where we used $\ln(a + ib) = \ln\sqrt{a^2 + b^2} + i\tan^{-1}(b/a)$. Substituting (6.6-8) in the second term of (6.6-7) and separating the exponent into its real and imaginary parts, we obtain

$$\exp\left[\frac{-ikr^2}{2(q_0 + z)}\right] = \exp\left\{\frac{-r^2}{\omega_0^2\left[1 + \left(\dfrac{\lambda z}{\pi\omega_0^2 n}\right)^2\right]} - \frac{ikr^2}{2z\left[1 + \left(\dfrac{\pi\omega_0^2 n}{\lambda z}\right)^2\right]}\right\} \tag{6.6-10}$$

If we define the following parameters:

$$\omega^2(z) = \omega_0^2\left[1 + \left(\frac{\lambda z}{\pi\omega_0^2 n}\right)^2\right] = \omega_0^2\left(1 + \frac{z^2}{z_0^2}\right) \tag{6.6-11}$$

$$R = z\left[1 + \left(\frac{\pi\omega_0^2 n}{\lambda z}\right)^2\right] = z\left(1 + \frac{z_0^2}{z^2}\right) \tag{6.6-12}$$

$$\eta(z) = \tan^{-1}\left(\frac{\lambda z}{\pi\omega_0^2 n}\right) = \tan^{-1}\left(\frac{z}{z_0}\right) \tag{6.6-13}$$

$$z_0 \equiv \frac{\pi\omega_0^2 n}{\lambda}$$

we can combine (6.6-9) and (6.6-10) in (6.6-7) and, recalling that $E(x, y, z) = \psi(x, y, z)\exp(-ikz)$, obtain

$$\begin{aligned} E(x, y, z) &= E_0\frac{\omega_0}{\omega(z)}\exp\left\{-i[kz - \eta(z)] - i\frac{kr^2}{2q(z)}\right\} \\ &= E_0\frac{\omega_0}{\omega(z)}\exp\left\{-i[kz - \eta(z)] - r^2\left[\frac{1}{\omega^2(z)} + \frac{ik}{2R(z)}\right]\right\} \end{aligned} \tag{6.6-14}$$

so that if we use (6.5-9) and (6.6-4)

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda}{\pi n\omega^2(z)} \tag{6.6-14a}$$

This is our basic result. We refer to it as the fundamental Gaussian-beam solution since we have excluded the more complicated solutions of (6.5-3) (i.e., those with azimuthal variation) by limiting ourselves to transverse dependence involving $r = (x^2 + y^2)^{1/2}$ only. These higher-order modes will be discussed separately.

From (6.6-14), the parameter $\omega(z)$, which evolves according to (6.6-11), is the distance r at which the field amplitude is down by a factor $1/e$ compared to its value on the axis. We will consequently refer to it as the beam "spot size." The parameter ω_0 is the minimum spot size. It is the beam spot size at the plane $z = 0$. The parameter R in (6.6-14) is the radius of curvature of the very nearly spherical wavefronts[6] at z. We can verify this statement by deriving the radius of curvature of the constant phase surfaces (wavefronts) or, more simply, by considering the form of a spherical wave emitted by a point radiator placed at $z = 0$. It is given by

$$\begin{aligned} E &\propto \frac{1}{R}e^{-ikR} = \frac{1}{R}\exp\left(-ik\sqrt{x^2 + y^2 + z^2}\right) \\ &\simeq \frac{1}{R}\exp\left(-ikz - ik\frac{x^2 + y^2}{2R}\right) \qquad x^2 + y^2 \ll z^2 \end{aligned} \tag{6.6-15}$$

since z is equal to R, the radius of curvature of the spherical wave. Comparing (6.6-15) with (6.6-14), we identify R as the radius of curvature of the Gaussian beam. The convention regarding the sign of R is the same as that adopted in Table 6.1; that is, $R(z)$ is negative if the center of curvature occurs at $z' > z$ and vice versa.

The form of the fundamental Gaussian beam is, according to (6.6-14), uniquely determined once its minimum spot size ω_0 and its location—that is, the plane $z = 0$—are specified. Its spot size ω and radius of curvature R at any plane z are then found from (6.6-11) and (6.6-12). Some of these characteristics are displayed in Figure 6.5. The hyperbolas shown in this figure correspond to the ray direction and are intersections of planes that include the z axis and the hyperboloids

$$x^2 + y^2 = \text{const.}\ \omega^2(z) \tag{6.6-16}$$

They correspond to the direction of energy propagation. The spherical surfaces shown have radii of curvature given by (6.6-12). For large z, the hyperboloids $x^2 + y^2 = \omega^2$ are asymptotic to the cone

$$r = \sqrt{x^2 + y^2} = \frac{\lambda}{\pi\omega_0 n}z \tag{6.6-17}$$

whose half-apex angle, which we take as a measure of the angular beam spread, is

$$\theta_{\text{beam}} = \tan^{-1}\left(\frac{\lambda}{\pi\omega_0 n}\right) \simeq \frac{\lambda}{\pi\omega_0 n} \tag{6.6-18}$$

