Physical Properties

Reading Assignment:

1. J. F. Nye, Physical Properties of Crystals
2. R. E. Newnham, Properties of Materials
3. R. E. Newnham, Structure-property Relations

Contents

Physical Properties of Crystals

- first rank tensor
- pyroelectricity: develop an electric polarization when their temperature is changed

$$\Delta P_i = p_i \Delta T \quad (p_i : \text{pyroelectric coefficient})$$

- hydrostatic pressure instead of temperature change
- experiment- clamped- primary pyroelectricity
  free - secondary pyroelectricity

Pyroelectricity

- Neumann's principle

in a crystal possessing a center of symmetry

$$\vec{p} = 0$$

inversion center $p_1 \to -p_1, p_2 \to -p_2, p_3 \to -p_3$

property is unchanged after applying the symmetry

element $p_1 \to -p_1 = p_1 \quad p_1 = 0$

centrosymmetric (11 groups) → non-pyroelectric

$\bar{1}$, $\frac{2}{m}$, $\frac{2}{m}\frac{2}{m}\frac{2}{m}$, $\bar{3}$, $\bar{3}\frac{2}{m}$, $\frac{4}{m}$, $\frac{4}{m}\frac{2}{m}\frac{2}{m}$

$\frac{6}{m}$, $\frac{6}{m}\frac{2}{m}\frac{2}{m}$, $\frac{2}{m}\bar{3}$, $\frac{4}{m}\bar{3}\frac{2}{m}$

Pyroelectricity

- a pyroelectric moment can only lie along a direction in a crystal which is unique, in the sense that it is not repeated by any symmetry element.
- polar direction: 이 축에 수직인 2회 회전축이나 거울면을 갖지 않는 극성을 갖는 축

  ex) point group 32

  a diad axis is a polar, but not unique direction
- all unique directions are polar, but only some polar directions are unique.

Pyroelectricity

Triclinic

Class *1*: no symmetry restriction on the direction of **p**: (p_1, p_2, p_3).

Monoclinic. x_2 parallel to the diad axis, rotation or inverse, (y).

Class *2*: **p** parallel to the diad axis: $(0, p, 0)$.

Class *m*: **p** has any direction in the symmetry plane: $(p_1, 0, p_3)$.

Orthorhombic. x_1, x_2, x_3 parallel to crystallographic x, y, z respectively.

Class *mm2*: **p** parallel to the diad axis: $(0, 0, p)$.

Class *222*: $(0, 0, 0)$.

Tetragonal, trigonal, hexagonal. x_3 parallel to z.

Classes *4*, *4mm*, *3*, *3m*, *6*, *6mm*: **p** parallel to the 4, 3 or 6 axis: $(0, 0, p)$.

Classes $\bar{4}$, $\bar{4}2m$, *422*, *32*, $\bar{6}$, $\bar{6}m2$, *622*: $(0, 0, 0)$.

Cubic

Classes *432*, $\bar{4}3m$, *23*: $(0, 0, 0)$.

Pyroelectricity

- polar class- non-zero pyroelectric coefficients

1, 2, 3, 4, 6, m, 2mm, 3m, 4mm, 6mm

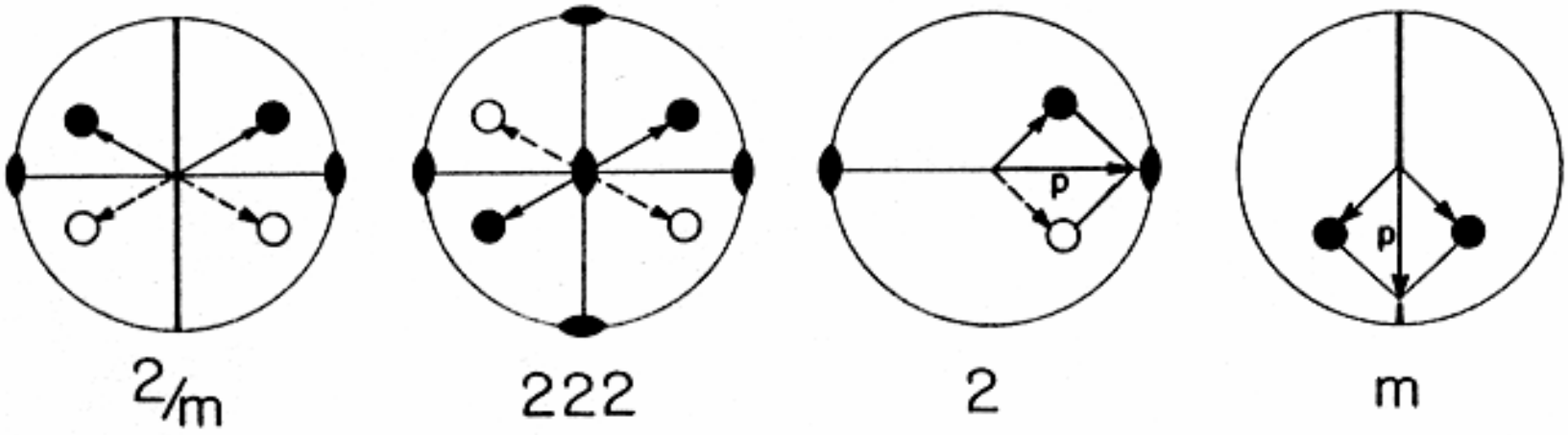


Fig. 31. Dipole configurations in two monoclinic and two orthorhombic point groups. In centric point groups like 2/*m*, every dipole is cancelled by an equal and opposite dipole. These point groups are neither pyroelectric nor piezoelectric. Dipole cancellation also occurs in certain acentric point groups such as 222. These point groups are generally piezoelectric but not pyroelectric. Dipoles do not cancel in point groups 2 and *m* which are both piezoelectric and pyroelectric. In 2, the net spontaneous polarization must be lie along the rotation axis. In *m*, P_s lies somewhere in the mirror plane

Pyroelectricity

-Wurtzite-hexagonal ZnS-6mm
polar axis and P are parallel to c, six-fold axis

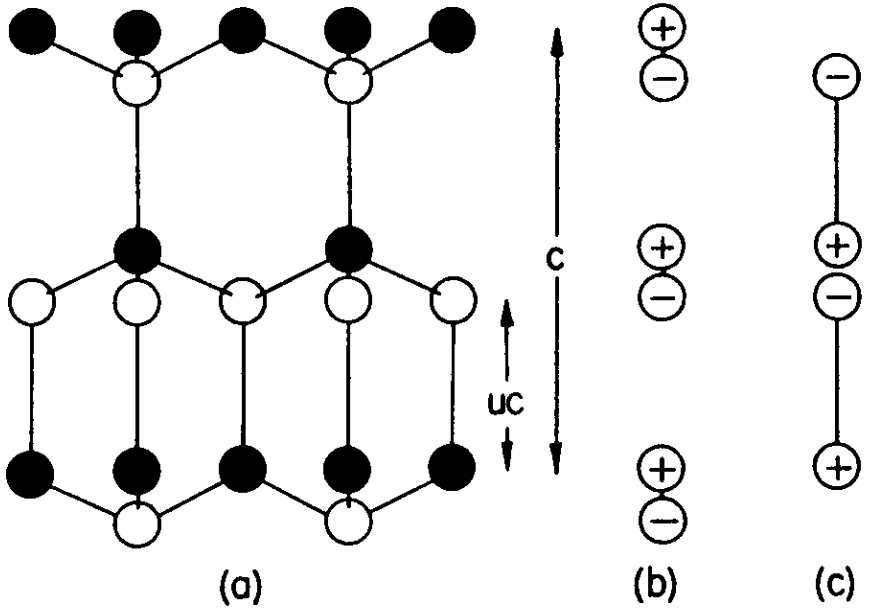


Fig. 32a–c. Wurtzite structure projected on (010), and equivalent polar chains showing two ways in which the crystal can terminate

u=3/8

$p=+(1/2-u)c$ $p=-quc$

Pyroelectricity

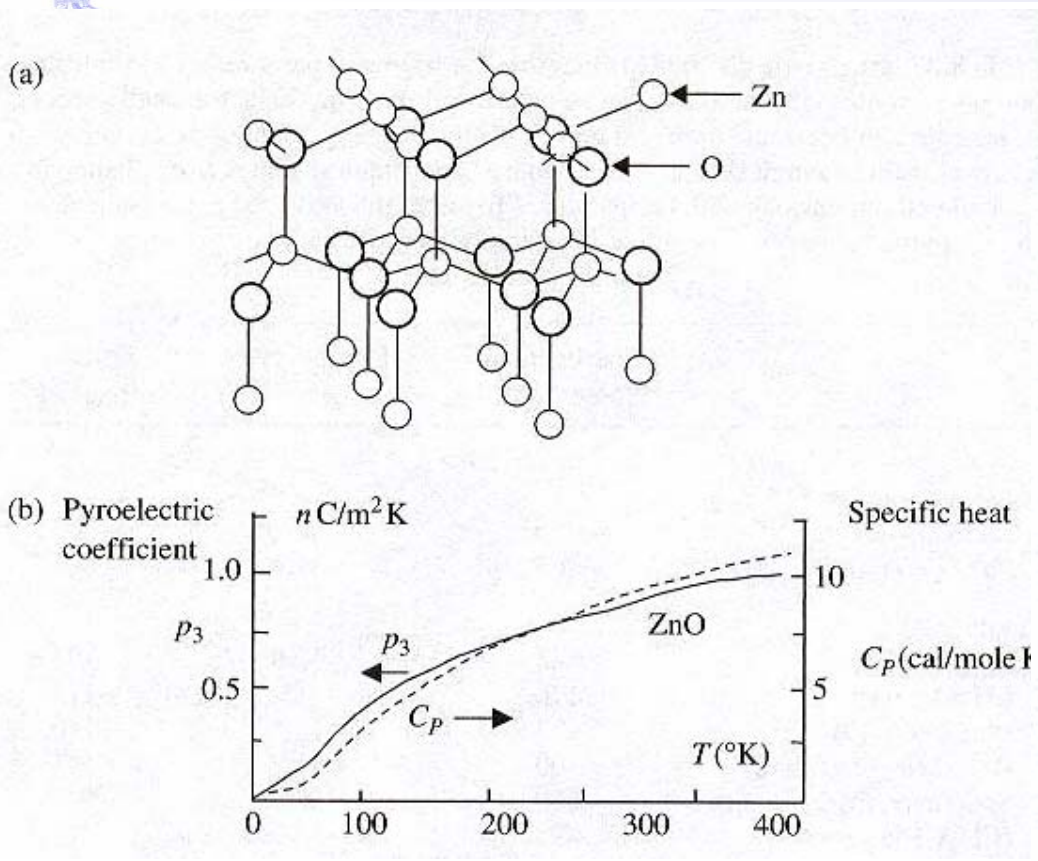


Fig. 8.4 (a) Crystal structure of zinc oxide showing the polar orientation of the zinc–oxygen bonds along the hexagonal c-axis. ZnO has the wurtzite structure and belongs to polar group $6mm$. (b) The pyroelectric coefficients of nonferroelectric crystals are generally small and correlate well with specific heat.

