Module #5:
Algorithms

Rosen 5th ed., §2.1

~31 slides, ~1 lecture

Chapter 2: More Fundamentals

- §2.1: Algorithms (Formal procedures)
- §2.2: Complexity of algorithms
  - Analysis using order-of-growth notation.
- §2.3: The Integers & Division
  - Some basic number theory.
- §2.6: Matrices
  - Some basic linear algebra.

§2.1: Algorithms

- The foundation of computer programming.
- Most generally, an *algorithm* just means a definite procedure for performing some sort of task.
- A computer *program* is simply a description of an algorithm in a language precise enough for a computer to understand, requiring only operations the computer already knows how to do.
- We say that a program *implements* (or "is an implementation of") its algorithm.

Algorithms You Already Know

- Grade school arithmetic algorithms:
  - How to add any two natural numbers written in decimal on paper using carries.
  - Similar: Subtraction using borrowing.
  - Multiplication & long division.
- Your favorite cooking recipe.
- How to register for classes at UF.

Programming Languages

- Some common programming languages:
  - **Newer:** Java, C, C++, Visual Basic, JavaScript, Perl, Tcl, Pascal
  - **Older:** Fortran, Cobol, Lisp, Basic
  - Assembly languages, for low-level coding.
- In this class we will use an informal, Pascal-like "*pseudo-code*" language.
- You should know at least 1 real language!

Algorithm Example (English)

- Task: Given a sequence $\{a_i\}=a_1,\ldots,a_n$, $a_i \in \mathbf{N}$, say what its largest element is.
- Set the value of a *temporary variable* v (largest element seen so far) to a_1's value.
- Look at the next element a_i in the sequence.
- If $a_i > v$, then re-assign v to the number a_i.
- Repeat previous 2 steps until there are no more elements in the sequence, & return v.

Executing an Algorithm

- When you start up a piece of software, we say the program or its algorithm are being *run* or *executed* by the computer.
- Given a description of an algorithm, you can also execute it by hand, by working through all of its steps on paper.
- Before ~WWII, “computer” meant a *person* whose job was to run algorithms!

Executing the Max algorithm

- Let $\{a_i\}$=7,12,3,15,8. Find its maximum…
- Set $v = a_1 = 7$.
- Look at next element: $a_2 = 12$.
- Is $a_2 > v$? Yes, so change v to 12.
- Look at next element: $a_2 = 3$.
- Is 3>12? No, leave v alone….
- Is 15>12? Yes, v=15…

Algorithm Characteristics

Some important features of algorithms:

- *Input*. Information or data that comes in.
- *Output*. Information or data that goes out.
- *Definiteness*. Precisely defined.
- *Correctness*. Outputs correctly relate to inputs.
- *Finiteness*. Won’t take forever to describe or run.
- *Effectiveness*. Individual steps are all do-able.
- *Generality*. Works for many possible inputs.
- *Efficiency*. Takes little time & memory to run.

Our Pseudocode Language: §A2

Declaration

STATEMENTS

procedure
    name(*argument*: *type*)

variable := *expression*

informal statement

begin *statements* **end**

{*comment*}

if *condition* then
    statement [else
    statement]

for *variable* := *initial*
    value to *final value*
    statement

while *condition*
    statement

procname(*arguments*)

Not defined in book:

return *expression*

procedure *procname*(*arg*: *type*)

- Declares that the following text defines a procedure named *procname* that takes inputs (*arguments*) named *arg* which are data objects of the type *type*.
  - Example:
    procedure *maximum*(*L*: list of integers)
      [statements defining *maximum*…]

variable := *expression*

- An *assignment* statement evaluates the expression *expression*, then reassigns the variable *variable* to the value that results.
  - Example:
    v := $3x+7$ (If x is 2, changes v to 13.)
- In pseudocode (but not real code), the *expression* might be informal:
  - x := the largest integer in the list L

Informal statement

- Sometimes we may write a statement as an informal English imperative, if the meaning is still clear and precise: "swap *x* and *y*"
- Keep in mind that real programming languages never allow this.
- When we ask for an algorithm to do so-and-so, writing "Do so-and-so" isn't enough!
  - Break down algorithm into detailed steps.

begin *statements* **end**

- Groups a sequence of statements together:

  begin

  statement 1

  statement 2

  …

  statement n

  end

- Allows sequence to be used like a single statement.
- Might be used:
  - After a **procedure** declaration.
  - In an **if** statement after **then** or **else**.
  - In the body of a **for** or **while** loop.

