노트 제목 2008-04-01

$$Y = g(X)$$

$$p_Y(y) = \sum_{\{x : g(x) = y\}} p_X(x)$$

$$g^{-1}(\{y\})$$

$$EY = \sum_y y\, p_Y(y) = \sum_x g(x)\, p_X(x)$$

$$E(aX+b) = aEX+b$$

$$E(a_1X+a_2X^2+\cdots+b)$$

$$= a_1EX+a_2EX^2+\cdots+b$$

$$E(X-\mu_x) = EX-\mu_x = 0$$

$$E\underbrace{(X-\mu_x)}^2 = \sigma_x^2 \quad \sigma_x: \text{std dev.}$$

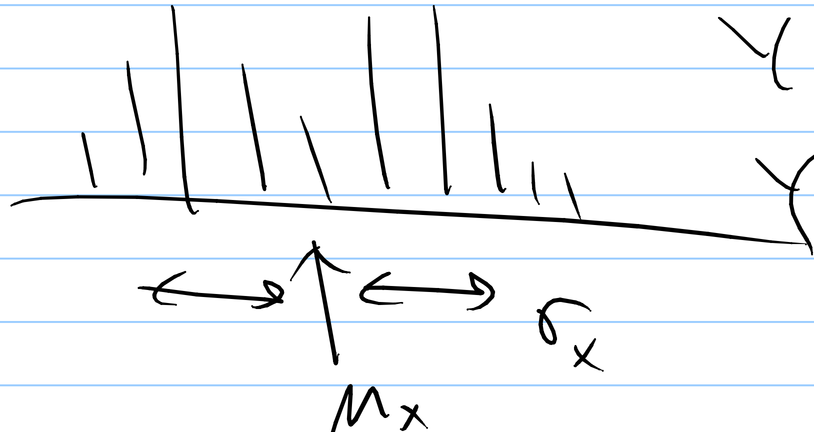
$$= \underbrace{EX^2-\mu_x^2}$$

$E\underbrace{(X-\underline{a})}^2$ is min when $a=\mu_x$

min mse estimate

$EX^n$ : nth moment

$E(X-\mu_x)^n$ : nth central moment

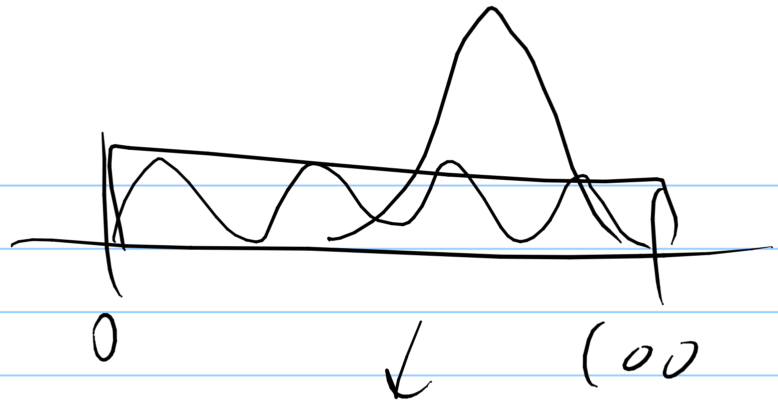



$Y = X + \underline{a}$

$Y = aX$

$$\mathrm{Var}(X) = \frac{1-p}{p^2}$$

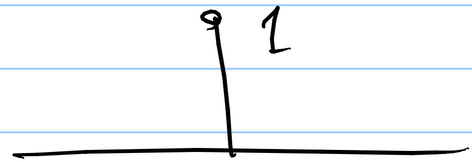
geo(p)




$$\mathrm{Var}(\underbrace{aX+b}_{Y}) = E\Big(aX+\cancel{b} - \underbrace{E(aX+b)}_{a\mu_X+\cancel{b}}\Big)^2$$

