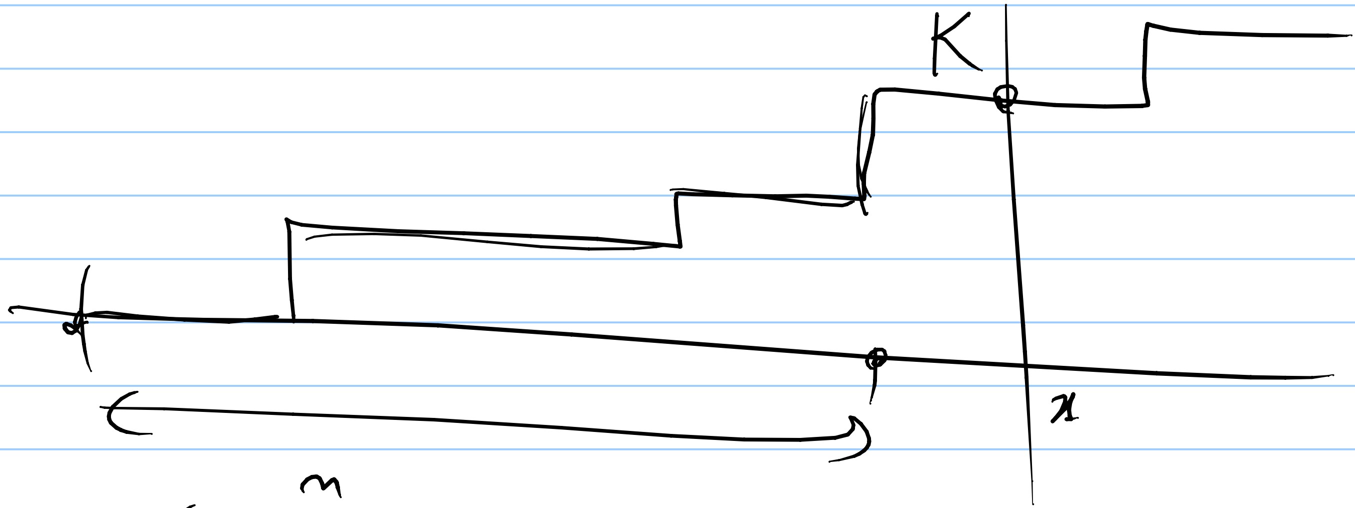


$$\left\{ \sum_{i=1}^{n} X_i \le x \right\} = \{ K \ge n \}$$

$$F_X(x) = 1 - F_K(n-1)$$

Gauss = Normal.

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$aX + b$

$$F_X(x) = \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}} du$$

$N(0,1)$

$$\boxed{Q(x)} = 1 - \Phi(x)$$

$$P(a < X \leq b) =$$

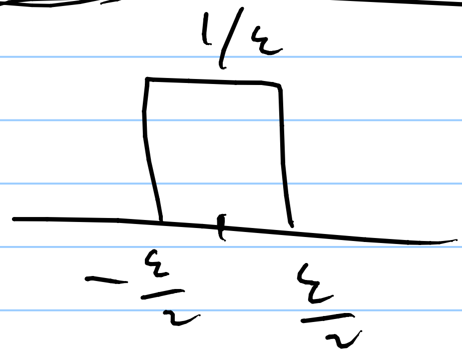
$N(\mu, \sigma^2)$

$erfc(x)$

Mixed r.v.

← Mixed

$$\delta(x) = \lim_{\epsilon \to 0}$$



$$\int_a^b \delta(x)\,dx = 1 \qquad \underline{\underline{a < 0 < b}}$$

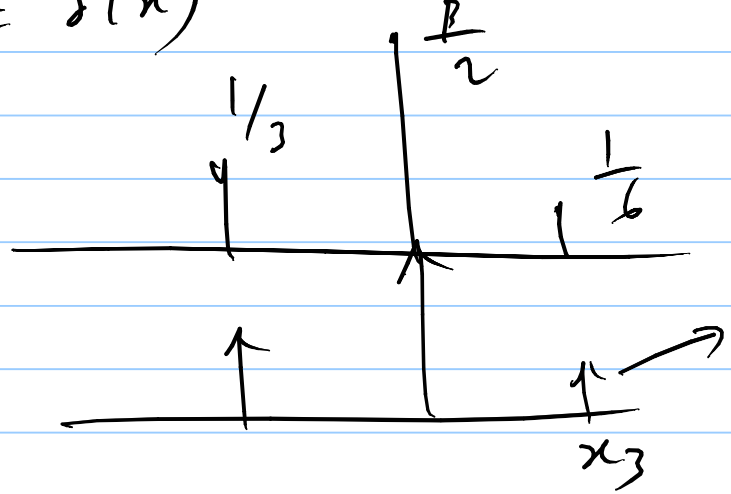
$$\int_a^b g(x)\,\delta(x - x_0)\,dx = \underline{\underline{g(x_0)}} \qquad a < x_0 < b$$

