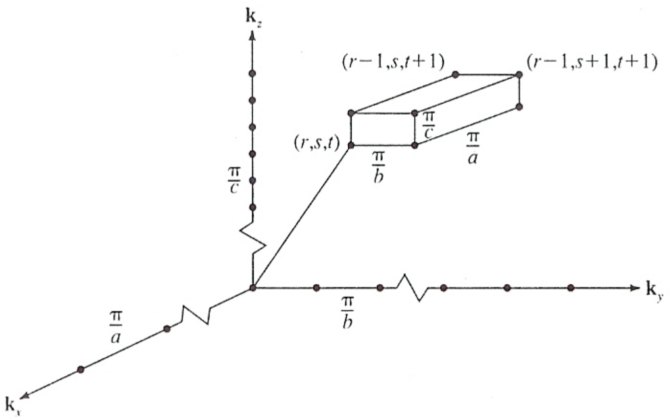

Figure 4-1 $\mathbf{k}$ space description of modes. Every positive triplet of integers r,s,t defines a unique mode. We can thus associate a primitive volume π^3/abc in $\mathbf{k}$ space with each mode.

To find the total number of modes with $\mathbf{k}$ values between 0 and k, we divide the corresponding volume in $\mathbf{k}$ space by the volume per mode:

$$N(k) = \frac{\left(\frac{1}{8}\right)\frac{4\pi}{3}k^3}{\frac{\pi^3}{V}} = \frac{k^3 V}{6\pi^2}$$

(The factor 1/8 is due to the restriction of $r,s,t > 0$.)

We next use (4.0-10) to obtain the number of modes with resonant frequencies between 0 and ν:

$$N(\nu) = \frac{4\pi\nu^3 n^3 V}{3c^3}$$

The mode density, that is, the number of modes per unit ν near ν in a resonator with volume $V(\gg\lambda^3)$, is thus

$$p(\nu) = \frac{dN(\nu)}{d\nu} = \frac{8\pi\nu^2 n^3 V}{c^3} \tag{4.0-11}$$

where we multiplied the final result by 2 to account for the two independent orthogonally polarized modes that are associated with each r,s,t triplet.

The number of modes that fall within the interval $d\nu$ centered on ν is thus

$$N \simeq \frac{8\pi n^3 \nu^2 V}{c^3}\, d\nu \tag{4.0-12}$$

where V is the volume of the resonator. For the case of $V = 1\ \text{cm}^3$, $\nu = 3 \times 10^{14}$ Hz and $d\nu = 3 \times 10^{10}$, as an example, (4.0-12) yields $N \sim 2 \times 10^9$ modes. If the resonator were closed, all these modes would have similar values of Q. This situation is to be avoided in the case of lasers, since it will cause the atoms to emit power (thus causing oscillation) into a large number of modes, which may differ in their frequencies as well as in their spatial characteristics.

This objection is overcome to a large extent by the use of open resonators, which consist essentially of a pair of opposing flat or curved reflectors. In such resonators the energy of the vast majority of the modes does not travel at right angles to the mirrors and will thus be lost in essentially a single traversal. These modes will consequently possess a very low Q. If the mirrors are curved, the few surviving modes will, as shown below, have their energy localized near the axis; thus the diffraction losses caused by the open sides can be made small compared with other loss mechanisms such as mirror transmission. (This point is considered in detail in Section 4.9. The subject of losses is also considered in Section 4.7.)

4.1 FABRY–PEROT ETALON

The Fabry–Perot etalon, or interferometer, named after its inventors [3], can be considered as the archetype of the optical resonator. It consists of a plane-parallel plate of thickness l and index n that is immersed in a medium of index n'.[1] Let a plane wave be incident on the etalon at an angle θ' to the normal, as shown in Figure 4-2(a). We can treat the problem of the transmission (and reflection) of the plane wave through the etalon by considering the infinite number of partial waves produced by reflections at the two end surfaces. The phase delay between two partial waves—which is attributable to one additional round trip—is given, according to Figure 4-2(a), by

$$\delta = \frac{4\pi n l \cos\theta}{\lambda} \tag{4.1-1}$$

where λ is the vacuum wavelength of the incident wave and θ is the internal angle of incidence. If the complex amplitude of the incident wave is taken as A_i, then the partial reflections, B_1, B_2, and so forth, are given by

