노트 제목 2008-04-22

$$Y = g(X) \qquad p_Y(y) = \sum_{x:\, g(x) = y} p_X(x)$$

$$Y = aX + b \qquad F_Y(y)$$

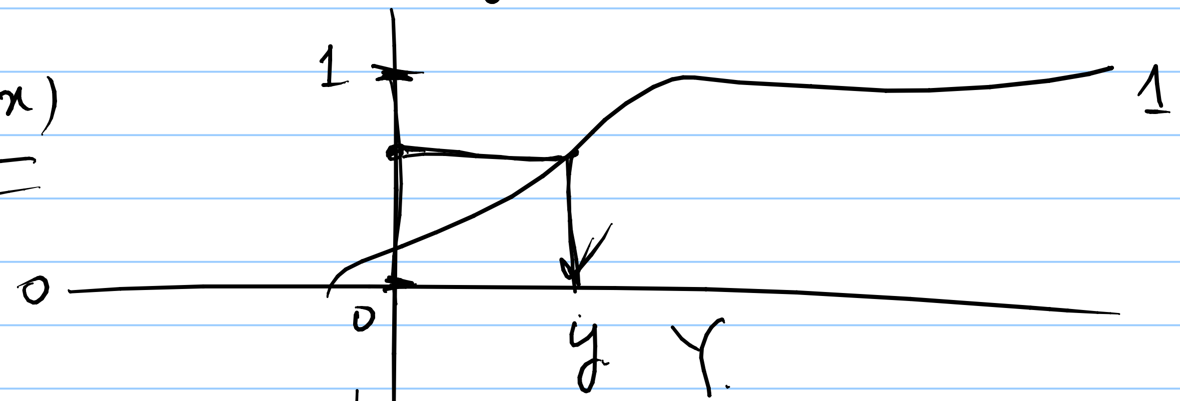
$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$$

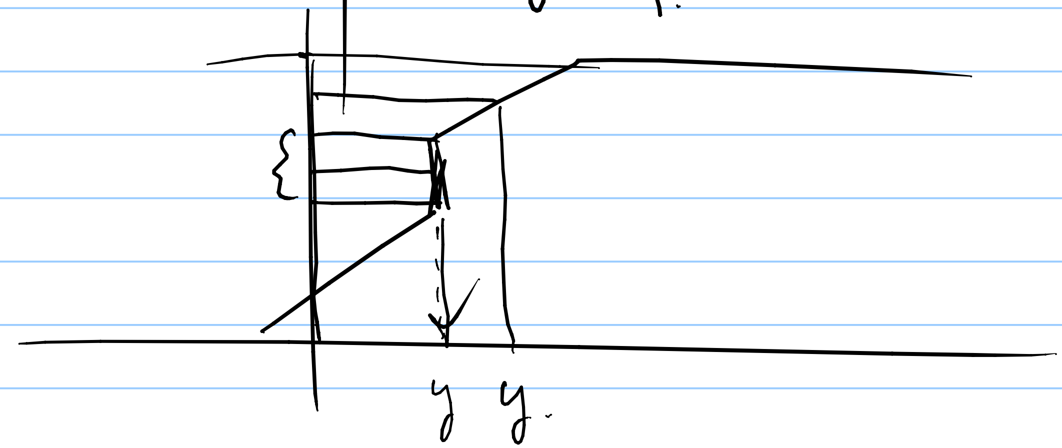
$$Y = g(X) \qquad f_Y(y) = \frac{1}{\left|\frac{dg(x)}{dx}\right|} f_X(g^{-1}(y))$$

$$\underline{\underline{F_Y(y)}} = \underline{\underline{P(g(x) \leq \underline{\underline{y}})}} \qquad f_Y(y)$$

