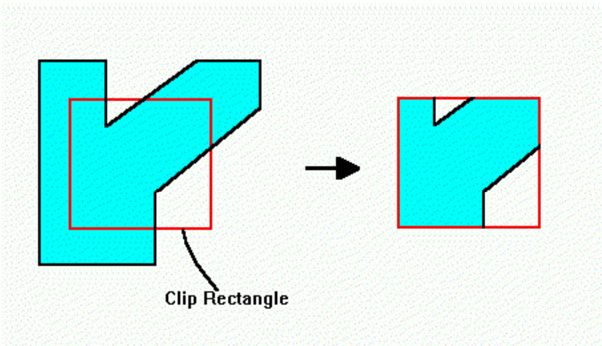
Graphics Primitives

Chapter 3 & 4
Intro. to Computer Graphics
Spring 2008, Y.G. Shin

Graphic Output and Input Pipeline

- Scan conversion
  - converts primitives such as lines, circles, etc. into pixel values
  - geometric description $\Rightarrow$ a finite scene area
- Clipping
  - the process of determining the portion of a primitive lying within a region called *clip region*



Graphic Output Pipeline

- Output pipeline (rendering process)

application model : descriptions of objects

↓

application program : generates a sequence of functions to display a model

↓

graphics package : clipping, scan conversion, shading, etc.

↓

display H/W

Graphic Input Pipeline

- Input pipeline
  - user interaction (e.g., mouse click)

    ↓
  - graphic package (by sampling or event-driven input functions)

    ↓
  - application program

    ↓
  - modify the model or the image on the screen

Graphic Output Pipeline

- displays with frame buffers and display controllers
  - common in pug-in graphics card
  - scan conversion by a graphic package and display processor
- displays with frame buffers only
  - scan conversion by a graphic package

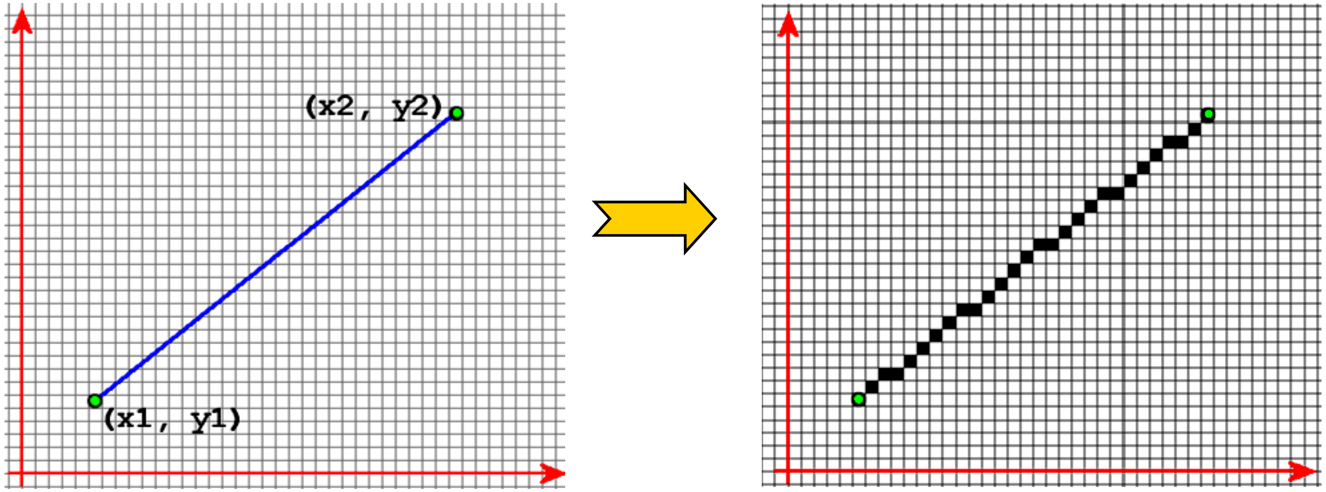
Output Pipeline in Software

When scan conversion and clipping?

- Clipping before scan conversion
  - for lines, rectangles, and polygons clipping after scan converting each primitive (scissoring)
- Clipping after scan converting the entire collection of primitives into a temporary canvas
  - for text

Scan Converting Lines

A line from (x0,y0) to (x1,y1) $\Rightarrow$ a series of pixels



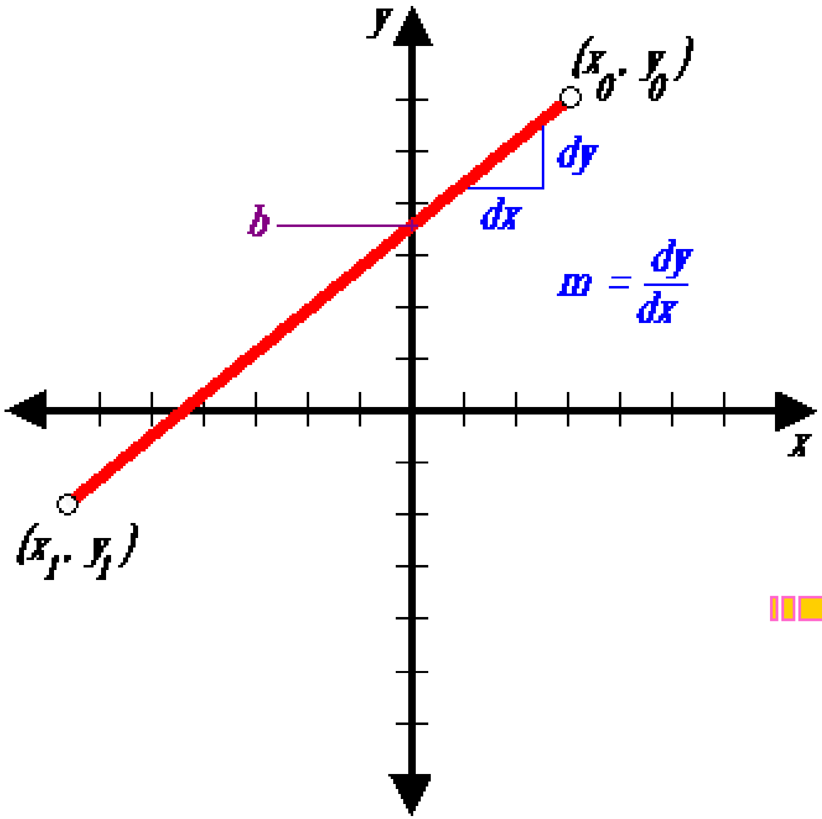
[Criteria]

- Straight lines should appear straight
- Line end-points should be constrained - grids, snaps
- Uniform density and intensity
- Line algorithms should be fast

Why Study
Scan Conversion Algorithms?

- Every high-end graphics card support this.
- You will never have to write these routines yourself, unless you become a graphics hardware designer.
- So why learn this stuff?
  - Maybe you *will* become a graphics hardware designer.
  - But seriously, the same basic tricks underlie lots of algorithms:
    - 3-D shaded polygons
    - Texture mapping
    - etc.

Simple Scan Converting Lines



Based on *slope-intercept algorithm* from algebra*:*

$y = mx + b$

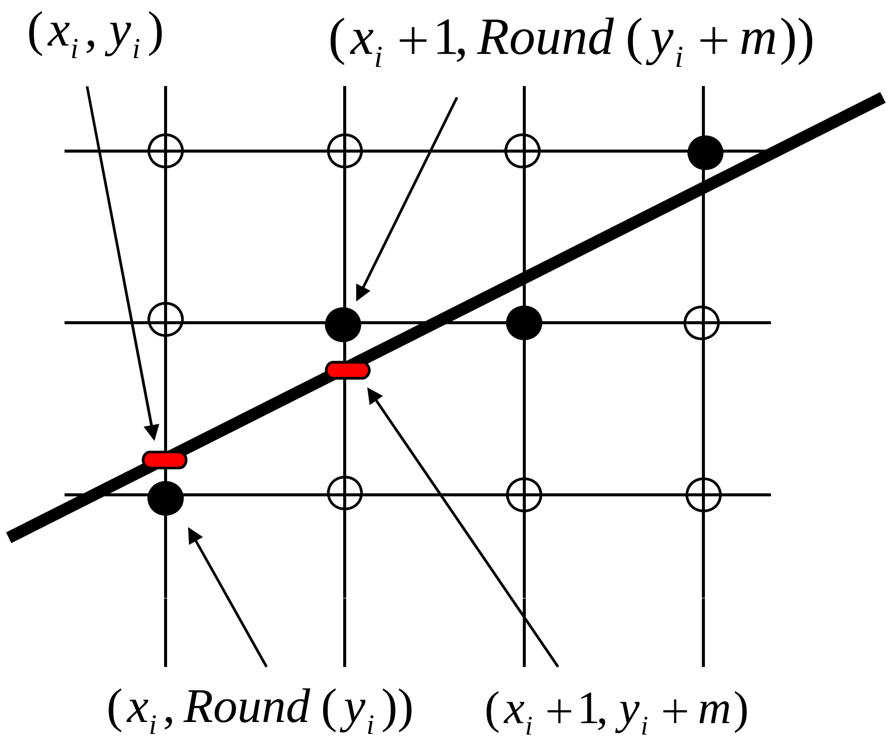
Simple approach:

increment x, solve for y

Floating point arithmetic required

Digital Differential Analyzer(DDA)

- Idea
  1. Go to starting end point
  2. Increment x and y values by constants proportional to x and y such that one of them is 1.
  3. Round to the closest raster position