$$B_1 = rA_i \qquad B_2 = tt'r'\,A_i e^{i\delta} \qquad B_3 = tt'r'^3\,A_i e^{2i\delta} \qquad \cdots$$

where r is the reflection coefficient (ratio of reflected to incident amplitude), t is the transmission coefficient for waves incident from n' toward n, and r' and t' are the corresponding quantities for waves traveling from n toward n'. The complex amplitude of the (total) reflected wave is $A_r = B_1 + B_2 + B_3 + \cdots$, or

$$A_r = \{r + tt'r'e^{i\delta}(1 + r'^2 e^{i\delta} + r'^4 e^{2i\delta} + \cdots)\}\, A_i \tag{4.1-2}$$

For the transmitted wave,

$$A_1 = tt'\,A_i \qquad A_2 = tt'r'^2 e^{i\delta}\,A_i \qquad A_3 = tt'r'^4 e^{2i\delta}\,A_i$$

[1] In practice, one often uses etalons made by spacing two partially reflecting mirrors a distance l apart so that $n = n' = 1$. Another common form of etalon is produced by grinding two plane-parallel (or curved) faces on a transparent solid and then evaporating a metallic or dielectric layer (or layers) on the surfaces.

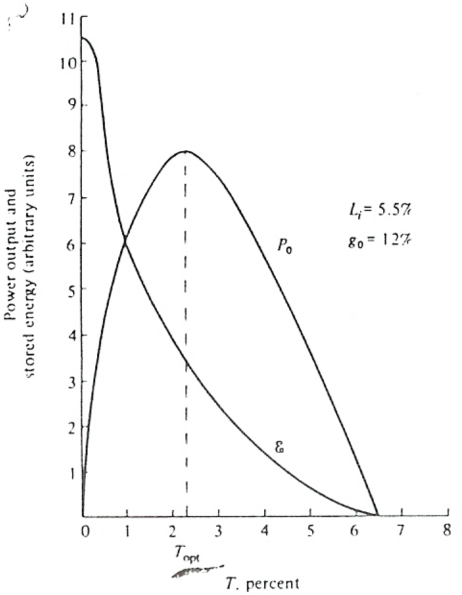

Figure 6-8 Power output P_o and stored energy $\mathscr{E}$ plotted against mirror transmission T.

$\mathscr{E} \propto P_o/T$ as a function of the coupling T is shown in Figure 6-8. As we may expect, $\mathscr{E}$ is a monotonically decreasing function of T.

6.6 MULTIMODE LASER OSCILLATION AND MODE LOCKING

In this section we contemplate the effect of homogeneous or inhomogeneous broadening (in the sense described in Section 5.1) on the laser oscillation.

We start by reminding ourselves of some basic results pertinent to this discussion:

1. The actual gain constant prevailing inside a laser oscillator *at the oscillation frequency* ν is clamped, at steady state, at a value

$$\gamma_t(\nu) = \alpha - \frac{1}{l} \ln r_1 r_2 \qquad (6.1\text{-}8)$$

where l is the length of the gain medium as well as the distance between the mirrors which are taken here to be the same.

2. The gain constant of a distributed laser medium is

$$\gamma(\nu) = (N_2 - N_1) \frac{c^2}{8\pi n^2 \nu^2 t_{spont}} g(\nu) \qquad (5.3\text{-}3)$$

3. The optical resonator can support oscillations, provided sufficient gai[n] is present to overcome losses, at frequencies[8] ν_q separated by

$$\nu_{q+1} - \nu_q = \frac{c}{2nl} \qquad (4.6\text{-}4)$$

Now consider what happens to the gain constant $\gamma(\nu)$ inside a lase[r] oscillator as the pumping is increased from some value below threshold. Operationally, we can imagine an extremely weak wave of frequency launched into the laser medium and then measuring the gain constant $\gamma(\nu)$ as "seen" by this signal as ν is varied.