$X \sim \text{unif}[0,1]$

$\underline{\underline{F(x)}}$

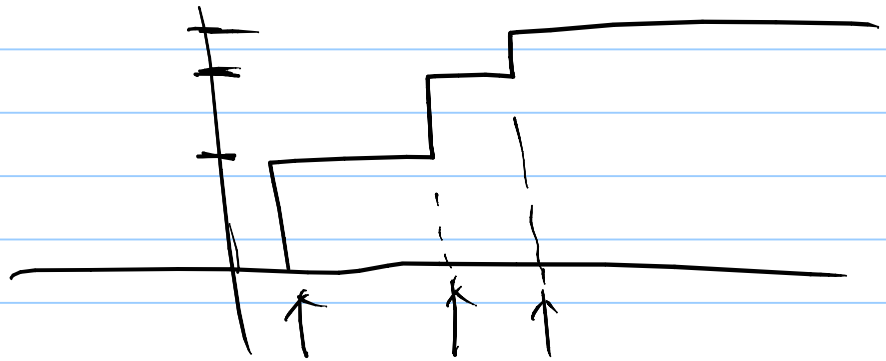




$$g(x) = F^{-1}(x) \qquad Y = g(X)$$

Pseudo-inverse

unif $[0,1]$

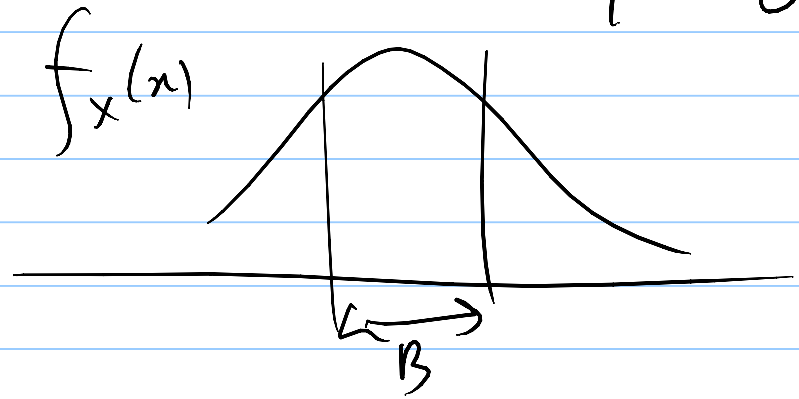


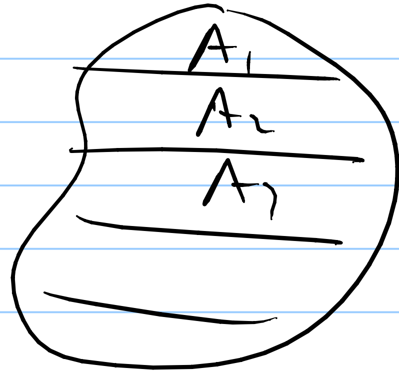
Condit cdf $\quad F_{X|A}(x) = P(X \le x \mid A)$

$$f_{X|A}(x) = \lim_{\Delta x \to 0} \frac{P(x < X < x + \delta x \mid A)}{\Delta x}$$

$$A = \{X \in B\}$$

$$f_{X|A}(x) = \begin{cases} \dfrac{f_X(x)}{P(A)}, & x \in B \\ 0, & \text{else.} \end{cases}$$





$$f_{\underline{X}}(x) = \sum_i \underline{f_{X|A_i}(x)}\, P(A_i)$$

A_i partition S

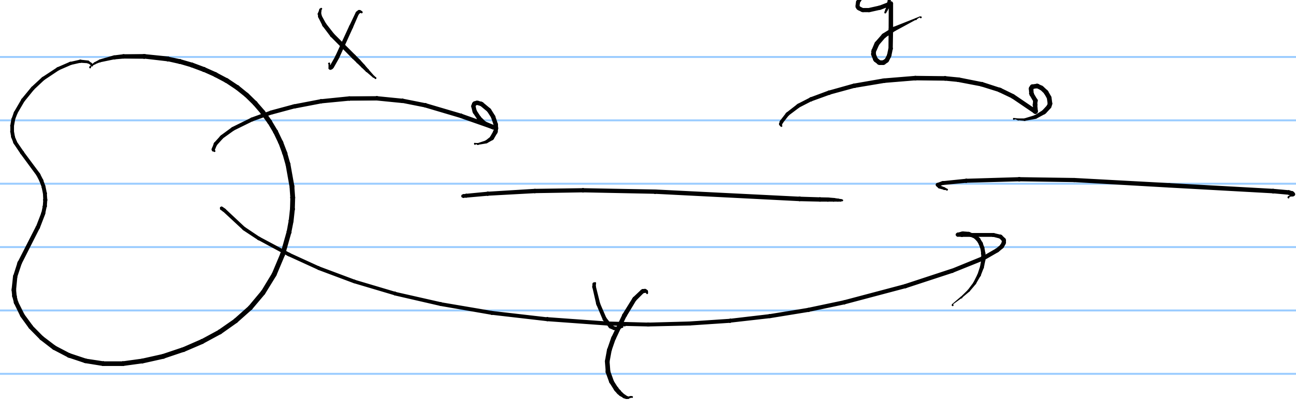
$$E(X|A) = \int x \, f_{X|A}(x) \, dx.$$

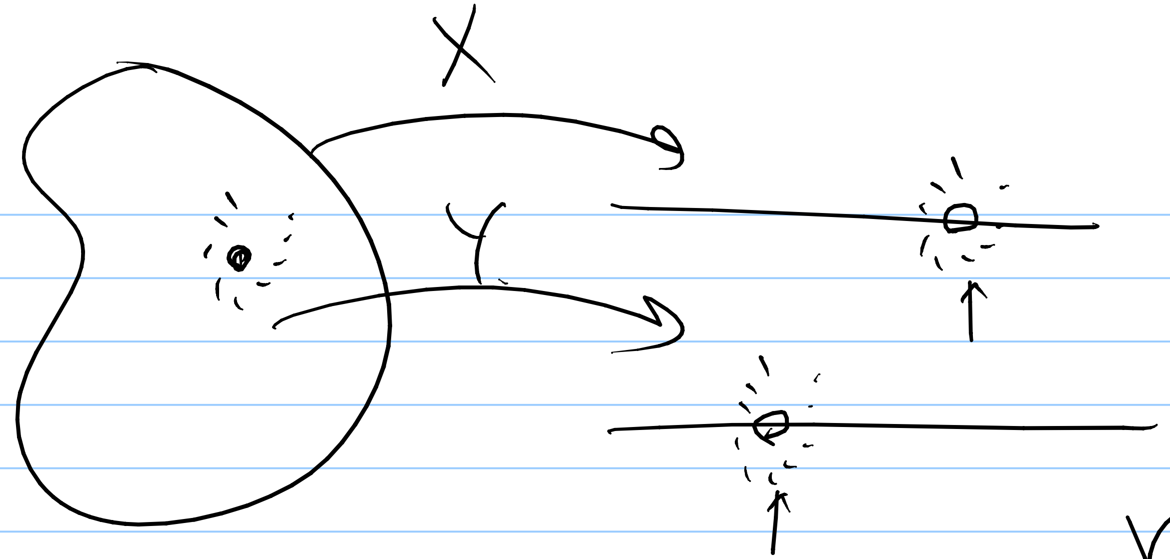
$$E(g(X)|A) = \int g(x) \, f_{X|A}(x) \, dx$$