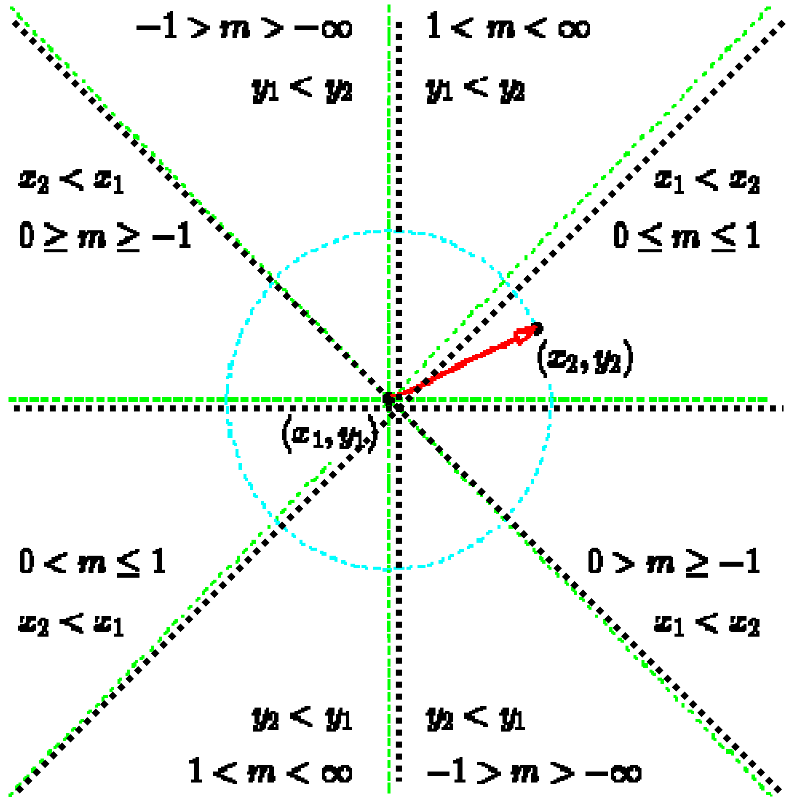
Digital Differential Analyzer(DDA)

- Drawbacks
  - rounding to an integer takes time
  - floating-point operations
- Is there a simpler way ?
- Can we use only integer arithmetic ?

  ⇐ Easier to implement in hardware.

Midpoint Line Algorithm (Bresenham's Line Algorithm)

- Assume a line from (x_1,y_1) to (x_2,y_2) that $0<\text{slope}<1$ and $x_1<x_2$.
- Use ***symmetry***



Midpoint Line Algorithm (Bresenham's Line Algorithm)

Suppose that we have just finished drawing a pixel P = (x_p, y_p) and we are interested in figuring out which pixel to draw next.

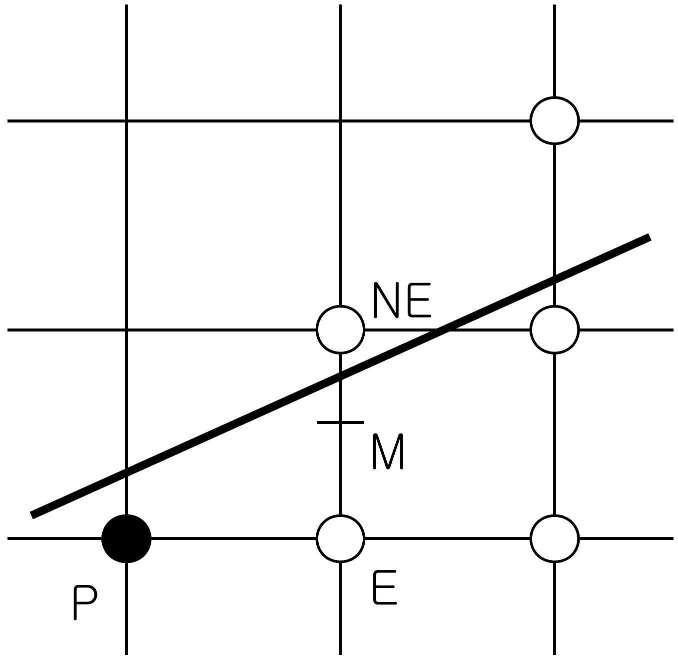
If distance(NE,M) >
distance(E,M)

then

select E = (x_p+1, y_p)

else

select NE = (x_p+1, y_p+1)



Midpoint Line Algorithm (Bresenham's Line Algorithm)

- A line eq. in the implicit form:

  $F(x, y) = ax + by + c = 0$

- Using $y = \Delta y/\Delta x \cdot x + B$,

  where $a = \Delta y$, $b = -\Delta x$, $c = B$.

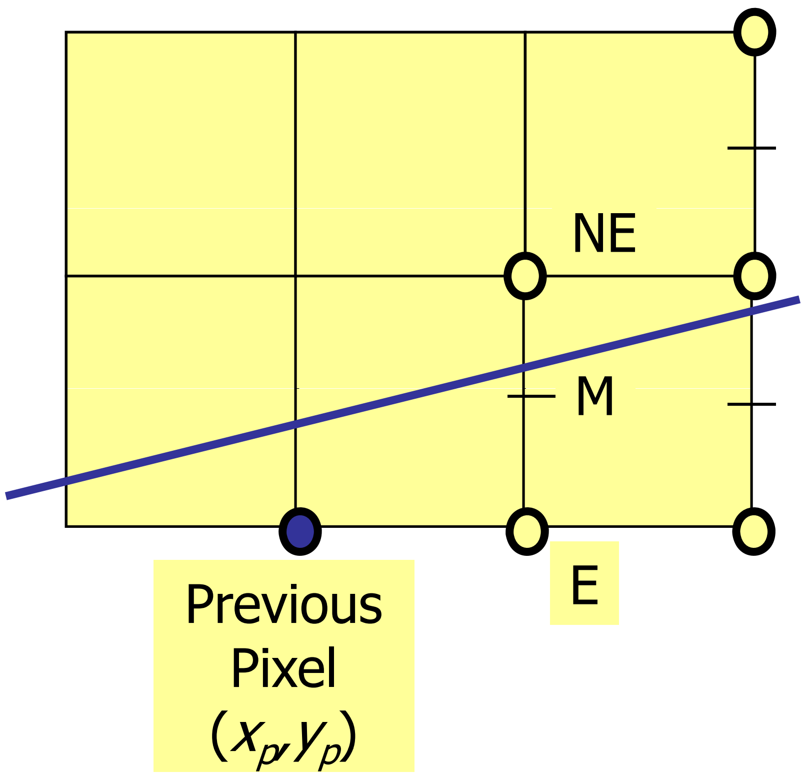
  $F(x,y) = \Delta y \cdot x - \Delta x \cdot y + B \cdot \Delta x = 0.$

- Let's use an equivalent representation:

  $F(x,y) = 2ax + 2by + 2c = 0.$

Midpoint Line Algorithm (Bresenham's Line Algorithm)

- Making slope assumptions, observe that b < 0, and this implies:
  - F(x,y) < 0 for points above the line
  - F(x,y) > 0 for points below the line
- To apply the midpoint criterion, we need only to compute $F(M) = F(x_p+1, y_p+\frac{1}{2})$ and to test its sign.



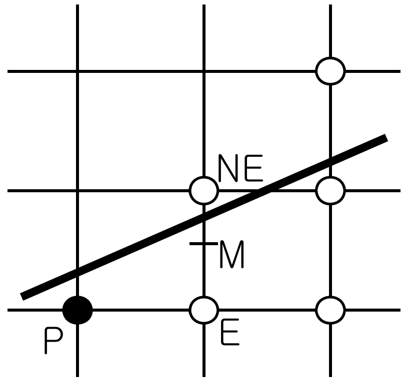
Midpoint Line Algorithm (Bresenham's Line Algorithm)

- To determine which one to pick up, we define a decision variable

$$D = F(x_p+1, y_p+½)$$

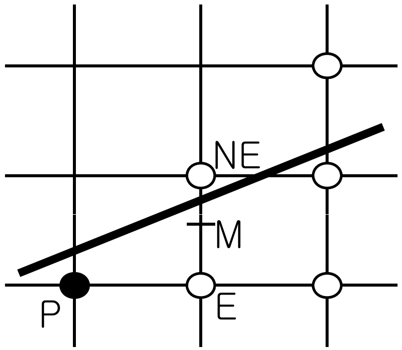
$$D = 2a(x_p+1) + 2b(y_p+½) + 2c$$
$$= 2ax_p + 2by_p + (2a + b + c)$$

- If D > 0 then M is below the line, so select NE, otherwise select E.



Midpoint Line Algorithm (Bresenham's Line Algorithm)

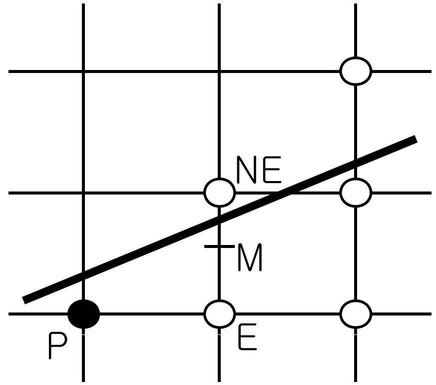
- *How to compute D incrementally?*
  - Suppose we know the current D value, and we want to determine the next D.
  - If we decide on going to E next,
    - $D_{new} = F(x_p + 2, y_p + ½)$
      $= 2a(x_p + 2) + 2b(y_p + ½) + c$
      $= D + 2a = D + 2\Delta y$
  - If we decide on going to NE next,
    - $D_{new} = F(x_p + 2, y_p + 1 + ½)$
      $= 2a(x_p + 2) + 2b(y_p + 1 + ½) + c$
      $= D + 2(a + b) = D + 2(\Delta y - \Delta x)$.



