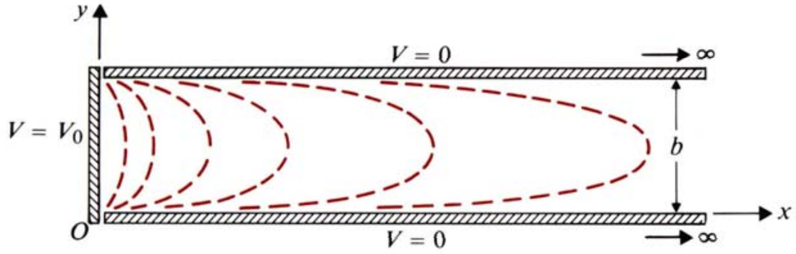
Ex 4-6) the potential distribution enclosed by the electrodes



① Find B.C. first and apply to D.E.

a) V independent of $z \rightarrow \frac{\partial}{\partial z} = 0,$ and $V(x, y, z) = V(x, y)$

$$\frac{d^2 Z(z)}{dz^2} + k_z^2 Z(z) = 0, \; k_z^2 = 0 \text{ and } Z(z) = A_0 z + B_0 = B_0$$

b) $x -$ direction

- $x \rightarrow \infty$ 일때 $V(\infty, y) = 0$: V is decreasing function in the x direction
- $x = 0$ 일때 $V(0, y) = V_0$

$\therefore \; k_x^2 + k_y^2 = 0 \Rightarrow k_y^2 = -k_x^2 = k^2 \quad \therefore k_x^2 = -k^2$ therefore, $k_x = \pm jk, \; k > 0$

then $X(x) = C_1 e^{kx} + D_2 e^{-kx} = D_2 e^{-kx}$

$cf)$ $V(0, y) = V_0$ will be applied later

Ex 4-6) the potential distribution enclosed by the electrodes

c) $y-$direction

$Y(x,0)=0, \quad Y(x,b)=0 \Rightarrow$ implies sine function,

from $Y(y)=A_1 \sin ky + B_1 \cos ky$

$$\therefore Y(0)=0, \quad Y(b)=A_1 \sin kb = 0 \quad \therefore k=\frac{n\pi}{b}$$

then $Y(y)=A_1 \sin \dfrac{n\pi}{b} y$

② combine $X(x), \ Y(y), \ Z(z)$

$$V(x,y)=X(x)Y(y)Z(z)=A_1 B_0 D_2 e^{-\frac{n\pi}{b}x} \sin \frac{n\pi}{b} y, \quad n=1,2,3,.....$$

cf) n cannot be negative since $X(x)$ should be decaying function

Ex 4-6) the potential distribution enclosed by the electrodes

③ $V_n(x,y)$ alone cannot satisfy the B.C that at $x=0$,

$V(0,y)=V_0$ for $0<y<b$

let $V(x,y)=\sum_n V_n(x,y)$, then $V(x,y)$ is a solution of Laplace equation

④ apply the last B.C

$$V(0,y)=\sum_{n=1}^{\infty}V_n(0,y)=\sum_{n=1}^{\infty}C_n\sin\frac{n\pi}{b}y=V_0,\quad \text{for } 0<y<b$$

i.e) $\sum_{n=1}^{\infty}C_n\sin\frac{n\pi}{b}y=V_0$, to find C_n

$$\sum_{n=1}^{\infty}\int_0^b C_n\sin\frac{n\pi}{b}y\sin\frac{m\pi}{b}y\,dy=\int_0^b V_0\sin\frac{m\pi}{b}y\,dy$$

Ex 4-6) the potential distribution enclosed by the electrodes

- left hand side

$$\int_0^b C_n \sin\frac{n\pi}{b}y \sin\frac{m\pi}{b}y dy = \frac{C_n}{2}\int_0^b [\cos\frac{(n-m)\pi}{b}y - \cos\frac{(n+m)\pi}{b}y]dy$$

$$= \begin{cases} \frac{C_n}{2}b \ , & \text{for } m=n \\ 0 \ , & \text{for } m \neq n \end{cases}$$

$$\boxed{\text{For } n \neq m, \int_0^b \sin\frac{n\pi}{b}y \cdot \sin\frac{m\pi}{b}y dy = 0}$$

