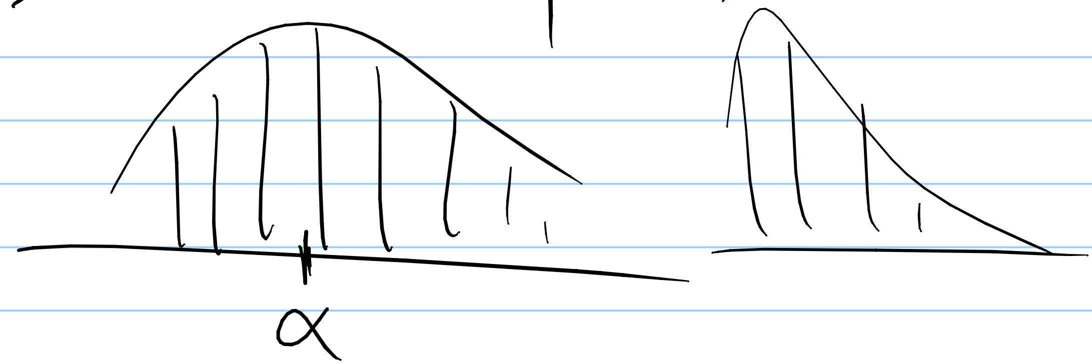
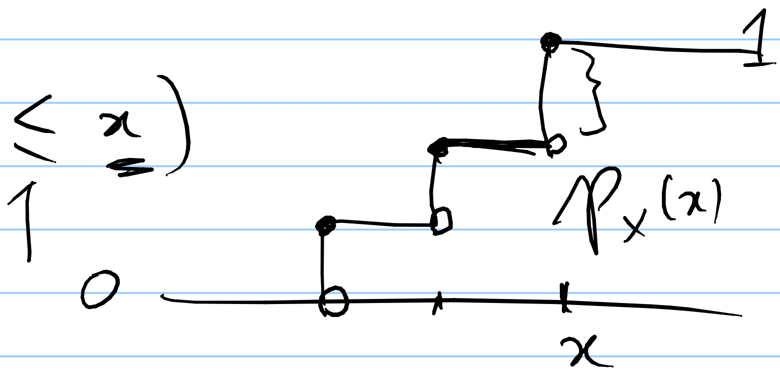
Poisson r.v. $p_X(x) = \begin{cases} \dfrac{\alpha^x}{x!} e^{-\alpha}, & x=0,1,\dots \\ 0, & \text{else} \end{cases}$

$\alpha > 0$

α



cdf $F_X(x) = P(X \leq x)$

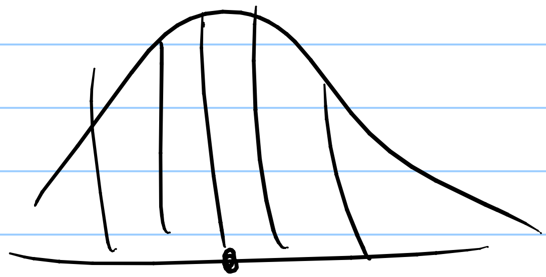


$a \le b \qquad P(a < X \le b) = F_X(b) - F_X(a)$

Averages   mean $\rightarrow$ EX

mode $\rightarrow$ x when $p_X(x)$ is the largest

median $\rightarrow$ x when



$$\sum_{u \le x} p_X(u) = \sum_{v > x} p_X(v)$$

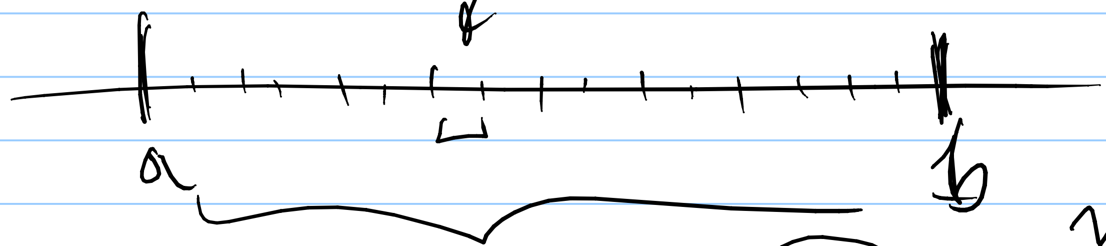
$$EX = \mu_X = \sum_{x \in S_X} x \, p_X(x) = \sum_{x \in S_X} x \frac{N_x}{n}$$

