노트 제목 2008-04-03

$$\mathrm{Var}(\underline{a}X+\underline{\underline{b}}) = a^2\,\mathrm{Var}(X)$$

$$|EX| \le E|X| \qquad |a+b| \le |a|+|b|$$

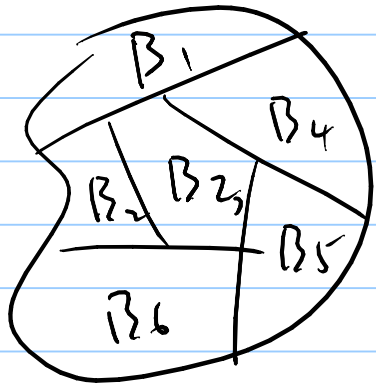
$$\left|\sum a_i\right| \le \sum |a_i|$$

$$\left|\int f(x)\,dx\right| \le \int |f(x)|\,dx$$

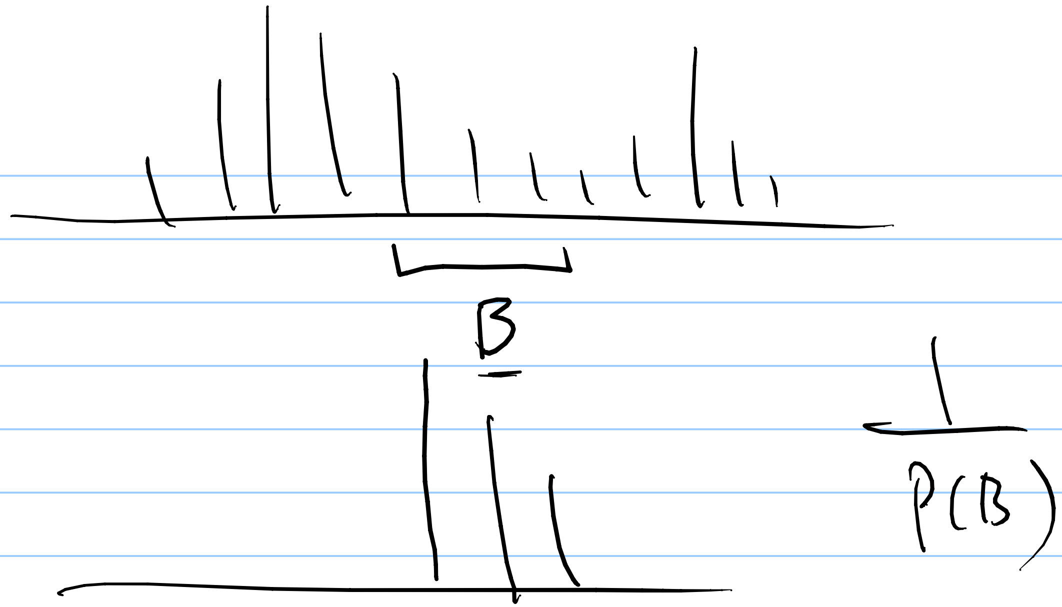
Condit pmf given an event B

$$p_{X|B}(x) = \frac{P(\{\underline{X=x}\}\cap B)}{P(B)}$$

$$p_X(x) = \sum_{i=1}^{m} \underbrace{p_{X|B_i}(x)\, P(B_i)}$$



$$p_{X|\{X\in B\}}(x) = \begin{cases} \dfrac{p_X(x)}{P(X\in B)}, & x \in B \\ 0, & \text{else} \end{cases}$$



$$E(X|B) = \sum_x x\, p_{X|B}(x)$$

$$\mu_{X|B}$$

elt of a part

$$EX = \sum_{i=1}^{n} E(X|B_i)\, P(B_i)$$

$$E\left(a_0 + a_1 X + a_2 X^2 + \cdots \mid \underline{B}\right)$$

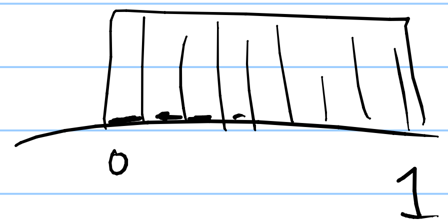
$$= a_0 + a_1 E(X \mid B) + a_2 E(X^2 \mid B) + \cdots$$