- right hand side

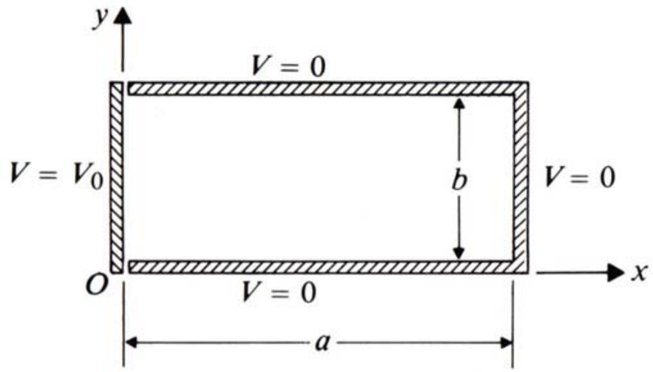
$$\int_0^b V_0 \sin\frac{m\pi}{b}y dy = V_0 \frac{b}{m\pi}[-\cos\frac{m\pi}{b}y]\Big|_0^b = \frac{V_0 b}{m\pi}[1-\cos m\pi]$$

$$\Rightarrow \begin{cases} 0 \ , & \text{for } m = even \\ \frac{2V_0 b}{m\pi} \ , & \text{for } m = odd \end{cases}$$

$$\therefore C_n = \begin{cases} \frac{4V_0}{n\pi} \ , & \text{for } n = odd \\ 0 \ , & \text{for } n = even \end{cases}$$

$$\therefore V(x,y) = \sum_{n=1}^{\infty} C_n e^{-\frac{n\pi}{b}x} \sin\frac{n\pi}{b}y = \sum_{\substack{n=1 \\ ,odd}}^{\infty} \frac{4V_0}{n\pi} e^{-\frac{n\pi}{b}x} \sin\frac{n\pi}{b}y, \quad \text{for } x>0 \text{ and } 0<y<b$$

Ex 4-7) the potential distribution enclosed by conducting planes



sol) ➢ $V(x, y, z) = V(x, y), \quad \dfrac{\partial}{\partial z} = 0, \quad k_z^2 = 0$

➢ $V(0, y) = V_0, \; V(a, y) = 0, \; V(x, 0) = V(x, b) = 0$

① $Z(z) = B_0, \quad k_y^2 = -k_x^2 = k^2$

② for $y = 0, \quad V(x, y) = 0 \rightarrow \quad \therefore \; Y(y) = A_1 \sin ky$

for $y = b, \quad V(x, y) = 0 \rightarrow \quad \therefore \; Y(y) = A_1 \sin \dfrac{n\pi}{b} y \quad i.e) k = \dfrac{n\pi}{b}$

Ex 4-7) the potential distribution enclosed by conducting planes

③ $V(0,y)=V_0$ and $V(a,y)=0,\ k_x = jk \Rightarrow$ not exponentially decay

$i.e)$ both e^{kx} and e^{-kx} should exist

$$\therefore\ X(x)=A_2\sinh kx + B_2\cosh kx\ :\ \text{for } x=a,\ X(x)=0 \ \text{ for all } y$$

$$\therefore\ X(a)=A_2\sinh ka + B_2\cosh ka \qquad \therefore B_2 = -A_2\frac{\sinh ka}{\cosh ka}$$

$$X(x)=A_2[\sinh kx - \frac{\sinh ka}{\cosh ka}\cosh kx]$$

$$=\frac{A_2}{\cosh ka}[\sinh kx\cosh ka - \cosh kx\sinh ka]$$

$$=\frac{A_2}{\cosh ka}\sinh k(x-a) = A_3\sinh k(x-a) \qquad cf)k=\frac{n\pi}{b}$$

$$\therefore V_n(x,y)=C_n^{'}\sinh\frac{n\pi}{b}(x-a)\sin\frac{n\pi}{b}y,\quad \text{for } n=1,2,3,....$$