$\frac{1}{n}\sum x_i$

$\boxed{EX = \text{mean}}$ $\quad EX^2 \quad E(X-a)^3$

Poiss $\qquad$ Bin $\quad K_n$

0, 1

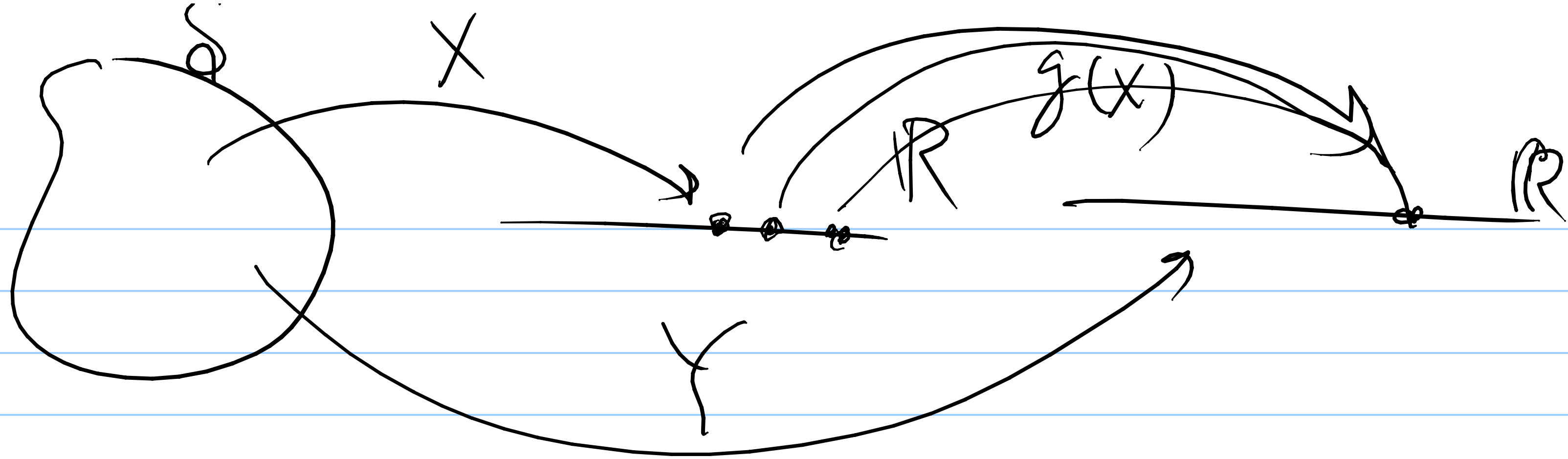


n

$p = \frac{\alpha}{n}$

$p_{K_n}(x) \Longrightarrow p_X(x)$

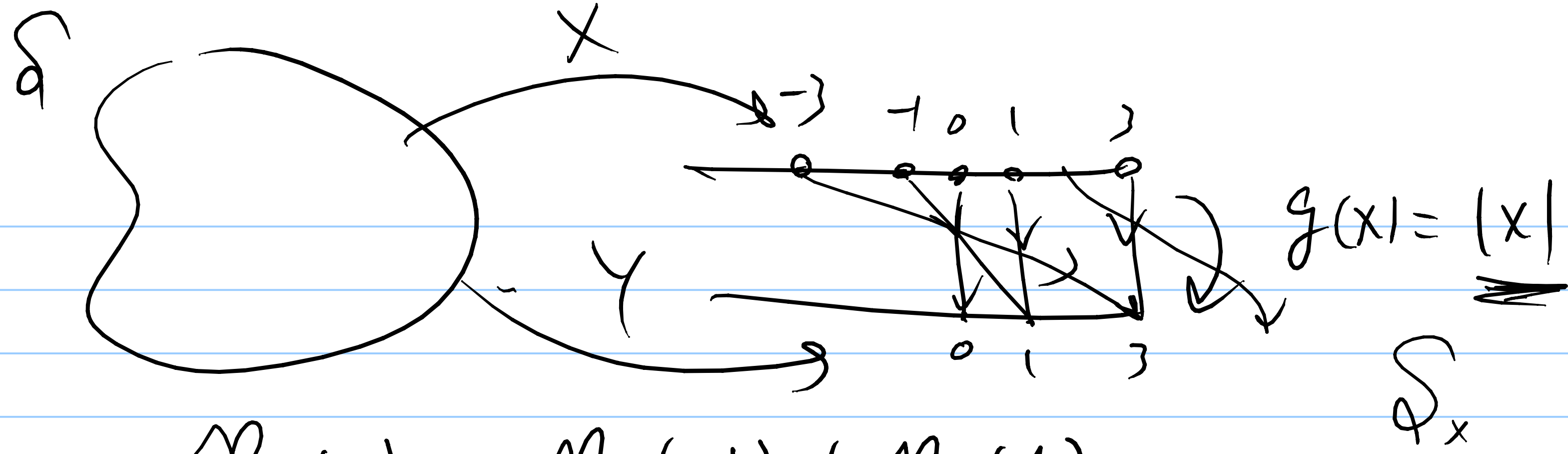
$\leftarrow$ Poisson (α)



