노트 제목 2008-05-13

$$p_{\underline{X}}(\underline{x}) = p_{X_1 X_2}(x_1, x_2)\, p_{X_3 X_4 X_5 | X_1 X_2}(x_3\, x_4\, x_5 | x_1, x_2)$$

$$p_{X_6 | X_1 \cdots X_5}(x_6 | x_1 \cdots x_5) \cdots$$

Indep $\quad p_{\underline{X}}(\underline{x}) = \prod p_{X_i}(x_i)$

$\quad\quad F \qquad\qquad F$

$$p_{\underline{X}}(\underline{x}) = p_{X_1 \cdots X_m}(x_1, \cdots, x_m)\, p_{X_{m+1} \cdots X_n}(\quad)$$

iid (indep & identically dist)

$$p_{\underline{X}}(\underline{x}) = \prod p_X(x_i)$$

$$W = g(\underline{X})$$

$$F_W(w) = P(g(\underline{X}) \le w) = \sum_{\underline{x}:\, g(\underline{x}) \le w} p_{\underline{X}}(\underline{x})$$

$$p_W(w) = P(g(\underline{X}) = w) = \sum_{\underline{x}:\, g(\underline{x}) = w} p_{\underline{X}}(\underline{x})$$

$$Y = \max(X_1, \dots, X_n)$$

iid $F_Y(y) = \left(F_X(y)\right)^n$ $\quad f_Y(y) = n\left(F_X(y)\right)^{n-1} f_X(y)$

$$W = \min(X_1, \dots, X_n)$$

iid $F_W(w) = 1 - \left(1 - F_X(w)\right)^n$

$$f_W(w) = n\left(1 - F_X(w)\right)^{n-1} f_X(w)$$

$$E\, g(\underline{X}) = \begin{cases} \sum_{\underline{x}} g(\underline{x})\, p_{\underline{X}}(\underline{x}) \\ \int g(\underline{x})\, f_{\underline{X}}(\underline{x})\, d\underline{x} \end{cases}$$

Indep rvs.

$$E\, g(X_1) \cdots g_n(X_n) = E\, g(X_1) \cdots E\, g(X_n)$$

$$\underline{Y} = \underline{g}(\underline{X})$$

cts, ctsly diff, one-to-one, invtbl

$$f_{\underline{Y}}(\underline{y}) = f_{\underline{X}}(\underset{\substack{\uparrow \\ g^{-1}}}{\underline{h}}(\underline{y}))\left\|\frac{\partial(\underline{x})}{\partial(\underline{y})}\right\| = \left|\frac{\partial(\underline{y})}{\partial(\underline{x})}\right|^{-1}$$

$$f_{\underline{Y}}(\underline{y})\,|\underline{\underline{\Delta y}}| = f_{\underline{X}}(\underline{x})\,|\underline{\underline{\Delta x}}|$$

$$\underline{Y} = A\underline{X} + b$$

$$f_{\underline{Y}}(\underline{y}) = \frac{f_{\underline{X}}(A^{-1}(y-\underline{b}))}{|\det A|}$$

$$\mu_{\underline{X}} = E\underline{X} = \begin{bmatrix} EX_1 \\ \vdots \\ EX_n \end{bmatrix}$$

$$R_{\underline{X}} = E\underline{X}\underline{X}^t = \begin{bmatrix} EX_1^2 & EX_1X_2 & \cdots \\ EX_2X_1 & EX_2^2 & \cdots \\ \vdots & \vdots & \\ & & EX_n^2 \end{bmatrix}$$

$$C_{\underline{X}} = E(\underline{X} - \underline{\mu}_x)(\underline{X} - \underline{\mu}_x)^t$$

$$= \begin{bmatrix} \sigma_1^2 & \mathrm{cov}(X_1X_2) & \cdots \\ \mathrm{cov}(X_1X_2) & \sigma_2^2 & \cdots \\ \vdots & & \ddots \\ & & & \sigma_n^2 \end{bmatrix} = R_{\underline{X}} - \mu_{\underline{X}}\mu_{\underline{X}}^t$$