Ex 4-7) the potential distribution enclosed by conducting planes

④ $V(0, y) = V_0$

$$V_0 = \sum_{n=1}^{\infty} V_n(0, y) = \sum_{n=1}^{\infty} C_n' \sinh\frac{n\pi(-a)}{b} \sin\frac{n\pi}{b} y = -\sum_{n=1}^{\infty} C_n' \sinh\frac{n\pi a}{b} \sin\frac{n\pi}{b} y$$

$$= \sum_{n=1}^{\infty} C_n \sin\frac{n\pi}{b} y, \quad \text{where } C_n = -C_n' \sinh\frac{n\pi}{b} a$$

$$C_n = \begin{cases} \dfrac{4V_0}{n\pi} & , \text{ for } odd\ n \\ 0 & , \text{ for } even\ n \end{cases} \qquad \therefore\ C_n' = \begin{cases} \dfrac{-4V_0}{n\pi \sinh\frac{n\pi}{b} a} & , \text{ for } odd\ n \\ 0 & , \text{ for } even\ n \end{cases}$$

$$\therefore\ V(x, y) = \sum_{n=1}^{\infty} C_n' \sinh\frac{n\pi}{b}(x-a) \sin\frac{n\pi}{b} y$$

$$= \sum_{n=1,\ odd}^{\infty} \frac{4V_0}{n\pi \sinh\frac{n\pi}{b} a} \sinh\frac{n\pi}{b}(a-x) \sin\frac{n\pi}{b} y, \quad \text{for } 0 < x < a,\ 0 < y < b$$

Boundary value problem in cylindrical coordinate

✓ $$\boxed{\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0}$$ → General solution has a form of Bessel function

① for simplicity,

assume the lengthwise dimension ≫ the radius dimension

then $\dfrac{\partial^2}{\partial z^2} = 0 \qquad \therefore \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial V}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 V}{\partial \phi^2} = 0$ let $V(r,\phi) = R(r)\Phi(\phi)$

② then $$\underbrace{\frac{r}{R(r)}\frac{d}{dr}\left(r\frac{dR(r)}{dr}\right)}_{\text{function of } r \text{ only}} + \underbrace{\frac{1}{\Phi(\phi)}\frac{d^2\Phi(\phi)}{d\phi^2}}_{\text{function of } \phi \text{ only}} = 0$$

let $\dfrac{r}{R(r)}\dfrac{d}{dr}\left(r\dfrac{dR(r)}{dr}\right) = k^2$ then $\dfrac{1}{\Phi(\phi)}\dfrac{d^2\Phi(\phi)}{d\phi^2} = -k^2$

(∵ periodic in ϕ for the most of case)

Boundary value problem in cylindrical coordinate

③ $\dfrac{d^2\Phi(\phi)}{d\phi^2} + k^2\Phi(\phi) = 0$, ($\Phi = A_0\phi + B_0$ or $\Phi = C_1 e^{jk\phi} + D_2 e^{-jk\phi}$)

but for circular cylindrical configurations, potential functions and therefore $\Phi(\phi)$ are periodic in ϕ and k is an integer n

$$\therefore \Phi(\phi) = A_\phi \sin n\phi + B_\phi \cos n\phi$$

④ for radial function,

$$r^2 \frac{d^2R(r)}{dr^2} + r\frac{dR(r)}{dr} - n^2R(r) = 0 \rightarrow R(r) = A_r r^n + B_r r^{-n}$$

cf) $\boxed{\text{Cauchy equation(Euler equation) : } x^2y'' + axy' + by = 0}$

then $y = x^m$ form

$$\Rightarrow m^2 + (a-1)m + b = 0 \quad \therefore m = m_1 \text{ or } m_2 \quad \text{then } y = C_1x^{m_1} + C_2x^{m_2}$$

Boundary value problem in cylindrical coordinate

⑤ $V_n(r,\phi) = r^n(A_n \sin n\phi + B_n \cos n\phi) + r^{-n}(A_n^{'} \sin n\phi + B_n^{'} \cos n\phi), \quad n \neq 0$

cf) · if region of interest includes the cylindrical axis where $r = 0$,

the terms containing the r^{-n} factor cannot exist.