We treat first the case of a homogeneous laser. Below threshold th[e] inversion $N_2 - N_1$ is proportional to the pumping rate and $\gamma(\nu)$, which i[s] given by (5.3-3), is proportional to $g(\nu)$. This situation is illustrated by curv[e] A in Figure 6-9(a). The spectrum (4.6-4) of the passive resonances is show[n] in Figure 6-9(b). As the pumping rate is increased, the point is reached a[t] which the gain per pass at the center resonance frequency ν_0 is equal to th[e] average loss per pass. This is shown in curve B. At this point, oscillation a[t] (cf. p180) ν_0 starts. An increase in the pumping cannot increase the inversion sinc[e]

[8] The high-order transverse modes discussed in Section 4.5 are ignored here.

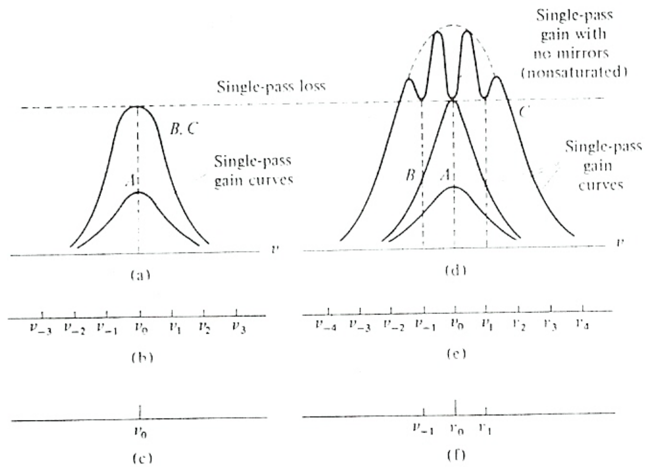

Figure 6-9 (a) Single-pass gain curves for a homogeneous atomic system (A—below threshold; B—at threshold; C—well above threshold). (b) Mode spectrum of optical resonator. (c) Oscillation spectrum (only one mode oscillates). (d) Single-pass gain curves for an inhomogeneous atomic system (A—below threshold; B—at threshold; C—well above threshold). (e) Mode spectrum of optical resonator. (f) Oscillation spectrum for pumping level C, showing three oscillating modes.

From (5.4-15) and (5.5-1) we obtain

$$\chi'(\nu) = \frac{2(\nu_0 - \nu)}{\Delta\nu}\chi''(\nu)$$
$$= \frac{(N_1 - N_2)\lambda^3}{4\pi^3 n(\Delta\nu)^2 t_{\text{spont}}} \frac{(\nu_0 - \nu)}{1 + [4(\nu - \nu_0)^2/(\Delta\nu)^2]} \tag{5.5-2}$$

This expression will be used to represent the dispersion of a homogeneously broadened transition with a Lorentzian lineshape.

6 GAIN SATURATION IN HOMOGENEOUS LASER MEDIA

In Section 5.3 we derived an expression (5.3-3) for the exponential gain constant due to a population inversion. It is given by

$$\gamma(\nu) = (N_2 - N_1)\frac{c^2}{8\pi n^2 \nu^2 t_{\text{spont}}} g(\nu) \tag{5.6-1}$$

where N_2 and N_1 are the population densities of the two atomic levels involved in the induced transition. There is nothing in (5.6-1) to indicate what causes the inversion $(N_2 - N_1)$, and this quantity can be considered as a parameter of the system. In practice the inversion is caused by a "pumping" agent, hereafter referred to as the pump, that can take various forms such as the electric current in injection lasers, the flashlamp light in pulsed ruby lasers, or the energetic electrons in plasma-discharge gas lasers.

Consider next the situation prevailing at some point *inside* a laser medium in the presence of an optical wave. The pump establishes a population inversion, which in the absence of any optical field has a value ΔN^0. The presence of the optical field induces $2 \rightarrow 1$ and $1 \rightarrow 2$ transitions. Since $N_2 > N_1$ and the induced rates for $2 \rightarrow 1$ and $1 \rightarrow 2$ transitions are equal, it follows that more atoms are induced to undergo a transition from level 2 to level 1 than in the opposite direction and that, consequently, the new equilibrium population inversion is smaller than ΔN^0.

The reduction in the population inversion and hence of the gain constant brought about by the presence of an electromagnetic field is called gain saturation. Its understanding is of fundamental importance in quantum electronics. As an example, which will be treated in the next chapter, we may point out that gain saturation is the mechanism that reduces the gain inside laser oscillators to a point where it just balances the losses so that steady oscillation can result.