$$\mathrm{Var}(X \mid B) = E\left((X - \mu_{X|B})^2 \mid B\right)$$

$$= E(X^2 \mid B) - \mu_{X|B}^2$$

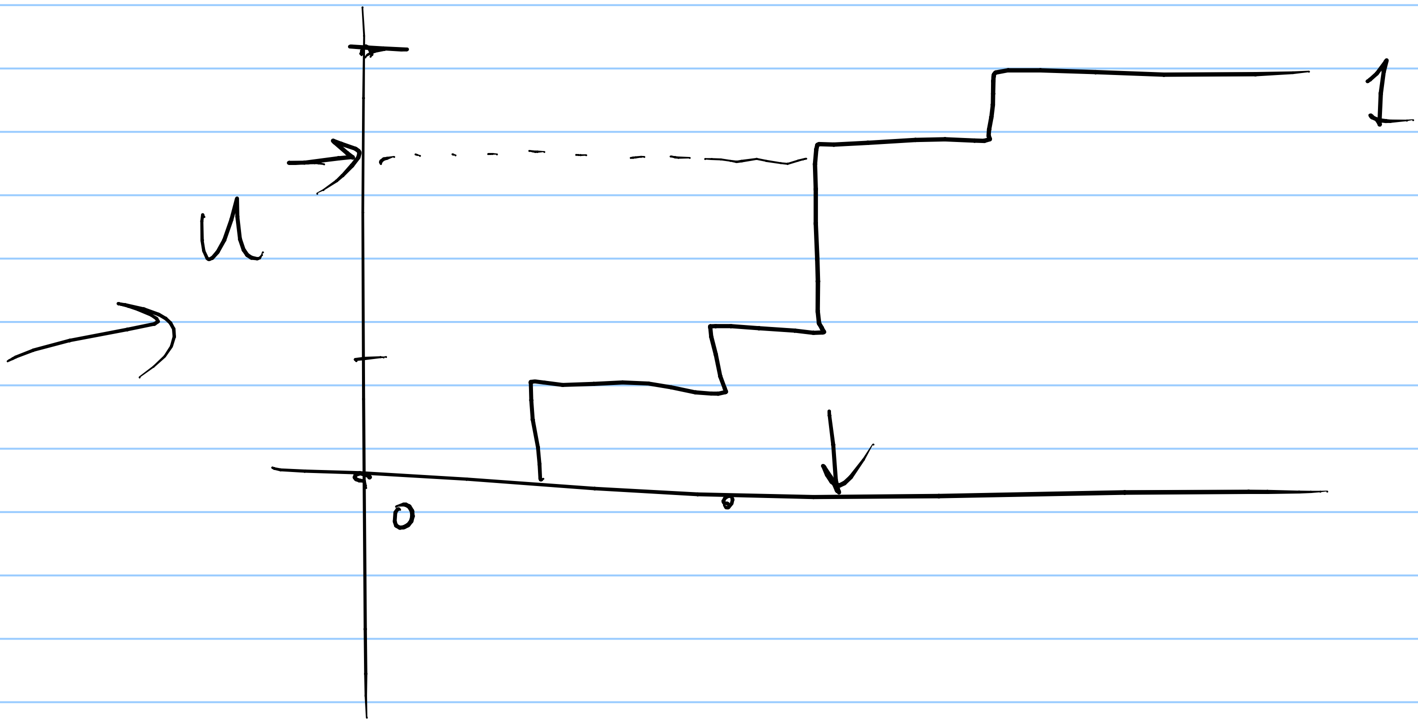
$$\sqrt{\mathrm{Var}(X \mid B)} \overset{d}{=} \sigma_{X|B}$$