Midpoint Line Algorithm (Bresenham's Line Algorithm)

- Since we start at (x_0, y_0), the initial value of D can be calculated by

$$D_{init} = F(x_0 + 1, y_0 + ½)$$
$$= (2ax_0 + 2by_0 + c) + (2a + b)$$
$$= 0 + 2a + b$$
$$= 2\Delta y - \Delta x$$



- Advantages
  - Only need add integers and multiply by 2 (which can be done by shift operations)
  - Incremental algorithm

Example code

```
void MidpointLine(int x0, int y0,
                int x1, int y1, int value) {
    int     dx = x1 - x0;
    int     dy = y1 - y0;
    int     d = 2 * dy - dx;
    int     incrE = 2 * dy;
    int     incrNE = 2 * (dy - dx);
    int     x = x0;
    int     y = y0;

    writePixel(x, y, value);

    while (x < x1) {
            if (d <= 0) {           // East Case
                    d = d + incrE;
            } else {                // Northeast Case
                    d = d + incrNE;
                    y++;
            }
            x++;
            writePixel(x, y, value);
    }               /* while */
}                   /* MidpointLine */
```

Midpoint Line Algorithm- Example

Line end points:

$(x_0,y_0) = (5,8);$

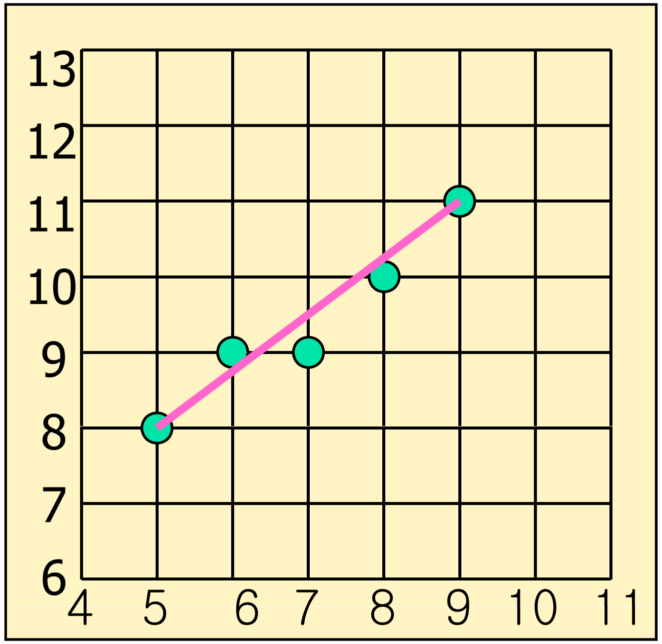
$(x_1,y_1) = (9,11)$

- $\Delta x = 4; \Delta y = 3$
- $D_{init} = 2\Delta y - \Delta x = 2 > 0$

  ➔ select NE
- $D_{new} = D + 2(\Delta y - \Delta x) = 0$

  ➔ Select E
- $D_{new} = D + 2\Delta y = 0 + 6 = 6$

  ➔ Select NE

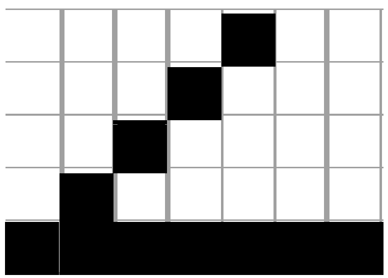


Scan Converting Lines (issues)

- Endpoint order
  - S_{01} is a set of pixels that lie on the line from P_0 to P_1
  - S_{10} is a set of pixels that lie on the line from P_1 to P_0

    $\Rightarrow$ S_{01} should be the same as S_{10}
- Varying intensity of a line as a function of slope
  - For the diagonal line, it is longer than the horizontal line but has the same number of pixels as the latter

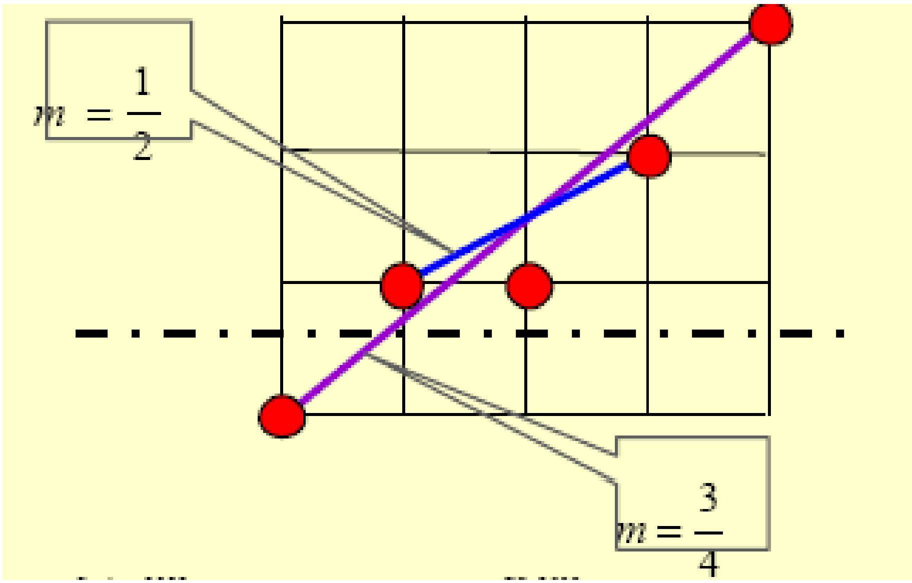
    $\Rightarrow$ needs antialiasing
- Outline primitives composed of lines
  - Care must be taken to draw shared vertices of polylines only once



Scan Converting Lines (issues)

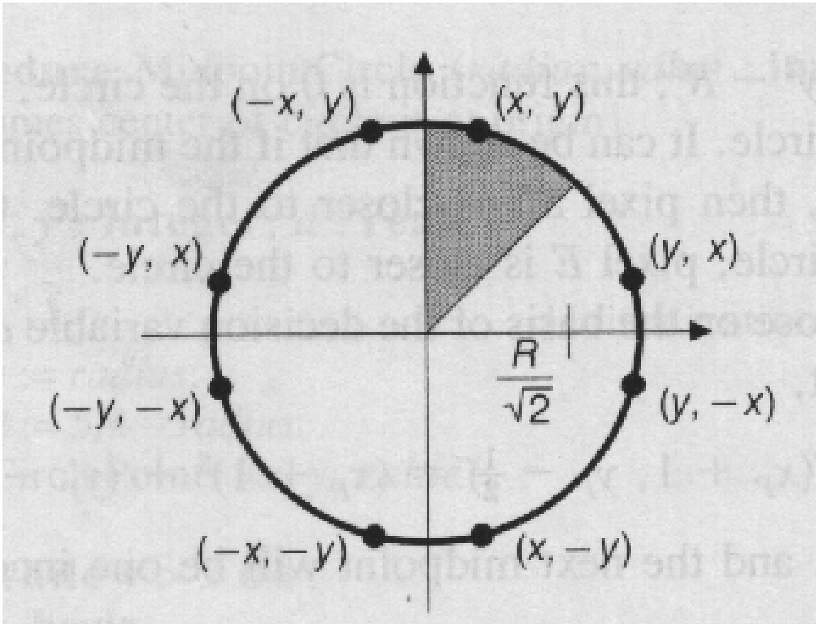
- Starting at the edge of a clip rectangle
  - Starting point is not the intersection point of the line with clipping edge

    $\Rightarrow$ Clipped line may have a different slope



Scan Converting Circles

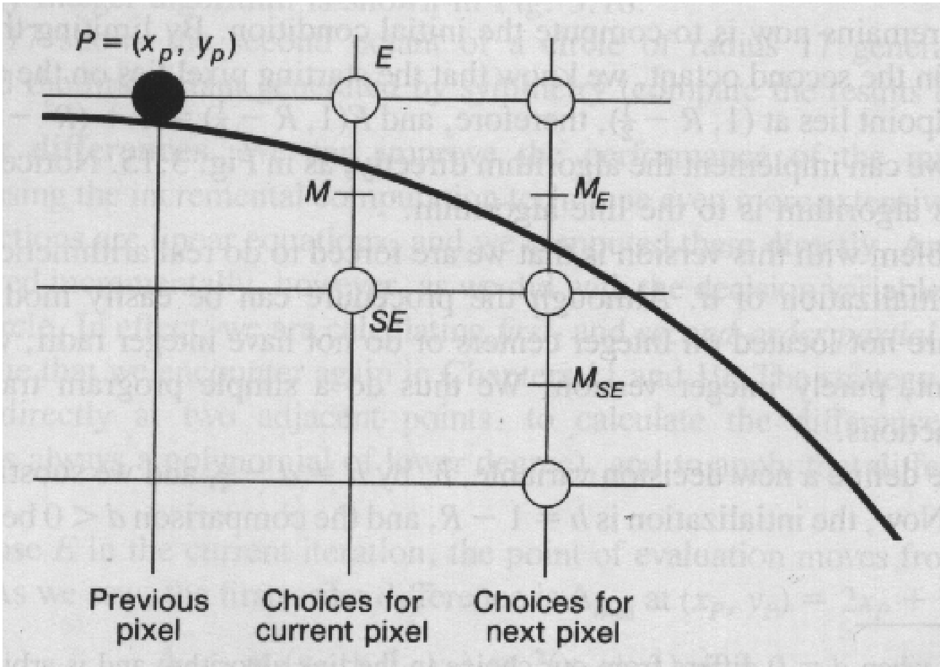
- Eight-way symmetry



We only consider 45° of a circle

Midpoint Circle Algorithm

- Suppose that we have just finished drawing a pixel (x_p, y_p) and we are interested in figuring out which pixel to draw next.