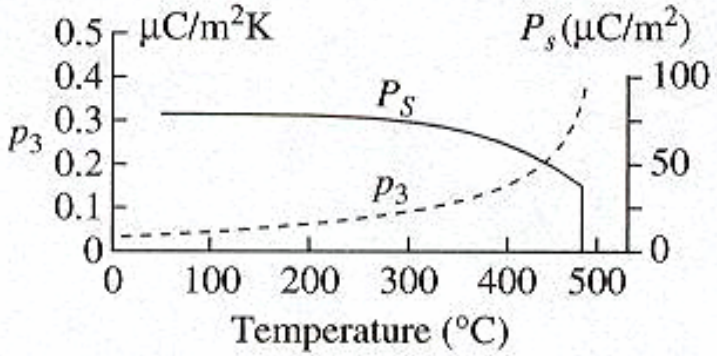
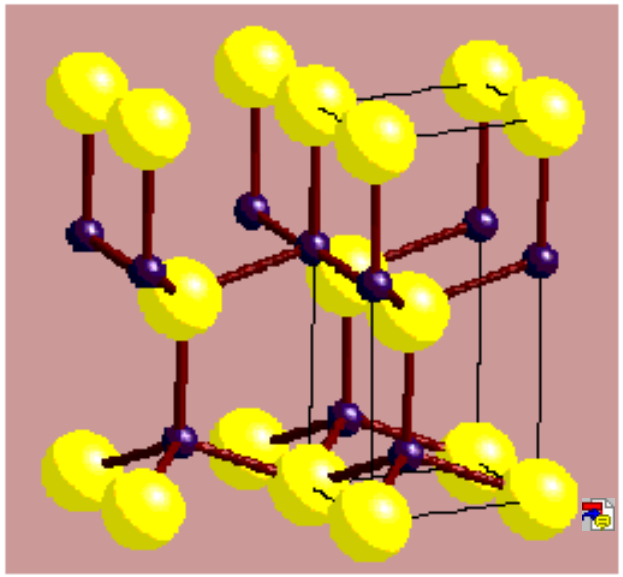
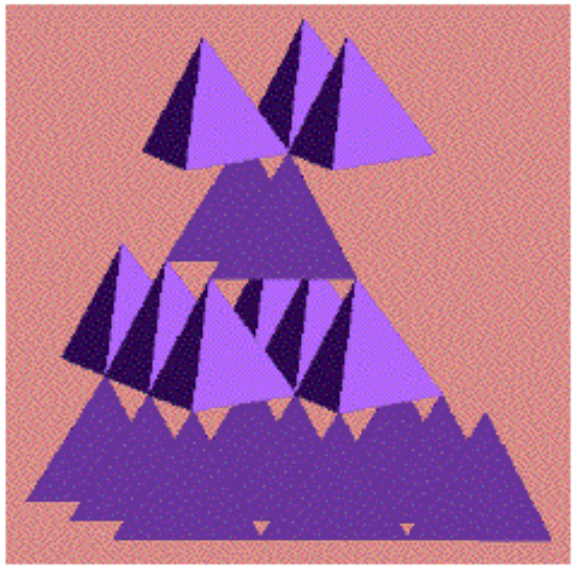
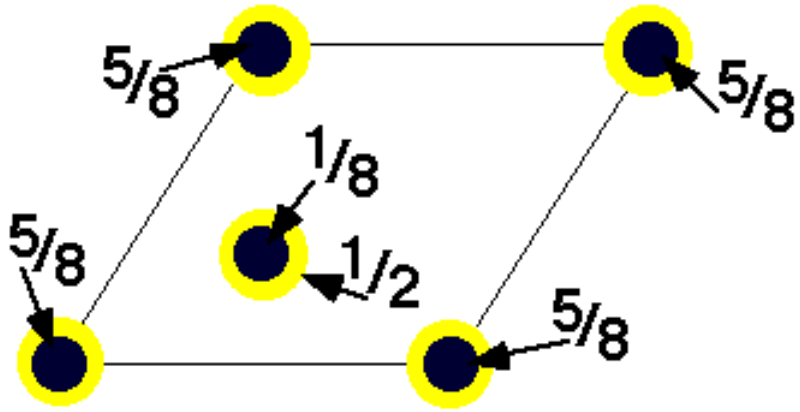


Fig. 8.5 In poled ferroelectric crystals the pyroelectric effect is the temperature derivative of the spontaneous polarization. $PbTiO_3$ polarizes along the tetragonal c-axis. Both P_s and the pyroelectric coefficient p_3 are zero above T_c where the symmetry changes to cubic at 490°C.

ZnS *Wurtzite*







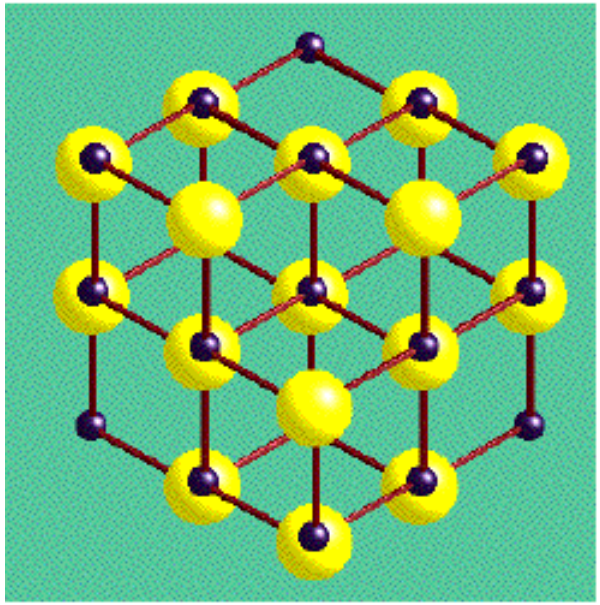
http://www.chem.ox.ac.uk/icl/heyes/structure_of_solids/Strucsol.html

ZnS *Wurtzite*

- HCP S^{2-} with Zn^{2+} in half Tetrahedral holes (only T+ {or T-} filled)
- *Lattice*: Hexagonal - P
- *a = b*, *c* Å Ã(8/3)*a*
- *Motif*: 2S at (0,0,0) & ($^{2}/_{3}$,$^{1}/_{3}$,$^{1}/_{2}$); 2Zn at ($^{2}/_{3}$,$^{1}/_{3}$,$^{1}/_{8}$) & (0,0,$^{5}/_{8}$)
- **2**ZnS in unit cell
- *Coordination*: 4:4 (tetrahedral)