This last result is a rigorous manifestation of wave diffraction according to which a wave that is confined in the transverse direction to an aperture

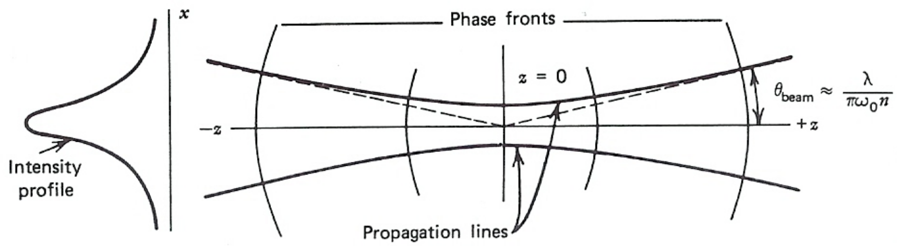


FIGURE 6.5 Propagating Gaussian beam.

[6] Actually, it follows from (6.6-14) that, with the exception of the immediate vicinity of the plane $z = 0$, the wavefronts are parabolic since they are defined by $k[z + (r^2/2R)] = \text{const.}$ For $r^2 \ll z^2$, the distinction between parabolic and spherical surfaces is not important.

where, according to (6.5-4),

$$k^2 = k^2(0) = \omega^2\mu\varepsilon(0)\left[1 - i\,\frac{\sigma(0)}{\omega\varepsilon(0)}\right]$$

and k_2 is some constant. Furthermore, we assume a solution whose transverse dependence is on $r = \sqrt{x^2 + y^2}$ only so that in (6.5-3) we can replace ∇^2 by

$$\nabla^2 = \nabla_t^2 + \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{6.5-6}$$

The kind of propagation we are considering is that of a nearly plane wave in which the flow of energy is predominantly along a single (e.g., z) direction so that we may limit our derivation to a single transverse field component E. Taking E as

$$E = \psi(x, y, z)e^{-ikz} \tag{6.5-7}$$

we obtain from (6.5-3) and (6.5-5) in a few simple steps

$$\nabla_t^2\psi - 2ik\psi' - kk_2r^2\psi = 0 \tag{6.5-8}$$

where $\psi' = \partial\psi/\partial z$ and where we assume that the longitudinal variation is slow enough that $k\psi' \gg \psi'' \ll k^2\psi$.

Next, we take ψ in the form of

$$\psi = \exp\{-i[P(z) + \tfrac{1}{2}Q(z)r^2]\} \tag{6.5-9}$$

that, when substituted into (6.5-8) and after using (6.5-6), gives

$$-Q^2r^2 - 2iQ - kr^2Q' - 2kP' - kk_2r^2 = 0 \tag{6.5-10}$$

If (6.5-10) is to hold for all r, the coefficients of the different powers of r must each be equal to zero. This leads to (Reference 7)

$$\begin{aligned} Q^2 + kQ' + kk_2 &= 0 \\ P' &= -\frac{iQ}{k} \end{aligned} \tag{6.5-11}$$

The wave equation (6.5-3) is thus reduced to Eqs. (6.5-11).

6.6 THE GAUSSIAN BEAM IN A HOMOGENEOUS MEDIUM

If the medium is homogeneous, we can, according to (6.5-5), put $k_2 = 0$, and (6.5-11) becomes

$$Q^2 + kQ' = 0 \tag{6.6-1}$$

Introducing the function $s(z)$ by the relation

$$Q = k\frac{s'}{s} \tag{6.6-2}$$

matrix with a unity determinant (unimodular) to obtain

$$\begin{aligned} A_T &= \frac{A\sin(s\theta) - \sin[(s-1)\theta]}{\sin\theta} \\ B_T &= \frac{B\sin(s\theta)}{\sin\theta} \\ C_T &= \frac{C\sin(s\theta)}{\sin\theta} \\ D_T &= \frac{D\sin(s\theta) - \sin[(s-1)\theta]}{\sin\theta} \end{aligned} \tag{6.8-2}$$

where

$$\cos\theta = \tfrac{1}{2}(A + D) = \left(1 - \frac{d}{f_2} - \frac{d}{f_1} + \frac{d^2}{2f_1f_2}\right) \tag{6.8-3}$$

and then use (6.8-2) in (6.7-6) with the result

$$q_{s+1} = \frac{\{A\sin(s\theta) - \sin[(s-1)\theta]\}q_1 + B\sin(s\theta)}{C\sin(s\theta)q_1 + D\sin(s\theta) - \sin[(s-1)\theta]} \tag{6.8-4}$$