$$= E\big(a(X-\mu_X)\big)^2 = a^2\,\mathrm{Var}(X)$$

$a = 0 \quad \mathrm{Var}(b) = 0$




$a = 1 \quad \mathrm{Var}(X+b) = \mathrm{Var}(X) \geq 0$

$$|EX| = \left|\sum x\, p_X(x)\right| \le \sum |x\, p_X(x)|$$

$\Delta$ ineq

$$= \sum |x|\, p_X(x) = E|X|$$

$$|EX| \le E|X|$$

Triangular inequality

Conditional pmf : cpmf

Event $B$

$$p_{X|B}(x) = P(X = x \mid B)$$

$$= \frac{P(\{X = x\} \cap B)}{P(B)}$$

$$X^{-1}(\{x\}) = \{s \mid X(s) = x\}$$

$$p_X(x) = \sum_{i=1}^{m} p_{X|B_i}(x)\, P(B_i)$$

Total prob law

$$S = \bigcup_{i=1}^{m} B_i$$

$$B_i \cap B_j \neq \phi$$

$$\left( P(A) = \sum \underline{P(\underline{A} \mid B_i)}\, \underline{P(B_i)} \right)$$

Example: $P(H) = 2/5$

$$p_{X|H}(x) = \begin{cases} \frac{1}{10}\left(\frac{9}{10}\right)^{x-1}, & x = 1, 2, \ldots \\ 0, & \text{else} \end{cases} \leftarrow 10 \text{ yrs}$$

$$p_{X|R}(x) = \begin{cases} \frac{1}{20}\left(\frac{19}{20}\right)^{x-1}, & x = 1, 2, \ldots \\ 0, & \text{else} \end{cases} \leftarrow 20 \text{ yrs}$$

$$p_X(x) = \frac{1}{10}\left(\frac{9}{10}\right)^{x-1}\frac{2}{5} + \frac{1}{20}\left(\frac{19}{20}\right)^{x-1}\frac{3}{5}$$

$B$ : $\{X \in B\}$

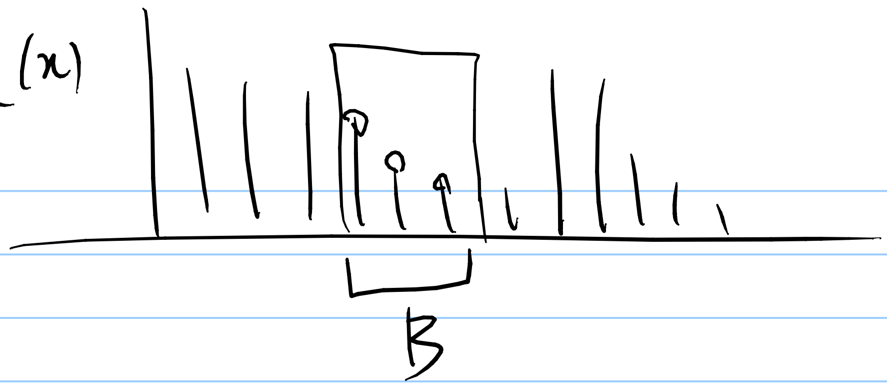
set of real numbers

$$p_{X|\{X\in B\}}(x) = P(X=x \mid X\in B)$$

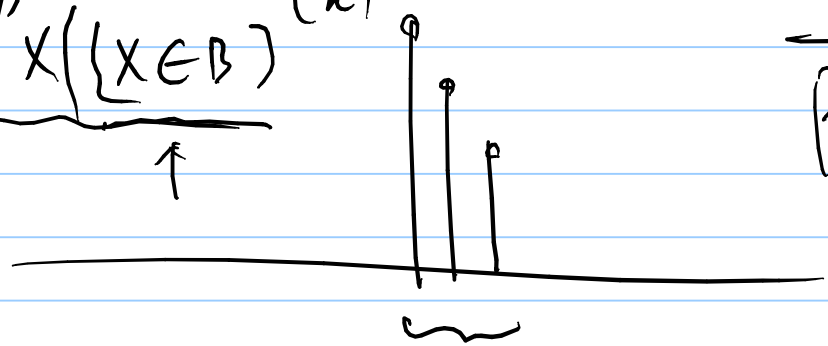
$$= \frac{P(X=x,\ X\in B)}{P(X\in B)}$$

$$= \begin{cases} \dfrac{P(X=x)}{P(X\in B)} & x\in B \\ 0, & \text{else} \end{cases}$$

$p_X(x)$



$p_{X|\{X\in B\}}(x)$



$$\frac{1}{P(X\in B)}$$

If $B$ is fixed $p_{X|B}(x)$ can be treated as a pmf.