· if region include $r = \infty$, the terms r^n cannot exist

· depending on the boundary condition, $V(r,\phi)$ may be is $\sum_n V_n(r,\phi)$

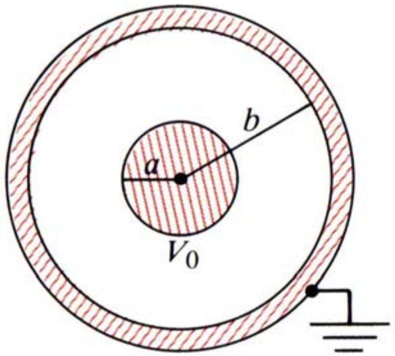
⑥ for $k = 0$, $\dfrac{d^2\Phi(\phi)}{d\phi^2} = 0 \rightarrow \Phi(\phi) = A_0\phi + B_0$

if no variation along ϕ, $\Phi = B_0$ for $k = 0$ $(i.e \;\; A_0 = 0)$

$\dfrac{d}{dr}[r\dfrac{dR(r)}{dr}] = 0, \quad R(r) = C_0 \ln r + D_0$ for $k = 0$

$V(r) = R(r)\Phi(\phi) \qquad \therefore V(r) = C_1 \ln r + C_2 \rightarrow$ independent of z, ϕ

Ex 4-8) a very long coaxial cable



< very long coaxial cable's cross-section >

sol) ① very long in z : no z variation

② by symmetry, no ϕ variation $\rightarrow \boxed{\Phi(\phi)=B_0 \text{ and } k=0}$

③ potential is only a function of $R \rightarrow R(r)=C_0 \ln r + D_0$

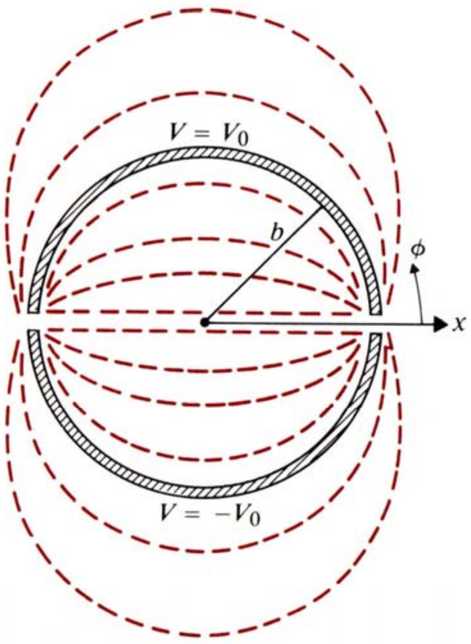
④ boundary condition, $\begin{cases} V(b)=0 \\ V(a)=V_0 \end{cases}$

$$\therefore V(r,\phi)=C_1 \ln r + C_2, \quad \begin{cases} V(a)=C_1 \ln a + C_2 = V_0 \\ V(b)=C_1 \ln b + C_2 = 0 \end{cases}$$

- $C_1 \ln \dfrac{a}{b} = V_0 \qquad \therefore C_1 = \dfrac{V_0}{\ln\left(\dfrac{a}{b}\right)} = -\dfrac{V_0}{\ln\left(\dfrac{b}{a}\right)}$
- $C_2 = -C_1 \ln b = \dfrac{V_0 \ln b}{\ln\left(\dfrac{b}{a}\right)}$

$$\therefore V(r,\phi)=V(r)=-\frac{V_0}{\ln\left(\frac{b}{a}\right)}\ln r + \frac{V_0 \ln b}{\ln\left(\frac{b}{a}\right)} = \frac{V_0}{\ln\left(\frac{b}{a}\right)}\ln\left(\frac{b}{r}\right)$$

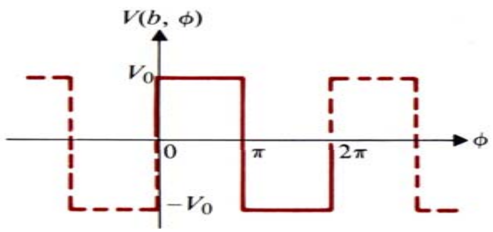
Ex 4-9) Infinitely long, thin, conducting circular tube.



sol) $\frac{d}{dz} = 0$

$$V(b,\phi) = V_0 \;(\text{for } 0 \le \phi \le \pi)$$
$$= -V_0 \;(\text{for } \pi \le \phi \le 2\pi)$$

i.e.



a) Inside tube

① $r < b \rightarrow R(r) = A_r r^n$ $\because$ $r = 0$ is included.