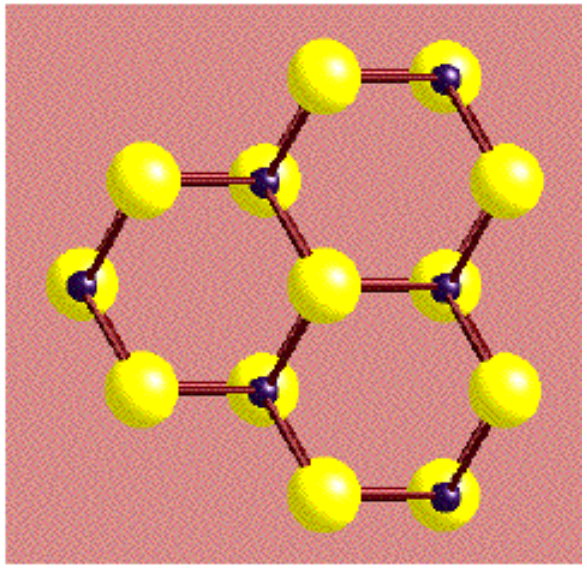
Comparison of Wurtzite and Zinc Blende

PLAN VIEWS



Zinc Blende

CCP ABC repeat



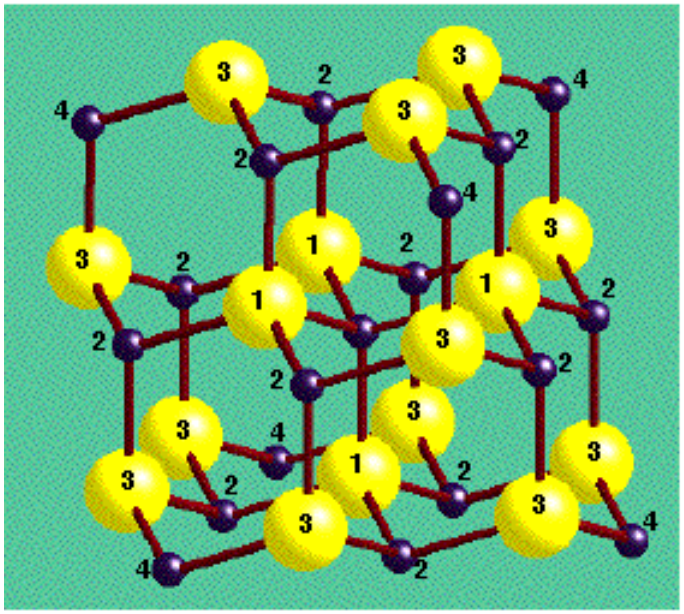
Wurtzite

HCP AB repeat

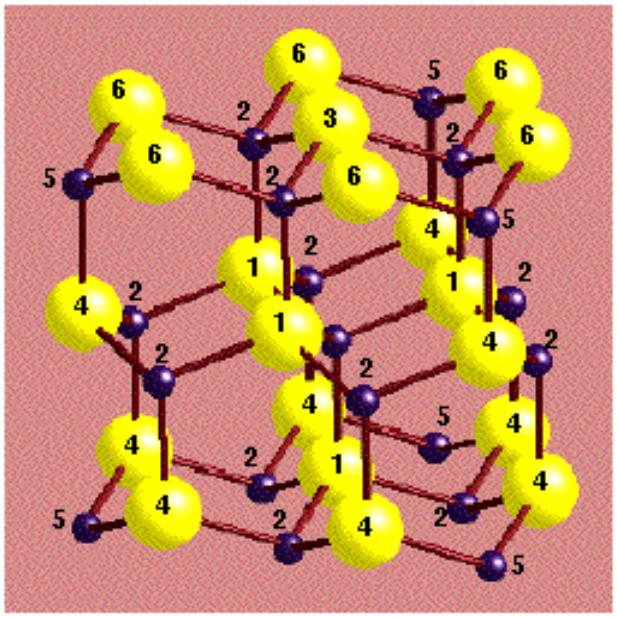
http://www.chem.ox.ac.uk/icl/heyes/structure_of_solids/Strucsol.html

Comparison of Wurtzite and Zinc Blende

COORDINATION ENVIRONMENTS



Zinc Blende



Wurtzite

4 Nearest Neighbours (***Tetrahedral***)

12 Next-Nearest Neighbours

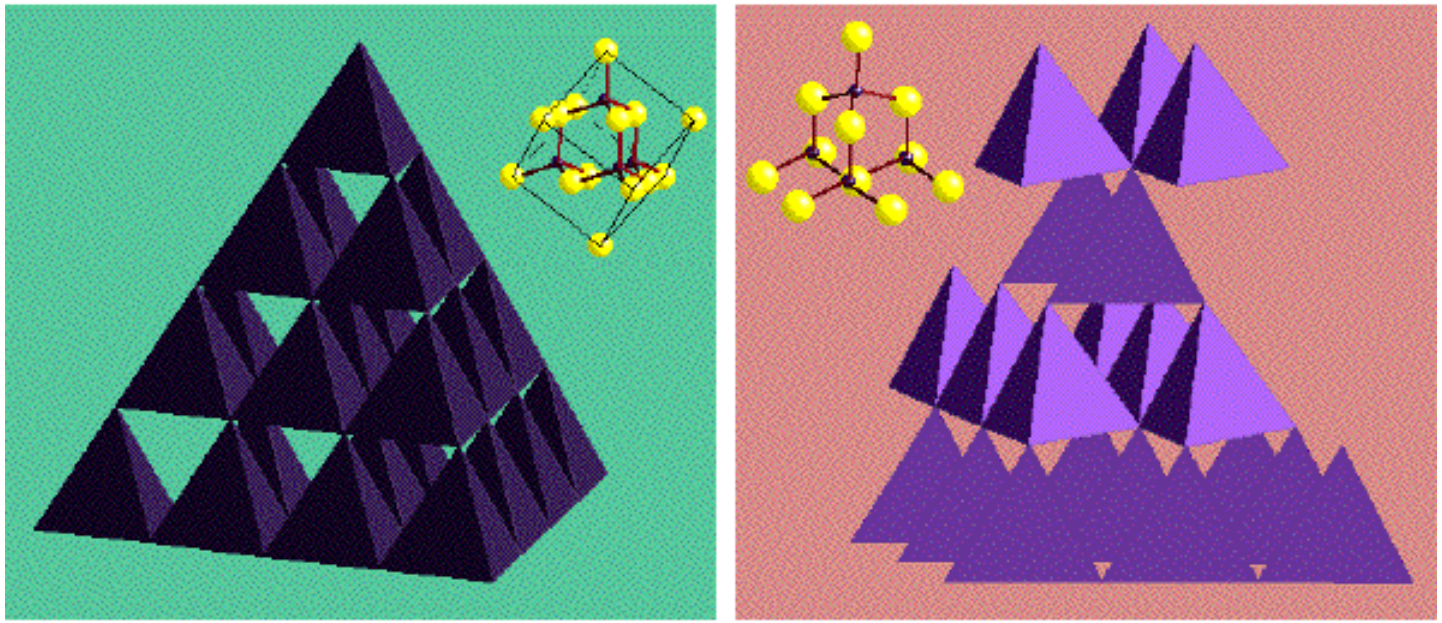
Cuboctahedral ← → ***Anti-Cuboctahedral***

Very different Next, Next-Nearest Neighbour Coordinations & beyond

http://www.chem.ox.ac.uk/icl/heyes/structure_of_solids/Strucsol.html

Comparison of Wurtzite and Zinc Blende

POLYHEDRAL REPRESENTATIONS



Zinc Blende | Wurtzite

Vertex-linked tetrahedra only, but layers skewed in Wurtzite, & not in Blende

http://www.chem.ox.ac.uk/icl/heyes/structure_of_solids/Strucsol.html

Pyroelectricity

Table 10. Comparison of pyroelectric detector materials [3]

$\frac{p}{\epsilon}$

Material	Operating temperature (°C)	Dielectric constant	Pyroelectric coefficient ($\times 10^{-7}$ C/°−cm^2)	Figure of merit ($\times 10^{-11}$)
Triglycine sulfate	40	35	1.3	300
Lithium sulfate	25	10.3	0.078	76
Rochelle salt	22	1000	0.2	2
Barium titanate	25	4100	0.6	1.5
Lead zirconate titanate	25	2200	0.6	3

*used as infrared radiation detectors

- ferroelectricity: spontaneous polarization
  reversability
  hysteresis

Second Rank Tensor

- **electrical conductivity**
- Ohm's law

$$j_i = -\sigma_{ik}\frac{\partial \phi}{\partial x_k} = \sigma_{ik} E_k$$

where j_i: current density, σ_{ik}: electrical conductivity tensor

ϕ: potential, E_k: electric field intensity

$$E_i = \rho_{ik} j_k$$

Electrical resistivities of metallic crystals

Values of principal resistivities at 20° C (unit = 10^{-8} ohm–m)

Crystal	*System*	ρ_1, ρ_2	ρ_3
Tin	tetragonal	9·9	14·3
Bismuth . . .	trigonal	109	138
Cadmium . . .	hexagonal	6·80	8·30
Zinc	,,	5·91	6·13
Tungsten . . .	cubic	5·48	
Copper . . .	,,	1·51	

The effect of crystal symmetry on properties represented by symmetrical second-rank tensors

Optical classification	*System*	*Characteristic symmetry* (see p. 280)†	*Nature of representation quadric and its orientation*	*Number of independent coefficients*	*Tensor referred to axes in the conventional orientation*‡
Isotropic (anaxial)	Cubic	4 3-fold axes	*Sphere*	1	$\begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix}$
Uniaxial	Tetragonal	1 4-fold axis	*Quadric of revolution* about the principal symmetry axis $(x_3)(z)$	2	$\begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_1 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$
	Hexagonal	1 6-fold axis			
	Trigonal	1 3-fold axis			
Biaxial	Orthorhombic	3 mutually perpendicular 2-fold axes; no axes of higher order	*General quadric* with axes $(x_1, x_2, x_3) \parallel$ to the diad axes (x, y, z)	3	$\begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$
	Monoclinic	1 2-fold axis	*General quadric* with one axis $(x_2) \parallel$ to the diad axis (y)	4	$\begin{bmatrix} S_{11} & 0 & S_{31} \\ 0 & S_2 & 0 \\ S_{31} & 0 & S_{33} \end{bmatrix}$
	Triclinic	A centre of symmetry or no symmetry	*General quadric.* No fixed relation to crystallographic axes	6	$\begin{bmatrix} S_{11} & S_{12} & S_{31} \\ S_{12} & S_{22} & S_{23} \\ S_{31} & S_{23} & S_{33} \end{bmatrix}$

Second Rank Tensor

- **thermal conductivity**

$$h_i = -K_{ij}\frac{\partial T}{\partial x_j}$$

where h_i: heat flow rate per unit area, K_{ij}: thermal conductivity tensor, $\frac{\partial T}{\partial x_j}$: temperature gradient

Thermal conductivities of crystals

Values of the principal conductivities in m.k.s. units [joule/(m sec °C)]

Crystal	*System*	*Temperature* °C	k_1, k_2	k_3
Quartz . .	trigonal	30	6·5	11·3
Calcite . .	,,	30	4·18	4·98
Bismuth . .	,,	18	9·24	6·65
Graphite . .	hexagonal	30	355	89
Aluminium . .	cubic	30	208	
Copper . .	,,	0	410	

Second Rank Tensor

- **electric polarization**

$$\vec{D} = \varepsilon_o \vec{E} + \vec{P}$$

where D: electric displacement, ε_o: permittivity of vacuum

P: polarization

$P_i = \varepsilon_o \chi_{ij} E_j$ (χ_{ij} : dielectric susceptibility)