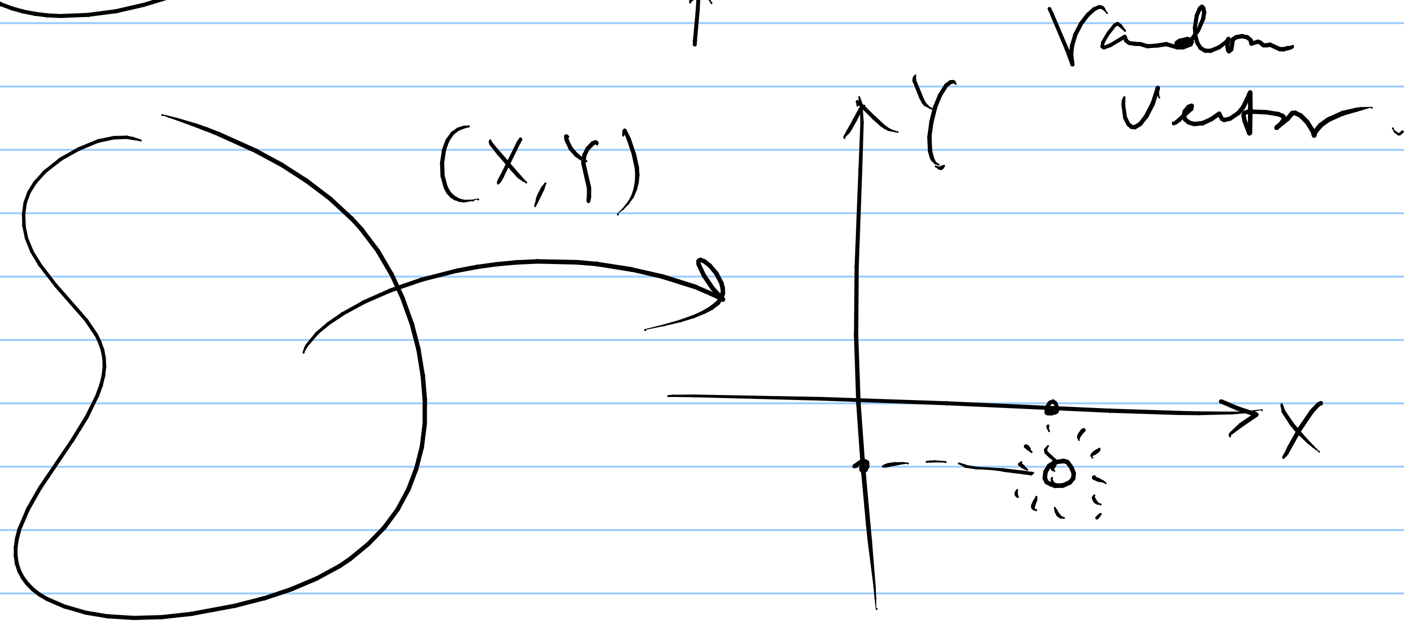
$$\mathrm{Var}(X|A) = E\left((X - \mu_{X|A})^2 \middle| A \right)$$