symm. $\rightarrow$ real. eival, orthonormal eivecs

non-neg def $\rightarrow$ non neg eival. ///

$$\underline{Y} = A\underline{X} + \underline{b}$$

$$E\underline{Y} \stackrel{d}{=} \underline{\underline{\mu_Y}} = E(A\underline{X} + \underline{b}) = A\,E\underline{X} + \underline{b}$$

$$= \underline{A\underline{\mu_x} + \underline{b}} \leftarrow$$

$$EY_1 = E(\underline{\underline{A_{11}}}X_1 + \cdots + \underline{\underline{A_{1m}}}X + b_1)$$

$\vdots$

Y_m

$$\underline{\underline{R_{\underline{Y}}}} = E\,\underline{Y}\,\underline{Y}^t = E(A\underline{X}+\underline{b})(A\underline{X}+\underline{b})^t$$

$$= E\left(\underline{A\underline{X}\,\underline{X}^t A^t} + A\underline{X}\,\underline{b}^t + \underline{b}\,\underline{X}^t A^t + \underline{b}\,\underline{b}^t\right)$$

$$= A\,\underbrace{E(\underline{X}\,\underline{X}^t)}_{\underline{\underline{R_{\underline{X}}}}}A^t + A\,\underbrace{E\underline{X}}_{\underline{\underline{\mu_{\underline{X}}}}}\,\underline{b}^t + \underline{b}\,\underbrace{E\underline{X}^t}_{\underline{\underline{\mu_{\underline{X}}}}}A^t + \underline{b}\,\underline{b}^t$$

$$C_{\underline{Y}} = E(\underline{Y}-\mu_{\underline{Y}})(\underline{Y}-\mu_{\underline{Y}})^t$$

$$= E\underbrace{A(\underline{X}-\mu_{\underline{X}})(\underline{X}-\mu_{\underline{X}})^t}A^t$$

$$= A\,C_{\underline{X}}\,A^t$$

Gauss vecs.

$$f_{\underline{X}}(\underline{x}) = \frac{1}{(2\pi)^{\frac{n}{2}} \left|\det(C_{\underline{X}})\right|^{1/2}} \exp\left(-\frac{1}{2}(\underline{x}-\underline{\mu}_{\underline{X}})^t C_{\underline{X}}^{-1} (\underline{x}-\underline{\mu}_X)\right)$$

$$f_X(x) = \frac{1}{(2\pi)^{1/2} \left|\sigma_X^2\right|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu_X)(\sigma_X^2)^{-1}(x-\mu_X)\right)$$

$\underline{X}$ uncorrd. $\Rightarrow$ $$C_{\underline{X}} = \begin{pmatrix} \sigma_1^2 & & & 0 \\ & \sigma_2^2 & & \\ & & \ddots & \\ 0 & & & \sigma_n^2 \end{pmatrix}$$

$$C_{\underline{X}}^{-1} = \begin{pmatrix} 1/\sigma_1^2 & & & 0 \\ & 1/\sigma_2^2 & & \\ & & \ddots & \\ 0 & & & 1/\sigma_n^2 \end{pmatrix}$$

$$\underline{\underline{\exp}}\left(-\frac{1}{2}\left(\underline{\sigma_1^{-2}(x_1-\mu_1)^2} + \underline{\sigma_2^{-2}(x_2-\mu_2)^2} + \cdots + \underline{\sigma_n^{-2}(x_n-\mu_n)^2}\right)\right)$$

$$f_{\underline{X}}(\underline{x}) = \underline{\prod f_{X_i}(x_i)}$$

indep $\Rightarrow$ uncorrel

$\not\Leftarrow$

$\Leftarrow$ (jG.)

$$\underline{Y} = A\underline{X} + \underline{b} \qquad \underline{X} = A^{-1}(\underline{Y} - \underline{b})$$

t jG.