{*comment*}

- Not executed (does nothing).
- Natural-language text explaining some aspect of the procedure to human readers.
- Also called a *remark* in some real programming languages.
- Example:
  - {Note that *v* is the largest integer seen so far.}

if *condition* **then** *statement*

- Evaluate the propositional expression *condition*.
- If the resulting truth value is **true**, then execute the statement *statement*; otherwise, just skip on ahead to the next statement.
- Variant: **if** *cond* **then** *stmt1* **else** *stmt2* Like before, but iff truth value is **false**, executes *stmt2*.

while *condition* *statement*

- *Evaluate* the propositional expression *condition*.
- If the resulting value is **true**, then execute *statement*.
- Continue repeating the above two actions over and over until finally the *condition* evaluates to **false**; then go on to the next statement.

while *condition* *statement*

- Also equivalent to infinite nested **if**s, like so:

if *condition*
    begin
        statement
        if *condition*
            begin
                statement
                …(continue infinite nested if's)
            end
    end

for *var* := *initial* to *final* *stmt*

- *Initial* is an integer expression.
- *Final* is another integer expression.
- Repeatedly execute *stmt*, first with variable *var* := *initial*, then with *var* := *initial*+1, then with *var* := *initial*+2, *etc*., then finally with *var* := *final*.
- What happens if *stmt* changes the value that *initial* or *final* evaluates to?

for *var* := *initial* to *final* *stmt*

- **For** can be exactly defined in terms of **while,** like so:

```
begin
    var := initial
    while var ≤ final
        begin
            stmt
            var := var + 1
        end
end
```

procedure(*argument*)

- A *procedure call* statement invokes the named *procedure*, giving it as its input the value of the *argument* expression.
- Various real programming languages refer to procedures as *functions* (since the procedure call notation works similarly to function application $f(x)$), or as *subroutines*, *subprograms*, or *methods*.

Max procedure in pseudocode

procedure *max*(a_1, a_2, …, a_n: integers)

$v := a_1$ {largest element so far}

for $i := 2$ **to** n {go thru rest of elems}

if $a_i > v$ **then** $v := a_i$ {found bigger?}

{at this point v's value is the same as the largest integer in the list}

return v

Another example task

- Problem of *searching an ordered list*.
  - Given a list L of n elements that are sorted into a definite order (*e.g.*, numeric, alphabetical),
  - And given a particular element x,
  - Determine whether x appears in the list,
  - and if so, return its index (position) in the list.
- Problem occurs often in many contexts.
- Let's find an *efficient* algorithm!

Search alg. #1: Linear Search

procedure *linear search*

(x: integer, $a_1, a_2, \ldots, a_n$: distinct integers)

$i := 1$

while $(i \leq n \wedge x \neq a_i)$

    $i := i + 1$

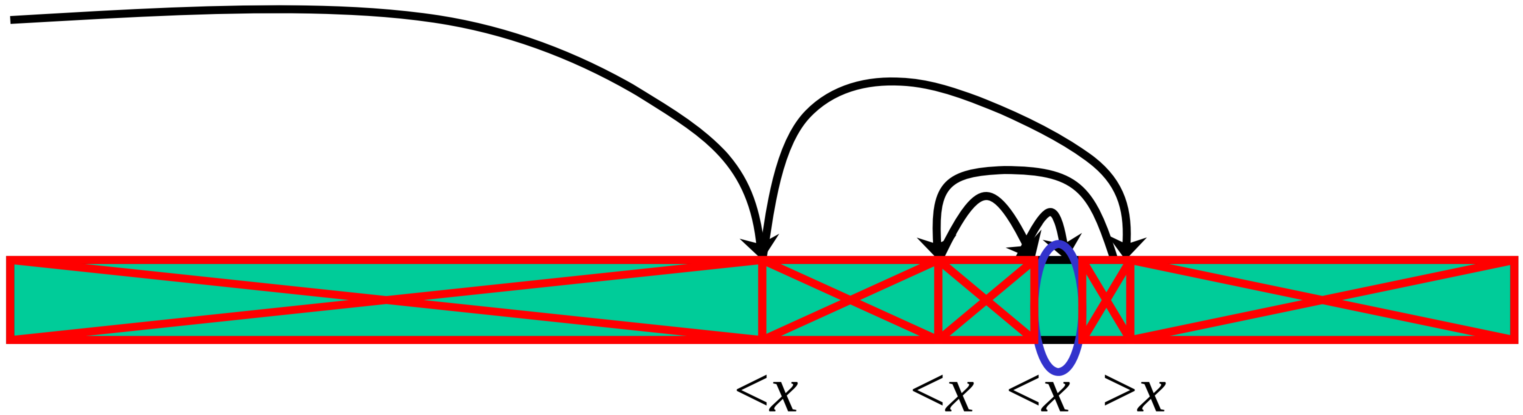
if $i \leq n$ **then** *location* $:= i$

else *location* $:= 0$

return *location* {index or 0 if not found}

Search alg. #2: Binary Search

- Basic idea: On each step, look at the *middle* element of the remaining list to eliminate half of it, and quickly zero in on the desired element.