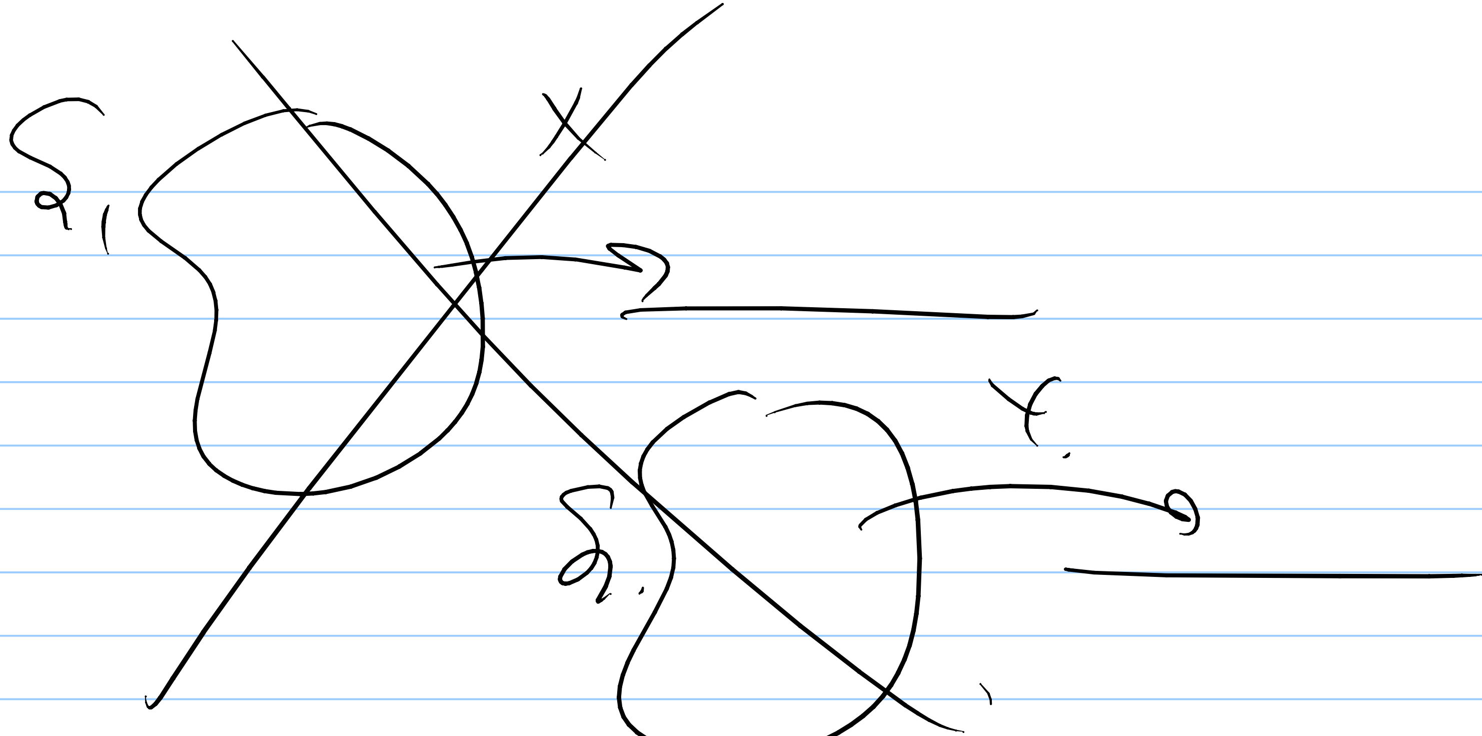
$$= E(X^2|A) - \mu_{X|A}^2$$

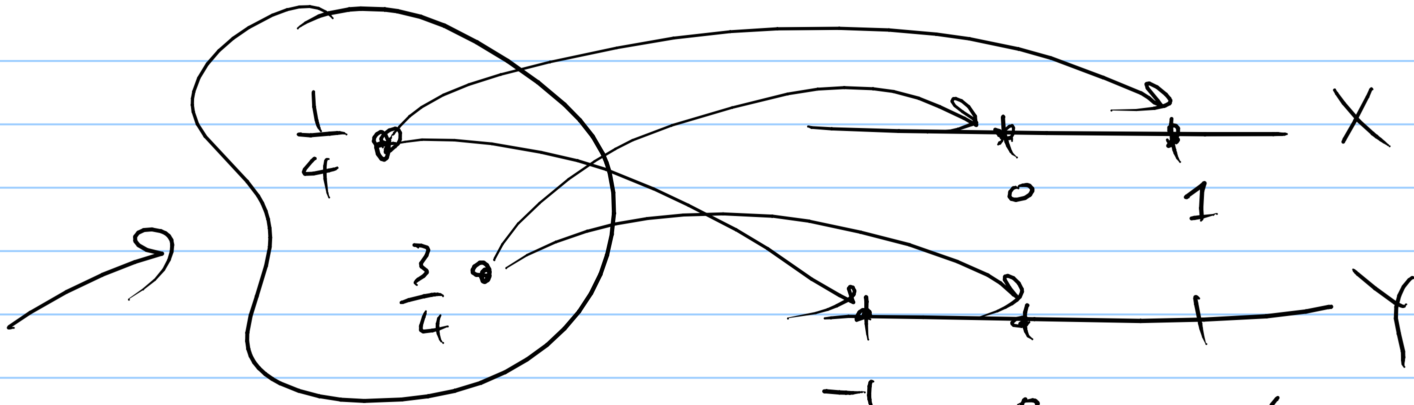
$g(X)$



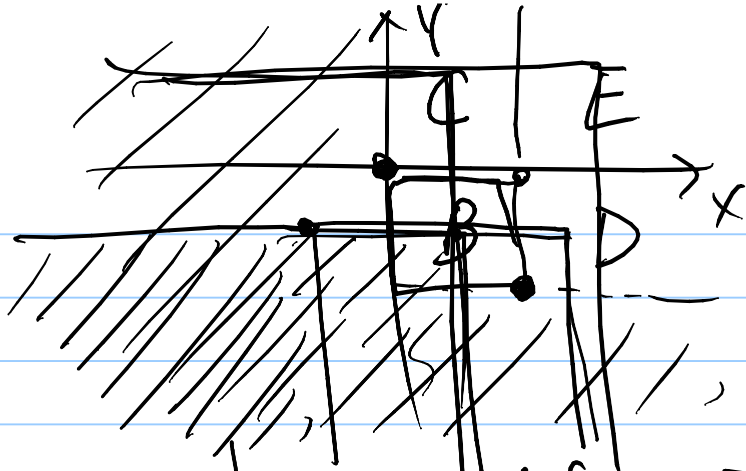

X
Y


(X, Y)
Y
X
random
vector.