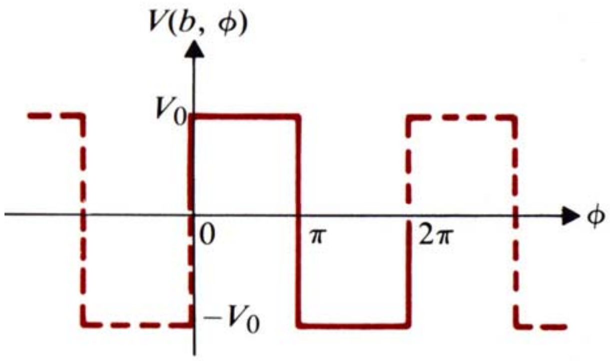
② $\Phi(\phi) = A_\phi \sin n\phi$ $\because$ odd function in ϕ

$$\therefore V_n(r,\phi) = A_n r^n \sin n\phi$$

③ In order to satisfy the periodic boundary condition,

$$V(r,\phi) = \sum_{n=1}^{\infty} A_n r^n \sin n\phi$$

Boundary-Value Problems in Cylindrical Coordinates



i.e. at $r = b$

rectangular periodic wave can be represented by Fourier Sine Series

$$\sum_{n=1}^{\infty} A_n b^n \sin n\phi = \begin{cases} V_0\,, & \text{for } 0<\phi<\pi \\ -V_0\,, & \text{for } \pi<\phi<2\pi \end{cases}$$

$f(x) = -f(-x)$: odd function and period L(Non-radian)

$$\therefore A_n b^n = \frac{2}{L}\int_0^L f(x)\sin\left(n\frac{2\pi x}{L}\right)dx$$

$$= \frac{2}{\pi}\int_0^{\pi} V_0 \sin n\phi\, d\phi = \frac{2V_0}{\pi}\left[-\frac{\cos n\phi}{n}\right]_0^{\pi}$$

$$= \frac{2V_0}{n\pi}\left[1-\cos n\pi\right]$$

$$\therefore A_n b^n = \begin{cases} \dfrac{4V_0}{n\pi} & \text{, for odd n} \\ 0 & \text{, for even n} \end{cases} \qquad \therefore A_n = \begin{cases} \dfrac{4V_0}{n\pi b^n} & \text{, for odd n} \\ 0 & \text{, for even n} \end{cases}$$

Boundary-Value Problems in Cylindrical Coordinates

i.e. at $r = b$

rectangular periodic wave is represented by Fourier Sine Series

$$\sum_{n=1}^{\infty} A_n b^n \sin n\phi = \begin{cases} V_0 & \text{for } 0<\phi<\pi \\ -V_0 & \text{for } \pi<\phi<2\pi \end{cases}$$

$$\int_0^{\pi} \sum_{n=1}^{\infty} A_n b^n \sin n\phi \sin m\phi \; d\phi = \int_0^{\pi} V_0 \sin n\phi \, d\phi$$

$$= \sum_{n=1}^{\infty} \int_0^{\pi} A_n b^n \sin n\phi \sin m\phi \, d\phi = \frac{V_0}{n}(-\cos n\phi)\Big|_0^{\pi}$$

$$= \sum_{n=1}^{\infty} \int_0^{\pi} A_n b^n \cdot \frac{1}{2}\Big[\overset{\neq 0 \text{ for } n=m}{\cos(n-m)\pi} - \overset{=0}{\cos(n+m)\pi}\Big] d\phi \;, \quad \frac{2V_0}{n} \; if \;\; n = odd, \; 0 \;\; if \;\; n = even$$

$$= A_n b^n \frac{1}{2} \cdot \pi = A_n \frac{\pi b^n}{2}, \qquad \therefore A_n \cdot \frac{\pi b^n}{2} = \frac{2V_0}{n}$$

$$\therefore \begin{cases} A_n = \dfrac{4V_0}{n\pi b^n} & , \; for \;\; odd \; n \\ A_n = 0 & , \; for \;\; even \, n \end{cases}$$

