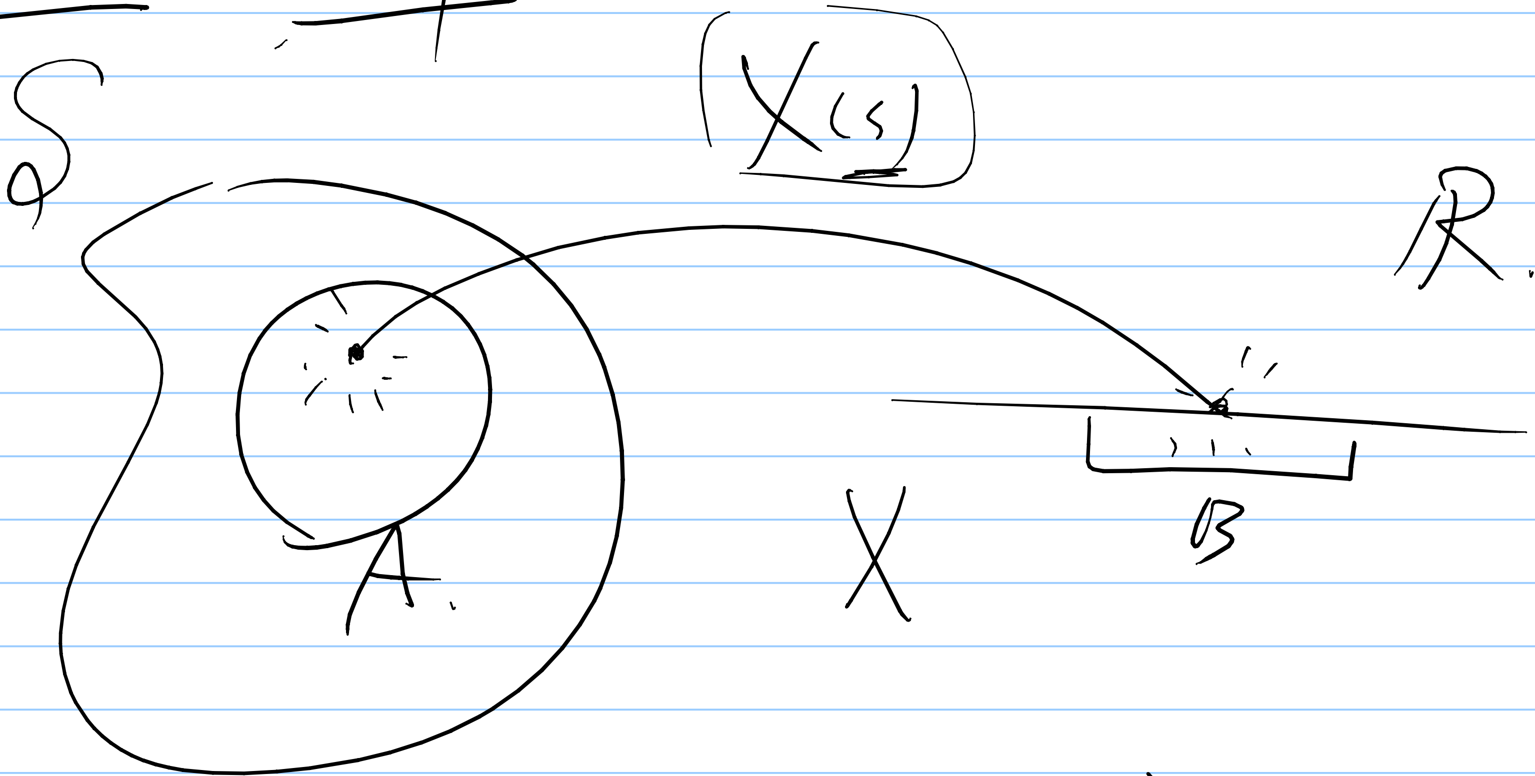
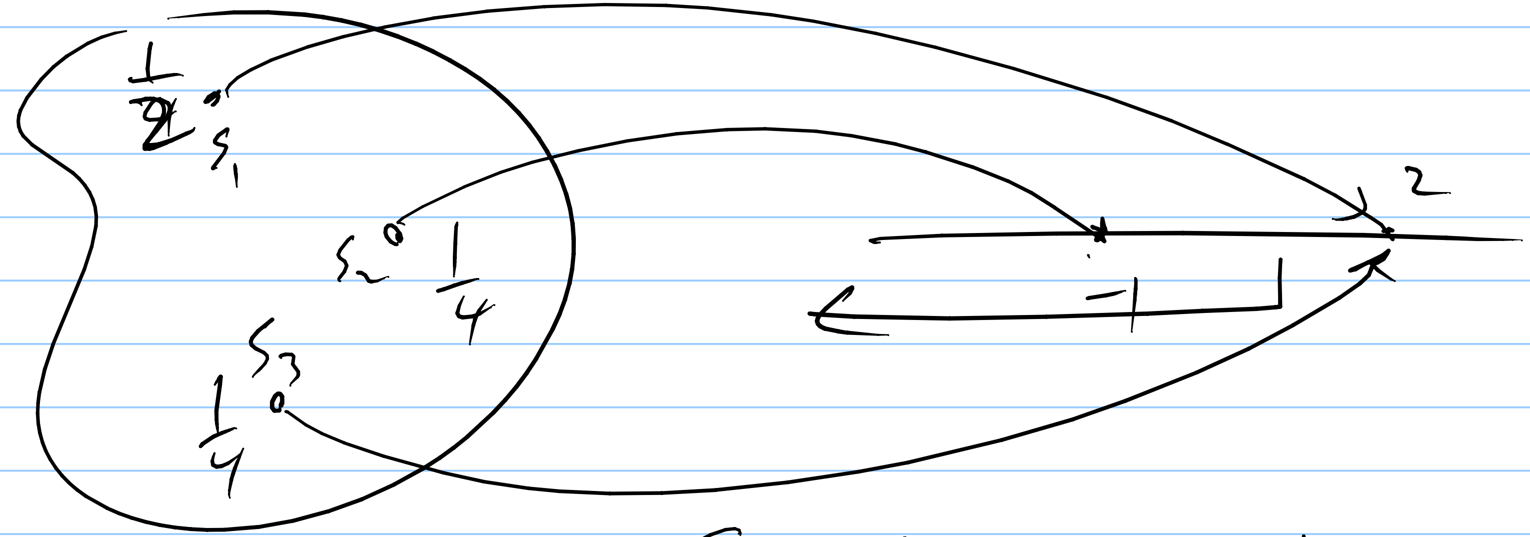
202 + 304