The condition for the confinement of the Gaussian beam by the lens sequence is, from (6.8-4), that θ be real; otherwise, the sine functions will yield growing exponentials. From (6.8-3), this condition becomes $|\cos\theta| \leq 1$, or

$$0 \leq \left(1 - \frac{d}{2f_1}\right)\left(1 - \frac{d}{2f_2}\right) \leq 1 \tag{6.8-5}$$

that is, the same as condition (6.1-16) for stable-ray propagation.

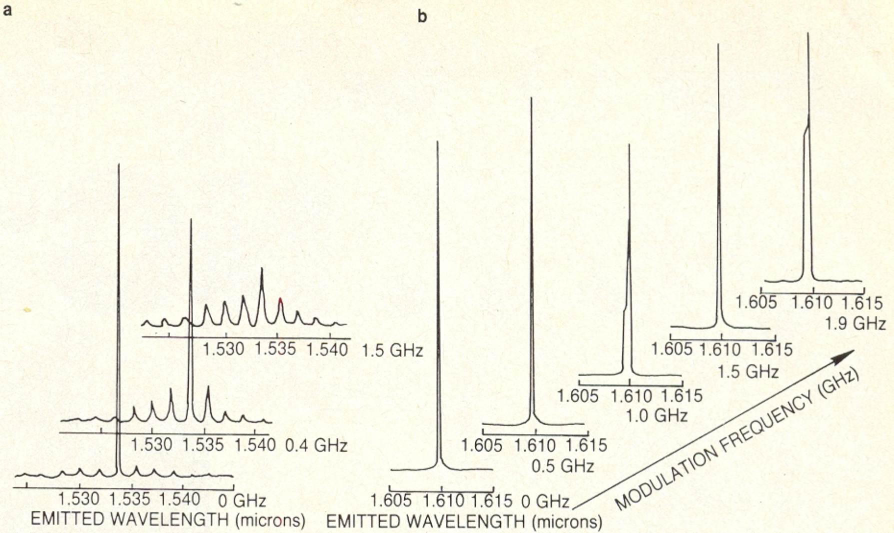
6.9 HIGH-ORDER GAUSSIAN BEAM MODES IN A HOMOGENEOUS MEDIUM

The Gaussian mode treated up to this point has a field variation that depends only on axial distance z and distance r from the axis. If we do not impose the condition $\partial/\partial\phi = 0$ [where ϕ is the azimuthal angle in a cylindrical coordinate system (r, ϕ, z)] and take $k_2 = 0$, the wave equation (6.5-3) has solutions in the form of (Supplementary Reference 1 and Reference 13).

$$\begin{aligned} E_{l,m}(x, y, z) &= E_0\frac{\omega_0}{\omega(z)}H_l\left[\sqrt{2}\frac{x}{\omega(z)}\right]H_m\left[\sqrt{2}\frac{y}{\omega(z)}\right] \\ &\quad\times\exp\left[-ik\frac{x^2+y^2}{2q(z)} - ikz + i(m+n+1)\eta\right] \\ &= E_0\frac{\omega_0}{\omega(z)}H_l\left[\sqrt{2}\frac{x}{\omega(z)}\right]H_m\left(\sqrt{2}\frac{y}{\omega(z)}\right] \\ &\quad\times\exp\left[-\frac{x^2+y^2}{\omega^2(z)} - \frac{ik(x^2+y^2)}{2R(z)} - ikz + i(l+m+1)\eta\right] \end{aligned} \tag{6.9-1}$$

where H_l is the Hermite polynomial of order l, and $\omega(z)$, $R(z)$, $q(z)$, and η are given by (6.6-11) through (6.6-13).

Laser spectra under direct modulation: (**a**) for a conventional buried heterostructure with a Fabry–Perot cavity, and (**b**) for a dynamic-single-mode laser diode. Both lasers are operating at a current 1.2 times their threshold, and the modulation depth is 100% for both. Note that the dynamic-single-mode laser diode continues to emit an extremely narrow spectrum at modulation frequencies that cause the conventional laser to emit over a very broad range of modes. Figure 5



taxial layers act as cladding layers to the quaternary $Ga_{1-x}In_xAs_{1-y}P_y$ active layer. The quaternary layer plays in this system the role played by GaAs in the GaAs/GaAlAs laser depicted in Figure 15-9. A typical quaternary laser structure is shown in Figure 15-18. Modern versions of these laser systems employ active regions with thicknesses in the 50Å → 100Å range. These are the so-called *quantum well lasers*. These lasers possess lower threshold currents and have a larger modulated bandwidth compared to earlier generations employing ''thick'' (~1000Å) active regions. They are discussed in detail in Chapter 16. Recent experiments [26, 39, 41, 42] have demonstrated propagation without repeaters at distances of ~ 150 km in optical fibers at 1.55 μm.