$$\underline{\underline{Y}}(s) = g(X(s)) \qquad p_Y(y) \qquad p_X(x)$$

Thm 2.9 $\quad Y = g(X)$

$$\boxed{p_Y(\underline{y}) = \sum_{\underbrace{\{x \,:\, g(x) = y\}}} p_X(x)} \quad \{x : g(x) = y\} = \underbrace{g^{-1}(\{y\})}$$

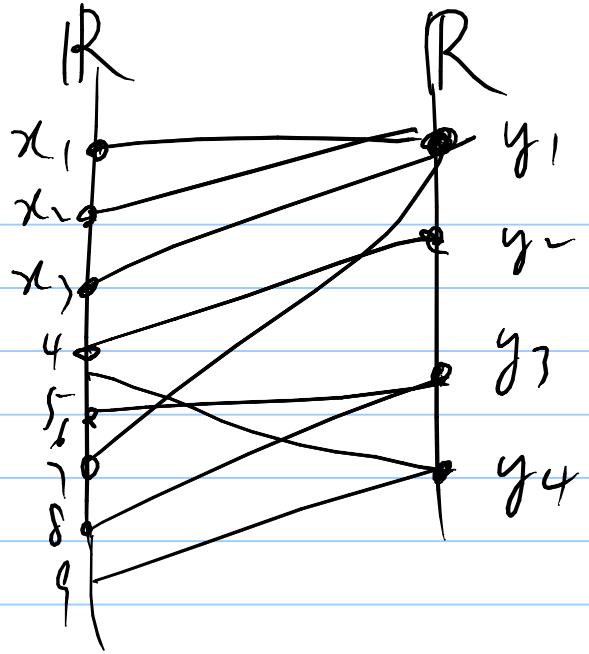


$$p_Y(1) = p_X(-1) + p_X(1)$$

$$P(Y \in B) = P(X \in g^{-1}(B)) = P(X^{-1}(g^{-1}(B)))$$

$$\boxed{EY} = \sum_{y \in S_Y} y\, p_Y(y) = \sum_{y \in S_Y} y \sum_{x \in g^{-1}(\{y\})} p_X(x)$$

$$= \sum_{y \in S_Y} \sum_{x \in g^{-1}(\{y\})} g(x)\, p_X(x) = \sum_{x \in S_X} \boxed{g(x)}\, p_X(x)$$



$$g^{-1}(\{y_1\}) = \{x_1, x_2, x_3, x_7\}$$

$$g^{-1}(\{y_2\}) = \{x_4\}$$

$$g^{-1}(\{y_3\}) = \{x_6, x_8\}$$

$$g^{-1}(\{y_4\}) = \{x_5, x_9\}$$

$$E(\underbrace{aX+b}_{Y}) = \sum (ax+b)\, p_X(x)$$

$$E(e^X) = \sum e^x p_X(x)$$

Thm 2.12 $\quad E(aX+b) = \sum_x (ax+b)\, p_X(x)$

$$= a\left(\underbrace{\sum_x x\, p_X(x)}_{EX}\right) + b\left(\underbrace{\sum_x p_X(x)}_{=1}\right)$$