$$\int_{-\infty}^{x} \delta(u)\,du = \underline{\underline{U(x)}} = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$$\frac{dU(x)}{dx} = \delta(x)$$

$p_X(x)$

$f_X(x)$



$\frac{1}{6}\delta(x - x_3)$

$$E(X) = \int_{-\infty}^{\infty} x\, f_X(x)\, dx.$$

Mixed

$$F_X(x) = a F_c(x) + (1-a) F_d(x)$$

$$f_X(x) = a f_c(x) + (1-a) f_d(x)$$

Derived rvs (functions of rvs)

esp in cts cases

$$\underline{\underline{Y}} = a \underline{\underline{X}} + \boxed{b}$$

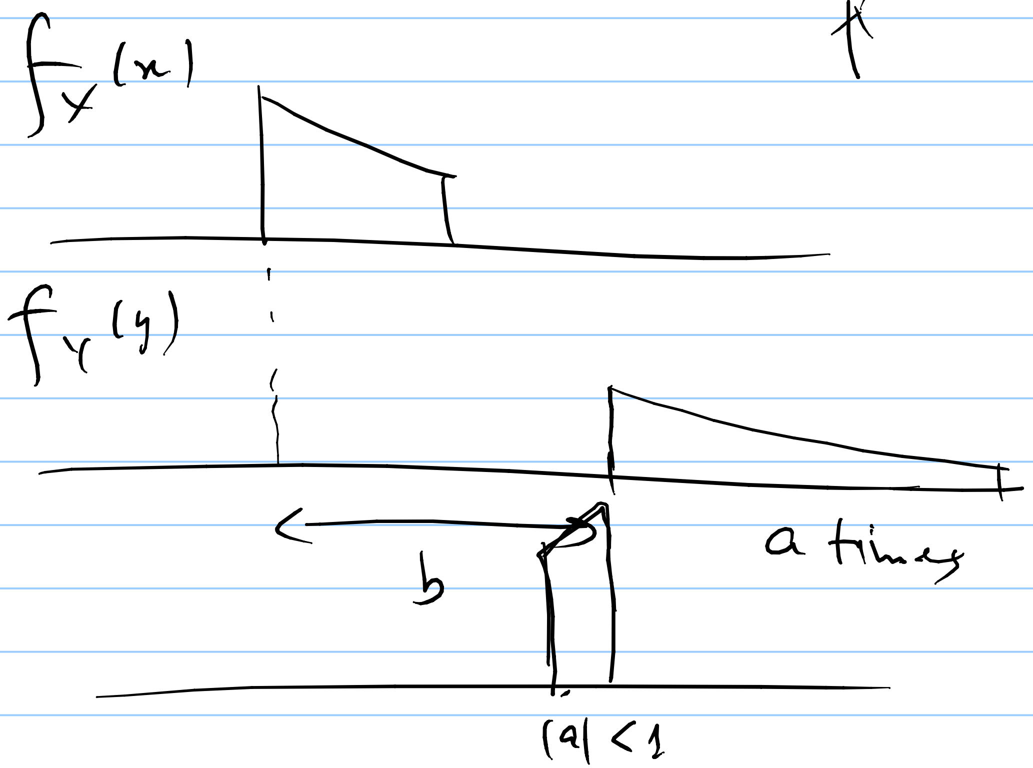
$$F_Y(y) = P(Y \leq y) = P(aX + b \leq y)$$

$$= \begin{cases} \underline{P\left(X \leq \frac{y-b}{a}\right)}, & \underline{a > 0} \\ P\left(X \geq \frac{y-b}{a}\right), & \underline{a < 0} \end{cases}$$