(a) $p_{X|B}(x) \geq 0$

(b) $\sum_{x \in \mathbb{R}} p_{X|B}(x) = 1$,

(c) $\sum_{x \in C} p_{X|B}(x) = P(X \in C \mid B)$

Condit expectation

$$E(X|B) = \mu_{X|B} = \sum_x x\, p_{X|B}(x)$$

$B_i$, $i = 1, \ldots, n$ partition

$$EX = \sum x\, p_X(x) = \sum_x x \left( \sum_i p_{X|B_i}(x) P(B_i) \right)$$

$$= \sum_i P(B_i)\Big(\underbrace{\sum_x x\, p_{X|B_i}(x)}_{E(X|B_i)}\Big)$$

$$EX = \sum_i E(X|B_i)\, P(B_i)$$

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$$EX = 10 \cdot \frac{2}{5} + 20 \cdot \frac{3}{5}$$

$$= \frac{80}{5} = \underline{16}$$

Given $B$ $\quad p_{X|B}(x)$ a pmf

$$E(X|B) = \mu_{X|B}$$

$$E(a_1 X + a_2 X^2 + \cdots + b \mid B)$$

$$= a_1 E(X \mid B) + a_2 E(X^2 \mid B) + \cdots + b$$

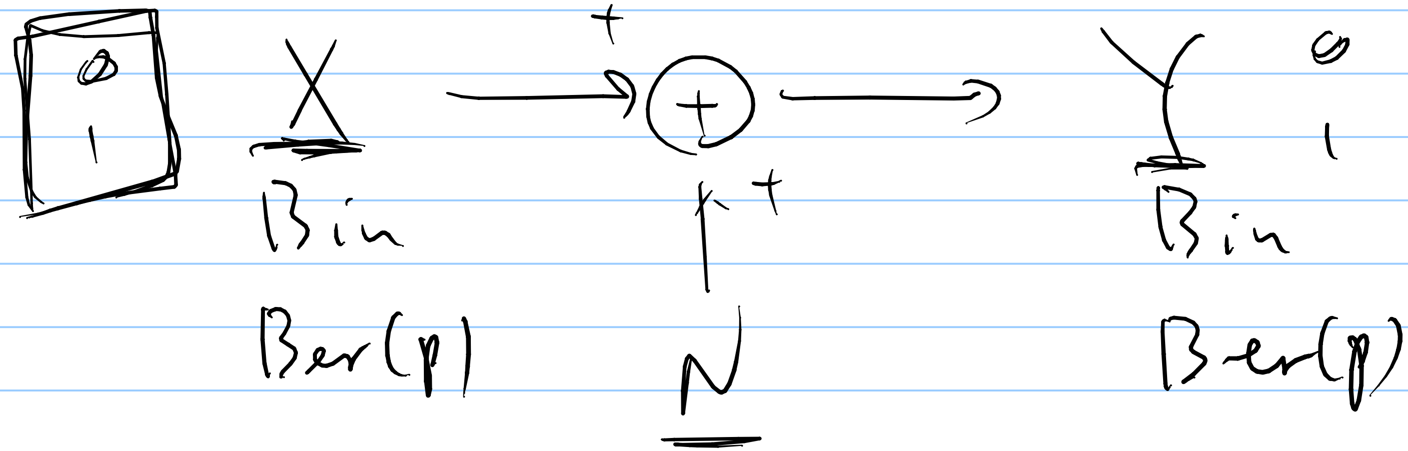
$$\mathrm{Var}(X \mid B) = E\left( (X - \mu_{X|B})^2 \mid B \right)$$