Midpoint Circle Algorithm

- $F(x,y) = x^2 + y^2 - R^2$

  $= 0$ on the circle

  > 0 outside the circle

  < 0 inside the circle

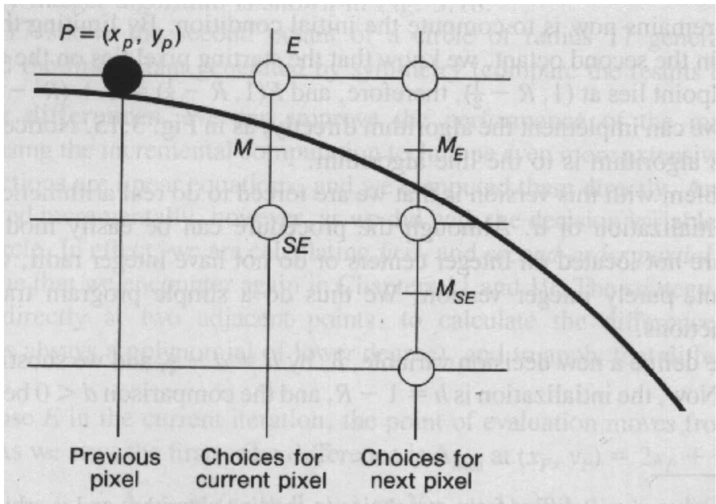
- If F(midpoint between E and SE) > 0

  then

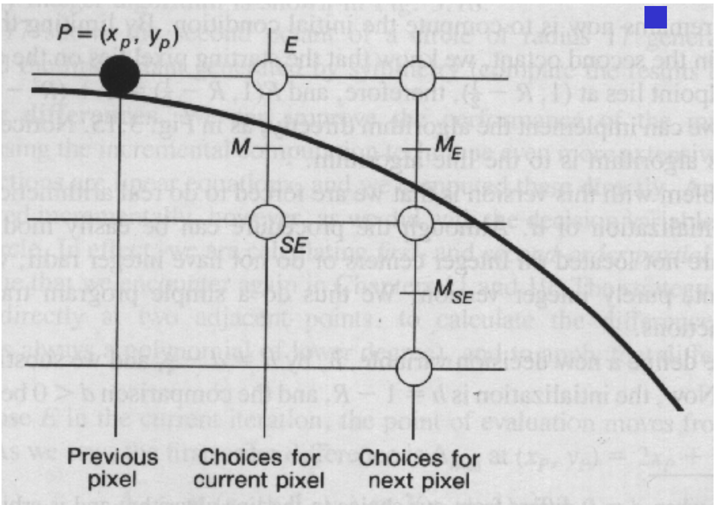
  select SE = (x_p+1, y_p-1)

  else

  select E = (x_p+1, y_p);



Midpoint Circle Algorithm



- Decision variable $d_{old} = F(x_p+1, y_p-½)$

  $= (x_p+1)^2 + (y_p-½)^2 - R^2$

  - If $d_{old} < 0$, select E.

    $d_{new} = F(x_p+2, y_p-½) = d_{old} + (2x_p + 3)$

  - If $d_{old} \geq 0$, select SE.

    $d_{new} = F(x_p+2, y_p-½-1) = d_{old} + (2x_p - 2y_p + 5)$

- We have to calculate d_{new} based on *the point of evaluation* $P=(x_p, y_p)$, but this is not expensive computationally.

Midpoint Circle Algorithm

- Since we start at (0,R), the initial value of d can be calculated by

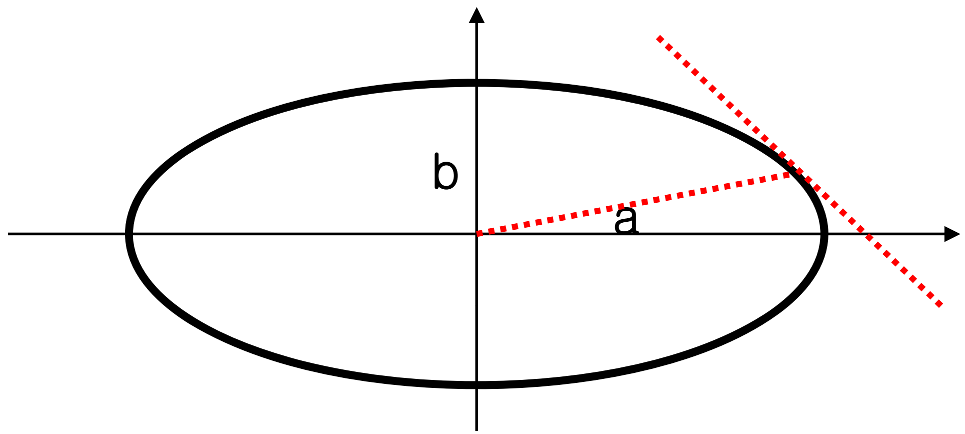
  $d_{init} = F(1, R - ½)$

  $= 5/4 - R.$

- By substituting d - 1/4 by h, we can get the integer midpoint circle scan-conversion algorithm.

Scan Converting Ellipses

- $F(x,y) = b^2x^2 + a^2y^2 - a^2b^2$
- Divide the quadrant into two regions; the boundary of two regions is the point at which the curve has a slope of -1.
- And then apply any midpoint algorithm.

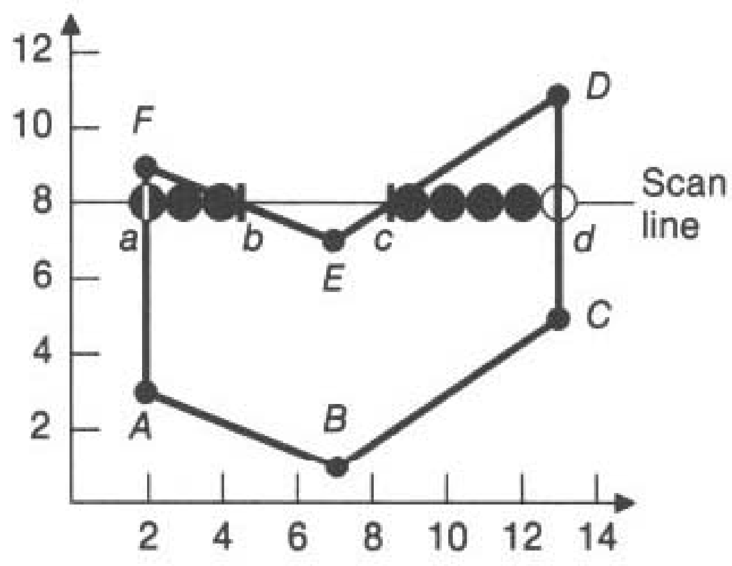


Area Filling

How to generate a solid color/patterned polygon area

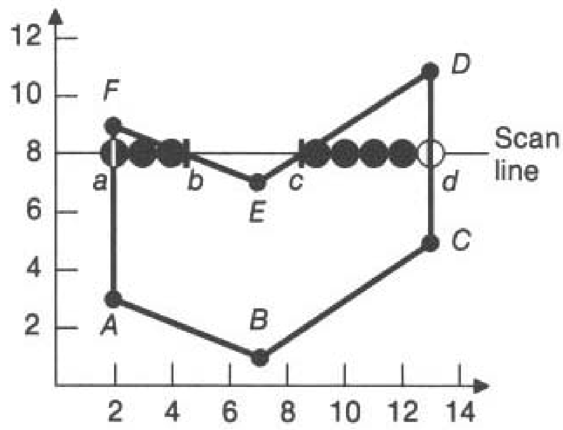
- Which pixels?
- What value?

