457.644 Advanced Bridge Engineering

Aerodynamic Design of Bridges

Part II : Estimation of Design Wind Speed

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Extreme Distribution

- Consider a random variable X with known probability distribution $f_X(x)$ or $F_X(x)$

- We have set of n observations $(x_1,\ x_2,\ \cdots, x_n)$ about X. Because the observed values are unpredictable, they are a specific realization of a set of random variables $(X_1,\ X_2,\ \cdots, X_n)$

- We are interested in the maximum and minimum of $(X_1,\ X_2,\ \cdots, X_n)$

$$Y_n = max(X_1,\ X_2,\ \cdots, X_n) \qquad Y_1 = min(X_1,\ X_2,\ \cdots, X_n)$$

- With the assumption that $X_1,\ X_2,\ \cdots, X_n$ are statically independent and identically distributed, i.e, $F_{X_1}(x) = F_{X_2}(x) = \cdots = F_{X_n}(x) = F_X(x)$

- The CDF of Y_n, therefore, is $F_{Y_n}(y) = P(X_1 \le y, X_2 \le y,\ \cdots, X_n \le y) = [F_X(y)]^n$

- The distribution of an extreme value converges asymptotically in distribution as n increases. According to Gumbel, there are three types of such asymptotic distribution.

Extreme Distribution

Type I(Gumbel) Distribution

PDF : $f_{Y_n}(y) = \alpha_n exp[-\alpha_n(y-u_n) - exp\{-\alpha_n(y-u_n)\}]$

CDF : $F_{Y_n}(y) = exp[-exp\{-\alpha_n(y-u_n)\}], -\infty < y < \infty$

Parameters : $\mu_{Y_n} = u_n + \frac{\gamma}{\alpha_n},\ \sigma_{Y_n} = \frac{\pi}{\alpha_n\sqrt{6}}$

Type II Distribution

PDF : $f_{Y_n}(y) = \left(\frac{k}{v_n}\right) \times \left(\frac{v_n}{y}\right)^{k+1} exp\left\{-\left(\frac{v_n}{y}\right)^k\right\}$

CDF : $F_{Y_n}(y) = exp\left\{-\left(\frac{v_n}{y}\right)^k\right\}, y > 0$

Parameters : $\mu_{Y_n} = v_n\Gamma\left(1-\frac{1}{k}\right), \sigma_{Y_n} = v_n\sqrt{\left\{\Gamma\left(1-\frac{2}{k}\right) - \Gamma^2\left(1-\frac{1}{k}\right)\right\}}$

Type III Distribution

PDF : $f_{Y_n}(y) = \frac{k}{u-w_n}\left(\frac{u-y}{u-w_n}\right)^{k-1} exp\left\{-\left(\frac{u-y}{u-w_n}\right)^k\right\}$

CDF : $F_{Y_n}(y) = exp\left\{-\left(\frac{u-y}{u-w_n}\right)^k\right\}, y \le u$

Parameters : $\mu_{Y_n} = u - (u-w_n)\Gamma\left(1-\frac{1}{k}\right), \sigma_{Y_n} = (u-w_n)\sqrt{\left\{\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)\right\}}$

About Basic Wind Speed

- Basic wind speed is defined by 10-min averaged wind speed of 100-yr return period at 10m height and open terrain(KBDC, 2012).
- Design Wind Speed is calculated from Basic Wind Speed.

$$V_D = 1.723\left(\frac{z}{z_G}\right)^{\alpha} V_{10}, \quad V_2 = \left(\frac{z_{G1}}{z_1}\right)^{\alpha 1} \cdot V_1 \cdot \left(\frac{z_2}{z_{G2}}\right)^{\alpha 2}$$

(*Ref*. KBDC, 2012)

지표조도 구분	지 표 상 황	α	α_2	β	c	ε	l(m)	z_b (m)	z_G (m)
I	• 해상, 해안	0.12	0.174	1.25	0.15	0.125	200	2	200
II	• 개활지, 농지, 전원 수목과 저층건축물이 산재하여 있는 지역	0.16	0.21	1.54	0.2	0.2	150	5	300
III	• 수목과 저층건축물이 밀집하여 있는 지역 • 중, 고층 건물이 산재하여 있는 지역 • 완만한 구릉지	0.22	0.286	2.22	0.3	0.333	100	10	400
IV	• 중, 고층 건물이 밀집하여 있는 지역 • 기복이 심한 구릉지	0.29	0.4	3.33	0.45	0.5	50	20	500

- Method for obtaining the Basic wind speed
  1) From annual maximum data of near weather station(Extreme Analy)
  2) Typhoon Simulation(Monte Carlo Simulation)

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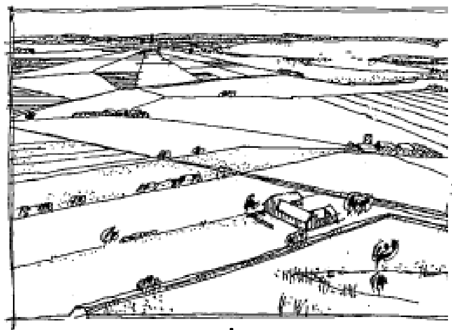
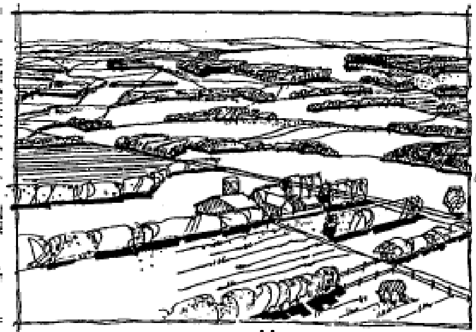
Extreme Analysis

- **Collect the annual maximum wind data from weather station.**
- **Transform the data to terrain category II and 10m height wind speed.**
  - The terrain category of weather station is determined by using map or picture.