Search alg. #2: Binary Search

procedure *binary search*
(x:integer, $a_1, a_2, \ldots, a_n$: distinct integers)
$i := 1$ {left endpoint of search interval}
$j := n$ {right endpoint of search interval}
while $i<j$ **begin** {while interval has >1 item}
$m := \lfloor (i+j)/2 \rfloor$ {midpoint}
if $x>a_m$ **then** $i := m+1$ **else** $j := m$
end
if $x = a_i$ **then** *location* $:= i$ **else** *location* $:= 0$
return *location*

Practice exercises

- 2.1.3: Devise an algorithm that finds the sum of all the integers in a list. [2 min]
- **procedure** *sum*(a_1, a_2, …, a_n: integers)
  - s := 0 {sum of elems so far}
  - **for** i := 1 **to** n {go thru all elems}
    - s := $s + a_i$ {add current item}
  - {at this point s is the sum of all items}
  - **return** s

Review §2.1: Algorithms

- Characteristics of algorithms.
- Pseudocode.
- Examples: Max algorithm, linear search & binary search algorithms.
- Intuitively we see that binary search is much faster than linear search, but how do we analyze the efficiency of algorithms formally?
- Use methods of *algorithmic complexity*, which utilize the order-of-growth concepts from §1.8.

Review: *max* algorithm

procedure $max(a_1, a_2, \ldots, a_n$: integers)

$v := a_1$ {largest element so far}

for $i := 2$ **to** n {go thru rest of elems}

if $a_i > v$ **then** $v := a_i$ {found bigger?}

{at this point v's value is the same as the largest integer in the list}

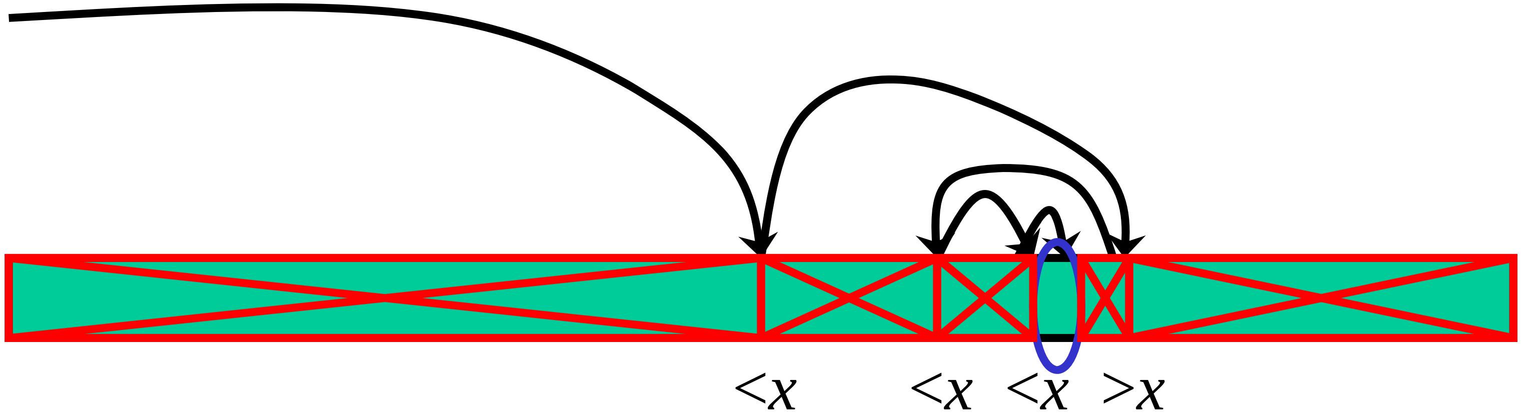
return v

Review: Linear Search

procedure *linear search*
(x: integer, $a_1, a_2, \ldots, a_n$: distinct integers)
$i := 1$
while ($i \leq n \wedge x \neq a_i$)
$i := i + 1$
if $i \leq n$ **then** $location := i$
else $location := 0$
return $location$ {index or 0 if not found}

Review: Binary Search

- Basic idea: On each step, look at the *middle* element of the remaining list to eliminate half of it, and quickly zero in on the desired element.



Review: Binary Search

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if $x = a_i$ **then** *location* $:= i$ **else** *location* $:= 0$

return *location*