Scan line approach

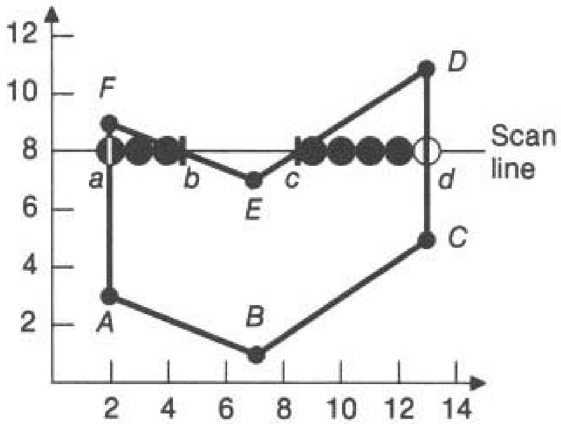


Area Filling (Scan line Approach)

- Take advantage of
  - span coherence - all pixels on a span are set to the same value
  - scan-line coherence - consecutive scan lines are identical
  - edge coherence - edges intersected by scan line *i* are also intersected by scan line *i+1*



Area Filling (Scan line Approach)



- For each scan line

  (1) Find intersections (the extrema of spans)

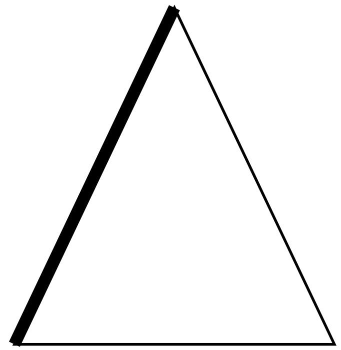
  - Use Bresenham's line-scan algorithm
  - Note that in a line drawing algorithm there is no difference between interior and exterior pixels
  - BUT it is better to draw interior only

  (2) Sort intersections (increasing x order)

  (3) Fill in between pair of intersections

Find intersections

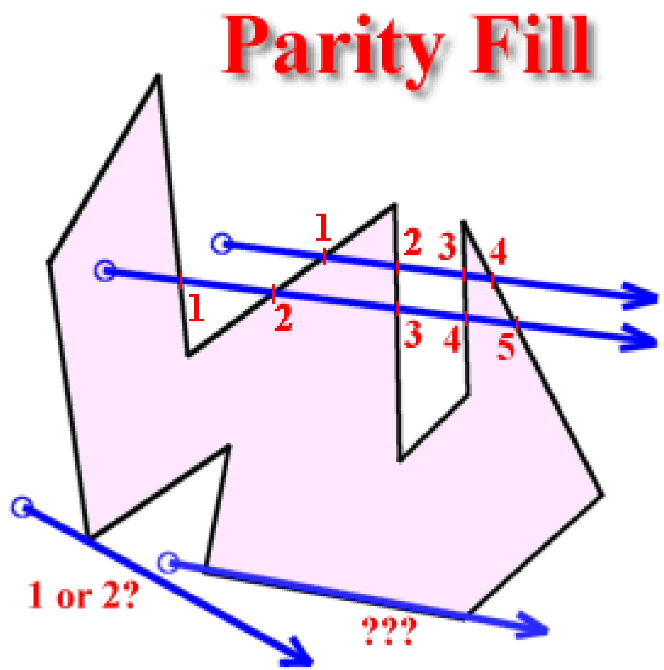
- $x_{k+1} = x_k + \Delta x / \Delta y$
  - example (left edge)
    - $m = 5/2$
    - $x_{min} = 3$
    - the sequence of x values
      - 3, 3+2/5, 3+4/5, 3+5/6=4+1/5



y	1	2	3	4
x	3	3+2/5	3+4/5	4+1/5
pixel	(3,1)	(4,2)	(4,3)	(5,3)

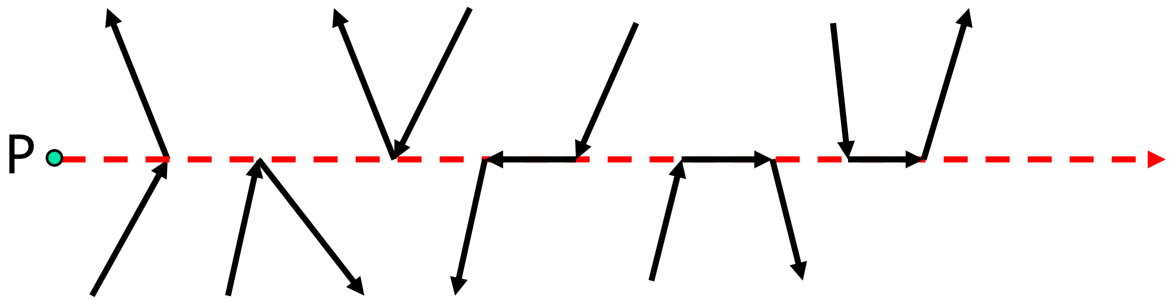
How to decide interior

Parity Fill Approach



- For each pixel, determine if it is inside or outside of a given polygon.
- **Approach**
  - from the point being tested cast a ray in an arbitrary direction
  - if the number of crossings is odd then the point is inside
  - if the number of crossings is even then the point is outside

Parity Fill Approach



- **Edge Crossing Rules**
  - an upward edge includes its starting endpoint, and excludes its final endpoint;
  - a downward edge excludes its starting endpoint, and includes its final endpoint;
  - horizontal edges are excluded;
  - the edge-ray intersection point must be strictly right of the point P.

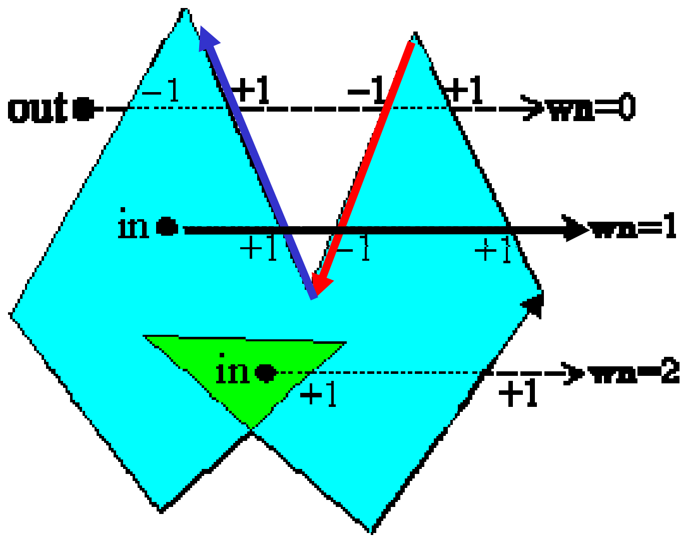
Parity Fill Approach

- Very fragile algorithm
  - Ray crosses a vertex
  - Ray is coincident with an edge
- Commonly used in ECAD
- Suitable for H/W

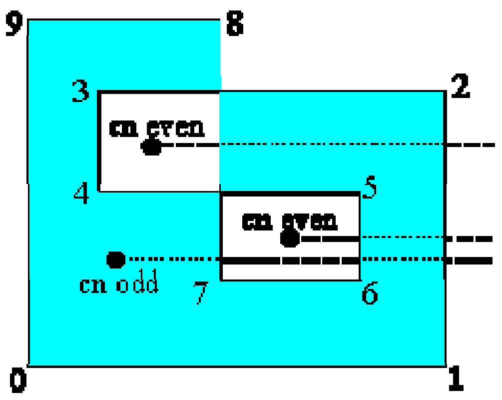
Winding Number Approach

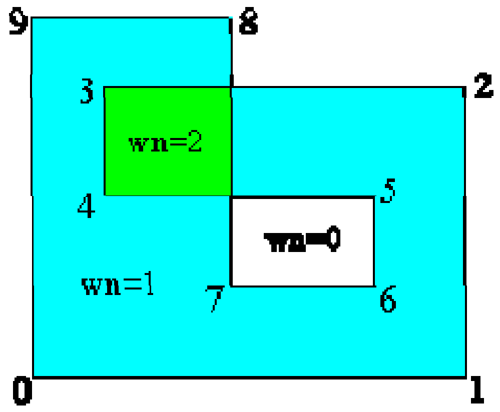
A winding number is an attribute of a point with respect to a polygon that tells us how many times the polygon encloses (or wraps around) the point. It is an integer, greater than or equal to 0. Regions of winding number 0 (unenclosed) are obviously outside the polygon, and regions of winding number 1 (simply enclosed) are obviously inside the polygon.

- Initially 0
  - +1: edge crossing the line from right to left
  - -1: left to right
- Use the sign of the cross product of the line and edge vectors
- The line does not cross any vertex