$D_i = \varepsilon_{ij} E_j, \qquad \varepsilon_{ij} = \varepsilon_o (\delta_{ij} + \chi_{ij})$

relative dielectric constant

$\varepsilon_{rij} = \varepsilon_{ij} / \varepsilon_o$

Second Rank Tensor

Dielectric constants of crystals

Crystal	*System*	K_1	K_2	K_3	*Frequency (cycles/sec.)*
Gypsum . .	monoclinic	9·9	5·1	5·0	3×10^8
Aragonite . .	orthorhombic	9·8	7·7	6·6	4×10^8
Quartz . .	trigonal	4·5		4·6	50 to 5×10^6
Calcite . .	,,	8·5		8·0	4×10^8
Rutile . .	tetragonal	89		173	4×10^8
Caesium chloride	cubic	6·3			2×10^5
Sodium chloride	,,	5·6			2×10^5

Second Rank Tensor

- **magnetization**

$$\vec{B} = \mu_o \vec{H} + \mu_o \vec{M}$$

where D: magnetic induction, ε_o: permeability of vacuum

M: magnetization

$M_i = \chi_{mij} H_j$ (χ_{mij} : magnetic susceptibility)

$B_i = \mu_{ij} H_j, \qquad \mu_{ij} = \mu_o(\delta_{ij} + \chi_{mij})$

relative permeability

$\mu_{rij} = \mu_{ij} / \mu_o$

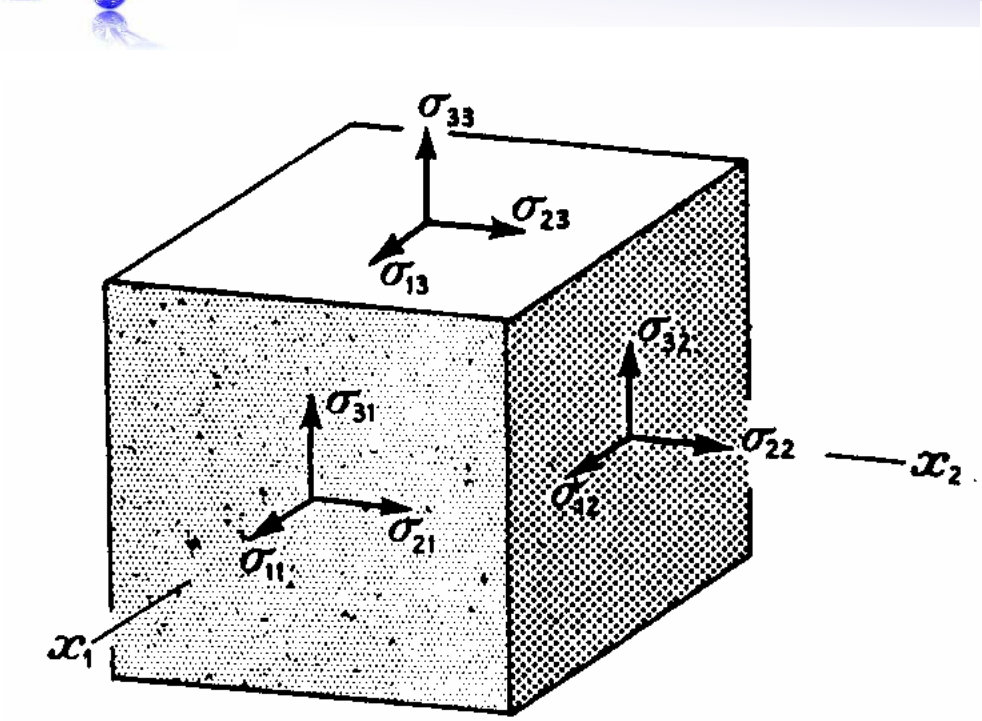
Second Rank Tensor

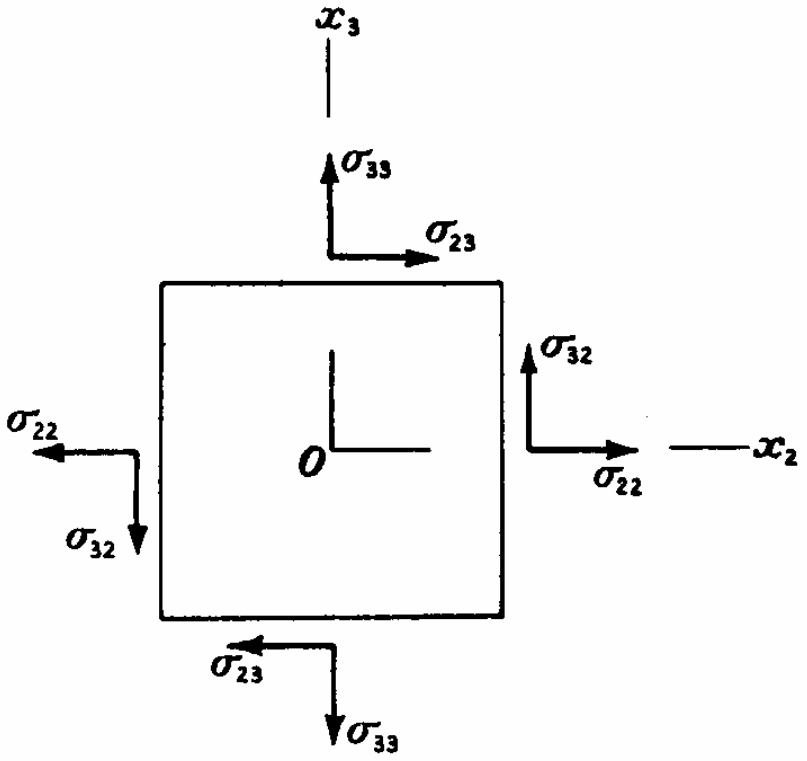
Susceptibilities of paramagnetic and diamagnetic crystals

When multiplied by 10^{-5} the numbers in the table are the volume susceptibilities in rationalized units.

Crystal	*System*	ψ_1	ψ_2	ψ_3	*Ref.*
Aragonite .	orthorhombic	−1·44	−1·42	−1·63	1
Quartz . .	trigonal	−1·51		−1·52	2
Calcite . .	,,	−1·24		−1·38	1
Cadmium . .	hexagonal	−1·74		−2·85	3
Beryl . .	,,	2·76		1·29	1
Rutile . .	tetragonal	10·5		11·2	1
Fluorite . .	cubic	−1·14			1
Sodium chloride	,,	−1·36			1

Stress and Strain





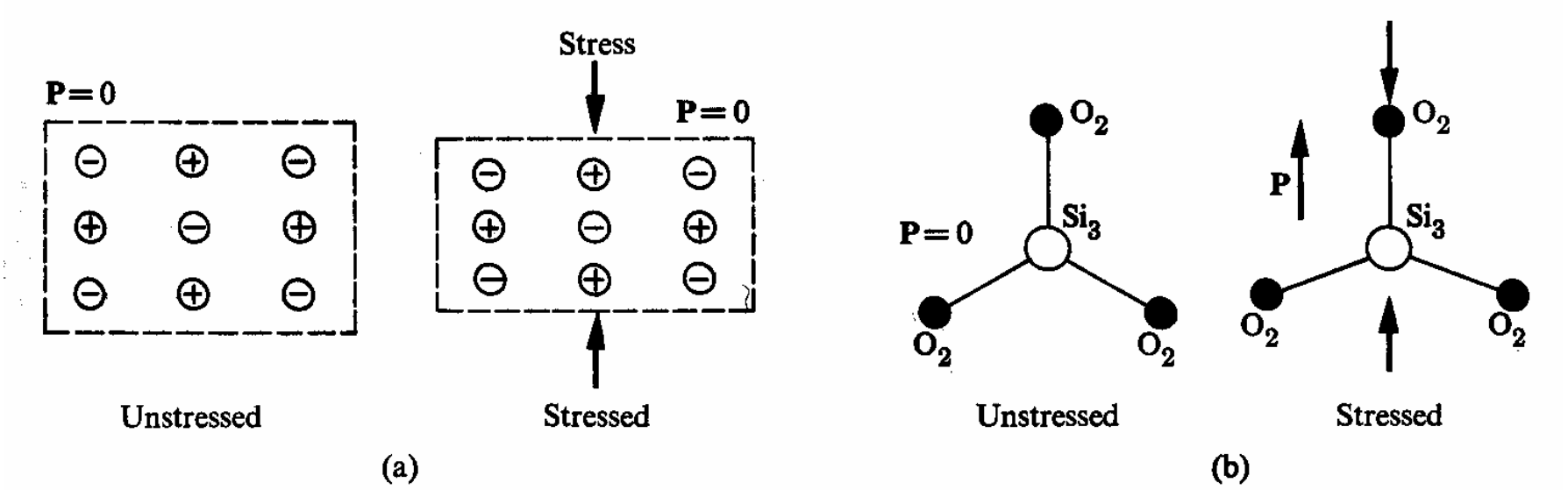
$$\begin{bmatrix} \sigma_{11} & \sigma_{21} & \sigma_{31} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

