project will be successful is just 0.50. President also knows that even if the project is successful, it will require the company building a new production facility at a cost of \$20 million to start large scale manufacturing. If the facility was built there would be demand uncertainty and consequently uncertainty about the profit. Another option is that if R&D project was successful the company could sell the right of the production for an estimated price of \$25 million. Under this option, the company would not build the \$20 million production facility. The revenue projections for each possible state of the market demand are shown in the following table:

Market Demand	Probability	Revenue (M\$)
High	0.5	59
Medium	0.3	45
Low	0.2	35

Assignment #2-2: Hemmingway, Inc.

- a. Develop and solve the decision tree for this problem
- b. Identify the optimal decision strategy
- c. How many decision strategies does this firm have?
- d. Develop a risk profile and a cumulative risk profile for each Hemmingway's decision strategy.
- e. Compare each decision strategy with the optimal decision strategy. Draw both cumulative risk profiles on the same graph. Is there any deterministic or stochastic dominance between each decision strategy and the optimal decision strategy?

Street Calculus

Q & A