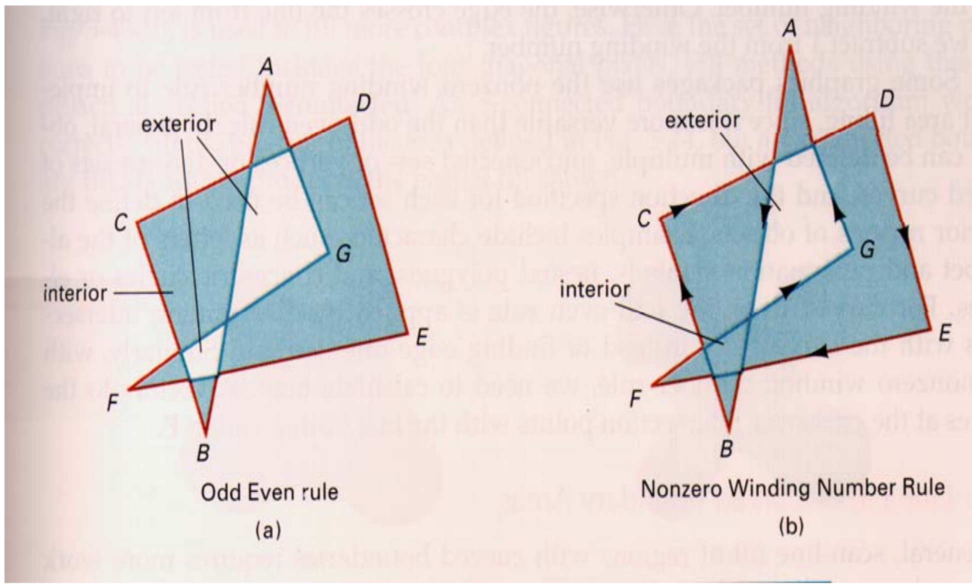


How to decide interior



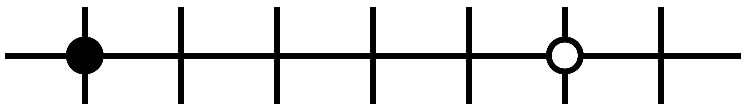


Vertices are numbered: 0 1 2 3 4 5 6 7 8 9



Span Rules

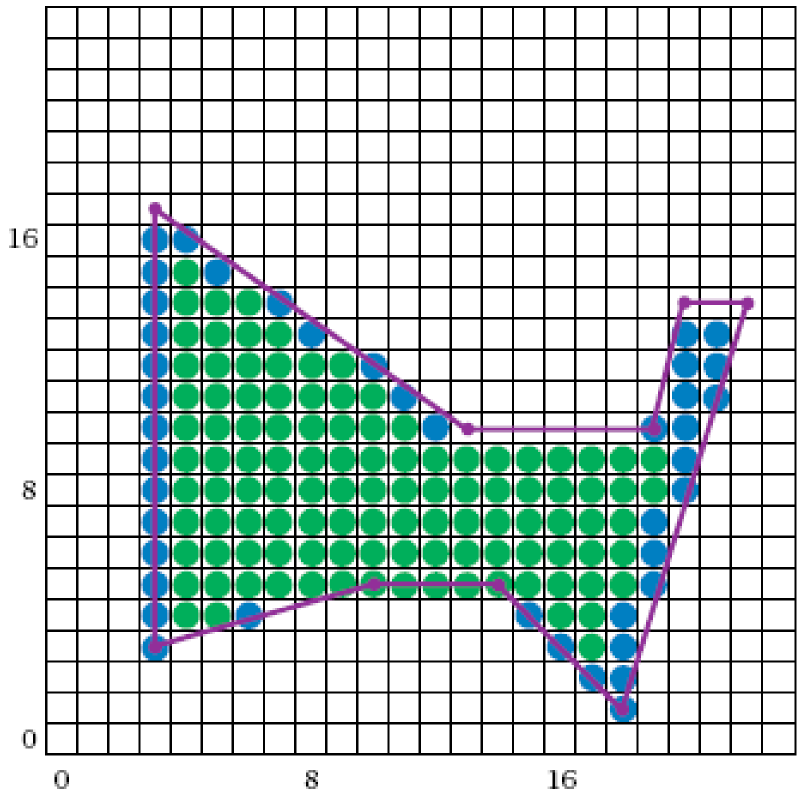
- intersection at integer coordinate
  - leftmost : interior
  - rightmost: exterior



- shared vertices
  - count parity at y_{min} vertices only
  - shorten edges
- horizontal edges
  - do not count vertices

A standard convention is to say that a point on a left or bottom edge is inside, and a point on a right or top edge is outside.

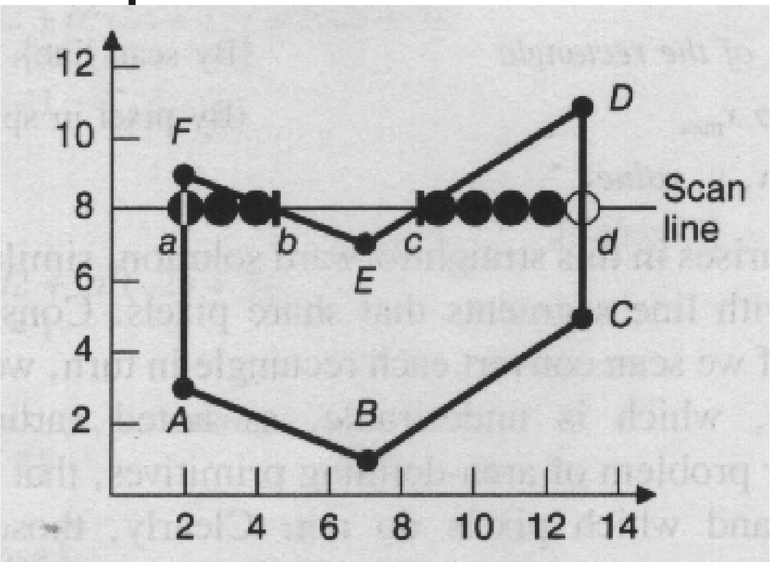
Span Rules

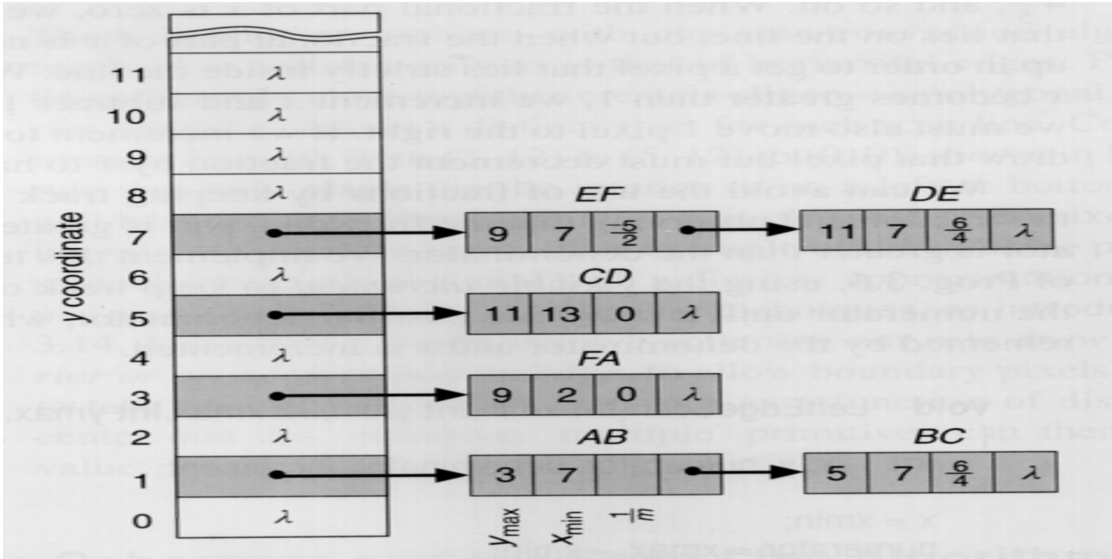


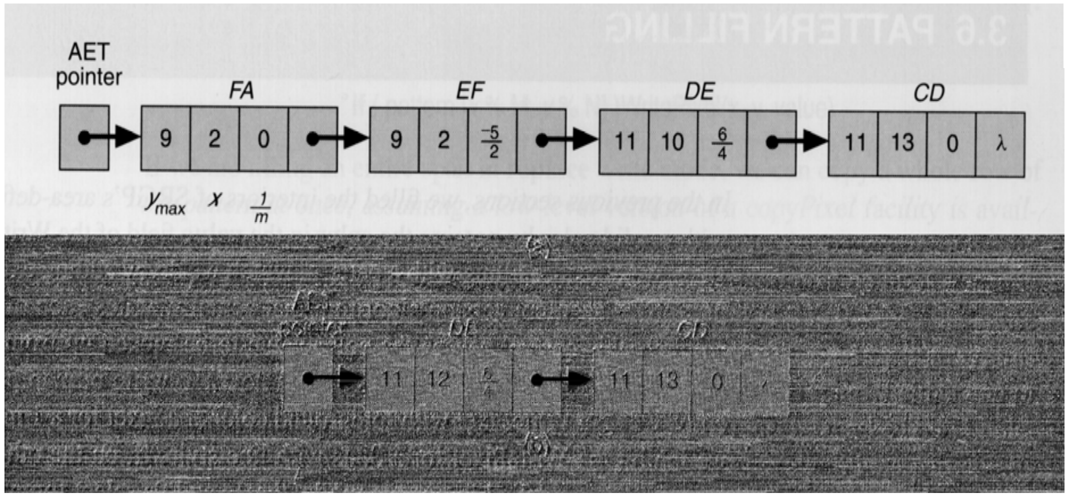
Area Filling

- Use edge coherence and the scan-line algorithm
  - ET
    - Contains all the non-horizontal edges.
    - Edges are sorted by their smaller y coordinates.
  - AET
    - Contains edges which intersect the current scan line.
    - Edges are sorted on their x intersection values.

Area Filling (Scan line method)





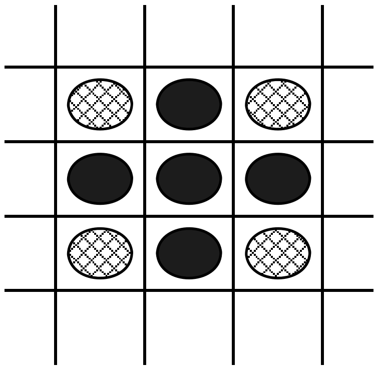


Scan line 9

Scan line 10

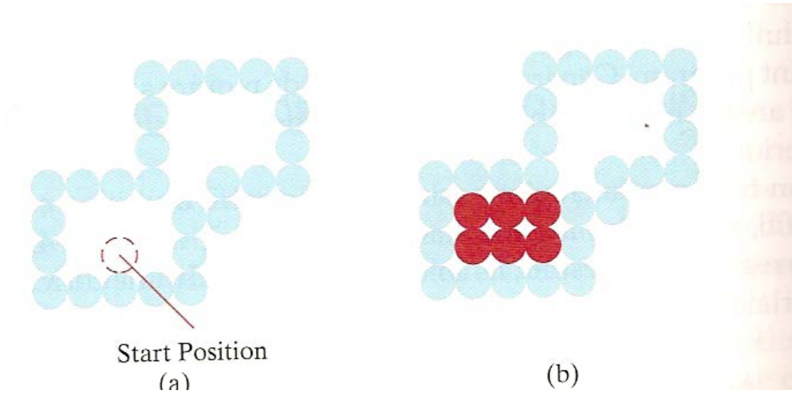
Area Filling(Filling Methods)

- Pixel Adjacency



4-connecte

8-connecte



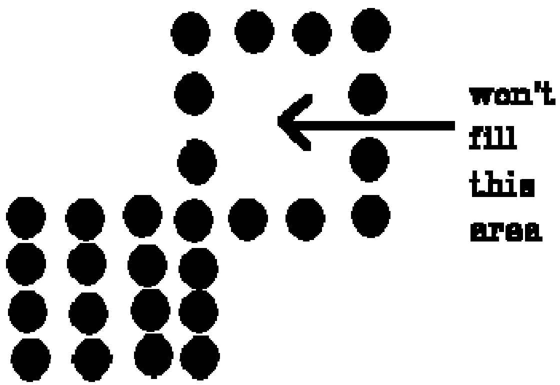
- Boundary-Fill Algorithm
  - starting a point inside the figure and painting the interior in a specified color or intensity.