Third Rank Tensor

- piezoelectricity
- direct piezoelectric effect

  stress → electric dipole moment

  $P = d\sigma$ (piezoelectirc modulus)



inversion center

quartz

Third Rank Tensor

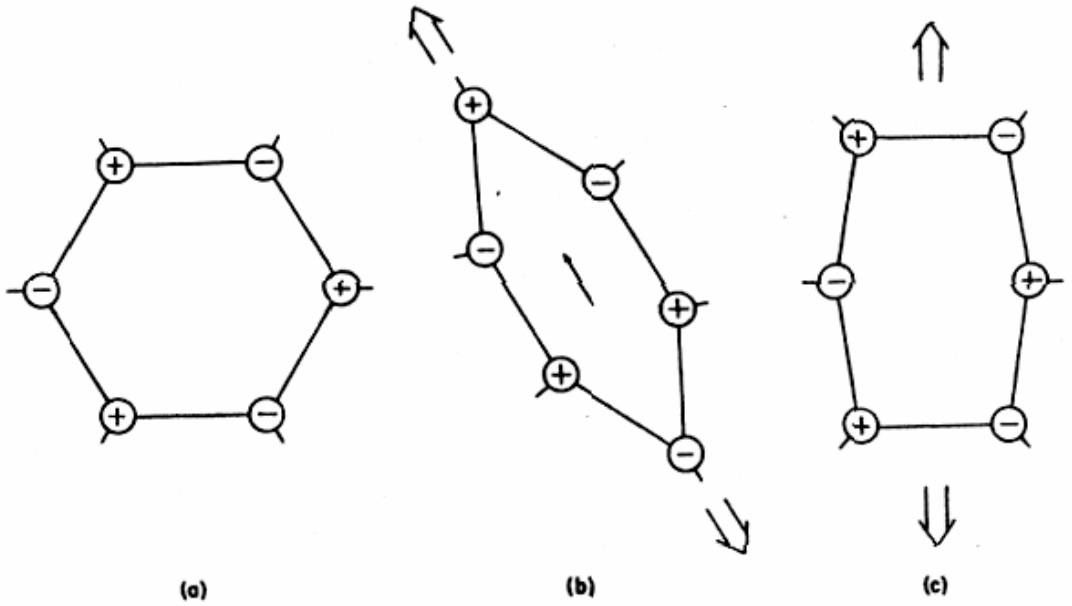


Fig. 33a–c. Unstressed (a) and stressed (b, c) structures containing positive and negative ions. Polarization, a separation of positive and negative charge centers, is achieved in (b) but not in (c), showing that not all directions are piezoelectric

Some Piezoelectric Crystals (in Decreasing Value of Piezoelectric Coefficient)

Crystal	Chemical formula	Relative strength
Rochelle salt	$NaKC_4H_4O_6 \cdot 4H_2O$	Very strong
ADP	$NH_4H_2PO_4$	Strong
KDP	KH_2PO_4	Moderate
α-Quartz	SiO_2	Weak

Third Rank Tensor

- $P_i = d_{ijk}\sigma_{jk}$
- $P_1 = d_{111}\sigma_{11} + d_{112}\sigma_{12} + d_{113}\sigma_{13} + d_{121}\sigma_{21} + d_{122}\sigma_{22} + d_{123}\sigma_{23}$
  $+ d_{131}\sigma_{31} + d_{132}\sigma_{32} + d_{133}\sigma_{33}$
- d_{ijk}: 27 coefficients
- third rank tensor: $d_{ijk} = a_{il}a_{jm}a_{kn}d_{lmn}$

Third Rank Tensor

1st layer	2nd layer	3rd layer
$i = 1$	$i = 2$	$i = 3$
$d_{111}\ d_{112}\ d_{113}$	$d_{211}\ d_{212}\ d_{213}$	$d_{311}\ d_{312}\ d_{313}$
$(d_{121})\ d_{122}\ d_{123}$	$(d_{221})\ d_{222}\ d_{223}$	$(d_{321})\ d_{322}\ d_{323}$
$(d_{131})(d_{132})\ d_{133}$	$(d_{231})(d_{232})\ d_{233}$	$(d_{331})(d_{332})\ d_{333}$

- $d_{ijk} = d_{ikj}$ (18 independent components)

- matrix notation

$d_{11}\ \frac{1}{2}d_{16}\ \frac{1}{2}d_{15}$	$d_{21}\ \frac{1}{2}d_{26}\ \frac{1}{2}d_{25}$	$d_{31}\ \frac{1}{2}d_{36}\ \frac{1}{2}d_{35}$
$d_{12}\ \frac{1}{2}d_{14}$	$d_{22}\ \frac{1}{2}d_{24}$	$d_{32}\ \frac{1}{2}d_{34}$
d_{13}	d_{23}	d_{33}

Third Rank Tensor

- tensor notation: 11 22 33 23,32 31,13, 12,21

  matrix notation: 1 2 3 4 5 6

- $\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \rightarrow \begin{bmatrix} \sigma_1 & \sigma_6 & \sigma_5 \\ \sigma_6 & \sigma_2 & \sigma_4 \\ \sigma_5 & \sigma_4 & \sigma_3 \end{bmatrix}$

- $P_1 = d_{11}\sigma_1 + \frac{1}{2}d_{16}\sigma_6 + \frac{1}{2}d_{15}\sigma_5 + \frac{1}{2}d_{16}\sigma_6 + d_{12}\sigma_2 + \frac{1}{2}d_{14}\sigma_4$

$$+ \frac{1}{2}d_{15}\sigma_5 + \frac{1}{2}d_{14}\sigma_4 + d_{13}\sigma_3$$

$$= d_{11}\sigma_1 + d_{12}\sigma_2 + d_{13}\sigma_3 + d_{14}\sigma_4 + d_{15}\sigma_5 + d_{16}\sigma_6$$

- $p_i = d_{ij}\sigma_j \quad (i = 1,2,3,\ j = 1,2,..,6)$

Third Rank Tensor

- array of d_{ij}

$$\begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{pmatrix}$$

- the d_{ij} do not transform like the components of a second rank tensor

- converse piezoelectric effect

electric field $\rightarrow$ strain

$$\varepsilon_{jk} = d_{ijk} E_i$$

(matrix notation: $\varepsilon_j = d_{ij} E_i (i = 1,2,3,\ j = 1,2,..,6)$)

Third Rank Tensor

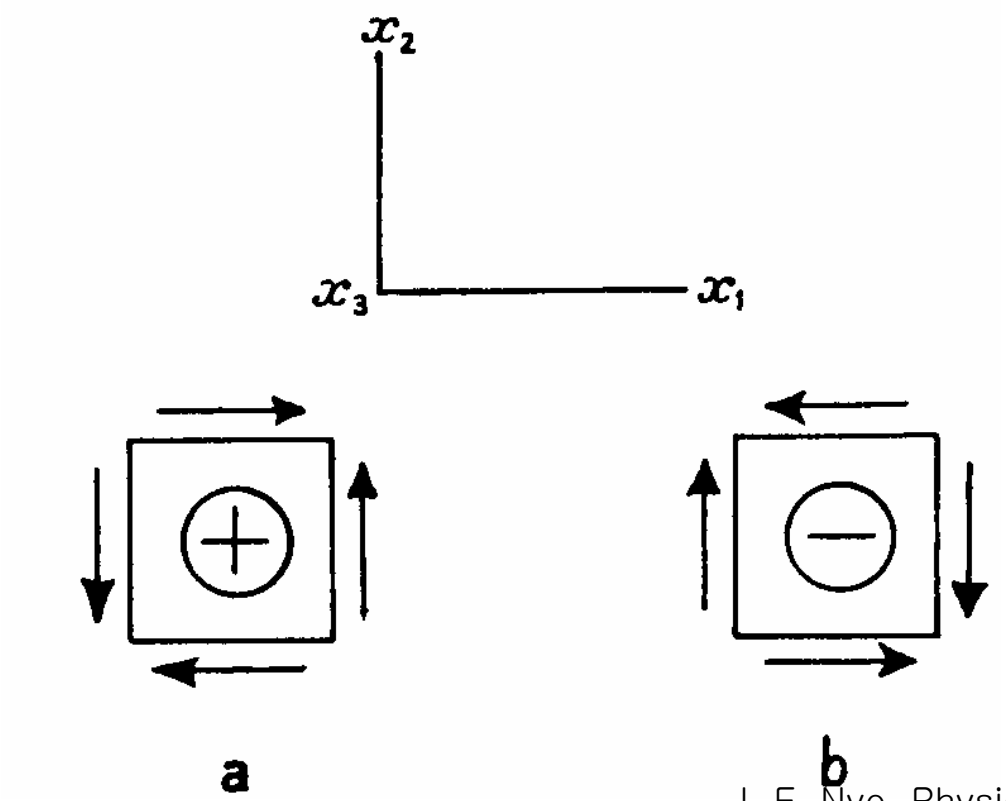
		ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6
		σ_1	σ_2	σ_3	σ_4	σ_5	σ_6
E_1	P_1	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}
E_2	P_2	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}
E_3	P_3	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}

Defining equations for the direct and converse piezoelectric effects

	Tensor notation $(i, j, k = 1, 2, 3)$	*Matrix notation* $(i = 1, 2, 3;\ j = 1, 2, \ldots, 6)$
Direct effect	$P_i = d_{ijk}\,\sigma_{jk}$	$P_i = d_{ij}\,\sigma_j$
Converse effect	$\epsilon_{jk} = d_{ijk}\,E_i$	$\epsilon_j = d_{ij}\,E_i$

Third Rank Tensor

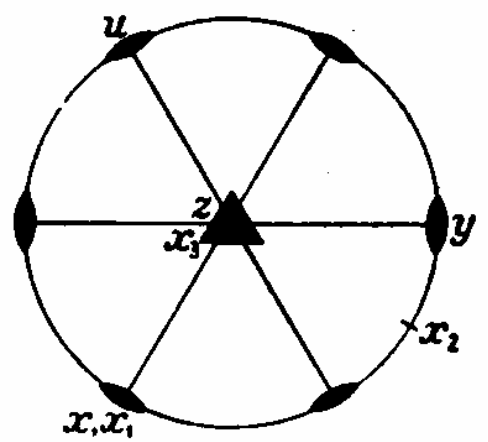
- reduction in the number of independent moduli by crystal symmetry
- a crystal with a center of symmetry cannot be piezoelectric
- modulus d_{36} in class 4



$$d_{36} = 0$$

Third Rank Tensor

- piezoelectric: 20 point groups
  (32 groups- centro-symmetric groups (11)-432)
- quartz (point group: 32)



$$\begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{cccccc} \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \sigma_5 & \sigma_6 \end{array} \\ \begin{pmatrix} d_{11} & -d_{11} & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14} & -2d_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