RAND(1) $[0, 1]$

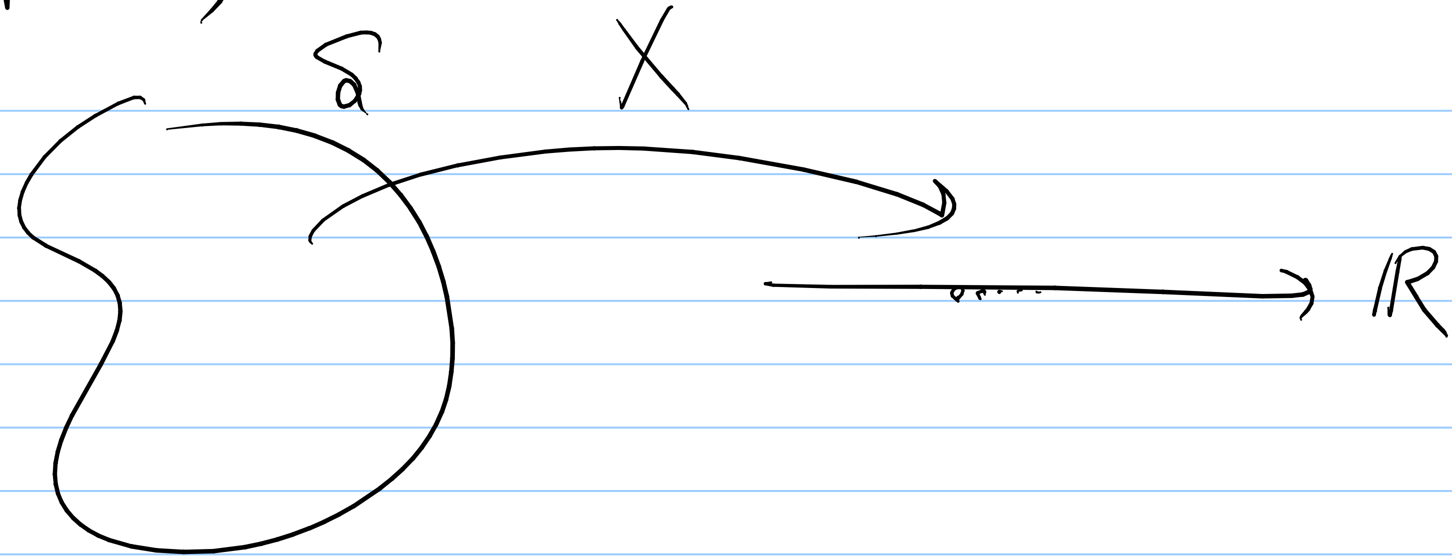


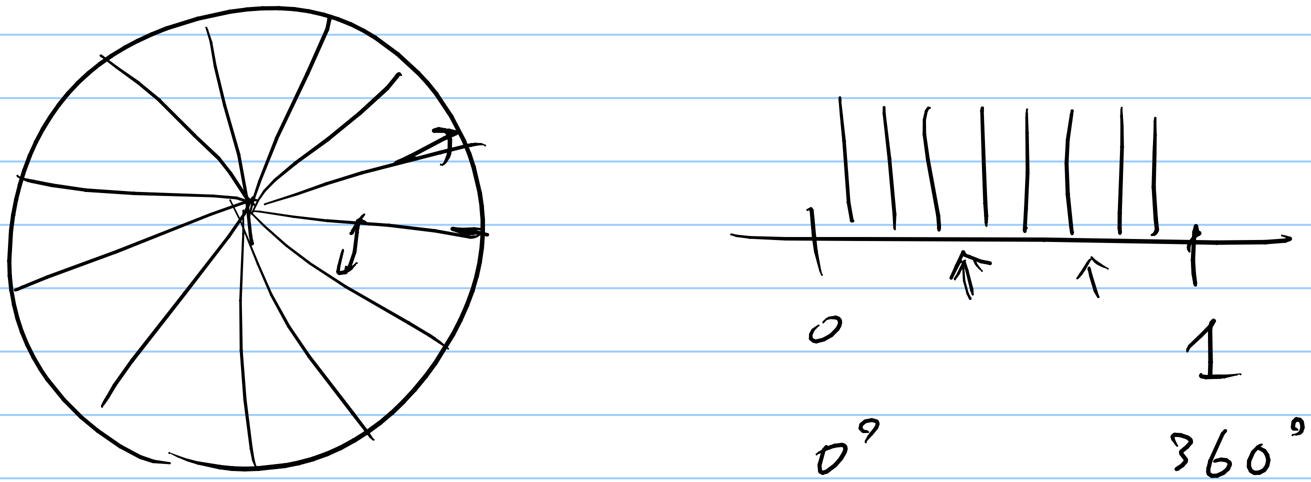
$$X = \mathrm{rand}(1) \qquad \boxed{Y} = \begin{cases} 1, & X \geq 1-p \\ 0, & X < 1-p \end{cases}$$