$P(X \in B)$ $P(X \leq 3.5)$

real number

$$P(\underbrace{\{s \mid X(s) \in B\}}_{X^{-1}(B)}) \qquad B = (-\infty, 3.5]$$



$$X \in \overbrace{\{-1, 2\}}^{S_X} \quad \text{Alphabet of } X$$

$$P(X \leq 1) = P(X^{-1}(B)) = P(\{s_2\}) = \frac{1}{4}$$

$$= P(X \in B) \qquad B = (-\infty, 1]$$

A disc set = finite elts in a finite interval

S_X is disc $\Rightarrow$ X is a discrete rv.

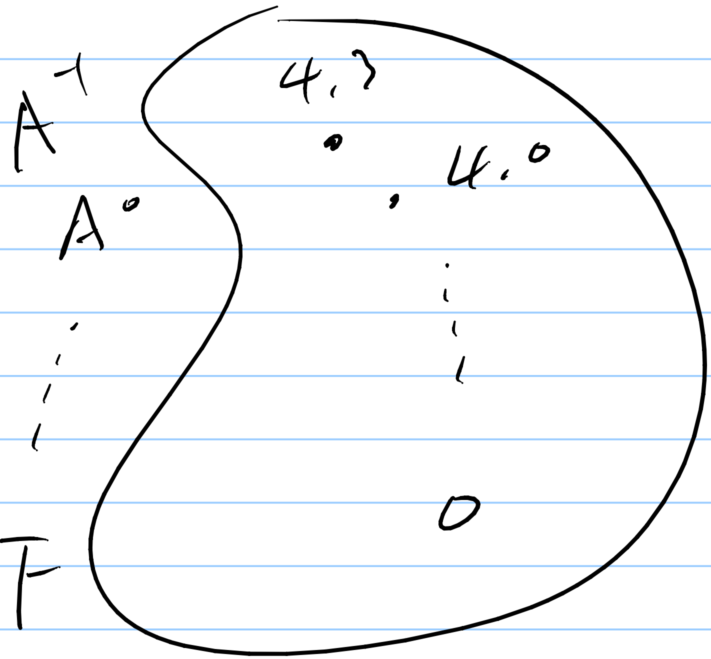
X is not disc $\Leftarrow$ X is a continuous rv
$\not\Rightarrow$