I



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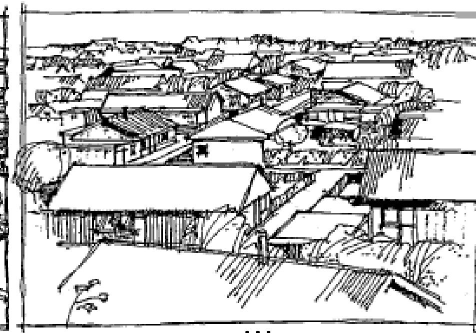
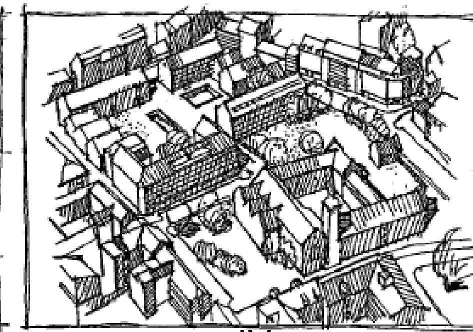


III



IV

- **Obtain the basic wind speed.**
  - The annual maximum data is assumed as <u>Type I distribution</u>.

$$F_{Y_n}(y) = exp[-exp\{-\alpha_n(y-u_n)\}], \quad \mu_{Y_n} = u_n + \frac{\gamma}{\alpha_n}, \quad \sigma_{Y_n} = \frac{\pi}{\alpha_n\sqrt{6}}$$

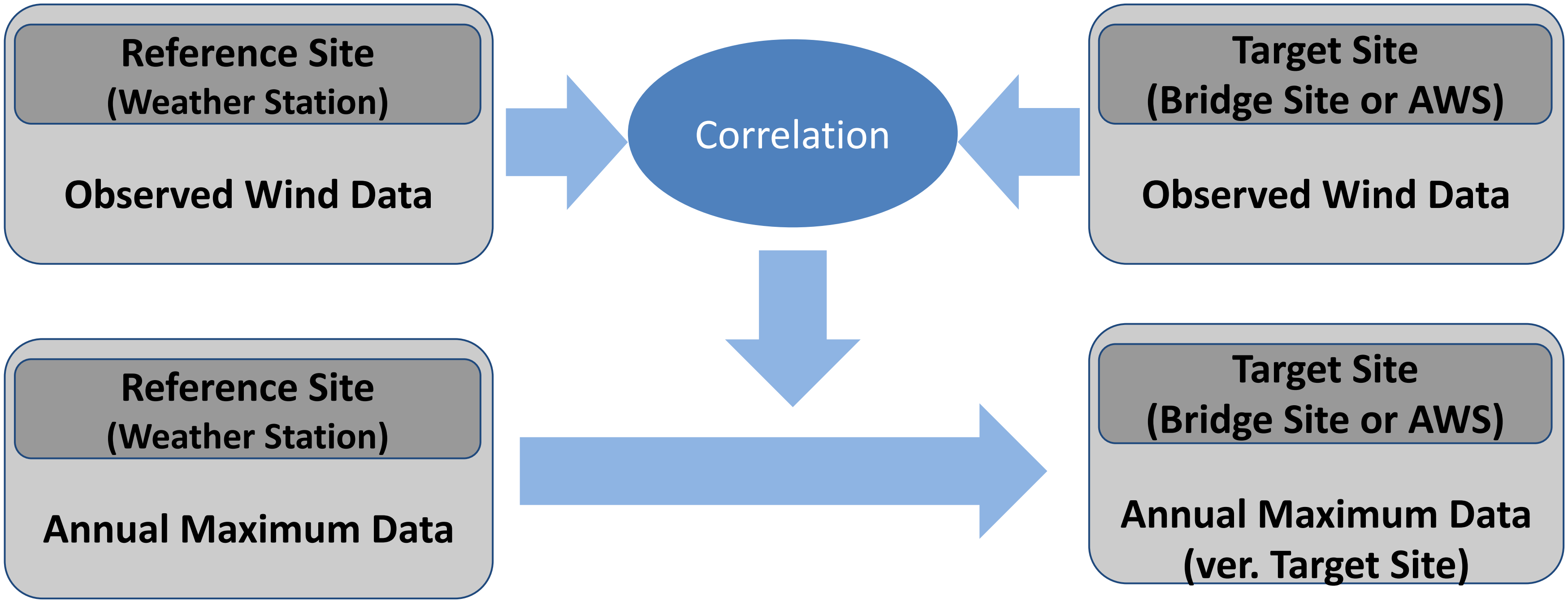
  - R-yr return period wind means that the wind occur once in R-yr. This wind will occur 1/R in 1-yr.

$$1-\frac{1}{R} = exp[-exp\{-\alpha_n(V_b-u_n)\}] \quad \Rightarrow \quad V_b = u_n - \frac{1}{\alpha_n} ln\left(-ln\left(1-\frac{1}{R}\right)\right)$$

- **Transform the basic wind speed to the target site**
  - Consider the terrain category and height of the target site.

Measure-Correlate-Predict(MCP) Method

Basic Concept



Theoretical

$$V_{target} = \left(\frac{\sigma_V}{\sigma_U}\right) U_{ref} + \bar{V} - \left(\frac{\sigma_V}{\sigma_U}\right) \bar{U}$$

- *U, V* are observed wind speed of reference site and target site, respectively.
- U_{ref}, V_{target} is annual wind speed of ref. site and target site.
- $\overline{(\ )}$, σ mean the mean and standard deviation.

Example

- **Target site is Red mark (35° 07’, 126° 19’)**
  - There are one weather station and 3 AWS(Automatic Weather Station).

Example – Extreme Analysis

- **Collect the annual maximum wind data of weather station.**
  - Terrain Category is determined by Engineer's judgment.

Year	Wind Speed (m/s)	Height (m)	Terrain Category	Year	Wind Speed (m/s)	Height (m)	Terrain Category
1964	25	15.8	III	1986	25.7	15.8	III
1965	22.5	15.8	III	1987	24.8	15.8	III
1966	23.3	15.8	III	1988	20	15.8	III
1967	21	15.8	III	1989	18.8	15.8	III
1968	22.3	15.8	III	1990	21.3	15.8	III
1969	20	15.8	III	1991	19.3	15.8	III
1970	26.5	15.8	III	1992	17.3	15.8	III
1971	30	15.8	III	1993	18.7	15.8	III
1972	26	15.8	III	1994	20	15.8	III
1973	25	15.8	III	1995	20.3	15.8	III
1974	18.2	15.8	III	1996	18.5	15.8	III
1975	19.2	15.8	III	1997	19	15.8	III
1976	19.7	15.8	III	1998	18.7	15.5	III
1977	18.3	15.8	III	1999	20.5	15.5	III
1978	19.7	15.8	III	2000	21.4	15.5	III
1979	21	15.8	III	2001	17.2	15.5	III
1980	23.7	15.8	III	2002	29.1	15.5	III
1981	18.3	15.8	III	2003	20.8	15.5	III
1982	18.3	15.8	III	2004	17.2	15.5	III
1983	19.3	15.8	III	2005	19.3	15.5	III
1984	15.2	15.8	III	2006	17	15.5	III
1985	22.7	15.8	III	2007	16.6	15.5	III

Example – Extreme Analysis

Condition of Weather Station

① Past(1964~1997) ② Current(1997~Current) Condition of Mokpo Weather Station

Correction

- Conversion the wind speed to condition of 10m height, Terrain Category II
- The effect of orography should be considered because the weather station is located at the hill(suggested in Eurocode 1 : Actions of Structures)