- tensile stress σ_1 parallel to x_1

$$P_1 = d_{11}\sigma_1, \quad P_2 = 0, \quad P_3 = 0$$

- tensile stress σ_2 parallel to x_2

$$P_1 = -d_{11}\sigma_1, \quad P_2 = 0, \quad P_3 = 0$$

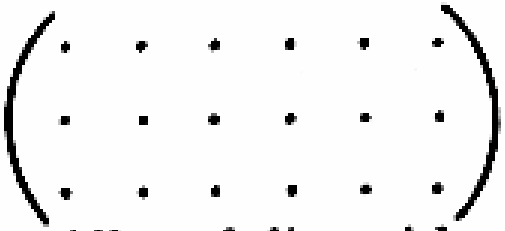
$$\begin{pmatrix} -2{\cdot}3 & 2{\cdot}3 & 0 & -0{\cdot}67 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0{\cdot}67 & 4{\cdot}6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \times 10^{-12} \ \text{C/N}$$

Third Rank Tensor

- zinc blende ZnS (point group: $\bar{4}3m$)

CUBIC

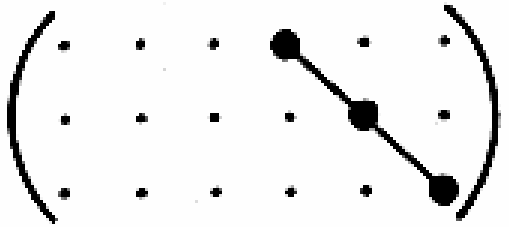
Class 432



(0)

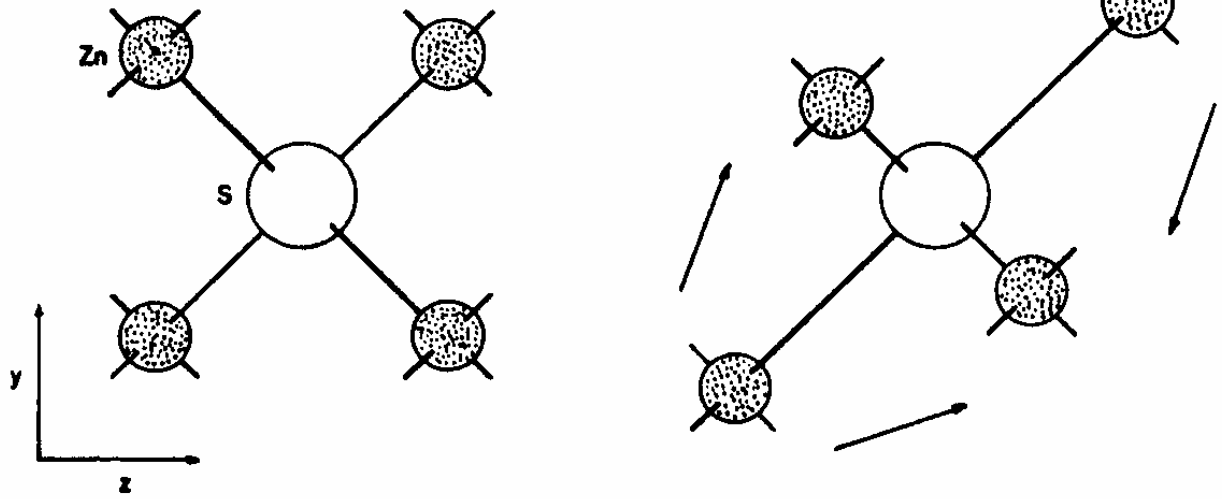
All moduli vanish

Classes $\bar{4}3m$ and 23



(1)

- $P_1 = d_{14}\sigma_4$



(a) (b)

Fig. 35a and b. A single tetrahedron of the zincblende structure, unstressed (a) and under shear stress (b), viewed along the polarization direction

Form of the (d_{ij}) matrix

KEY TO NOTATION

- zero modulus
- non-zero modulus
- equal moduli
- moduli numerically equal, but opposite in sign
- a modulus equal to minus 2 times the heavy dot modulus to which it is joined.

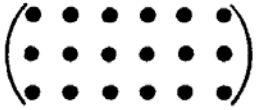
Centrosymmetrical classes

All moduli vanish

Non-centrosymmetrical classes

TRICLINIC

Class *1*



(18)

MONOCLINIC

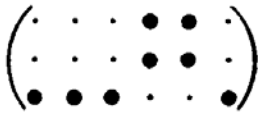
Class *2*

$2 \| x_2$ (standard orientation)

(8)

Class *2*

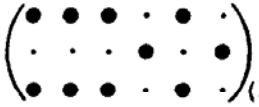
$2 \| x_3$



(8)

Class *m*

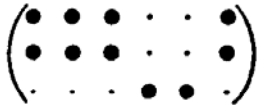
$m \perp x_2$ (standard orientation)



(10)

Class *m*

$m \perp x_3$



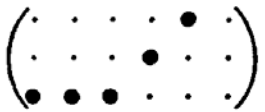
(10)

ORTHORHOMBIC

Class *222*

(3)

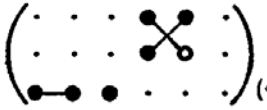
Class *mm2*



(5)