$\mathrm{Bin}(n, p)$ Pois, Unif



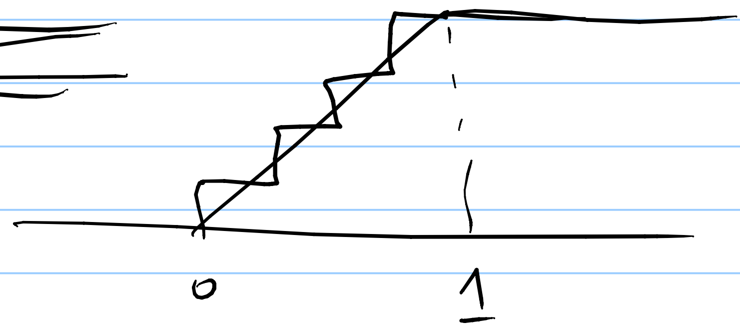
Continuous r.v.





$$P\left(0 \leq X < \frac{1}{n}\right) = \frac{1}{n}$$

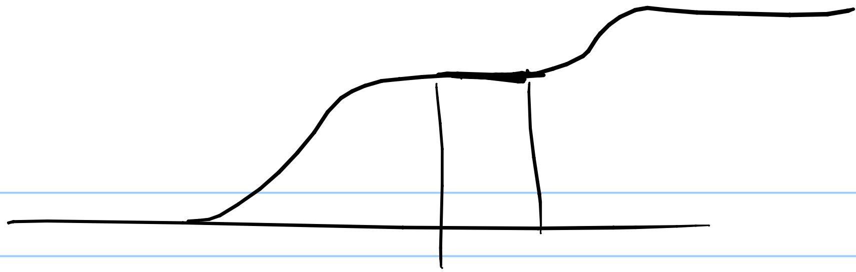
$$\underline{\underline{\underline{P(X = x) = 0}}}$$



$$\underline{\underline{F_X}}(x) = P(X \leq x)$$ cts fn of x

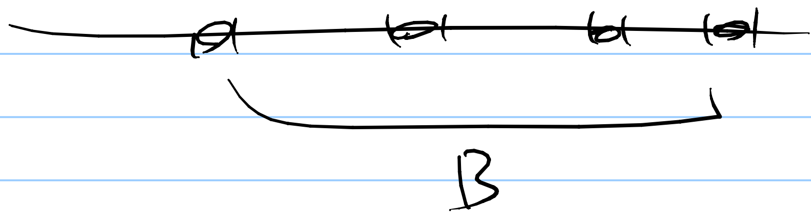
(a) $F_X(-\infty) = 0, \quad F_X(\infty) = 1$