Boundary-Value Problems in Cylindrical Coordinates

$$\therefore\ V(r,\phi) = \frac{4V_0}{\pi} \sum_{n=odd}^{\infty} \frac{1}{n} \left(\frac{r}{b} \right)^n \sin n\phi, \quad for\ \ r < b$$

b) out-side the tube $r > b$

$$\therefore\ R(r) = B_n r^{-n}$$

$$\therefore\ V(r,\phi) = \sum_{n=1}^{\infty} B_n^{'} r^{-n} \sin n\phi$$

$$at\ \ r = b\ ,\ \ V(b,\phi) = \sum_{n=1}^{\infty} B_n^{'} b^{-n} \sin n\phi = \begin{cases} V_0 & \text{for } 0<\phi<\pi \\ -V_0 & \text{for } \pi<\phi<2\pi \end{cases}$$

Following the same method to find coefficients of Fourier series.

$$B_n^{'} = \begin{cases} \dfrac{4V_0 b^n}{n\pi} & \text{, for odd n} \\ 0 & \text{, for even n} \end{cases}$$

$$\therefore\ V(r,\phi) = \frac{4V_0}{\pi} \sum_{n=odd}^{\infty} \frac{1}{n} \left(\frac{b}{r} \right)^n \sin n\phi, \quad r > b$$

Boundary-Value Problem in spherical Coordinates

✓ $$\nabla^2 V(r,\theta,\phi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = 0$$

Assuming symmetry in ϕ, then $\frac{\partial}{\partial\phi} = 0$

$$\therefore \nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) = 0$$

➢ let $V(r,\theta) = R(r)\Theta(\theta)$

then, $$\frac{1}{R(r)}\frac{d}{dr}\left[r^2\frac{dR}{dr}\right] + \frac{1}{\Theta(\theta)\sin\theta}\frac{d}{d\theta}\left[\sin\theta\frac{d\Theta(\theta)}{d\theta}\right] = 0$$

let $\frac{1}{R(r)}\frac{1}{dr}\left[r^2\frac{dR}{dr}\right] = k^2$: separation constant

$$\rightarrow r^2\frac{d^2R}{dr^2} + 2r\frac{dR}{dr} - k^2 R = 0 \Rightarrow$$ Cauchy's or Euler's equation

Boundary-Value Problem in spherical Coordinates

then $\dfrac{1}{\Theta(\theta)\sin\theta}\dfrac{d}{d\theta}\left[\sin\theta\dfrac{d\Theta(\theta)}{d\theta}\right]=-k^2$

cf) assume $R=r^\alpha,\ \alpha^2+\alpha-k^2=0$, let $k=n(n+1)$,

then $\alpha=n\ or\ -(n+1)$

General solution of $R(r)$ will be as follows

$$R(r)=A_n r^n+B_n r^{-(n+1)}$$

where $n(n+1)=k^2\ and\ n=0,1,2,\cdots$ positive integer

cf) $r^2\dfrac{d^2R}{dr^2}+r\dfrac{dR}{dr}-k^2R=0$, then, $R(r)=A_n r^n+B_n r^{-n}\rightarrow$ Cauchy's or Euler's equation

Boundary-Value Problem in spherical Coordinates

n	$P_n(\cos\theta)$
0	1
1	$\cos\theta$
2	$\frac{1}{2}(3\cos^2\theta-1)$
3	$\frac{1}{2}(5\cos^3\theta-3\cos\theta)$

cf) Legendre's D.E.

$$\left(1-x^2\right)y''-2xy'+n(n+1)y=0$$

$$\therefore \frac{d}{d\theta}\left[\sin\theta\frac{d\Theta(\theta)}{d\theta}\right]+n(n+1)\Theta(\theta)\sin\theta=0$$

$\rightarrow$ Turns out to be a Legendre's equation form

c.f.) let $\cos\theta=x$, then $\frac{d}{d\theta}=-\sin\theta\frac{d}{dw}$

Solution of Legendre's equation : Legendre's functions $P(\cos\theta)$

When n is integer $\Theta_n(\theta)=P_n(\cos\theta)\rightarrow Legendre\ Polynomials.$