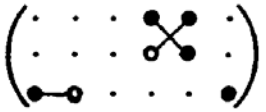
TETRAGONAL

Class *4*

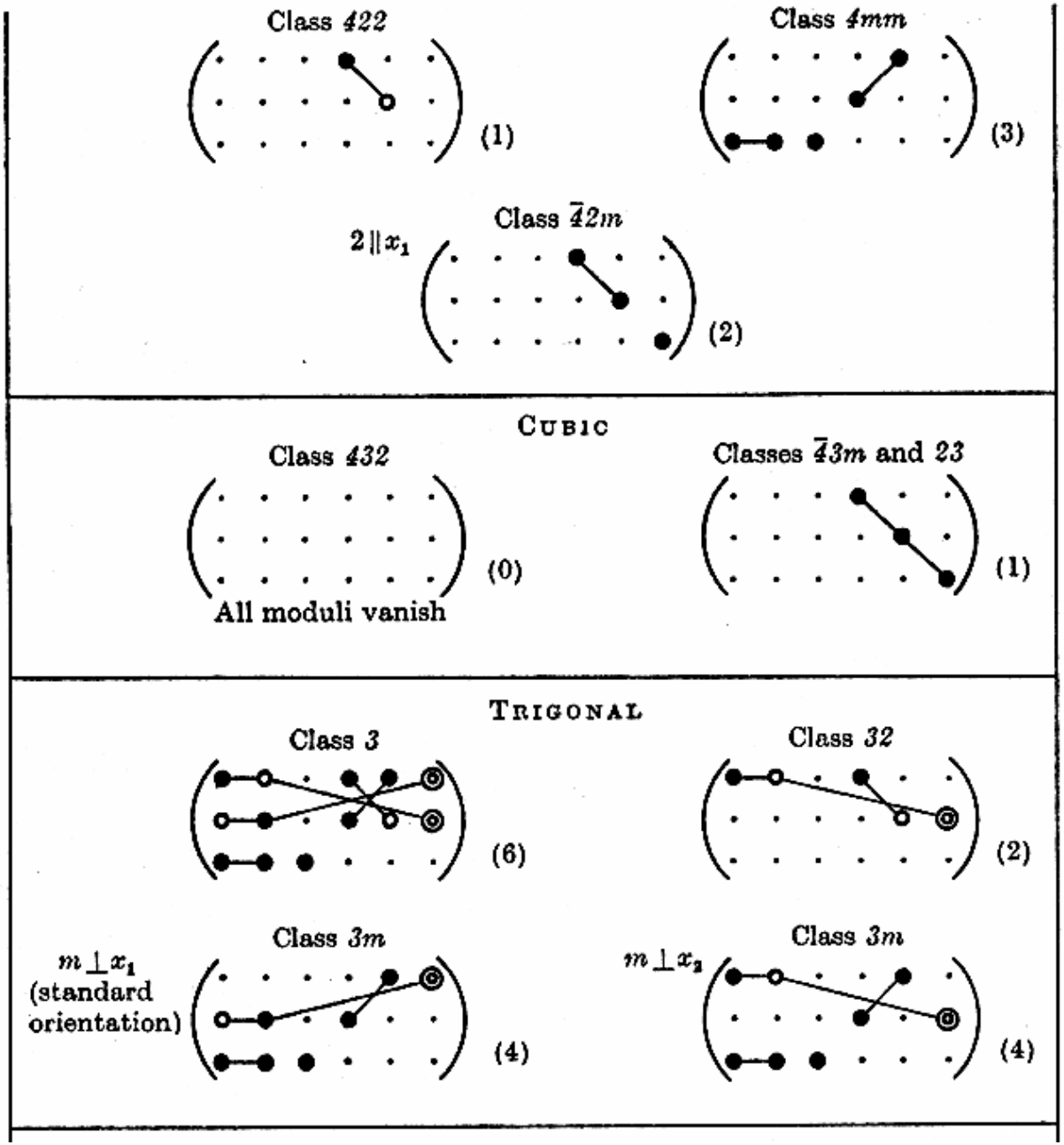


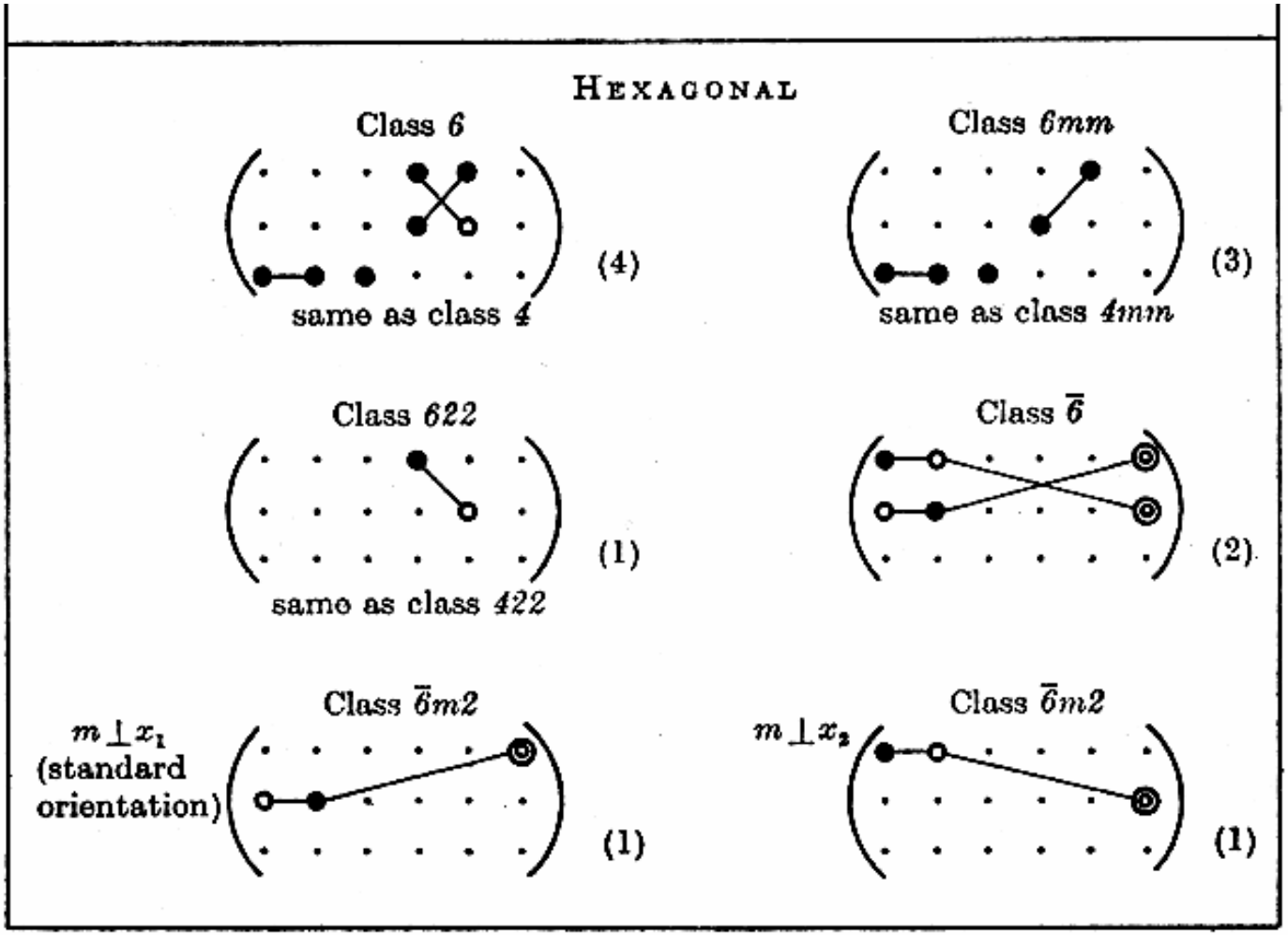
(4)

Class $\bar{4}$



(4)


Class 422
(1)
Class 4mm
(3)
Class $\bar{4}2m$
$2 \| x_1$
(2)
CUBIC
Class 432
(0)
All moduli vanish
Classes $\bar{4}3m$ and 23
(1)
TRIGONAL
Class 3
(6)
Class 32
(2)
Class 3m
$m \perp x_1$
(standard orientation)
(4)
Class 3m
$m \perp x_2$
(4)


HEXAGONAL
Class 6
(4)
same as class 4
Class 6mm
(3)
same as class 4mm
Class 622
(1)
same as class 422
Class $\bar{6}$
(2)
Class $\bar{6}m2$
$m \perp x_1$
(standard orientation)
(1)
Class $\bar{6}m2$
$m \perp x_2$
(1)

Fourth Rank Tensor

- Hook's law

$\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$ (c_{ijkl}: stiffness, Young modulus)

$\varepsilon_{ij} = s_{ijkl}\sigma_{kl}$ (s_{ijkl}: compliance)

- 81 components

$c_{ijkl} = c_{ijlk}$, $c_{ijkl} = c_{jikl}$ (36 components)

- matrix notation

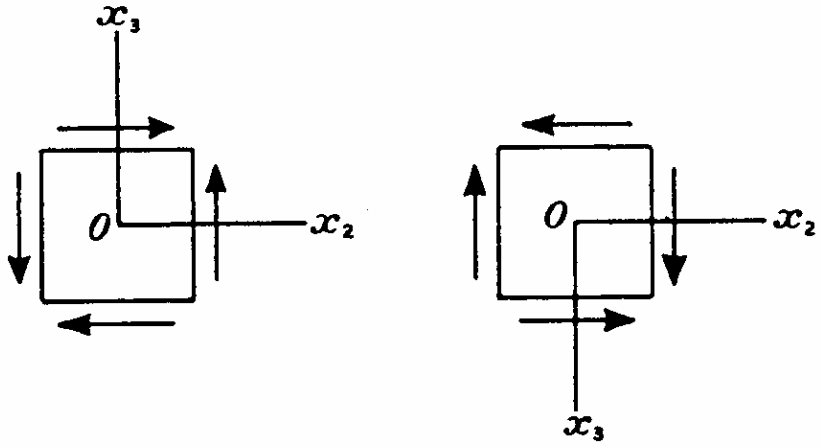
$\sigma_i = c_{ij}\varepsilon_j$ $(i, j = 1, 2, .., 6)$

6x6 matrix

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{pmatrix}$$

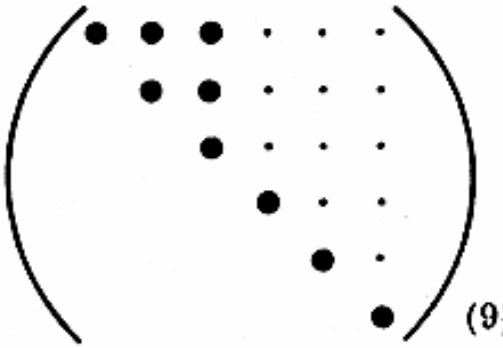
Fourth Rank Tensor

- orthorhombic class 222: s_{34}



ORTHORHOMBIC

All classes

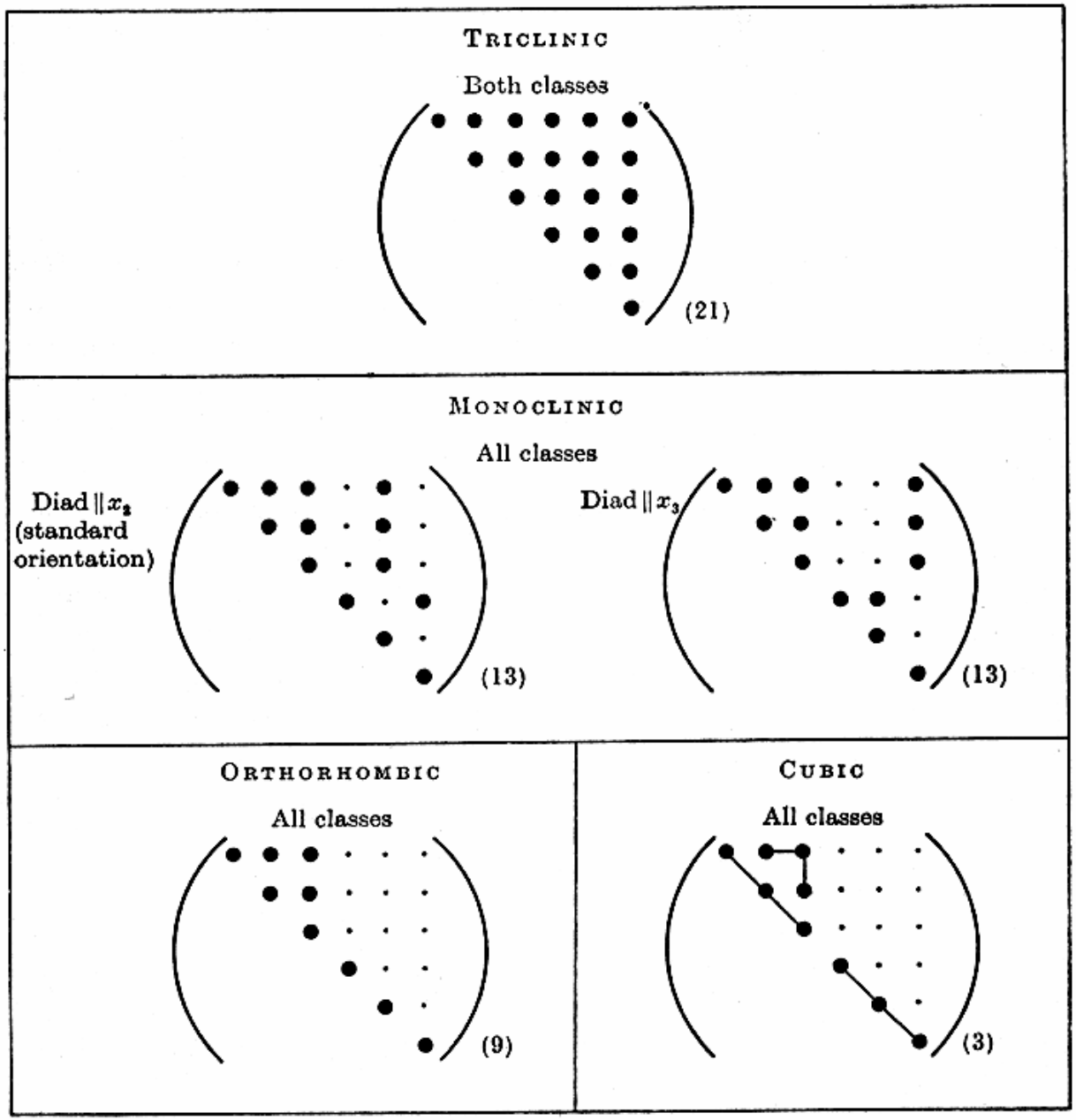


(9)

Elasticity of Crystals

Compliances at room temperature (unit = 10^{-11} m²/newton)

Crystal	*Class*	s_{11}	s_{12}	s_{44}	s_{33}	s_{13}	s_{14}	s_{66}
Sodium chloride	*m3m*	2·21	−0·45	7·83	..	..	..	..
Aluminium .	*m3m*	1·59	−0·58	3·52	..	..	..	..
Copper . .	*m3m*	1·49	−0·63	1·33	..	..	..	..
Nickel . .	*m3m*	0·799	−0·312	0·844	..	..	..	..
Tungsten . .	*m3m*	0·257	−0·073	0·660	..	..	..	..
Sodium chlorate	*23*	2·2	−0·6	8·6	..	..	..	..
Tin . . .	*4/mmm*	1·85	−0·99	5·70	1·18	−0·25	..	13·5
ADP . .	*$\bar{4}$2m*	1·8	0·7	11·3	4·3	−1·1	..	16·2
Zinc . .	*6/mmm*	0·84	0·11	2·64	2·87	−0·78	..	..
Cadmium . .	*6/mmm*	1·23	−0·15	5·40	3·55	−0·93	..	..
Quartz . .	*32*	1·27	−0·17	2·01	0·97	−0·15	−0·43	..
Tourmaline .	*3m*	0·40	−0·10	1·51	0·63	−0·016	0·058	..