(b) $x_1 \leq x_2 \Rightarrow F_X(x_1) \leq F_X(x_2)$



(c) $P(x_1 < X \le x_2) = F_X(x_2) - F_X(x_1)$

$\underline{B}$



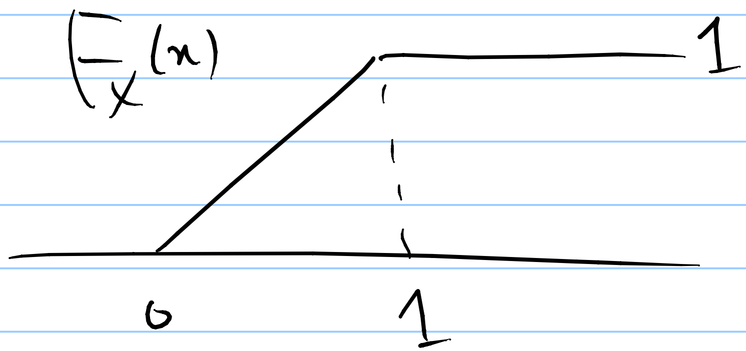
$$P(X \in B) = \sum_{B} p_X(x)$$

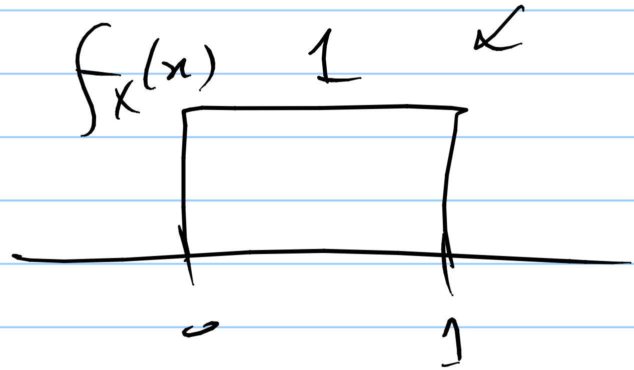
$$P(x < X \le x + \Delta x) = F_X(x+\Delta x) - F_X(x)$$

$P(X = x)$

$$\lim_{\Delta x \to 0} \boxed{\frac{P(x < X \le x+\Delta x)}{\Delta x}} = \lim_{\Delta x \to 0} \frac{F_X(x+\Delta x) - F_X(x)}{\Delta x}$$

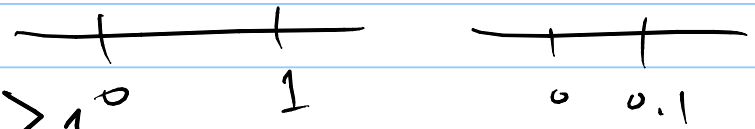
$$= \frac{dF_X(x)}{dx} \stackrel{d}{=} f_X(x)$$ prob density fn.
pdf