$Y = -X$

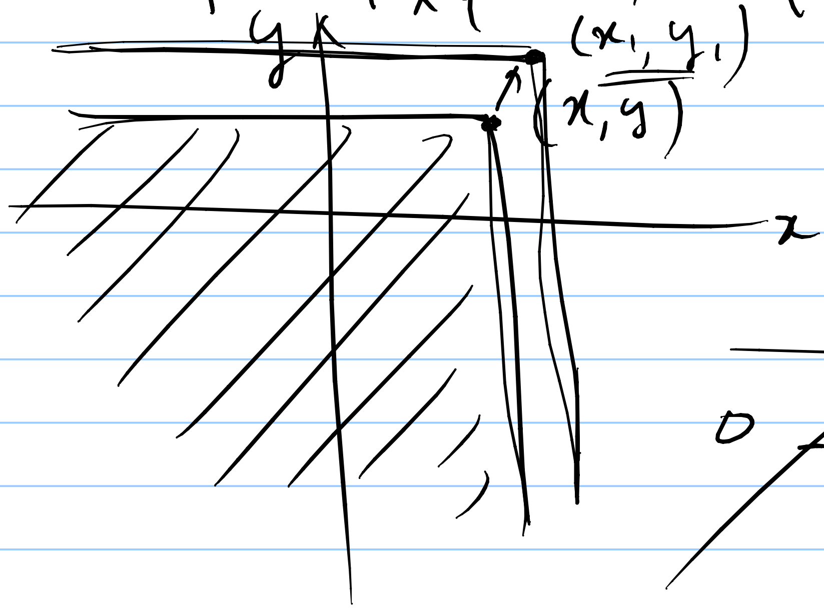


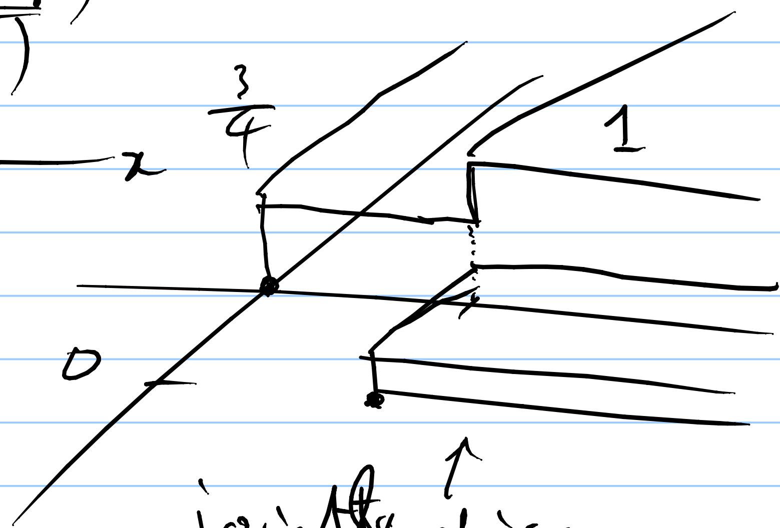
$$F_{XY}(x,y) = \begin{cases} 0 & x < 0 \text{ or } y < -1 \\ & \text{or } (x,y) \in B \\ \frac{3}{4} & (x,y) \in C \\ 1/4 & (x,y) \in D \\ 1 & (x,y) \in E \end{cases}$$

Joint cdf

$$F_{XY}(x,y) \triangleq P\left(\underline{X \le x}, \underline{\cancel{Y \le y}}\right)$$

$-\infty$





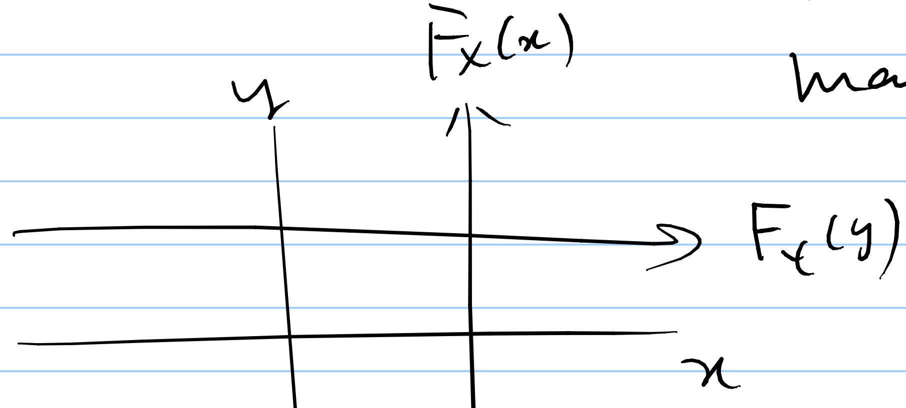
~~jointly~~ disc

Properties:

(a) $0 \le F_{XY}(x,y) \le 1$

(b) $F_{XY}(y,\infty) = F_X(x)$, $F_{XY}(\infty,y) = F_Y(y)$

marginal cdf



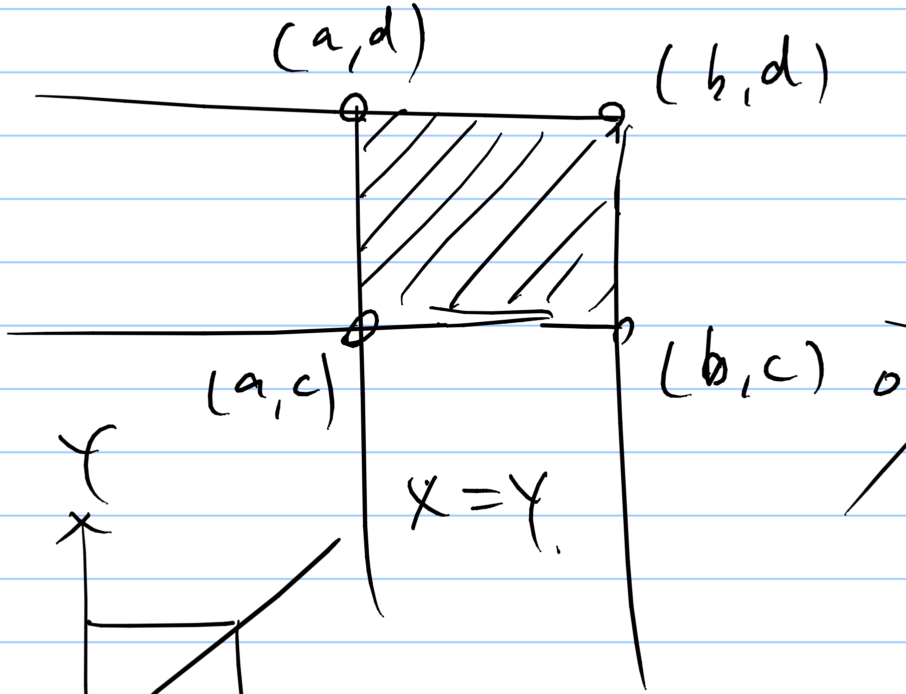
(c) $F_{XY}(-\infty,y) = F_{XY}(x,-\infty) = 0$

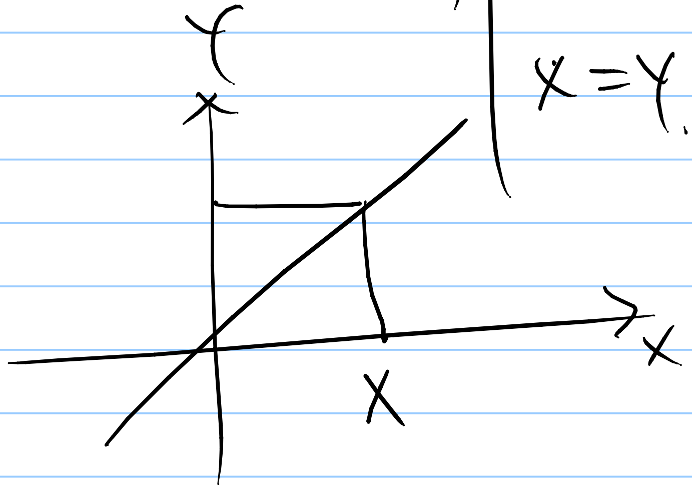
(d) $x \le x_1,\ y \le y_1 \Rightarrow F_{XY}(x,y) \le F_{XY}(x_1,y_1)$

(e) $F_{XY}(\infty,\infty) = 1$

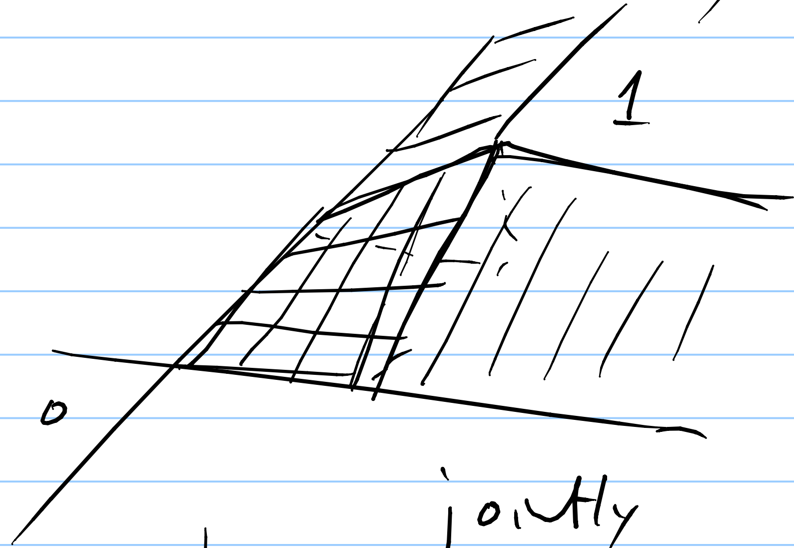
(f) $P(a < X \le b,\ c < Y \le d)$

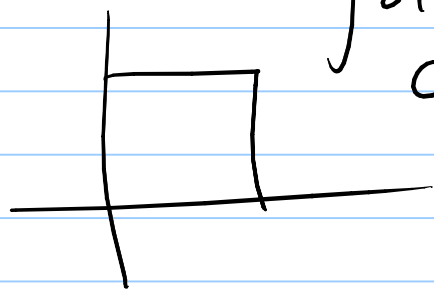
$$= \underline{F_{XY}(b,d) - F_{XY}(a,d) - F_{XY}(b,c) + F_{XY}(a,d)}$$