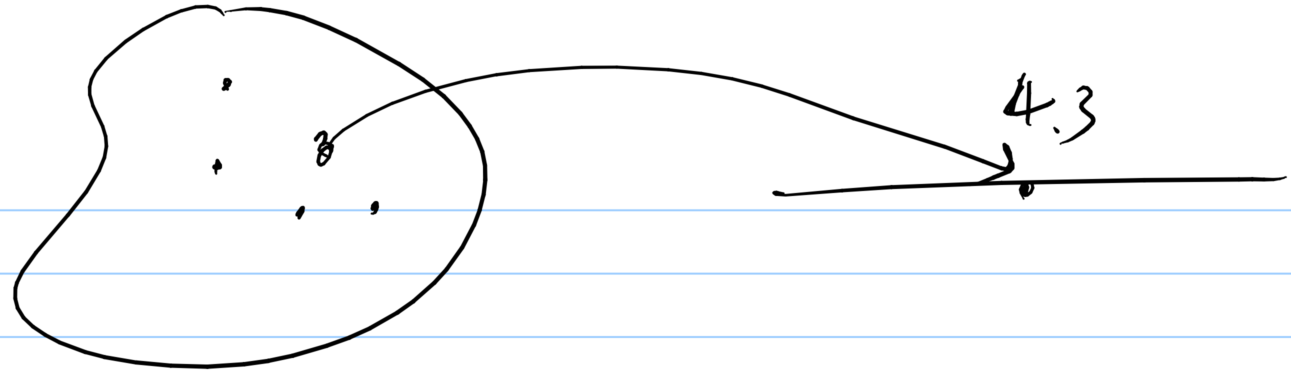
X disc rv if it has a <u>prob mass fcn</u>

pmf

X cts rv " " " <u>prob density fn</u>

pdf





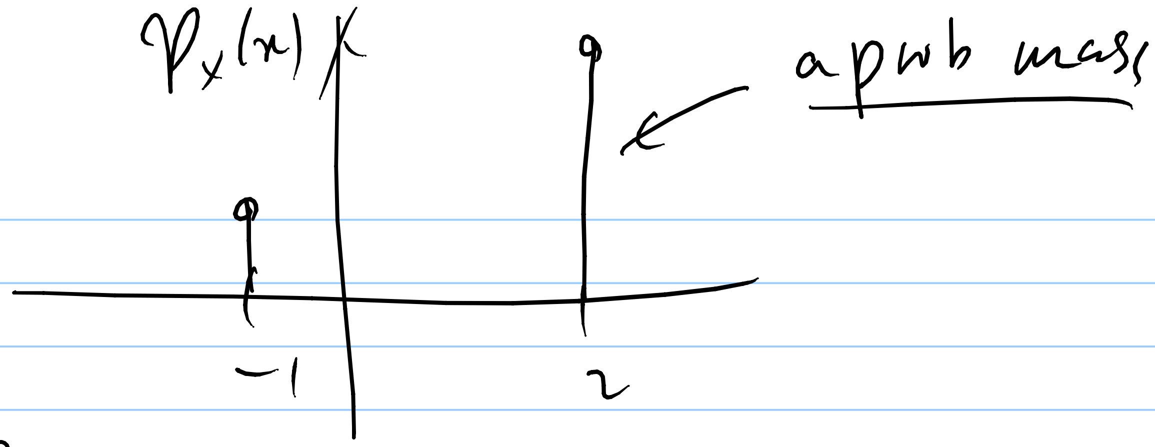
prob mass fn. (disc rv)

$$p_X(x) \triangleq P(X=x) = P\left(\{\underbrace{s \mid X(s)=x}_{X^{-1}(B)}\}\right)$$

rv.

$B = \{x\}$

$$p_X(x) = \begin{cases} \frac{1}{4}, & x = \cancel{0} -1 \\ \frac{3}{4}, & x = 2 \\ 0, & \text{else} \end{cases}$$



Prop: (a) $0 \le p_X(x) \le 1$.