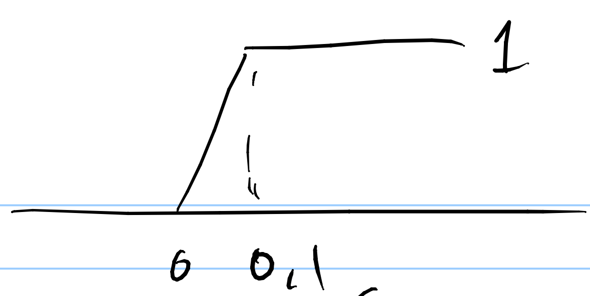


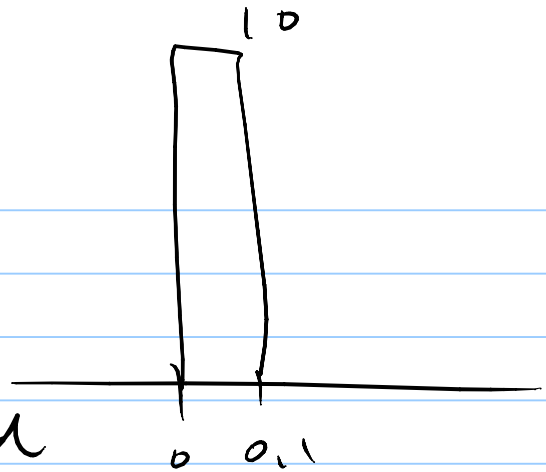


(a) $f_X(x) \geq 0$

$f_X(x)$ can be > 1







(b) $F_X(x) = \int_{-\infty}^{x} f_X(u)\,du$

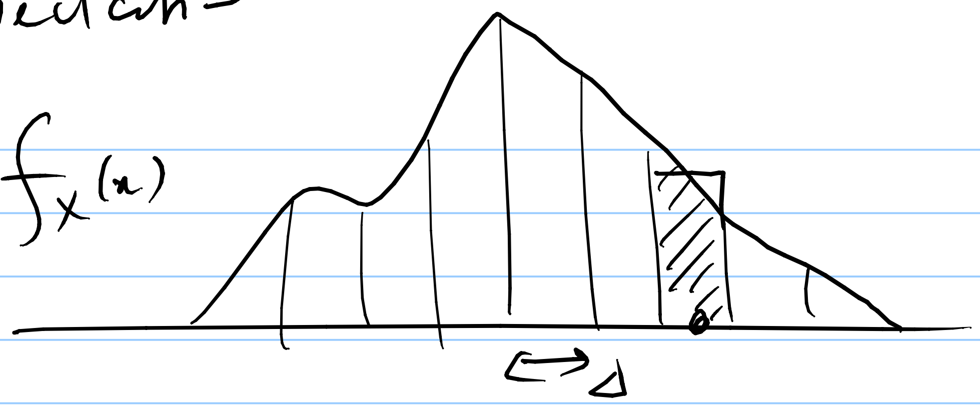
(c) $\int_{-\infty}^{\infty} f_X(u)\,du = 1$

(d) $P(X \in B) = \int_B f_X(x)\,dx$.

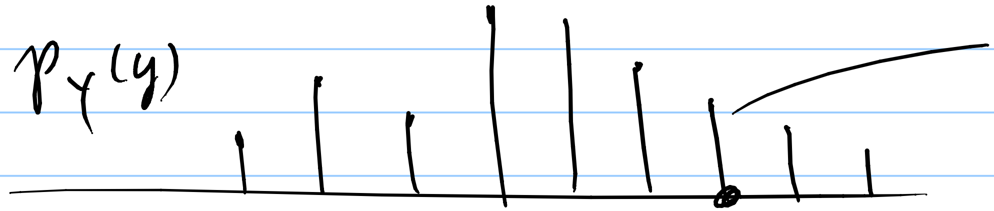
$P(x_1 < X \leq x_2) = \int_{x_1}^{x_2} f_X(x)\,dx$.

Expectation

$f_X(x)$



$p_Y(y)$



$$p_Y(y) = p_Y(x) = \underline{f_X(x)\Delta}$$

$$EY = \sum_y y\, p_X(y) = \boxed{\sum \underset{\substack{\| \\ x}}{y}\, \underline{f_X(x)\Delta}}$$