Example – Extreme Analysis

- **Transform the data to terrain category II and 10m height wind speed.**
  - Mean : 24.9m/s, Standard Deviation : 3.97m/s

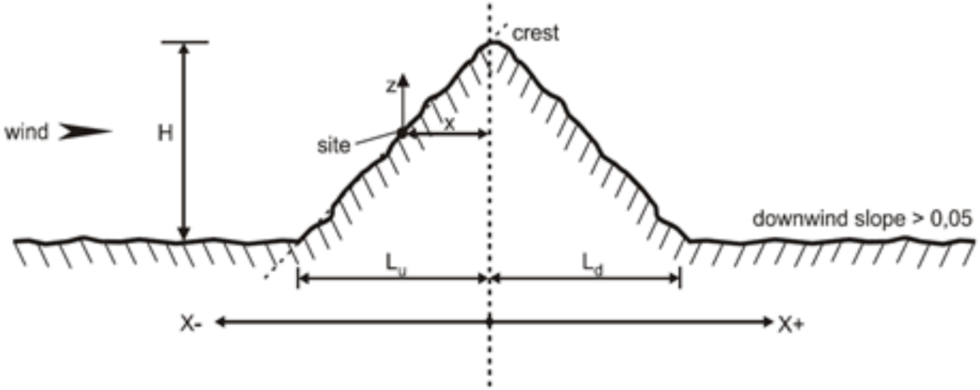
Year	Wind Speed (m/s)	Wind Speed (II, 10m)	Year	Wind Speed (m/s)	Wind Speed (II, 10m)
1964	25	29.9	1986	25.7	30.7
1965	22.5	26.9	1987	24.8	29.7
1966	23.3	27.9	1988	20	23.9
1967	21	25.1	1989	18.8	22.5
1968	22.3	26.7	1990	21.3	25.5
1969	20	23.9	1991	19.3	23.1
1970	26.5	31.7	1992	17.3	20.7
1971	30	35.9	1993	18.7	22.4
1972	26	31.1	1994	20	23.9
1973	25	29.9	1995	20.3	24.3
1974	18.2	21.8	1996	18.5	22.1
1975	19.2	23.0	1997	19	22.7
1976	19.7	23.6	1998	18.7	22.5
1977	18.3	21.9	1999	20.5	24.6
1978	19.7	23.6	2000	21.4	25.7
1979	21	25.1	2001	17.2	20.7
1980	23.7	28.3	2002	29.1	34.9
1981	18.3	21.9	2003	20.8	25.0
1982	18.3	21.9	2004	17.2	20.7
1983	19.3	23.1	2005	19.3	23.2
1984	15.2	18.2	2006	17	20.4
1985	22.7	27.1	2007	16.6	19.9

Example – Extreme Analysis

Orography Correction

- $V_{plain} = \frac{V_{hill}}{c_0}$

Slope	c_0
ϕ<0.05	1
0.05<ϕ<0.3	$1 + 2 \cdot s \cdot \phi$
0.3<ϕ	$1 + 0.6 \cdot s$



- $s = A \cdot e^{\left(B \cdot \frac{X}{L_u}\right)}$

$$A = 0.1552\left(\frac{z}{L_e}\right)^4 - 0.8575\left(\frac{z}{L_e}\right)^3 + 1.8133\left(\frac{z}{L_e}\right)^2 - 1.9115\left(\frac{z}{L_e}\right) + 1.0124$$

$$B = 0.3542\left(\frac{z}{L_e}\right)^2 - 1.0577\left(\frac{z}{L_e}\right) + 2.6456$$

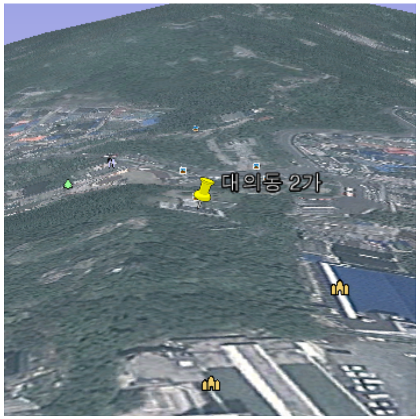
- ϕ : Slope(=H/L_u)
- X : Horizontal distance of the site from the crest
- z : Vertical distance from the ground level of the site
- H : Height
- Le : Effective length

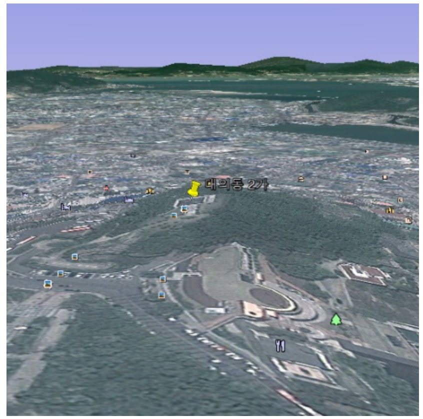
Shallow(0.05<ϕ<0.3)	Steep(0.3<ϕ)
$L_e = L_u$	$L_e = H/0.3$

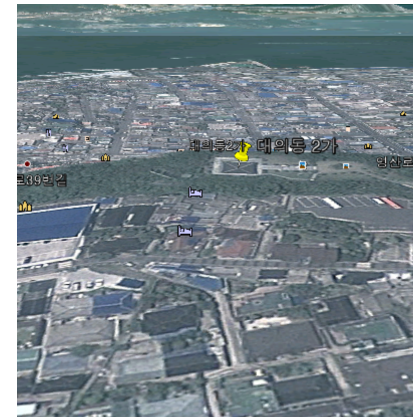
Example – Extreme Analysis

Orography Correction

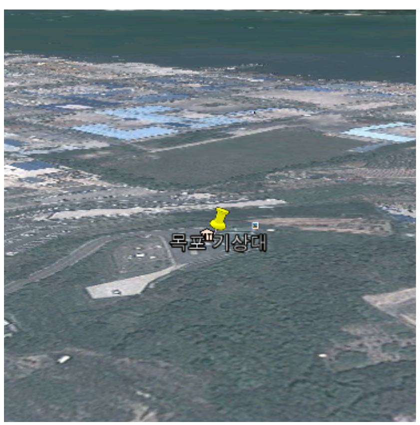
- Past(1964~1997yr)