$$= E\left( \left( X^2 - 2\mu_{X|B} X + \mu_{X|B}^2 \right) \mid B \right)$$

$$= E(X^2 \mid B) - \underline{\underline{\mu_{X|B}^2}}$$

$$\sigma_{X|B} = \sqrt{\mathrm{Var}(X \mid B)}$$

# Simulation



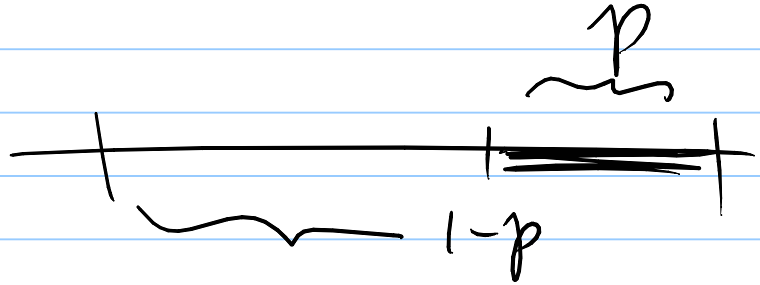

Error $P_e$

## Monte carlo simulation

$u = \text{rand}(1)$ $\quad [0, 1]$

$p$

- $\mathrm{Ber}(p)$ $\quad X = \begin{cases} 1 & \text{if } \underline{U} \geq 1-p \\ 0 & \text{otherwise} \end{cases}$




- $\mathrm{Bin}(p)$

$$X = \underset{\substack{\uparrow \\ \mathrm{Ber}(p)}}{X_1} + \cdots + X_n$$

- Geo $(p)$

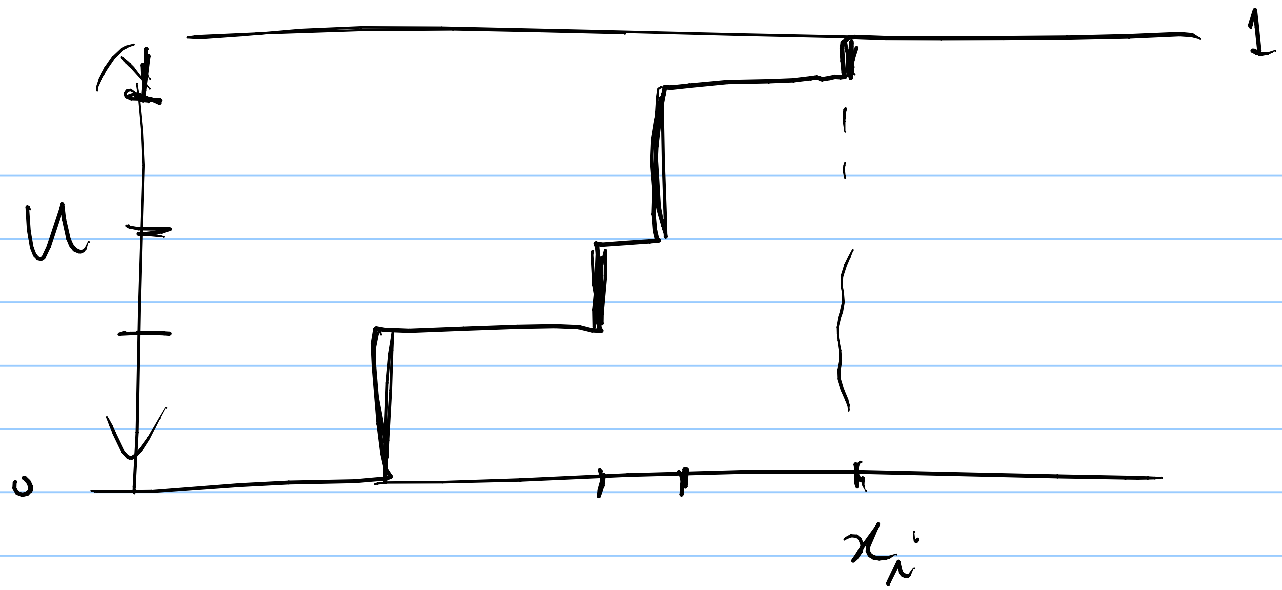
$$X = \leftarrow X_1, X_2, \ldots$$

$$(1-p)^n$$

- Pas $(k, p)$
- Poi $(\alpha)$ $\leftarrow$
- $F_X(x)$

$$\in [0, 1]$$

$$X = x \text{ st. } F_X(x-1) < u \leq F_X(x)$$



1
u
0
$x_i$