$\Delta \to 0$

$$EY \cong \int x f_X(x)\, dx = EX$$

◎ $Y = g(X)$ $\quad EY = \int y f_Y(y)\, dy$

$$= \int g(x) f_X(x)\, dx$$

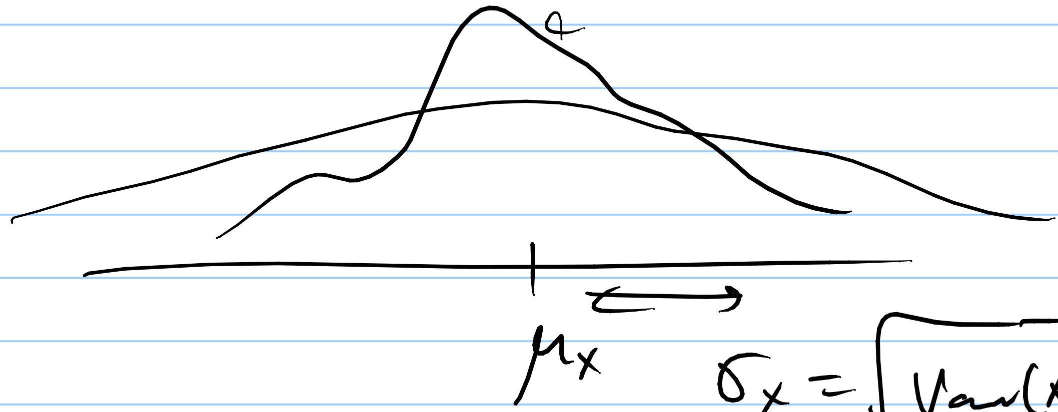
◎ $E(a_0 + a_1 X + a_2 X^2 + \cdots)$

$$= a_0 + a_1 EX + a_2 EX^2 + \cdots$$

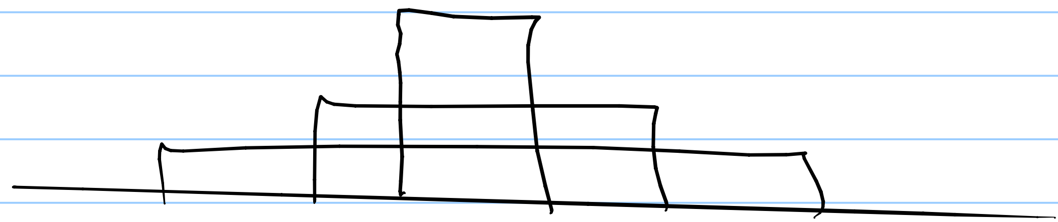
◎ $\mu_X = EX$ $\quad E(X - \mu_X) = 0$

$$E(X-\mu_X)^2 = Var(X) = EX^2 - \mu_X^2$$

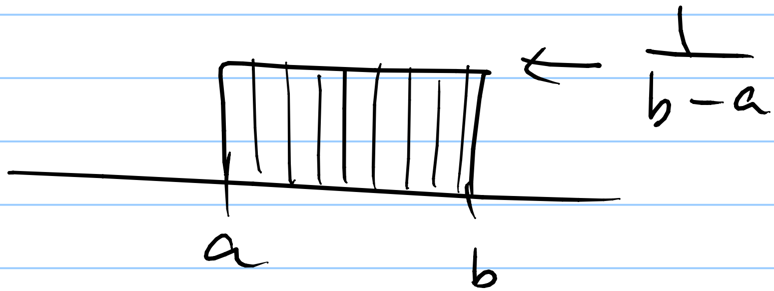
$$\mathrm{Var}(aX+b) = a^2\,\mathrm{Var}(X)$$



$$\sigma_X = \sqrt{\mathrm{Var}(X)}$$

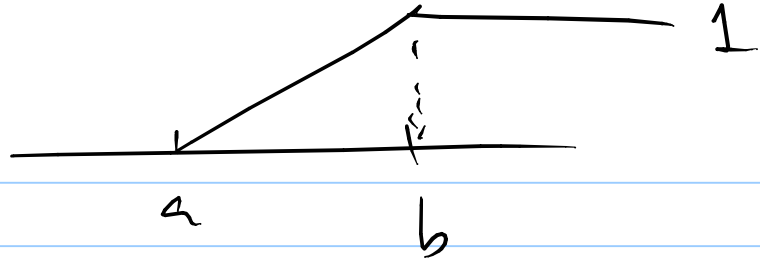


⊙ Unif(a, b)



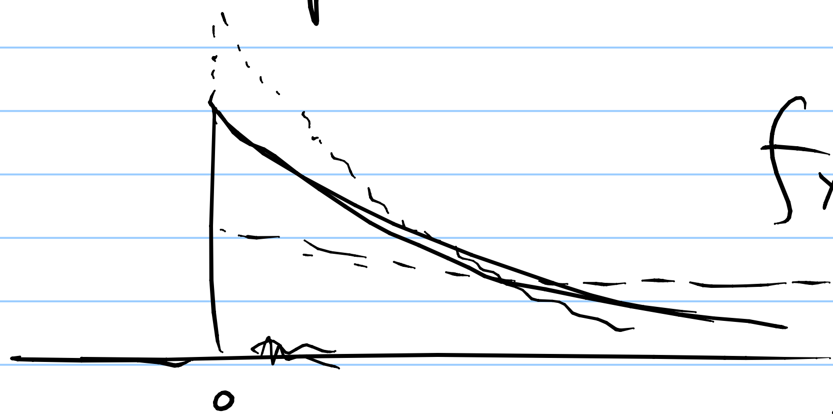
$$EX = \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)^2}{12}$$