In Figure 5-5 we show the ground state 0 as well as the two laser levels 2 and 1 of a four-level laser system. The density of atoms pumped per unit time into level 2 is taken as R_2, and that pumped into 1 is R_1. Pumping into 1 is, of course, undesirable since it leads to a reduction of the inversion. In many practical situations it cannot be avoided. The actual "decay" lifetime

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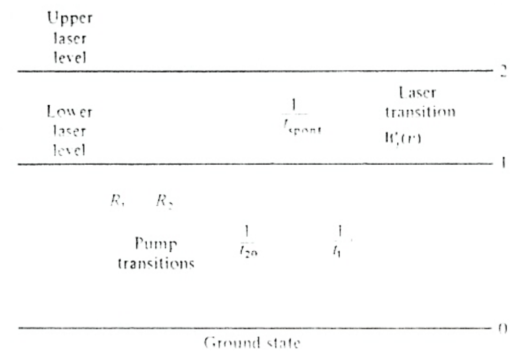

Figure 5-5 Energy levels and transition rates of a four-level laser system. (The fourth level, which is involved in the original excitation by the pump, is not shown and the pumping is shown as proceeding directly into levels 1 and 2.) The total lifetime of level 2 is t_2, where $1/t_2 = 1/t_{\text{spont}} + 1/t_{20}$.

of atoms in level 2 at the absence of any radiation field is taken as t_2. This decay rate has a contribution t_{spont}^{-1} that is due to spontaneous (photon emitting) $2 \rightarrow 1$ transitions as well as to additional nonradiative relaxation from 2 to 1. The lifetime of atoms in level 1 is t_1. The induced rate for $2 \rightarrow 1$ and $1 \rightarrow 2$ transitions due to a radiation field at frequency ν is denoted by $W_i(\nu)$ and, according to (5.2-15), is given by

$$W_i(\nu) = \frac{\lambda^2 g(\nu)}{8\pi n^2 h\nu t_{\text{spont}}} I_\nu \tag{5.6-2}$$

where $g(\nu)$ is the normalized lineshape of the transition and I_ν is the intensity (watts per square meter) of the optical field.

The equations describing the populations of level 2 and 1 in the combined presence of a radiation field at ν and a pump are:

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{t_2} - (N_2 - N_1)W_i(\nu) \tag{5.6-3}$$

$$\frac{dN_1}{dt} = R_1 - \frac{N_1}{t_1} + \frac{N_2}{t_{\text{spont}}} + (N_2 - N_1)W_i(\nu) \tag{5.6-4}$$

N_2 and N_1 are the population densities (m^{-3}) of levels 2 and 1 respectively. R_2 and R_1 are the pumping rates ($m^{-3}-s^{-1}$) into these levels. N_2/t_2 is the change per unit time in the population of 2 due to decay out of level 2 to all levels. This includes spontaneous transitions to 1 but *not* induced transitions. The rate for the latter is $N_2 W_i(\nu)$ so that the net change in N_2 due to induced transitions is given by the last term of (5.6-3). At steady state the populations are constant with time, so putting $d/dt = 0$ in the two preceding equations, 179

we can solve for N_1, N_2, and obtain[4]

$$N_2 - N_1 = \frac{R_2 t_2 - (R_1 + \delta R_2) t_1}{1 + [t_2 + (1 - \delta) t_1] W_i(\nu)} \tag{5.6-5}$$

where $\delta = t_2/t_{\text{spont}}$. If the optical field is absent, $W_i(\nu) = 0$, and the inversion density is given by

$$\Delta N^0 = R_2 t_2 - (R_1 + \delta R_2) t_1 \tag{5.6-6}$$

we can use (5.6-6) to rewrite (5.6-5) as

$$N_2 - N_1 = \frac{\Delta N^0}{1 + \phi t_{\text{spont}} W_i(\nu)} \tag{5.6-7}$$

where the parameter ϕ is defined by

$$\phi = \delta \left[1 + (1 - \delta) \frac{t_1}{t_2} \right]$$

We note that in efficient laser systems $t_2 \simeq t_{\text{spont}}$, so $\delta \simeq 1$, and that $t_1 \ll t_2$, so $\phi \simeq 1$. Substituting (5.6-2) for $W_i(\nu)$ the last equation becomes