(b) $\sum_{x} p_X(x) = 1$

(c) $\underline{B} \subset \mathbb{R}$ Not necessarily $B \subset S_X$

$$P(\underline{X} \in B) = \sum_{x \in B} p_X(x)$$

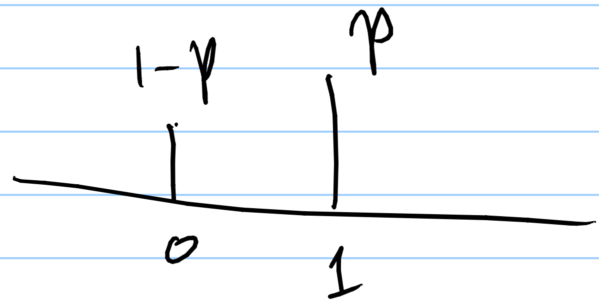
$\cancel{P(\underline{X} \in B)}$

$$P(X \le 1) = \frac{1}{4} \quad \left(= \sum_{x \le 1} p_X(x) = p_X(-1) = \frac{1}{4} \right)$$

Families of disc rv's

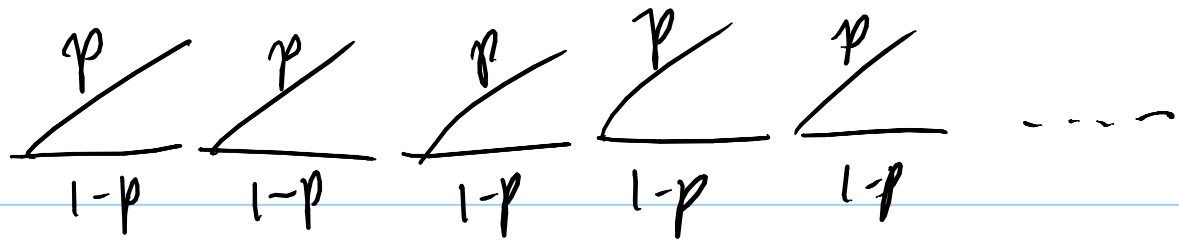
① Bernoulli rv. Ber(p) $\quad 0 \le p < 1.$

$$p_X(x) = \begin{cases} 1-p, & x=0 \\ p, & x=1 \\ 0, & \text{else} \end{cases}$$