Boundary Filling

```
procedure boundary_fill4(
        x,y : integer  starting point in region
        boundaryValue value that defines boundary
        newvalue : color); replacement value
    var
        c : color
    begin
        c := readPixel(x,y);
        if c <> boundaryValue and
            c <> newValue  then
             begin
                 writePixel(x,y,newValue);
                 boundary_fill4(x,y-1,boundaryValue,newValue);
                 boundary_fill4(x,y+1,boundaryValue,newValue);
                 boundary_fill4(x-1,y,boundaryValue,newValue);
                 boundary_fill4(x+1,y,boundaryValue,newValue);
             end
    end;
```

Boundary Filling

- There is the following problem with boundary_fill4:



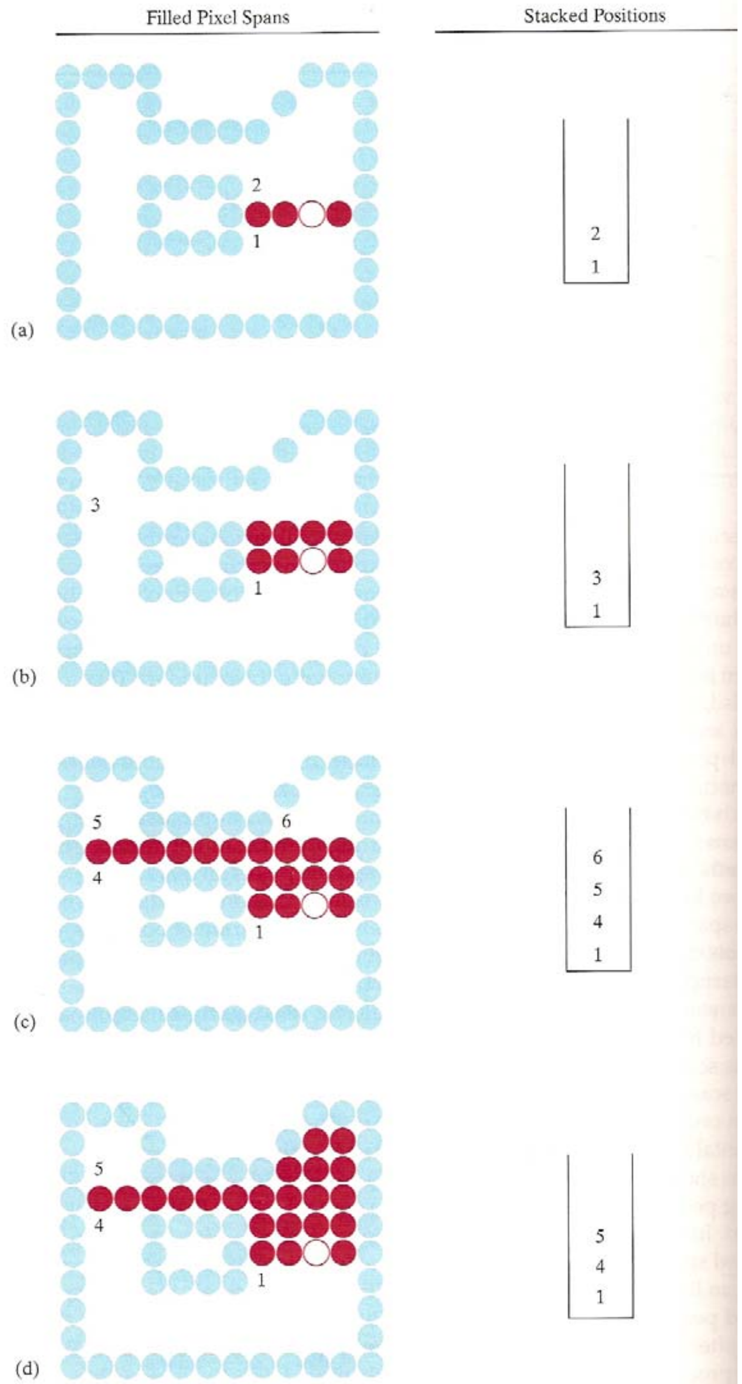
Solve with 8-connected

- Involve heavy duty recursion which may consume memory and time

Boundary Filling

Efficiency in space!

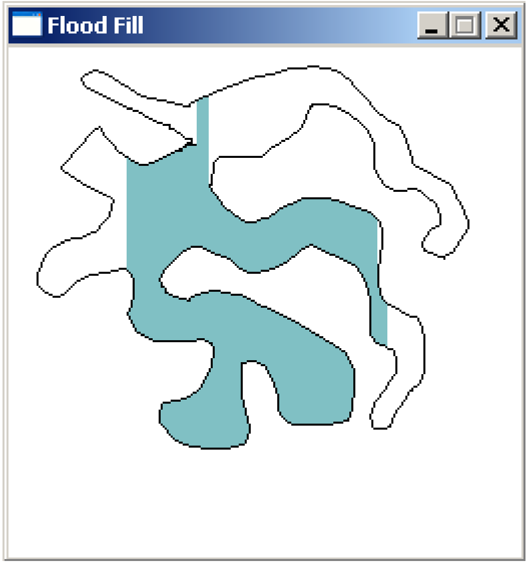
- finish the scan line containing the starting position
- process all lines below the start line
- process all lines above the start line



Flood Filling

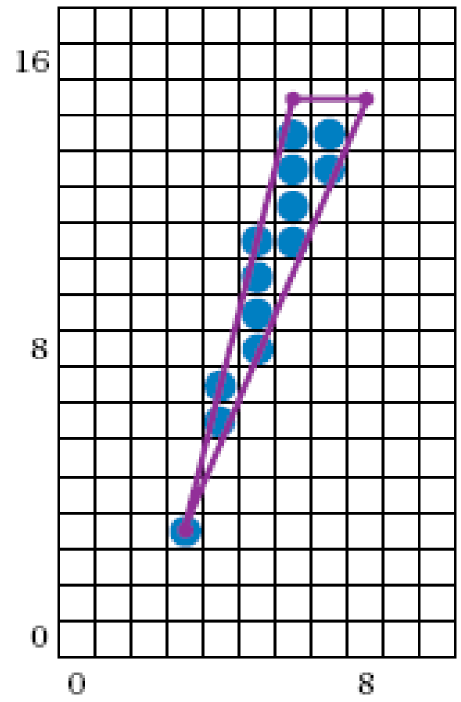
: Start a point inside the figure, replace a specified interior color only.

```
procedure flood_fill4(
   x,y : integer  starting point in region
   oldValue   value that defines interior
   newvalue : color); replacement value
begin
   if readPixel(x,y) = oldValue then
     begin
        writePixel(x,y,newValue);
        flood_fill4(x,y-1,oldValue,newValue);
        flood_fill4(x,y+1,oldValue,newValue);
        flood_fill4(x-1,y,oldValue,newValue);
        flood_fill4(x+1,y,oldValue,newValue);
     end
end;
```



Problems of Filling Algorithm

- What happens if a vertex is shared by more than one polygon, e.g. three triangles?
- What happens if the polygon intersects itself?
- What happens for a “sliver”?

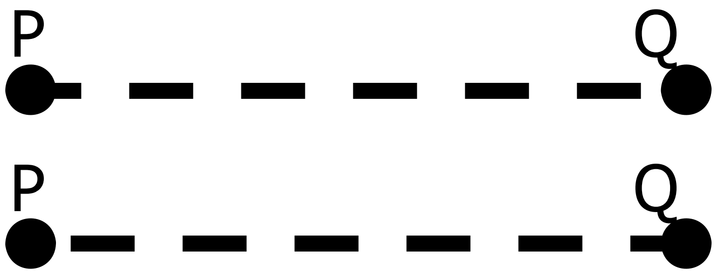


A sliver

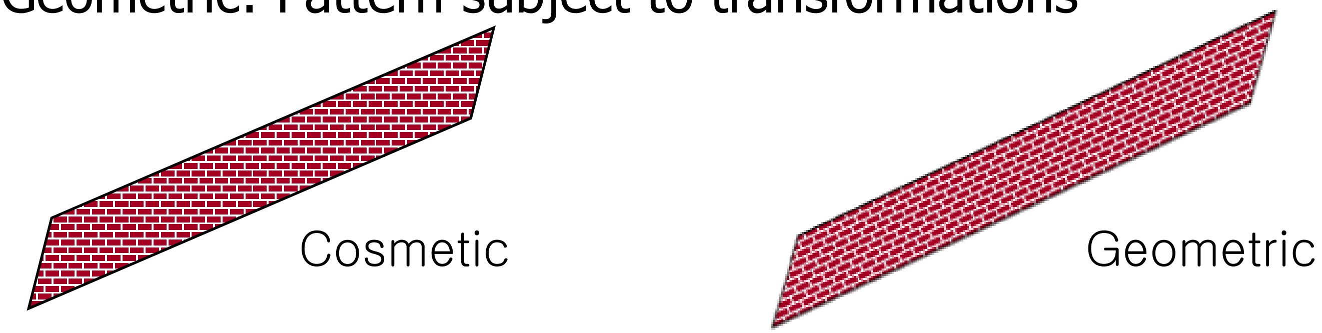
- Solutions?
  - Redefine what it means to be inside of a triangle
  - Different routines for nasty little triangles

Patterned Lines

- Patterned line from *P* to *Q* is <u>not</u> same as patterned line from *Q* to *P*.