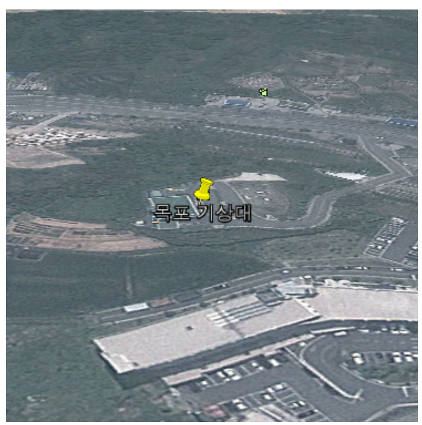


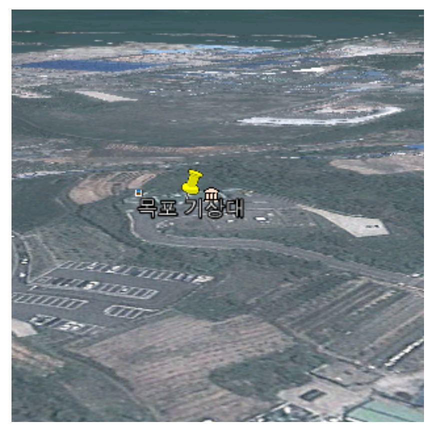


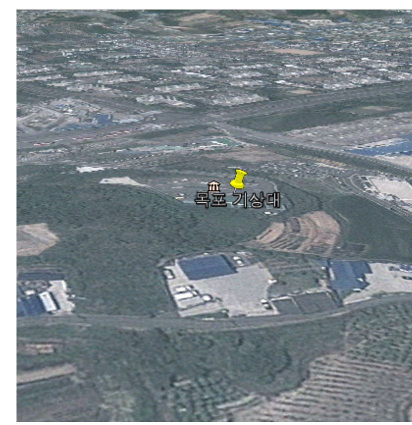



- Current(1997~)









Example – Extreme Analysis

Orography Correction

	1964~1997yr	1998~Current
Upwind Length, L_u	200	190
Effective Height, H	11	23
Horizontal distance from top, X	-25	-30
Vertical distance from ground, z	10	10
Slope, $\phi(=H/L_u)$	0.055	0.121
Effective Length, L_e	$200(=L_u)$	$190(=L_u)$

	1964~1997yr	1998~Current
A	0.921	0.917
B	2.594	2.591
s	0.666	0.609
c_0	$(1+2 \cdot s \cdot \phi)=1.073$	$(1+2 \cdot s \cdot \phi)=1.147$

Example – Extreme Analysis

Orography Correction

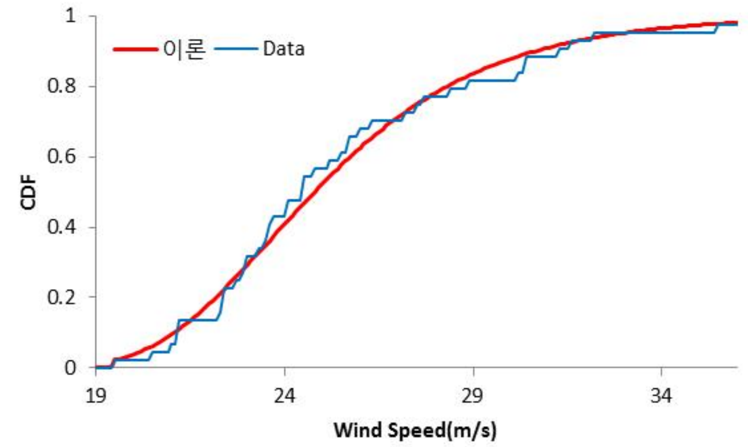
- c_0=1.147(1964~1997), c_0=1.073(1997~Current)

Year	Wind Speed (II, 10m)	Wind Speed (II, 10m, Plain)	Year	Wind Speed (II, 10m)	Wind Speed (II, 10m, Plain)
1964	29.9	25.5	1986	30.7	26.2
1965	26.9	23.0	1987	29.7	25.3
1966	27.9	23.8	1988	23.9	20.4
1967	25.1	21.4	1989	22.5	19.2
1968	26.7	22.8	1990	25.5	21.8
1969	23.9	20.4	1991	23.1	19.7
1970	31.7	27.1	1992	20.7	17.7
1971	35.9	30.6	1993	22.4	19.1
1972	31.1	26.6	1994	23.9	20.4
1973	29.9	25.5	1995	24.3	20.7
1974	21.8	18.6	1996	22.1	18.9
1975	23.0	19.6	1997	22.7	19.4
1976	23.6	20.1	1998	22.5	21.0
1977	21.9	18.7	1999	24.6	23.0
1978	23.6	20.1	2000	25.7	24.0
1979	25.1	21.4	2001	20.7	19.3
1980	28.3	24.2	2002	34.9	32.7
1981	21.9	18.7	2003	25.0	23.4
1982	21.9	18.7	2004	20.7	19.3
1983	23.1	19.7	2005	23.2	21.7
1984	18.2	15.5	2006	20.4	19.1
1985	27.1	23.2	2007	19.9	18.6

Example – Extreme Analysis

Obtain the basic wind speed.

- From mean and standard deviation, we can calculate the parameters of the Type I distribution



$$\alpha_n = \frac{\pi}{\sqrt{6}\sigma_{Y_n}} = 0.323$$

$$u_n = \mu_{Y_n} - \frac{\gamma}{\alpha_n} = 23.15$$

- Basic wind speed by return period

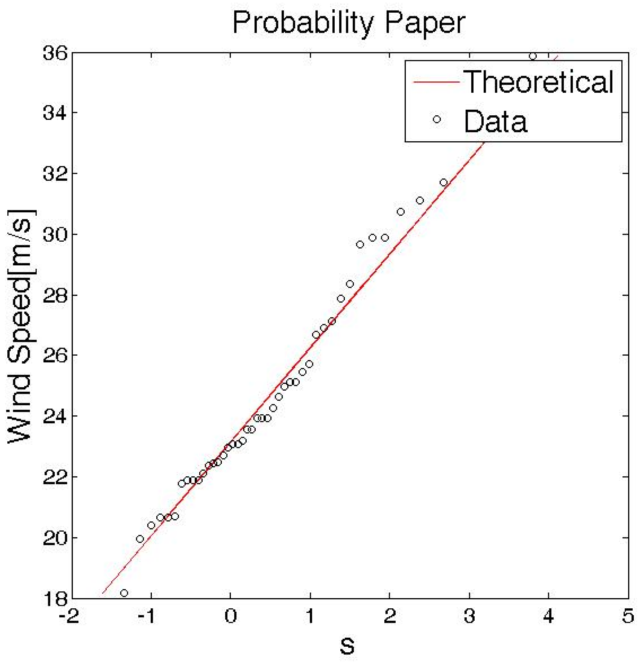
$$V_b = u_n - \frac{1}{\alpha_n} ln\left(-ln\left(1 - \frac{1}{R}\right)\right)$$