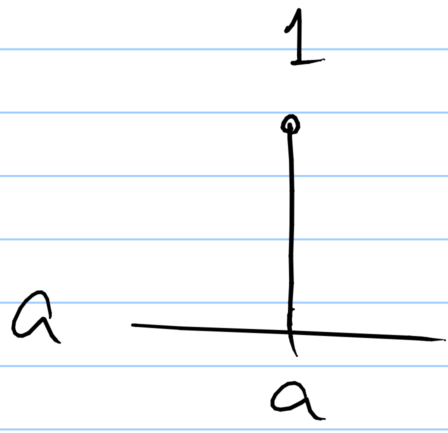
$$= \boxed{aEX + b}$$ Linear operator.

$$Q(ax_1 + bx_2) = aQ(x_1) + bQ(x_2)$$

$Y = aX + \boxed{b}$ Affine

$Y = aX$ Linear.

$$E(X - \mu_X) = EX - E\mu_X$$



$$= EX - \mu_X = 0 \qquad \mu_X \cdot 1$$

$$Ea = a$$

$$EX = \mu_X \qquad E\underbrace{EX}_{\mu_X} = \mu_X \qquad EEE\cdots EX = \mu_X$$

$$E\left(g_1(X) + g_2(X) + \cdots\right) = Eg_1(X) + Eg_2(X) + \cdots$$

$E\boxed{X^n}$: nth moment

$E(X-\mu_X)^n$: nth central moment

$\underline{E(X-\mu_X)^2}$: Variance $\mathrm{Var}(X)$

$$= E\left(X^2 - 2\mu_X X + \mu_X^2\right)$$

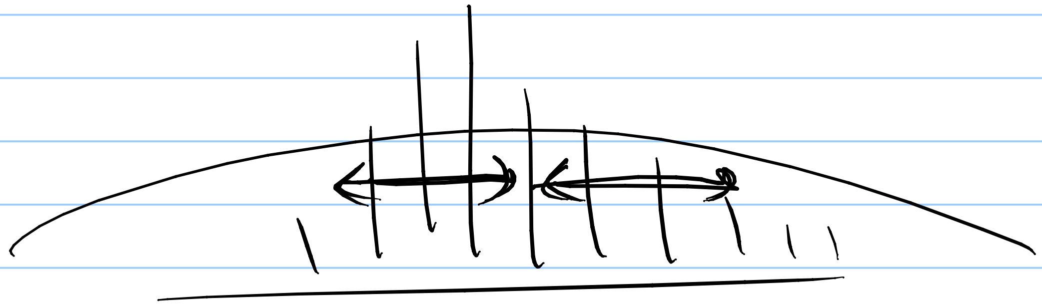
$$= EX^2 - 2\mu_x \underbrace{EX}_{\mu_x} + \mu_x^2 = \boxed{EX^2} - \mu_x^2$$

$$(X - \mu_x)^2 \geq 0$$

$$Y \geq 0 \qquad P(Y \geq 0) = 1$$

$$P(Y < 0) = 0$$

$$Var(X) \geq 0 \qquad EX^2 \geq \mu_x^2 \qquad EX^2 \geq Var(X)$$



$\uparrow \mu_x$

$$\mathrm{Var}(X) = E(X-\mu_X)^2 = \sigma_X^2$$

σ_X is std deviation.

$$g(a) = E(X-a)^2 \text{ smallest?}$$

$$= a^2 - 2\underbrace{(EX)}_{\mu_X} a + EX^2$$

$$= (a-\mu_X)^2 + \boxed{EX^2 - \mu_X^2}$$

$a = \mu_X$ minimizes $g(a)$

with minimum $\mathrm{Var}(X)$

μ_X is min. mean squared error estimation of X

$$\mathrm{Geo}(p) \quad X \quad EX = \frac{1}{p}$$

$$\mathrm{Var}(X) = EX^2 - \frac{1}{p^2}$$

$$EX^2 = \sum_{x=1}^{\infty} x^2 p(\underbrace{1-p}_{q})^{x-1} = p \sum_{x=1}^{\infty} x^2 q^{x-1}$$

$$= p \sum_{x=1}^{\infty} \frac{d}{dq}\left(x q^x\right) = p \frac{d}{dq}\left(\underbrace{\sum_{x=1}^{\infty} x q^x}_{\frac{q}{(1-q)^2}}\right)$$

$$= p \cdot \frac{1+q}{(1-q)^3} = \frac{2-p}{p^2}$$

$$\mathrm{Var}(X) = \frac{2-p}{p^2} - \frac{1}{p^2} = \frac{1-p}{p^2}$$