- Patterns can be geometric or cosmetic
  - Cosmetic: Texture applied after transformations
  - Geometric: Pattern subject to transformations

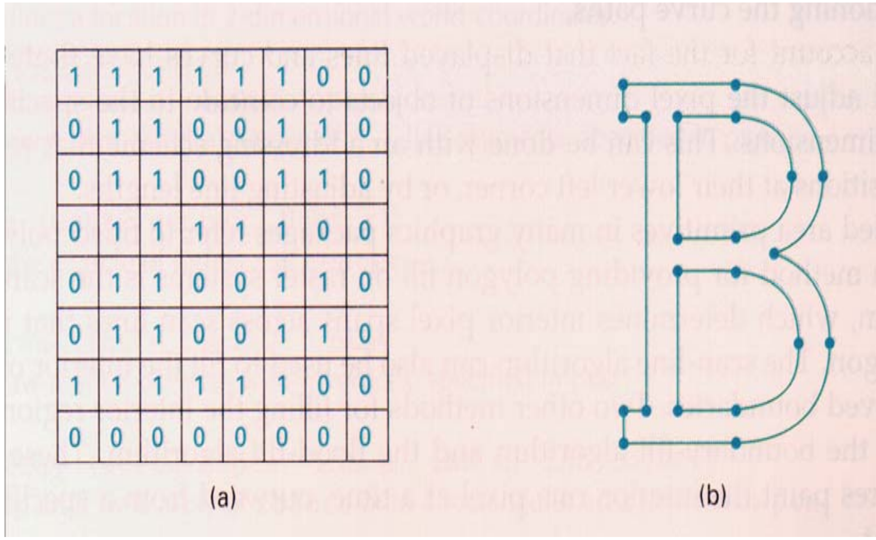


Character, Symbols

- Stroke tables : a set of vectors which are scan converted as lines

(Example) outline font

```
move  0   0
draw  1   1
move  0   1
draw  1   0
move  1   1
....
```



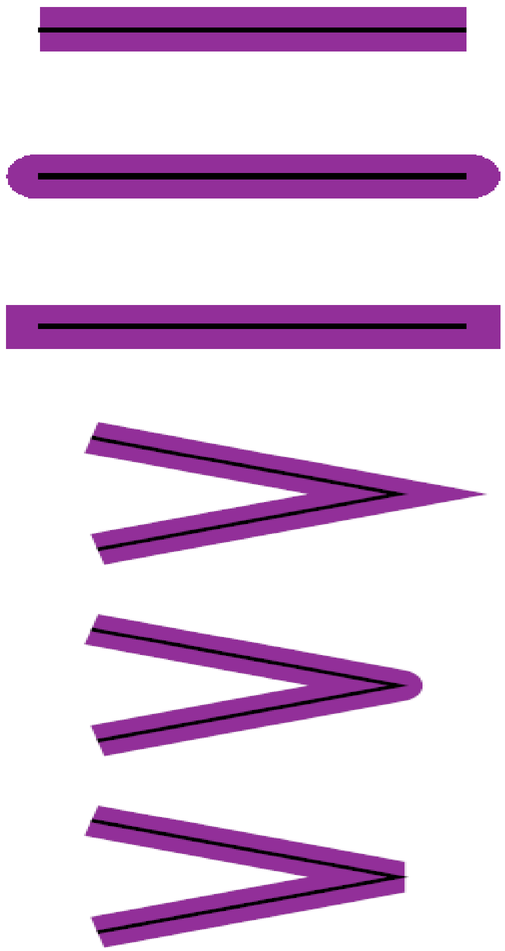
- Bitmaps : array of 0's and 1's, scan converted as points

Character, Symbols

Comparison of Methods

Stroke table	**Bitmap**
easy to rotate	rotate by multiples of 90°
easy to scale	scale by powers of 2
variable length storage	fixed length storage
Scan convert lines	scan convert points
fill if polygons	draw as filled or outline
may be anti-aliased or smoothed via curve fitting	may be pre-anti-aliased
best for linear designs	arbitrary patterns with many colors

Line Attributes



Butt cap

Round cap

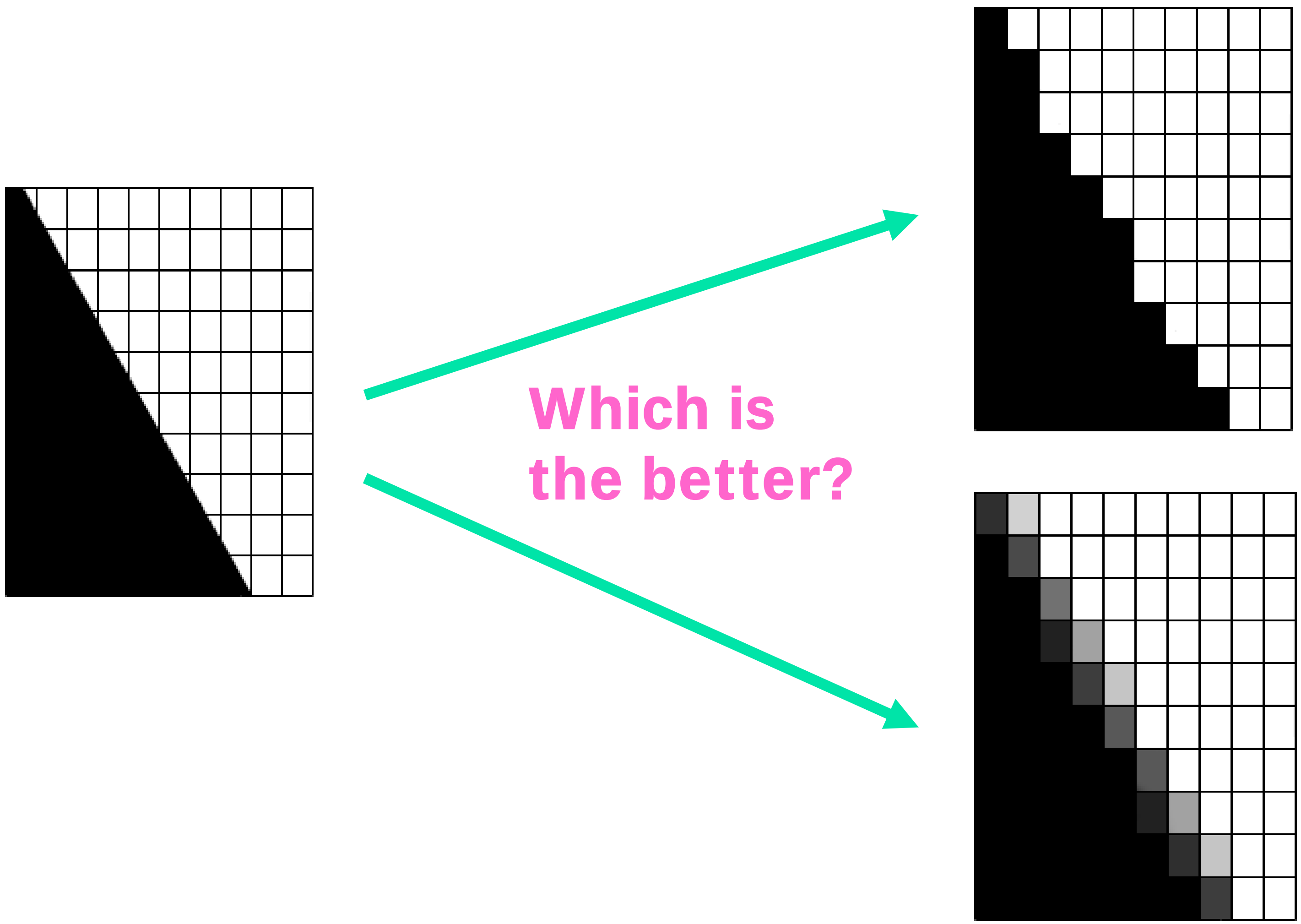
Projecting square cap

Miter join

Round Join

Bevel join

Aliasing in CG



Aliasing in CG

- Digital technology can only *approximate* analog signals through a process known as *sampling.*
- Aliaising : the distortion of information due to low-frequency sampling (undersampling).
- Choosing an appropriate *sampling rate* depends on data size restraints, need for accuracy, the cost per sample…
- Errors caused by aliasing are called artifacts. Common aliasing artifacts in computer graphics include jagged profiles, disappearing or improperly rendered fine detail, and disintegrating textures.