$X = Y$

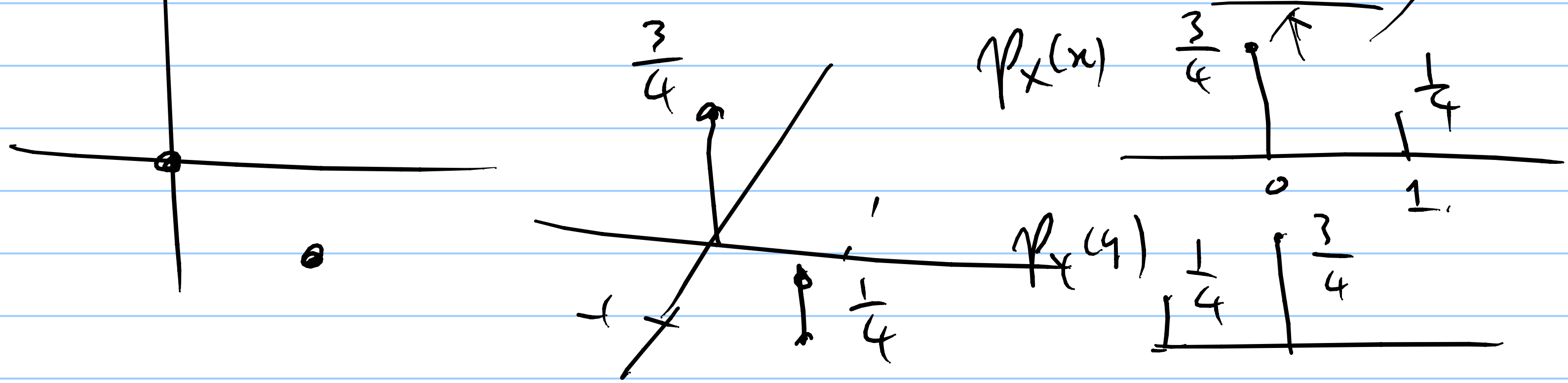




jointly cts

Jointly disc   joint pmf

$$p_{XY}(x,y) = P(X=x,\ Y=y)$$



Properties

(a) $0 \le p_{XY}(x,y) \le 1$.

(b) $$\sum_x \sum_y p_{XY}(x,y) = 1$$

(c) $$\sum_{y \in S_Y} p_{XY}(x,y) = p_X(x)$$

$$\sum_x \qquad = p_Y(y)$$

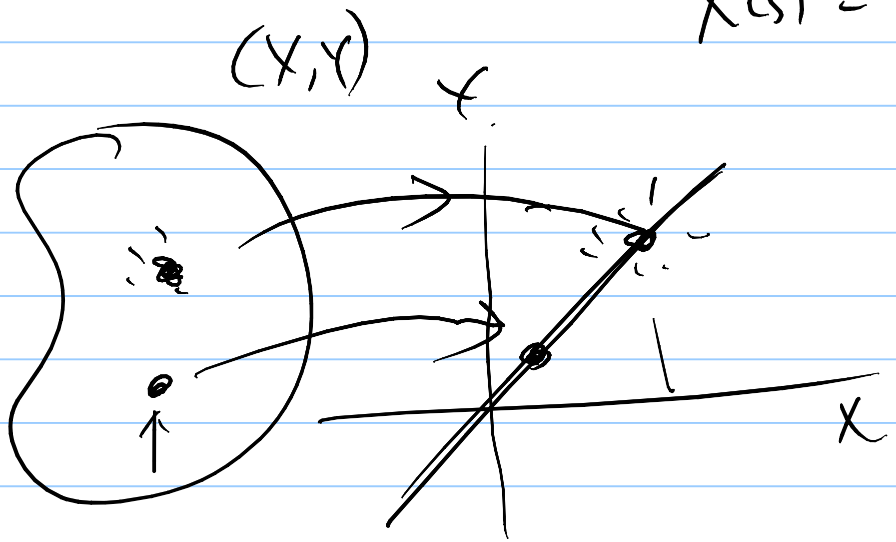
(d) $P((X,Y)\in B) = \sum_{(x,y)\in B} p_{XY}(x,y)$

X, Y are equal: $P(X=Y) = 1$.

$X(s) = Y(s)$ for almost all s

for all $s \in A$

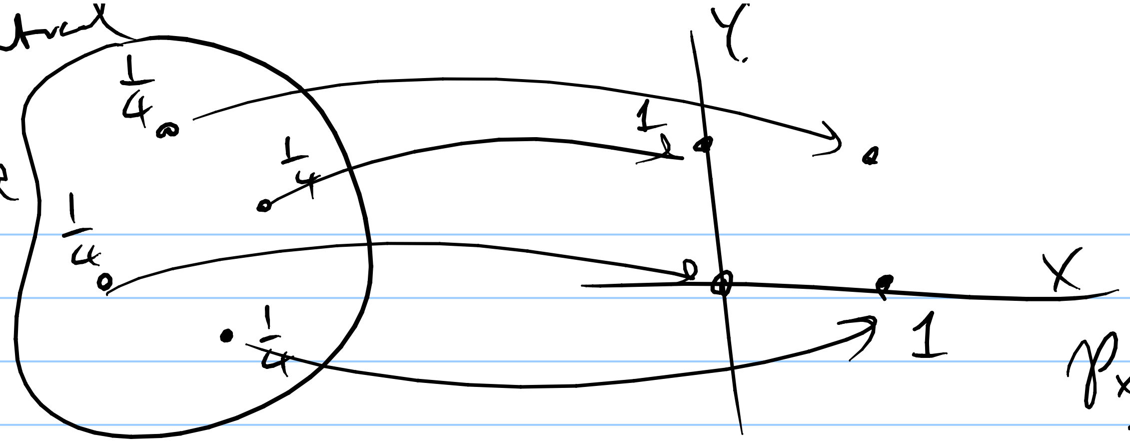
st $P(A) = 1$.