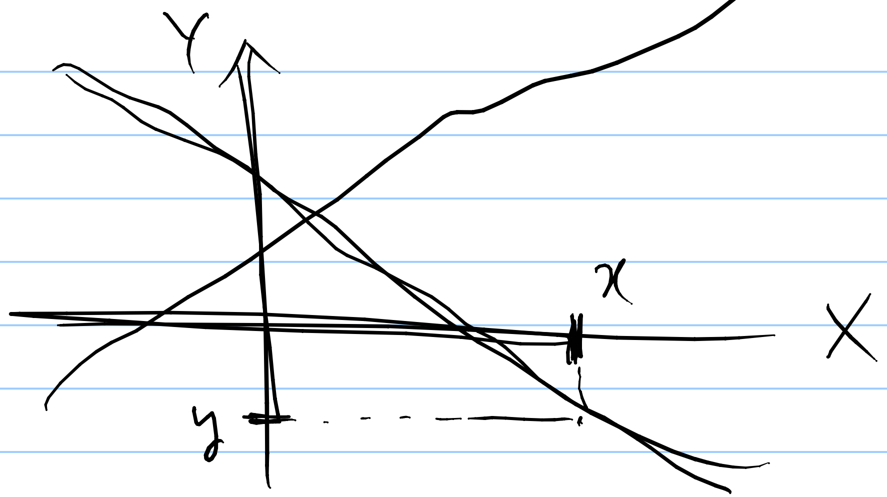
$$= \begin{cases} \underline{F_X\left(\frac{y-b}{a}\right)}, & a > 0 \\ \underline{1 - F_X\left(\frac{y-b}{a}\right)}, & a < 0 \\ \left(1 - F_X\left(\frac{y-b}{a}\right) + P\left(X = \frac{y-b}{a}\right) \quad \text{for disc or mixed.}\right) & \end{cases}$$

$$f_Y(y) = \frac{dF_Y(y)}{dy} = \begin{cases} f_X\left(\frac{y-b}{a}\right)\cdot\frac{1}{a}, & a>0 \\ -f_X\left(\frac{y-b}{a}\right)\cdot\frac{1}{a}, & a<0 \end{cases}$$

$$= f_X\left(\frac{y-b}{a}\right)\frac{1}{|a|} \quad \text{cts rv}$$



$Y = g(X)$ $\quad g$: mono, cts, ctsly diffbl.



$y = g(x)$

$$F_Y(y) = P(Y \le y) = P(g(X) \le y)$$

$$= \begin{cases} P(X \le g^{-1}(y)) & g\uparrow \\ P(X \ge g^{-1}(y)) & g\downarrow \end{cases}$$

$$= \{ F_X(g^{-1}(y)), \quad g\uparrow$$

$$\left[\, 1 - F_X(g^{-1}(y)), \quad g \downarrow \right.$$

$$\left(+ P(X = g^{-1}(y)) \right)$$

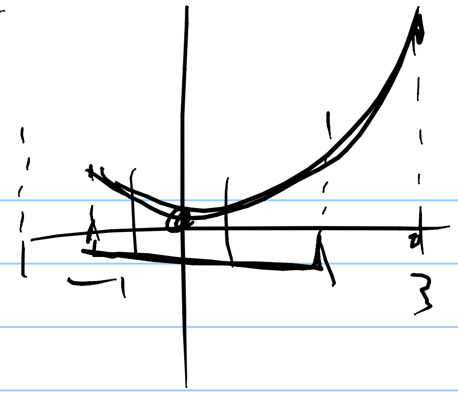
disc, mixed.

$$f_Y(y) = \begin{cases} f_X(g^{-1}(y)) \dfrac{d}{dy}(g^{-1}(y)), & g \uparrow \\ -f_X(g^{-1}(y)) \dfrac{d}{dy}(g^{-1}(y)), & g \downarrow \end{cases}$$

$$= f_X(g^{-1}(y)) \left| \frac{d}{dy}(g^{-1}(y)) \right|$$

$$= \frac{f_X(x)}{\left| \dfrac{dg(x)}{dx} \right|} \qquad x = g^{-1}(y)$$

Example $X \sim \text{unif}[-1,3]$, $Y = X^2$



$$F_Y(y) = P(Y \le y)$$

$$= P(X^2 \le y)$$

$$= \begin{cases} P(-\sqrt{y} \le X \le \sqrt{y}), & y \ge 0 \\ 0, & y < 0 \end{cases}$$

$$= \begin{cases} 0, & y < 0 \\ \boxed{\sqrt{y}/2}, \quad \frac{1}{2} & 0 \le y < 1 \\ \frac{\sqrt{y}+1}{4}, \quad \frac{2}{4} & 1 \le y < 9 \\ 1, & y \ge 9 \end{cases}$$

$$=$$

$$f_Y(y) = \begin{cases} \dfrac{1}{4\sqrt{y}}, & 0 \le y < 1 \\ \dfrac{1}{8\sqrt{y}}, & 1 \le y < 9 \\ 0, & \text{else.} \end{cases}$$