Return Period(yr)	Basic Wind Speed (m/s)
50	30.45
100	32.35
200	34.24
400	36.13

Example – Extreme Analysis

Probability Paper

- Visual method of verifying whether a theoretical distribution fits an empirical distribution
- The main advantage is that the plot tells us how well a theoretical distribution fits a sequence of data
- x-axis is the cdf or standard variate of a particular distribution
- y-axis is the value of data
- Ex) $F_{Y_n}(y) = exp[-exp\{-\alpha_n(y - u_n)\}]$
- Standard variate s $= \alpha_n(y - u_n)$
- Set of random variables $(\boldsymbol{X_1},\ \boldsymbol{X_2},\ \cdots, \boldsymbol{X_n})$
- $E[F(X_i)] = \frac{i}{n+1}$

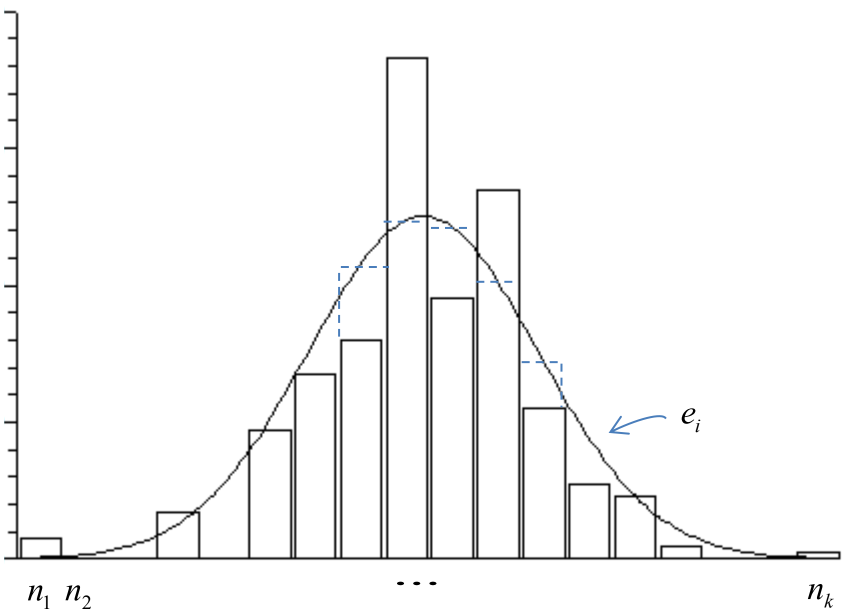


The Chi-Square Test for Goodness-of-Fit

- Compare the observed frequencies $n_1, n_2, \cdots, n_k$ of k values of the variate with the corresponding theoretical frequencies $e_1, e_2, \cdots, e_k$ calculated from the assumed theoretical distribution model.

$$\sum_{i=1}^{k} \frac{(n_i - e_i)^2}{e_i} < c_{1-\alpha, f}$$

where α is the significance level and f is the degrees-of-freedom.



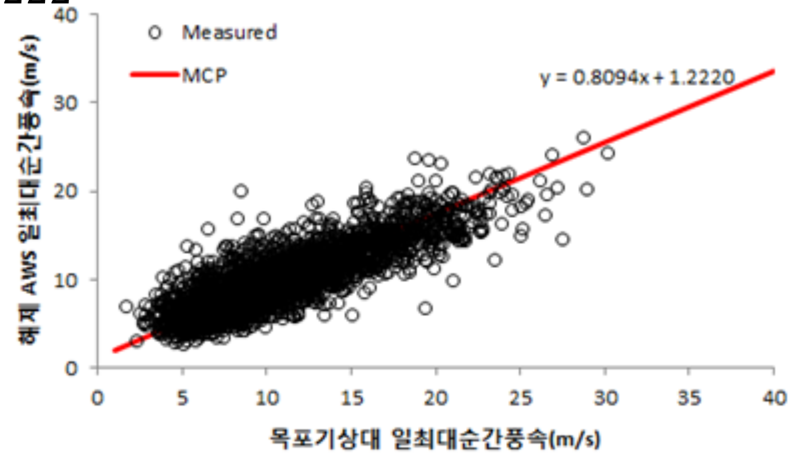
Example – MCP

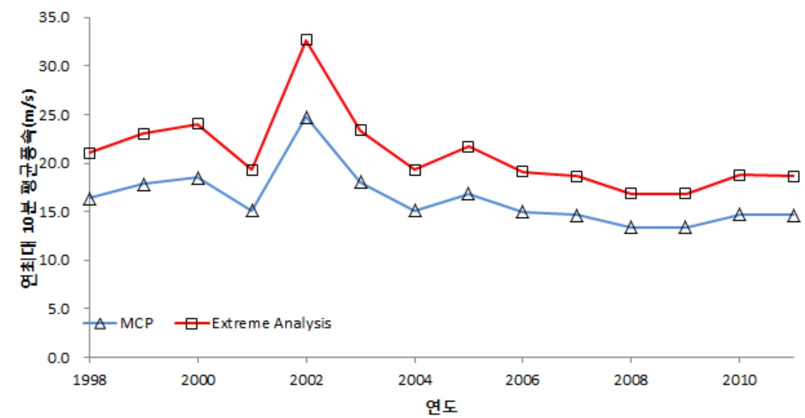
- **Obtain the Mokpo Weather Station(Reference) and Haeje AWS data(Target).**

$$V_{target} = \left(\frac{\sigma_V}{\sigma_U}\right) U_{ref} + \bar{V} - \left(\frac{\sigma_V}{\sigma_U}\right) \bar{U} = 0.809 U_{ref} + 1.222$$

	Reference Site, U	Target Site, V
Mean	10.24	9.51
Standard Deviation	4.27	3.46

Year	Annual Maximum Wind Speed(m/s)	Wind Speed (MCP)	Wind Speed (II, 10m, Plain)
1998	18.7	16.4	21.0
1999	20.5	17.8	23.0
2000	21.4	18.5	24.0
2001	17.2	15.1	19.3
2002	29.1	24.8	32.7
2003	20.8	18.1	23.4
2004	17.2	15.1	19.3
2005	19.3	16.8	21.7
2006	17.0	15.0	19.1
2007	16.6	14.7	18.6
2008	15.0	13.4	16.9
2009	15.0	13.4	16.9
2010	16.7	14.7	18.8
2011	16.6	14.7	18.6





Return Period(yr)	Basic Wind Speed(MCP)	Basic Wind Speed(Cor.)
100	25.53	33.74
200	27.12	35.95

Typhoon Simulation

- **For estimating the effect of typhoon, Monte Carlo simulation of typhoon is suggested(Russell, 1971)**
- **Various wind model of typhoon are existed.**
- **Simulation Process**
  - Collect the observed typhoon data
  - Determine the model of typhoon
  - Generate the typhoon parameters
  - Calculate the wind speed of target site
  - Estimate the wind speed applying the return period

Example – Typhoon Simulation

Collect the observed typhoon data

- Using Regional Specialized Meteorological Center data(Typhoon data)
- Observed year : 1951 ~ Current(total 62-yr)

51021906	002	2	200	1385	1010
51021912	002	2	200	1385	1010
51021918	002	2	230	1421	1000
51022000	002	9	250	1460	994
51022006	002	9	276	1506	994
51022012	002	9	289	1533	994
51022018	002	9	313	1575	992
51022100	002	9	326	1621	990
51022106	002	6	339	1660	990
51022112	002	6	360	1700	990
①			②	③	④

Format of Data

① Time

② Latitude of typhoon(0.1˚)

③ Longitude of typhoon(0.1˚)

④ Center Pressure(hpa)

- The typhoons data which pass the 500km range from target site are used to construct the probability model of parameters.