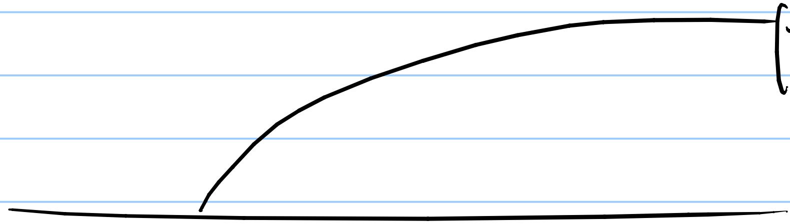


$\Rightarrow$ disc Unif

9 Exponential. $Exp(\lambda)$, $\lambda > 0$

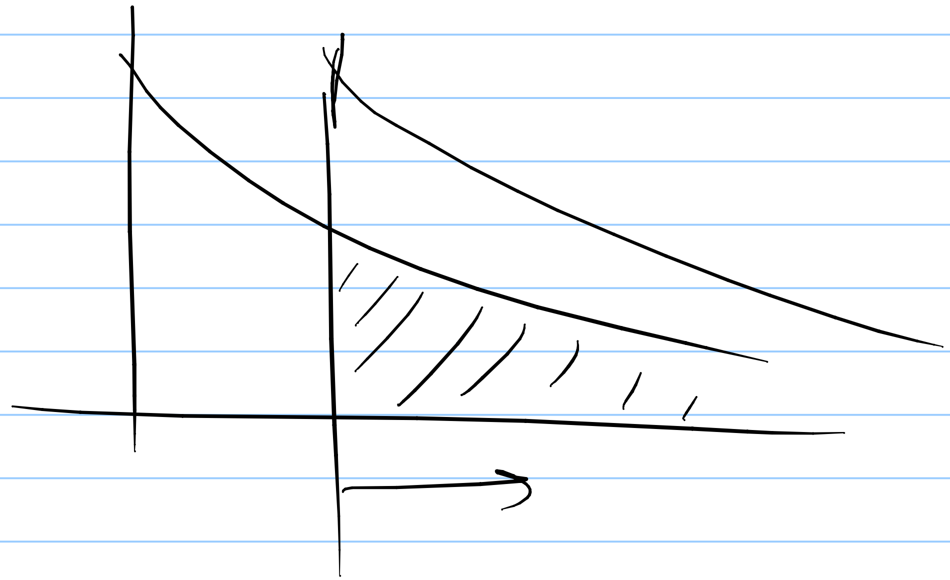


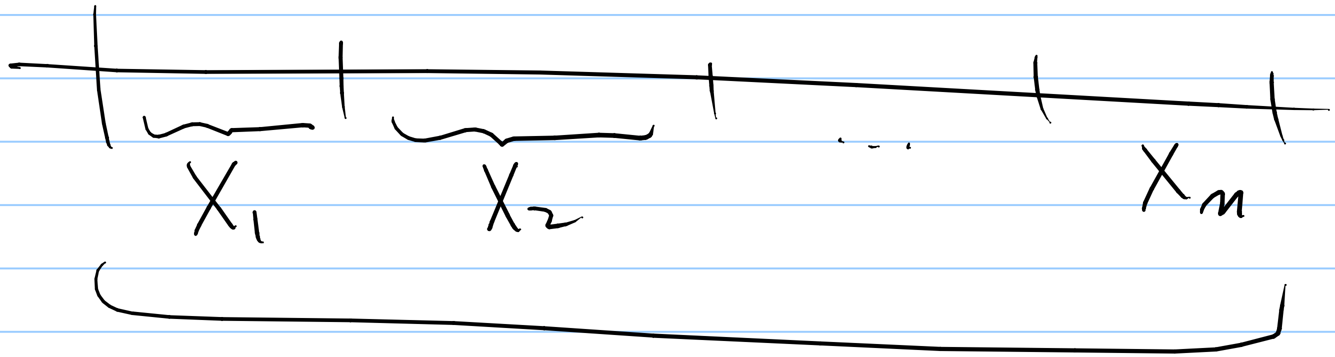
$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$



$$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

$$EX = \frac{1}{\lambda} \qquad Var(X) = \frac{1}{\lambda^2}$$





- Erlang r.v. $Er(\underline{n}, \underline{\lambda})$

$$f_X(x) = \begin{cases} \dfrac{\lambda^n x^{n-1} e^{-\lambda x}}{(n-1)!}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

$$\underline{X} = X_1 + \cdots + X_n$$

$X_1 \leftarrow Exp(\lambda)$ $\underline{\text{iid}}$ (indep & identically distributed)

$$EX = \frac{n}{\lambda} \qquad Var(X) = \frac{n}{\lambda^2}$$

@ Poiss ~ Exp