Y $\quad f_Y(y)$ $\quad \boxed{F_Y^*(y)}$ ← monotonic

unif $[0, 1]$

① ⟶ given

② Like to find $g(x)$ st $\quad Y = g(X)$

then $\quad F_Y(y) = F_Y^*(y)$

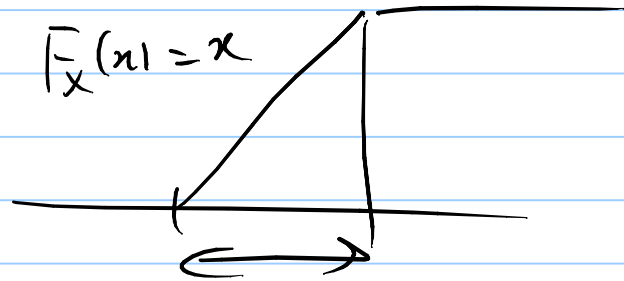
③ Let $\quad g(x) = F_Y^{*-1}(x)$ ←

$$\underline{Y = g(X)}$$

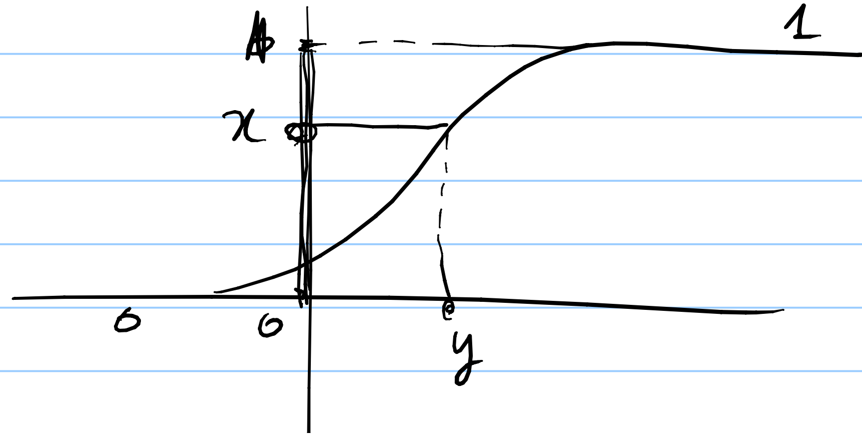
$$F_Y(y) = P(Y \le y) = P\left(F_Y^{*-1}(X) \le y\right)$$

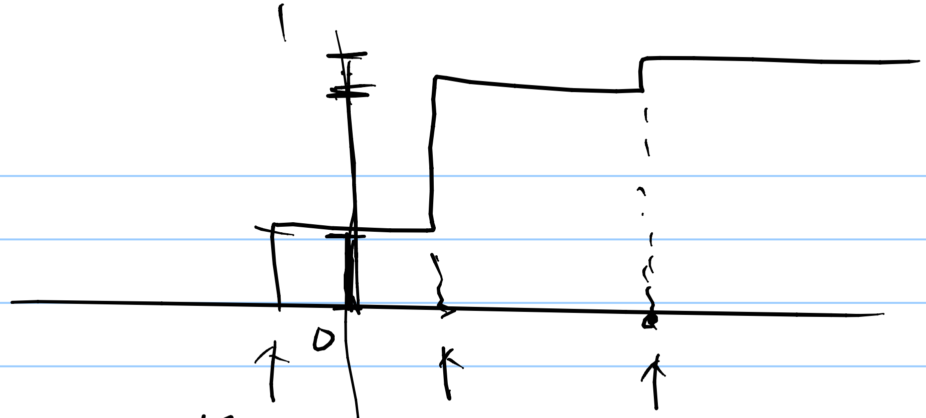
$$= P\left(X \le F_Y^{*}(y)\right) = F_X\left(F_Y^{*}(y)\right)$$