Example – Typhoon Simulation

Determine the model of typhoon

$$V_{g,1}(r) = \frac{V_T \cdot \sin\theta - f \cdot r}{2} + \sqrt{\left(\frac{V_T \cdot \sin\theta - f \cdot r}{2}\right)^2 + \frac{\Delta P \cdot B}{\rho}\left(\frac{r_0}{r}\right)^B e^{-\left(\frac{r_0}{r}\right)^B}}$$

:1-min averaged, Grdient height speed

Holland, G.J. (1980) An analytical model of the wind and pressure profiles in hurricane, *Monthly Weather Review, Vol. 108.*

V_T = Translation Velocity

θ = Approach angle

r = Distance of Closest Approach

ΔP = Central Pressure Difference (P_∞ - P_c)

r_0 = Radius of Maximum winds

Parameter

f = Coriolis Parameter **(=2Ωsinφ)**

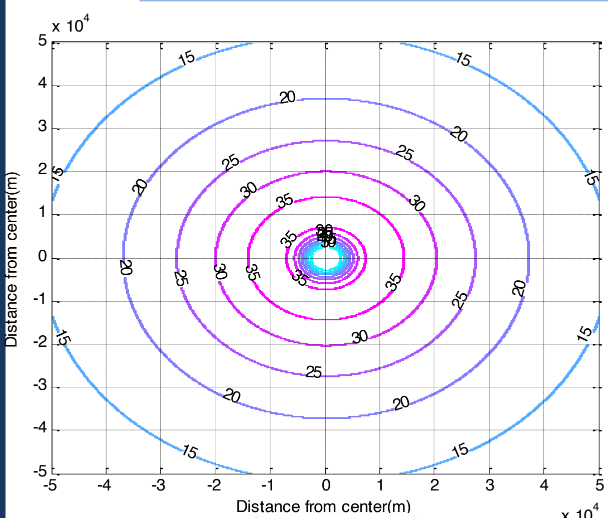
(Ω: Angular speed of the Earth,

φ: Latitude)