Bernoulli trials

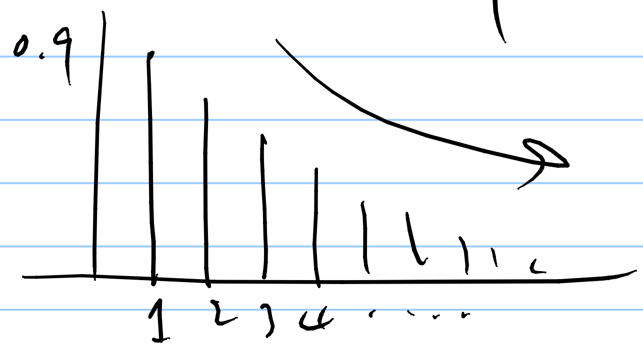
Repeat Ber trials

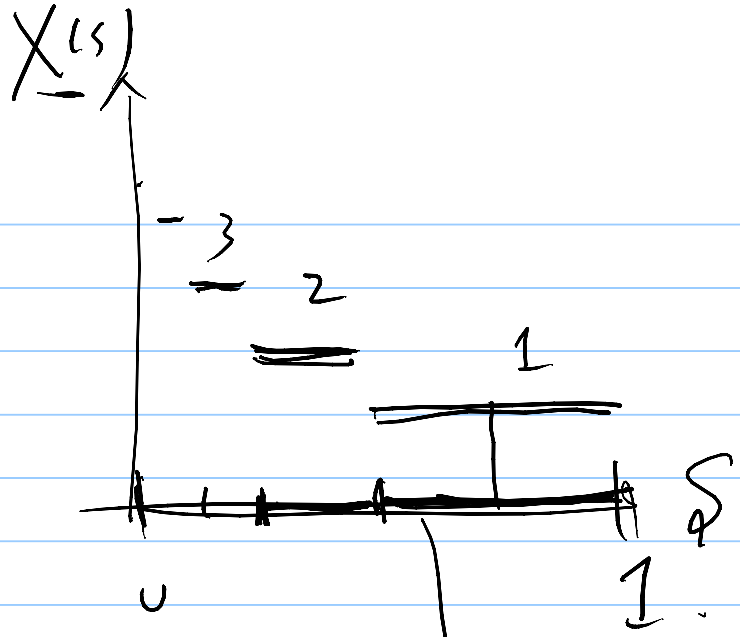


$p(1-p)^2 \sim X = 3$

n Geometric rv $\quad Geo(p) \quad 0 < p < 1$

$$p_X(x) = \begin{cases} p(1-p)^{x-1}, & x = 1, 2, \dots \\ 0, & \text{else} \end{cases}$$





$X : S \to \mathbb{R}$ $\boxed{1}$

000 . . . .

001 . . . .

$\boxed{1}$00 $\boxed{01}$

$\boxed{1}$10 . . .

1001 1 0 . . . . .

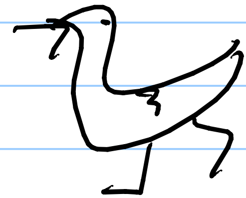
01 . . . .

n Ber trials, k successes
(p)

$$P(k \text{ successes}) = \binom{n}{k} p^k (1-p)^{n-k}$$

$n=5, \quad k=2.$

$$\binom{5}{2} \left\{ \begin{array}{l} 00011 \text{ — } p^2(1-p)^3 \\ 00101 \\ \vdots \\ 11000 \end{array} \right.$$



$$p_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 1, 2, \cdots, n \\ 0, & \text{else} \end{cases}$$

8 Binomial r.v. $\quad Bin(n, p) \quad 0 < p < 1$

$n \geq 1$

$\mathrm{Bin}(10, \tfrac{1}{2})$

$$P(4 \le \#\text{heads} \le 8)$$

$$= \binom{10}{4}\underbrace{\left(\tfrac{1}{2}\right)^4\left(\tfrac{1}{2}\right)^6}_{\left(\frac{1}{2}\right)^{10}} + \binom{10}{5}2^{-10} + \binom{10}{6}2^{-10} + \binom{10}{7}2^{-10} + \binom{10}{8}2^{-10}$$

$$=$$

Time until k successes (passes / fails / ---)
in repeated Ber trials

$$P(T=n) = P\left(\text{$k-1$ successes in $n-1$ trials, and success in nth} \right)$$

$$\binom{n-1}{k-1} p^{k-1} (1-p)^{n-k} \cdot p$$

@ Pascal rv (k, p), $0 < p < 1$,

$k = 1, 2, 3, \ldots$

$$p_X(x) = \begin{cases} \binom{x-1}{k-1} p^k (1-p)^{x-k}, & x = 1, 2, 3, \ldots \ (x = k, k+1, \ldots) \\ 0, & \text{else} \end{cases}$$

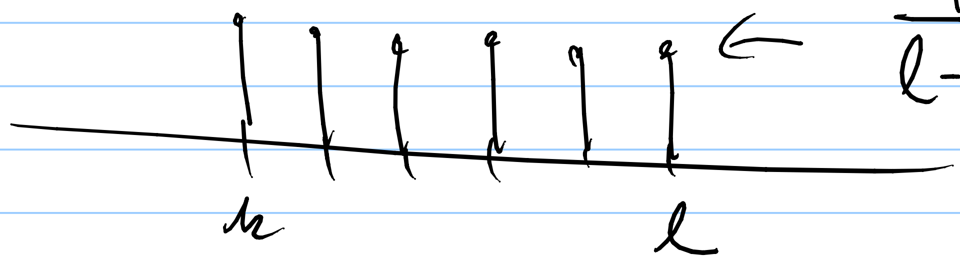
$X = 5, \ k = 3$

$$\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{array}$$

Negative Binomial r.v.

② Discrete Uniform r.v. $\quad \text{Unif}(k, l)$

$k \le l$



$\dfrac{1}{l-k+1}$

$$p_X(x) = \begin{cases} \dfrac{1}{l-k+1}, & x = k, k+1, \dots, l \\ 0, & \text{else} \end{cases}$$

$0, \underbrace{231\,0\,0 \dots\dots}_{132}$