X – unif $[0,1]$ $\quad F_X(x) = x$

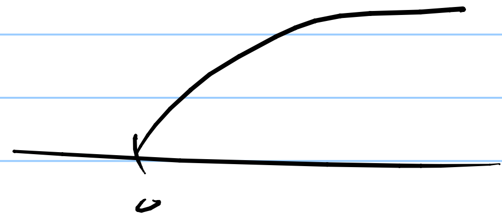


$$= \boxed{F_Y^{*}(y)}$$





$$F_Y(y) = \begin{cases} 1 - e^{-y}, & y \geq 0 \\ 0, & \text{else} \end{cases}$$



$$g(x) = F_X^{-1}(x) = -\ln(1-x)$$

$$\underline{\underline{Y}} = -\ln(1-\underline{\underline{X}})$$

$\text{Unif}[0,1]$

Condit cdf / pdf   Given an event $\boxed{A}$

$$\{X \in B\}$$

$$F_{X|A}(x) \triangleq P(X \le x \mid A)$$

$$= \frac{P(\{X \le x\} \cap A)}{P(A)}$$

$$f_{X|A}(x) \triangleq \frac{d F_{X|A}(x)}{dx}$$

cts.

$$= \lim_{\Delta x \to 0} \frac{P(x < X \le x + \Delta x \mid A)}{\Delta x}$$

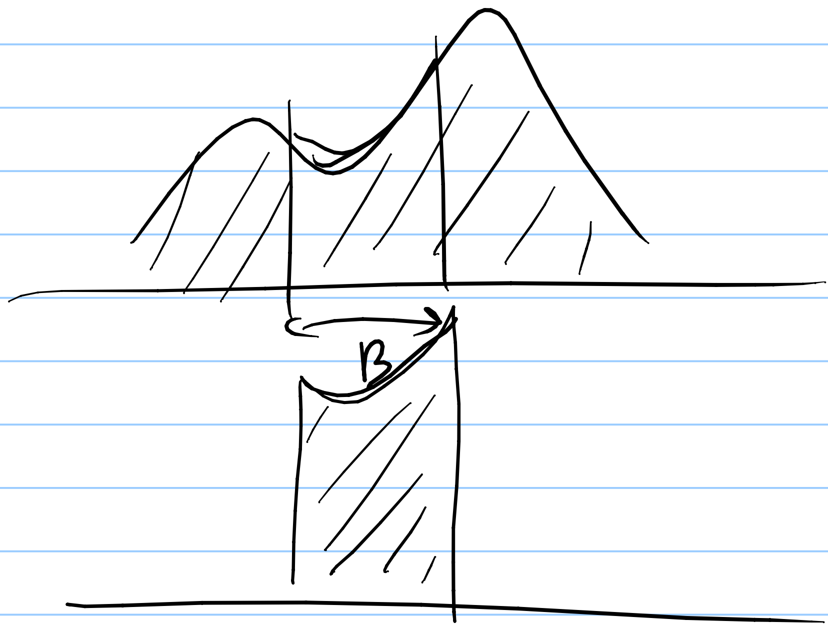
$$\longrightarrow \quad = \lim_{\Delta x \to 0} \frac{P(\{x < X \le x + \Delta x\} \cap A)}{}$$

$\Delta x \; P(A)$

If $A = \{X \in B\}$ set of real nos

$$f_{X|A}(x) = \begin{cases} \lim\limits_{\Delta x \to 0} \dfrac{P(x < X \le x + \Delta x)}{\Delta x \, P(A)}, & x \in B \\ 0, & \text{else} \end{cases}$$

$$= \boxed{\dfrac{f_X(x)}{P(A)}}$$



Total Prob law A_i's partition S

$$f_X(x) = \sum_i f_{X|A_i}(x) P(A_i)$$

Condit Exp.

$$\mu_{X|A} = E(X|A) = \int_{-\infty}^{\infty} x f_{X|A}(x) dx$$

condit mean

$$E(g(X)|A) = \int_{-\infty}^{\infty} \underline{g(x)} f_{X|A}(x) dx$$

$$Var(X|A) = E\left((X - \mu_{X|A})^2 | A\right)$$

$$= E(X^2|A) - \mu_{X|A}^2$$