Power Output of Injection Lasers

The considerations of saturation and power output in an injection laser are basically the same as that of conventional lasers, which were described in Sections 5.6 and 6.4. As the injection current is increased above the threshold value, the laser oscillation intensity builds up. The resulting stimulated emission shortens the lifetime of the inverted carriers to the point where the magnitude of the inversion is clamped at its threshold value. Taking the probability that an injected carrier recombine radiatively within the active region as η_i,[2] we can write the following expression for the power emitted by stimulated emission:

$$P_e = \frac{(I - I_t)\eta_i}{e} h\nu \tag{15.4-1}$$

Part of this power is dissipated inside the laser resonator, and the rest is coupled out through the end reflectors. These two powers are, according to (15.3-4), proportional to the effective internal loss $\alpha \equiv \alpha_n \Gamma_n + \alpha_p \Gamma_p + \alpha_s$ and to $-L^{-1} \ln R$, respectively. We can thus write the output power as

$$P_0 = \frac{(I - I_t)\eta_i h\nu}{e} \frac{(1/L) \ln (1/R)}{\alpha + (1/L) \ln (1/R)} \tag{15.4-2}$$

The external differential quantum efficiency η_{ex} is defined as the ratio of the photon output rate that results from an increase in the injection rate (carriers per second) to the increase in the injection rate:

$$\eta_{ex} = \frac{d(P_0/h\nu)}{d[(I - I_t)/e]} \tag{15.4-3}$$

Using (15.4-2) we obtain

$$\eta_{ex}^{-1} = \eta_i^{-1} \left(\frac{\alpha L}{\ln (1/R)} + 1 \right) \tag{15.4-4}$$

[2]The reason for a quantum efficiency η_i that is less than unity is, mostly, the existence of a leakage current component that bypasses the active p-n junction region.

By plotting the dependence of η_{ex} on L we can determine η_i, which in GaAs is around 0.9–1.0.

Since the incremental efficiency of converting electrons into useful output photons is η_{ex}, the main remaining loss mechanisms degrading the conversion of electrical to optical power is the small discrepancy between the energy eV_{appl} supplied to each injected carrier and the photon energy $h\nu$. This discrepancy is due mostly to the series resistance of the laser diode. The efficiency of the laser in converting electrical power input to optical power is thus

$$\eta = \frac{P_0}{VI} = \eta_i \frac{I - I_t}{I} \frac{h\nu}{eV_{appl}} \frac{\ln(1/R)}{\alpha L + \ln(1/R)} \tag{15.4-5}$$

In practice $eV_{appl} \sim 1.4E_g$ and $h\nu \simeq E_g$. Values of $\eta \sim 30$ percent at 300 K have been achieved.

We conclude this section by showing in Figures 15-16 and 15-17 typical plots of the power output versus current and the far field of commercial low-threshold GaAs semiconductor lasers.

15.5 DIRECT-CURRENT MODULATION OF SEMICONDUCTOR LASERS

Since the main application of semiconductor lasers is as sources for optical communication systems, the problem of high-speed modulation of their output by the high-data-rate information is one of great technological importance.

A unique feature of semiconductor lasers is that, unlike other lasers that are modulated externally (see Chapter 9), the semiconductor laser can be modulated directly by modulating the excitation current. This is especially important in view of the possibility of monolithic integration of the laser and the modulation electronic circuit, as will be discussed in Section 15.7. The following treatment follows closely that of Reference [27].

If we denote the photon density inside the active region of a semiconductor laser by P and the injected electron (and hole) density by N, then we can write

$$\begin{aligned}\frac{dN}{dt} &= \frac{I}{eV} - \frac{N}{\tau} - A(N - N_{tr})P \\ \frac{dP}{dt} &= A(N - N_{tr})P\Gamma_a - \frac{P}{\tau_p}\end{aligned} \tag{15.5-1}$$

where I is the total current, V the volume of the active region, τ the spontaneous recombination lifetime, τ_p the photon lifetime as limited by absorption in the bounding media, scattering and coupling through the output mirrors.

The term $A(N - N_{tr})P$ is the net rate per unit volume of induced transitions. N_{tr} is the inversion density needed to achieve transparency as defined by (15.2-17), and