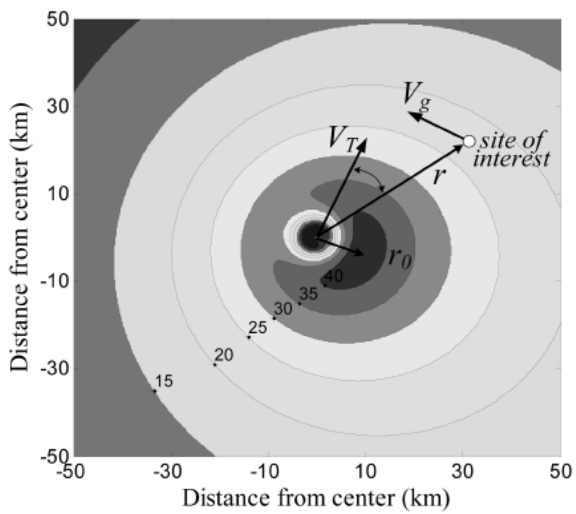
B = Shape Parameter **(=1)**

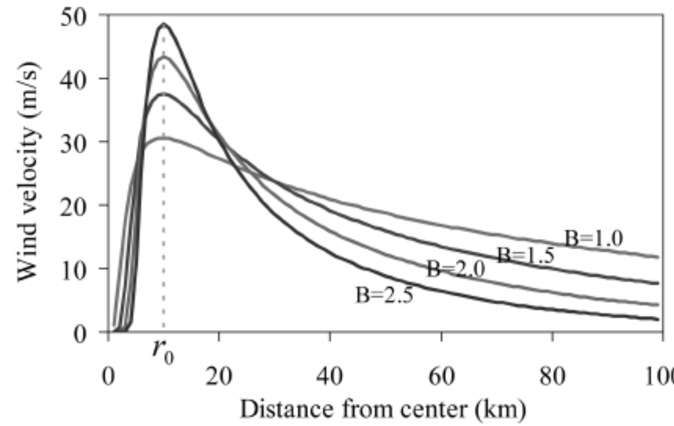
ρ = Air density **(=1.15kg/m³)**

Constant



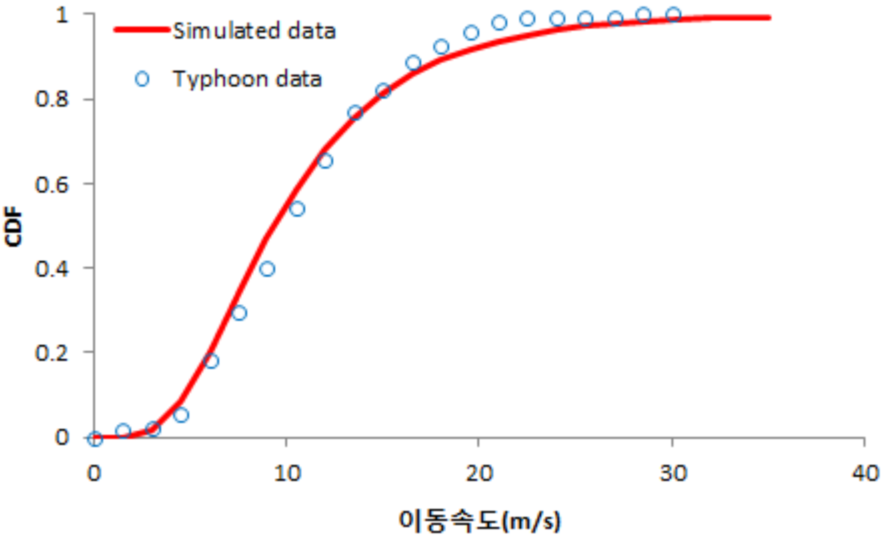
$V_T = 0$





Example – Typhoon Simulation

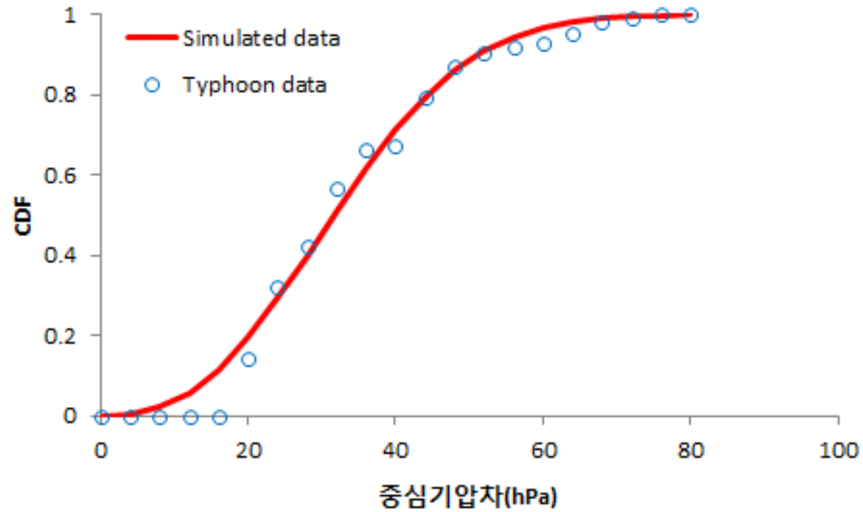
- **Determine the model of parameter,** Translation Velocity(V_T)
  - Obtained from RSMC data
  - Lognormal Distribution
  - μ=2.236, σ=0.529



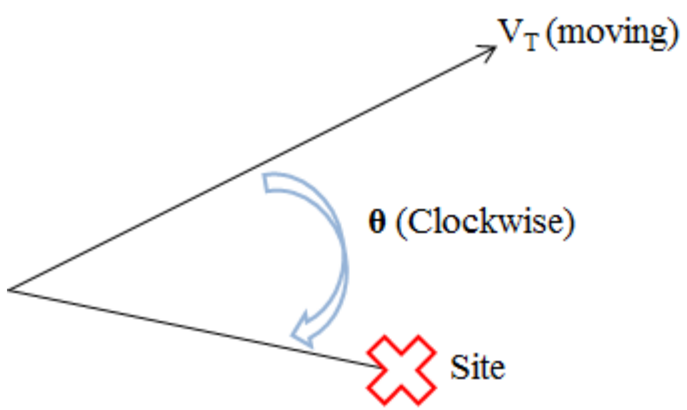
- **Determine the model of parameter,** Distance of Closest Approach(r)
  - Calculating the distance of closest approach(Typhoon ↔ Target site)
  - Uniform Distribution
  - Min : 25.672, Max=497.791

Example – Typhoon Simulation

- **Determine the model of parameter,** Central Pressure Difference(Δp)
  - Difference between pressure of typhoon and out of typhoon
  - Weibull Distribution
  - w_n=36.689, k=2.522



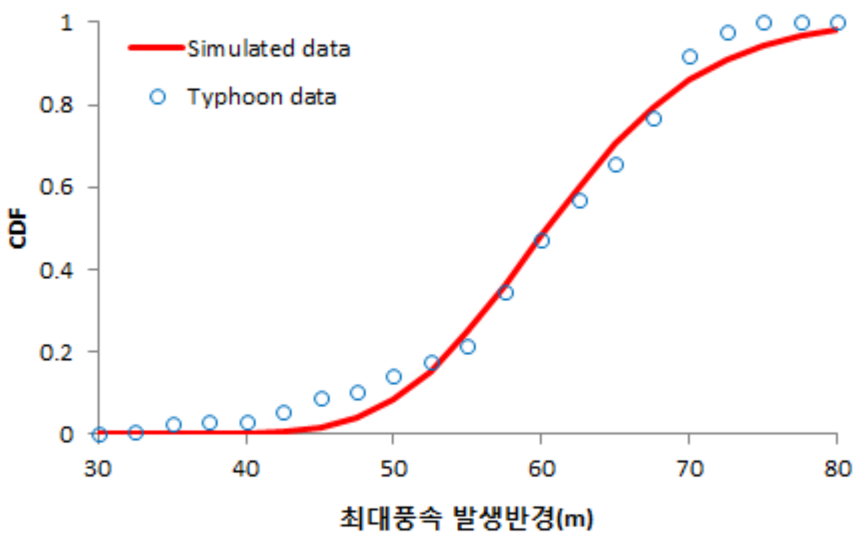
- **Determine the model of parameter,** Approach angle(θ)
  - Calculate the probability by sections
  - Linear Step Function



Example – Typhoon Simulation

- **Determine the model of parameter,** Radius of Maximum winds(r_0)
  - Lognormal Distribution
  - $r_0 = 15 + 4\left(\frac{p_c}{33.9} - 25\right)^{1.8} + k_1\left(\frac{\alpha-35}{k_2}\right)^{k_3}$
  - μ=4.101, σ=0.137