TRICLINIC
Both classes
(21)
MONOCLINIC
All classes
Diad ∥ x_2 (standard orientation)
(13)
Diad ∥ x_3
(13)
ORTHORHOMBIC
All classes
(9)
CUBIC
All classes
(3)

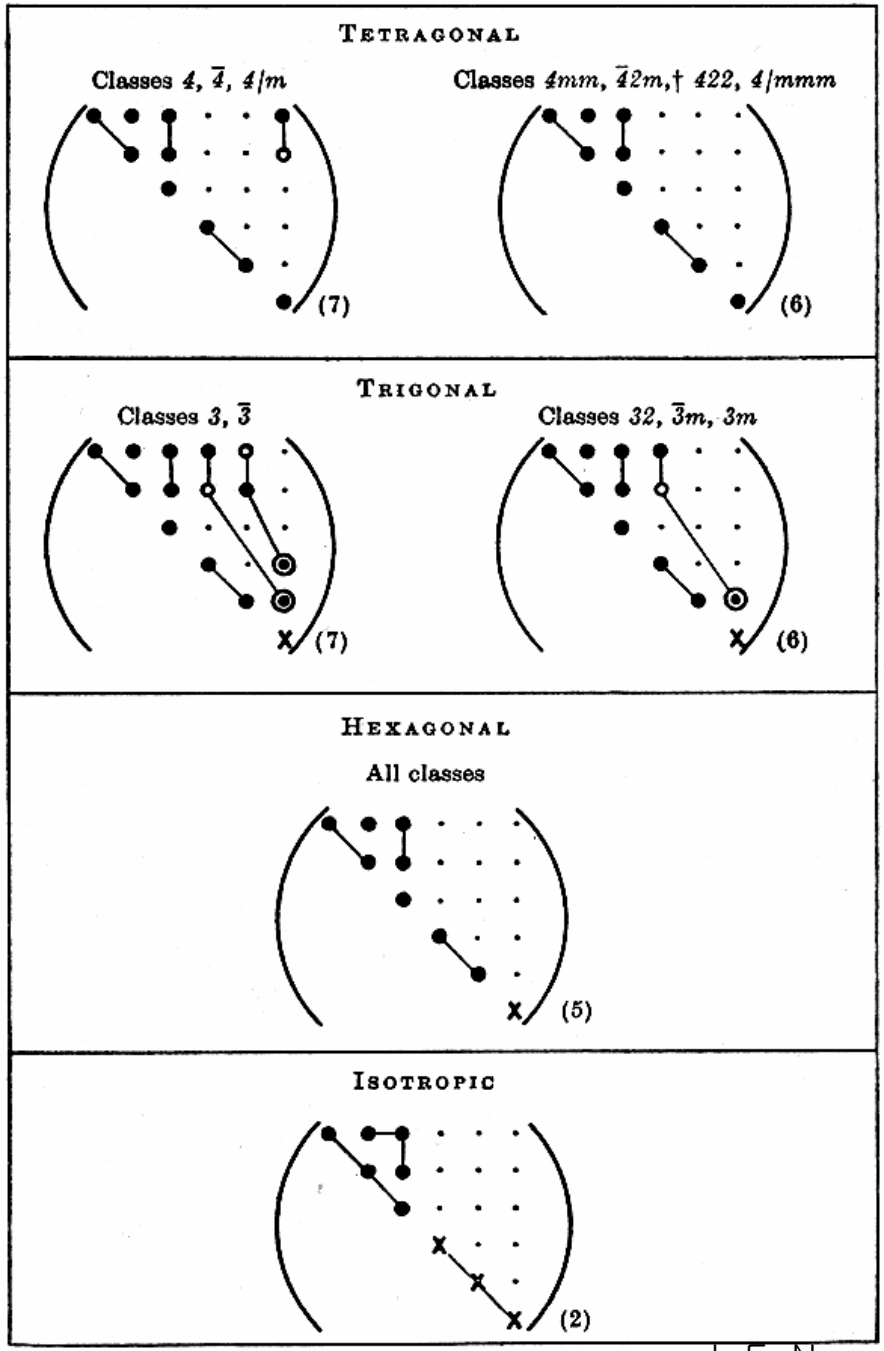

TETRAGONAL
Classes 4, $\bar{4}$, 4/m
(7)
Classes 4mm, $\bar{4}$2m,† 422, 4/mmm
(6)
TRIGONAL
Classes 3, $\bar{3}$
(7)
Classes 32, $\bar{3}$m, 3m
(6)
HEXAGONAL
All classes
(5)
ISOTROPIC
(2)

Table 4. Tensor properties for the 32 crystal classes [8]. Zero means the effect is absent. Other numbers indicate the number of non-zero coefficients, and the number of independent non-zero coefficients is given in parentheses. Third- and fourth-rank tensors are referred to the shortened two-subscript matrix notation

Crystal class	Tensor rank				Example
	1	2	3	4	
$1 = C_1$	3 (3)	9 (6)	18 (18)	36 (21)	Kaolinite $Al_2Si_2O_5(OH)$
$\bar{1} = C_i$	0	9 (6)	0	36 (21)	Copper sulfate $CuSO_4 \cdot 5H_2O$
$2 = C_2$	1 (1)	5 (4)	8 (8)	20 (13)	Sucrose $C_{12}H_{22}O_{11}$
$m = C_s$	2 (2)	5 (4)	10 (10)	20 (13)	Potassium nitrite KNO_2
$2/m = C_{2h}$	0	5 (4)	0	20 (13)	Orthoclase $KAlSi_3O_8$
$222 = D_2$	0	3 (3)	3 (3)	12 (9)	Iodic acid HIO_3
$mm2 = C_{2v}$	1 (1)	3 (3)	5 (5)	12 (9)	Sodium nitrite $NaNO_2$
$mmm = D_{2h}$	0	3 (3)	0	12 (9)	Forsterite Mg_2SiO_4
$3 = C_3$	1 (1)	3 (2)	13 (6)	24 (7)	Nickel tellurate Ni_3TeO_6
$\bar{3} = C_{3i}$	0	3 (2)	0	24 (7)	Ilmenite $FeTiO_3$
$32 = D_3$	0	3 (2)	5 (2)	18 (6)	Low-quartz SiO_2
$3m = C_{3v}$	1 (1)	3 (2)	8 (4)	18 (6)	Lithium niobate $LiNbO_3$
$\bar{3}m = D_{3d}$	0	3 (2)	0	18 (6)	Corundum Al_2O_3
$4 = C_4$	1 (1)	3 (2)	7 (4)	16 (7)	Iodosuccinimide $C_4H_4INO_2$
$\bar{4} = S_4$	0	3 (2)	7 (4)	16 (7)	Boron phosphate BPO_4
$4/m = C_{4h}$	0	3 (2)	0	16 (7)	Scheelite $CaWO_4$
$422 = D_4$	0	3 (2)	2 (1)	12 (6)	Nickel sulfate $NiSO_4 \cdot 6H_2O$
$4mm = C_{4v}$	1 (1)	3 (2)	5 (3)	12 (6)	Barium titanate $BaTiO_3$
$\bar{4}2m = D_{2d}$	0	3 (2)	3 (2)	12 (6)	Potassium dihydrogen phosphate KH_2PO_4
$4/mmm = D_{4h}$	0	3 (2)	0	12 (6)	Rutile TiO_2
$6 = C_6$	1 (1)	3 (2)	7 (4)	12 (5)	Nepheline $NaAlSiO_4$
$\bar{6} = C_{3h}$	0	3 (2)	6 (2)	12 (5)	Lead germanate $Pb_5Ge_3O_{11}$
$6/m = C_{6h}$	0	3 (2)	0	12 (5)	Apatite $Ca_5(PO_4)_3F$
$622 = D_6$	0	3 (2)	2 (1)	12 (5)	High-quartz SiO_2
$6mm = C_{6v}$	1 (1)	3 (2)	5 (3)	12 (5)	Zincite ZnO
$\bar{6}m2 = D_{3h}$	0	3 (2)	3 (1)	12 (5)	Benitoite $BaTiSi_3O_9$
$6/mmm = D_{6h}$	0	3 (2)	0	12 (5)	Beryl $Be_3Al_2Si_6O_{18}$
$23 = T$	0	3 (1)	3 (1)	12 (3)	Sodium chlorate $NaClO_3$
$m\bar{3} = T_h$	0	3 (1)	0	12 (3)	Pyrite FeS_2
$432 = O$	0	3 (1)	0	12 (3)	Manganese β-Mn
$\bar{4}3m = T_d$	0	3 (1)	3 (1)	12 (3)	Zincblende ZnS
$m\bar{3}m = O_h$	0	3 (1)	0	12 (3)	Rocksalt NaCl