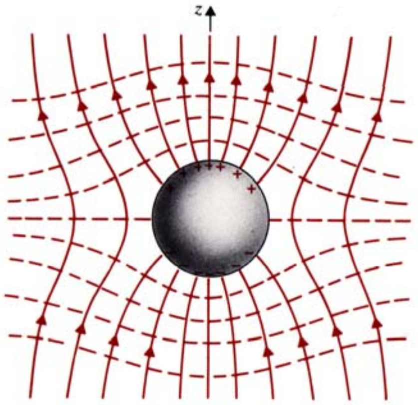
$$\therefore V_n(r,\theta)=\left[A_nr^n+B_nr^{-(n+1)}\right]P_n(\cos\theta)$$

Ex 4-10) Conducting sphere in a uniform electric field

❖ An uncharged conducting sphere of radius b is placed in an initially uniform electric field $\overrightarrow{E_0} = \hat{z}E_0$. Find $V(r,\theta)$ and $\vec{E}(r,\theta)$?

sol) (a) ① Surface of the sphere is maintained equipotential

$\Rightarrow$ i.e. Separation of charges and redistribution take place.



② $\vec{E}$ inside the sphere is zero.

③ Outside the sphere ;

the field line on the suface is normal to the surface

④ The field intensity at points for away from the sphere

$\Rightarrow$ No change.

⑤ Potential is independent of ϕ

Boundary-Value Problem in spherical Coordinates

$V(r,\theta)$ for $r \geq b$

B.C.

$V(b,\theta)=0$ (i.e. Assuming $V=0$(may not be 0) in the equipotential plane)

$V(r,\theta)=-E_0 z=-E_0 r\cos\theta$ for $r \gg b$

(E_0 is not distributed at points for away from the sphere.)

⑥ $V(r,\theta)=\sum_{n=0}^{\infty}\left[A_n r^n + B_n r^{-(n+1)}\right]P_n(\cos\theta)$

$r \gg b,\ V(r,\theta)=-E_0 r\cos\theta$

$\therefore A_1=-E_0$ and all other A_n's are zero.

$$\therefore V(r,\theta)=-E_0 r P_1(\cos\theta)+\sum_{n=0}^{\infty}B_n r^{-(n+1)}P_n(\cos\theta)$$

$$= B_0 r^{-1} + \left(B_1 r^{-2}-E_0 r\right)\cos\theta+\sum_{n=2}^{\infty}B_n r^{-(n+1)}P_n(\cos\theta)$$

form of $\dfrac{Q}{4\pi\varepsilon_0 r}$ $\Rightarrow$ Charged sphere form. But we have uncharged sphere $\therefore B_0=0.$

Boundary-Value Problem in spherical Coordinates

$$V(r,\theta)=\left(\frac{B_1}{r^2}-E_0 r\right)\cos\theta+\sum_{n=2}^{\infty}B_n r^{-(n+1)}P_n(\cos\theta)$$ and at $r=b,\ V(b,\theta)=0$

$$\therefore 0=\left(\frac{B_1}{b^2}-E_0 b\right)\cos\theta+\sum_{n=2}^{\infty}B_n b^{-(n+1)}P_n(\cos\theta)$$

$$\therefore B_1=E_0 b^3$$

$$B_n=0,\quad n\geq 2$$

$$\therefore V(r,\theta)=-E_0\left[1-\left(\frac{b}{r}\right)^3\right]r\cos\theta,\quad r\geq b$$

(b) $\vec{E}=-\nabla V=-\frac{\partial V}{\partial r}\hat{r}-\frac{\partial V}{r\partial\theta}\hat{\theta}$

$$\therefore E_r=-\frac{\partial V}{\partial r}=E_0\left[1+2\left(\frac{b}{r}\right)^3\right]\cos\theta,\quad r\geq b$$

$$\therefore E_\theta=-\frac{\partial V}{r\partial\theta}=-E_0\left[1-\left(\frac{b}{r}\right)^3\right]\sin\theta,\quad r\geq b$$

$$\rho_S(\theta)=\varepsilon_0 E_r\big|_{r=b}=3\varepsilon_0 E_0\cos\theta$$