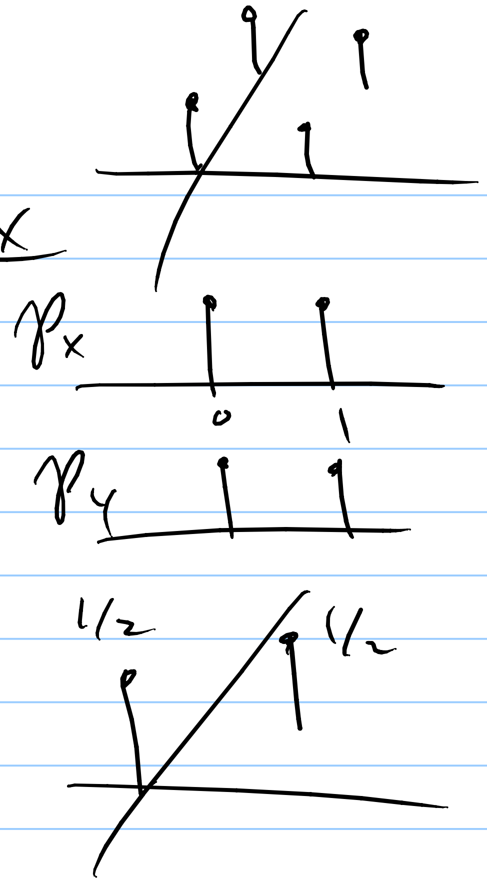


X, Y are identical. $\underline{F_X(u)} = \underline{F_Y(u)}$ $\forall u$.

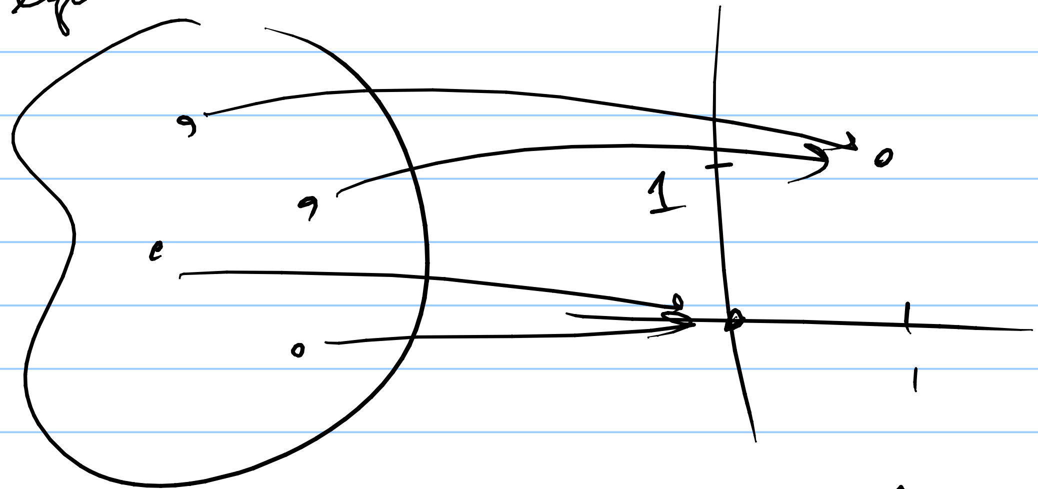
X, Y Ber.

X, Y identical
not equal



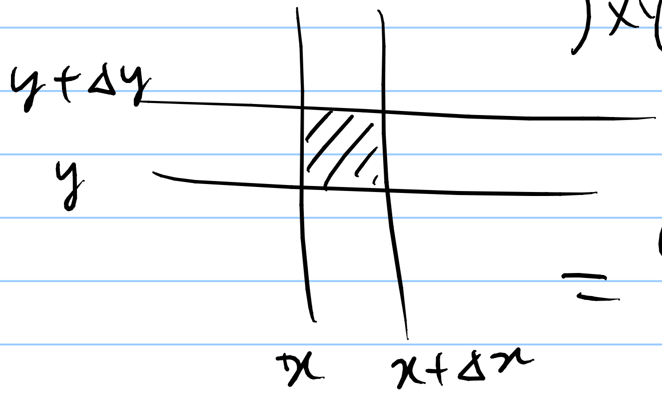


X, Y equal.



Equal $\Rightarrow$ identical
$\not\Leftarrow$

Joint pdf (jointly cts case)



$$f_{XY}(x,y) = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \frac{P(x < X \le x+\Delta x,\ y < Y \le y+\Delta y)}{\Delta x \Delta y}$$

$$= \lim \frac{\left(F_{XY}(x+\Delta x, y+\Delta x) - F_{XY}(x+\Delta x, y)\right) - \left(F_{XY}(x, y+\Delta y) - F_{XY}(x,y)\right)}{\Delta x \Delta y}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{\partial F_{XY}(x+\Delta x, y)}{\partial y} - \dfrac{\partial F_{XY}(x,y)}{\partial y}}{\Delta x}$$

$$= \frac{\partial^2 F_{XY}(x,y)}{\partial x \partial y}.$$

Properties. (a) $f_{XY}(x,y) = \dfrac{\partial^2 F(x,y)}{\partial x \partial y}$.

(b) $F_{XY}(x,y) = \displaystyle\int_{-\infty}^{x}\int_{-\infty}^{y} f_{XY}(u,v)\,dv\,du$

(c) $f_{XY}(x,y) \geq 0$ can be ≥ 1.

(d) $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy = 1$.

(e) $\displaystyle\int_{-\infty}^{\infty} f_{XY}(x,y)\,dy = f_X(x)$

$\displaystyle\int \qquad dx = f_Y(y)$

(f) $P((X,Y) \in B) = \iint_B f_{XY}(x,y)\,dx\,dy$