$$N_2 - N_1 = \frac{\Delta N^0}{1 + [\phi \lambda^2 g(\nu)/8\pi n^2 h\nu] I_\nu} = \frac{\Delta N^0}{1 + I_\nu / I_s(\nu)} \tag{5.6-8}$$

where $I_s(\nu)$, the saturation intensity, is given by

$$I_s(\nu) = \frac{8\pi n^2 h\nu}{\phi \lambda^2 g(\nu)} = \frac{8\pi n^2 h\nu}{(t_2/t_{\text{spont}}) \lambda^2 g(\nu)} = \frac{8\pi n^2 h\nu \Delta\nu}{(t_2/t_{\text{spont}}) \lambda^2} \tag{5.6-9}$$

and corresponds to the intensity level (watts per square meter) that causes the inversion to drop to one half of its nonsaturated value (ΔN^0). By using (5.6-8) in the gain expression (5.6-1), we obtain our final result

$$\gamma(\nu) = \frac{1}{1 + I_\nu / I_s(\nu)} \left(\frac{\Delta N^0 \lambda^2}{8\pi n^2 t_{\text{spont}}} \right) g(\nu)$$

$$= \frac{\gamma_0(\nu)}{1 + I_\nu / I_s(\nu)} \tag{5.6-10}$$

which shows the dependence of the gain constant on the optical intensity.

In closing we recall that (5.6-10) applies to a homogeneous laser system. This is due to the fact that in the rate equations (5.6-3) and (5.6-4) we considered all the atoms as equivalent and, consequently, experiencing the same transition rates. This assumption is no longer valid in inhomogeneous laser systems. This case is treated in the next section.

[4] Levels 1 and 2 are assumed to be high enough (in energy) that the role of thermal processes in populating them can be neglected.

5.7 GAIN SATURATION IN INHOMOGENEOUS LASER MEDIA

In Section 5.6 we considered the reduction in optical gain—that is, saturation—due to the optical field in a homogeneous laser medium. In this section we treat the problem of gain saturation in inhomogeneous systems.

According to the discussion of Section 5.1, in an inhomogeneous atomic system the individual atoms are distinguishable, with each atom having a unique transition frequency $(E_2 - E_1)/h$. We can thus imagine the inhomogeneous medium as made up of classes of atoms each designated by a continuous variable ξ.[5] Furthermore, we define a function $p(\xi)$ so that the *a priori* probability that an atom has its ξ parameter between ξ and $\xi + d\xi$ is $p(\xi)\, d\xi$. It follows that

$$\int_{-\infty}^{\infty} p(\xi)\, d\xi = 1 \tag{5.7-1}$$

since any atom has a unit probability of having its ξ value between $-\infty$ and ∞.

The atoms within a given class ξ are considered as homogeneously broadened, having a lineshape function $g^\xi(\nu)$ that is normalized so that

$$\int_{-\infty}^{\infty} g^\xi(\nu)\, d\nu = 1 \tag{5.7-2}$$

In Section 5.1 we defined the transition lineshape $g(\nu)$ by taking $g(\nu)\, d\nu$ to represent the *a priori* probability that a spontaneous emission will result in a photon whose frequency is between ν and $\nu + d\nu$. Using this definition we obtain

$$g(\nu)\, d\nu = \left[\int_{-\infty}^{\infty} p(\xi) g^\xi(\nu)\, d\xi \right] d\nu \tag{5.7-3}$$

which is a statement of the fact that the probability of emitting a photon of frequency between ν and $\nu + d\nu$ is equal to the probability $g^\xi(\nu)\, d\nu$ of this occurrence, given that the atom belongs to class ξ, summed up over all the classes.

Next we proceed to find the contribution to the inversion that is due to a single class ξ. The equations of motion [9] are

$$\frac{dN_2^\xi}{dt} = R_2 p(\xi) - \frac{N_2^\xi}{t_2} - [N_2^\xi - N_1^\xi] W_i^\xi(\nu)$$

$$\frac{dN_1^\xi}{dt} = R_1 p(\xi) - \frac{N_1^\xi}{t_1} + \frac{N_2^\xi}{t_{\text{spont}}} + [N_2^\xi - N_1^\xi] W_i^\xi(\nu) \tag{5.7-4}$$

[5] The variable ξ can, as an example, correspond to the center frequency of the lineshape function $g^\xi(\nu)$ of atoms in group ξ.