Latitude	k_1	k_2	k_3
>35°	27.0	8.0	2.0
<35°	-5.4	-12.0	1.3

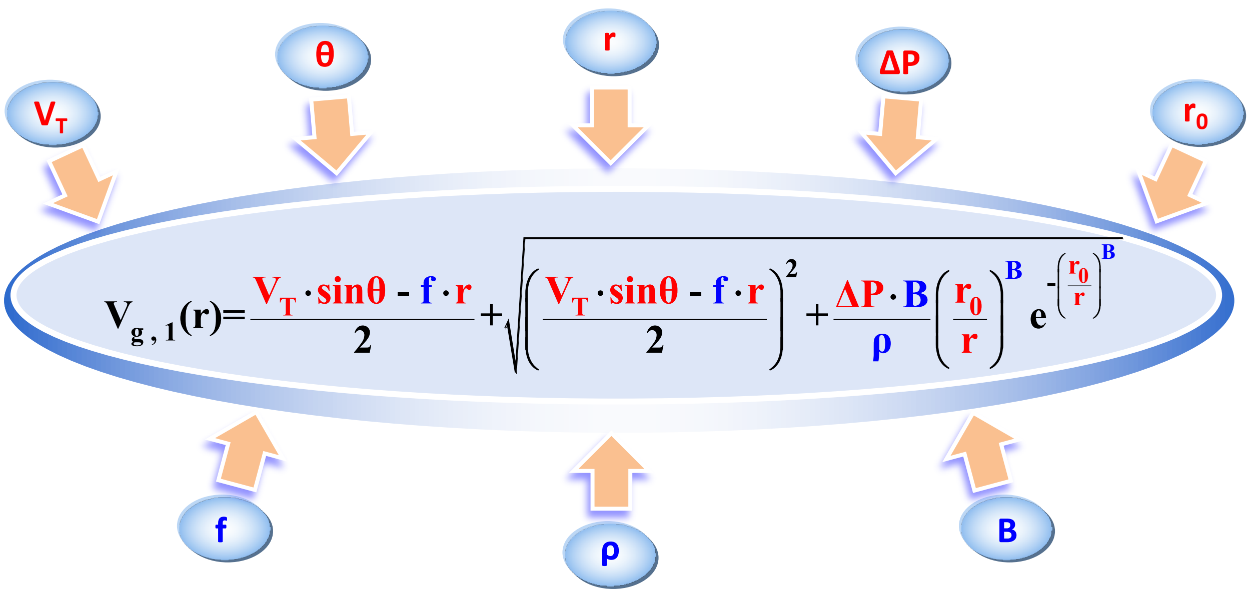


Example – Typhoon Simulation

Determine the model of typhoon

Parameter	Distribution	Value
Central Pressure Difference, Δp	Weibull	w_n=36.689, k=2.522
Translation Velocity, V_T	Lognormal	μ=2.236, σ=0.529
Radius of Maximum winds, r_0	Lognormal	μ=4.101, σ=0.137
Approach angle, θ	Linear Step Function	-
Distance of Closest Approach, r	Uniform	Min : 25.672, Max=497.791

Generate the typhoon parameters and typhoon



$$V_{g,1}(r)=\frac{V_T\cdot\sin\theta - f\cdot r}{2}+\sqrt{\left(\frac{V_T\cdot\sin\theta - f\cdot r}{2}\right)^2+\frac{\Delta P\cdot B}{\rho}\left(\frac{r_0}{r}\right)^B e^{-\left(\frac{r_0}{r}\right)^B}}$$

Example – Typhoon Simulation

Calculate the wind speed of target site

- Transformation Process

Grdient Height, 1-min averaged

↓

**Terrain Category I,
10m height, 1-min averaged**

↓

**Terrain Category I,
10m height, 10-min averaged**

↓

**Terrain Category II,
10m height, 10min averaged**

$$V_{ows,1} = \begin{cases} 0.825 \cdot V_{g,1}(r \le 2r_0) \\ \left(0.825 - \frac{0.075}{2r_0} \cdot r\right) \cdot V_{g,1}(2r_0 < r < 4r_0) \\ 0.750 \cdot V_{g,1}(r \ge 4r_0) \end{cases}$$

Vickery, P.J. and Twisdale, L.A. (1995b) Wind field and filling models for hurricane wind-speed predictions, J. Structural Engineering, ASCE, Vol. 121, No. 11.

$$V_{ows,10} = \frac{V_{ows,1}}{1.18}$$

Simiu, E. and Scanlan, R.H (1996) Wind Effects on Structures, Wiley Interscience.

$$V_{s,10} = \left(\frac{z_{G1}}{z_1}\right)^{\alpha 1} \cdot V_{ows,10} \cdot \left(\frac{z_2}{z_{G2}}\right)^{\alpha 2}$$

KBDC, 2012

Example – Typhoon Simulation

- **Estimate the wind speed applying the return period**

$$R = \frac{1}{1 - exp\left(-n_{rate}\left(1 - \frac{i}{m+1}\right)\right)}$$

n_{rate} : Yearly Occurrence Rate

i : i-th wind speed

m : Number of Simulated Typhoon

Batts, M.E., Russell, L.R., and Simiu, E. (1980) Hurricane wind speeds in the united states, *J. Structural Division, ASCE, Vol.*106, No. 10.

- **Basic wind speed.**
  - 100-yr Return period : 27.03m/s
  - 200-yr Return period : 28.74m/s

Proving

- **Let the probability that the wind speed in any one storm is less than v be denoted by F_v**
  - $P(V < v|1) = F_v$
- **The probability that the highest wind speed V in n storms is less than v**
  - $P(V < v \mid n) = (F_v)^n = F_v^n$
  - The wind speed of each n storm must be less than v
- **Let the probability that $V < v$ in τ years be denoted by $P(V < v,\ \tau)$**
  - $P(V < v,\ \tau) = \sum_{n=0}^{\infty} P(V < v \mid n) P(n,\ \tau)$

Proving

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Probabillity that n storms occur in τ year

Considering all n

Probability that the highest wind speed V in n storms is less than v

Proving

- **If a Poisson process is assumed to describe $P(n,\ \tau)$**

$$P(V<v,\ \tau)=\sum_{n=0}^{\infty}P(V<v\mid n)P(n,\ \tau)=\sum_{n=0}^{\infty}F_v^n\frac{(\nu\tau)^n}{n\,!}e^{-\nu\tau}$$

$$=e^{-\nu\tau}\sum_{n=0}^{\infty}\frac{(\nu\tau\ F_v)^n}{n\,!}=e^{-\nu\tau}e^{\nu\tau F_v}=e^{-\nu\tau(1-F_v)}$$

τ : How many years considered
n : The number of Hurricanes occurred in τ years
F_v : the probability that the wind speed in one storm is less than $\boldsymbol{v}$

  - ν is the annual rate of occurrence of hurricanes in the area of interest

- **Poisson Process**
  - Stochastic process which counts the number of events and the time that these events occur in a given time interval t(from *Wikipedia*)

$$P(X=n)=\frac{(\nu\ t)^n}{n!}e^{-\nu\ t}$$

  - ν is the mean occurrence rate
  - X is the number of occurrences in a time t

Proving

- For $\tau = 1$ yr, $P(V < v,\ 1)$ is the probability of occurrence of wind speeds less than v in 1 year

- Consider now the wind speed v_i . Its probability of occurrence can be written as $F_{v_i} = \frac{i}{m+1}$

- $$P(V < v,\ 1) = \exp[-v(1-F_v)] = \exp\left[-v\left(1-\frac{i}{m+1}\right)\right]$$

- Wind speed of return period R year means that this wind speed will occur $\frac{1}{R}$ in 1 year and not occur $1 - \frac{1}{R}$

- If v is the wind speed of return period R year,

$$P(V < v,\ 1) = 1 - \frac{1}{R}$$

Proving

- **Put the equal between two equations**
  - $$P(V < v,\ 1) = \exp[-v(1-F_v)] = \exp\left[-v\left(1-\frac{i}{m+1}\right)\right]$$
  - $$P(V < v,\ 1) = 1-\frac{1}{R}$$
- $$\exp\left[-v\left(1-\frac{i}{m+1}\right)\right] = 1-\frac{1}{R}$$
- **By arrangement the equation in terms of *R*, you can see the equation in 1st page**

$$R = \frac{1}{1-\exp\left[-v\left(1-\frac{i}{m+1}\right)\right]}$$

THANK YOU

for your attention!