$\underline{Y}$ - jG. $\quad f_{\underline{Y}}(\underline{y}) \quad f_{\underline{X}}(\underline{x})$

$\underline{X}$ iid $N(0,1)$ $\quad \mu_{\underline{X}} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$

$$C_{\underline{X}} = \begin{pmatrix} 1 & & 0 \\ & 1 & \\ & & \ddots \\ 0 & & & 1 \end{pmatrix} = I_n.$$

$\underline{Y} = A\underline{X} + \underline{b}$

$$\mu_{\underline{Y}} = A\mu_{\underline{X}} + \underline{b} = \boxed{\underline{b}} \leftarrow$$

$$C_{\underline{Y}} = A\,\underline{C}_X A^t = \boxed{AA^t} \leftarrow$$

If we want to have $\underline{Y}$ jG w/ mean $\underline{\mu}$ cov C

$\underline{b} = \underline{\mu}$, A st $AA^t = C$

$$C = UDU^t \qquad D = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$

eigen val of C

$$\underline{\underline{U}} = \begin{bmatrix} | & & | \\ \underline{u}_1 & \cdots & \underline{u}_n \\ | & & | \end{bmatrix}$$

orthonormal.

eigen vecs of C

Unitary

$$U^t = U^{-1} \qquad UU^t = \underline{I}$$

$$D^{1/2} = \begin{bmatrix} \sqrt{\lambda_1} & & & 0 \\ & \sqrt{\lambda_2} & & \\ & & \ddots & \\ 0 & & & \sqrt{\lambda_n} \end{bmatrix}$$

$$\underline{\underline{C}} = U D^{1/2} D^{1/2} U^t$$

$$= \underbrace{U D^{1/2}}_{A} \underbrace{\left(U D^{1/2} \right)^t}_{A^t}$$

$$= \underline{\underline{U D^{1/2} U^t}} \left(\underline{\underline{U D^{1/2} U^t}} \right)^t$$

$U D^{1/2}$

$U D^{1/2} U^t$

$U D^{1/2} U^t U^t$

Sums of rvs $W_n = X_1 + \cdots + X_n$

$EW_n = EX_1 + \cdots + EX_n$

$$\mathrm{Var}(W_n) = E\left(X_1 + \cdots + X_n - EX_1 - \cdots - EX_n\right)^2$$

$$= E\left(\sum_{i=1}^{n} \underline{(X_i - EX_i)}\right)^2$$

$$= \sum_{i=1}^{n} \underline{\mathrm{var}(X_i)} + \sum_{i \neq j}\sum \mathrm{cov}(X_i, X_j)$$

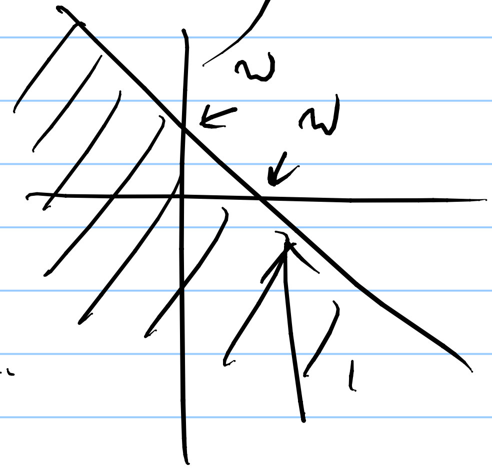
$$\left(\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\mathrm{Cov}(X,Y)\right)$$

X_i's uncorrel. $\quad \mathrm{Var}(W_n) = \sum \mathrm{Var}(X_i)$

$W = X+Y$, $\quad \underline{g(X,Y)}$

$$F_W(w) = P(X+Y \le w)$$

$$= \iint f_{X,Y}(x,y)\, dx\, dy.$$



$$x+y \le w$$

$$= \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\underline{w-y}} f_{XY}(x,y)\,dx\,dy}$$

$$f_W(w) = \frac{d}{dw} F_W(w)$$

$$= \int_{-\infty}^{\infty} \underbrace{\frac{d}{dw}\left(\int_{-\infty}^{\underline{\underline{w-y}}} f_{XY}(x,y)\,dx\right)}\,dy$$

$$= f_{XY}(w-y,\, y)$$

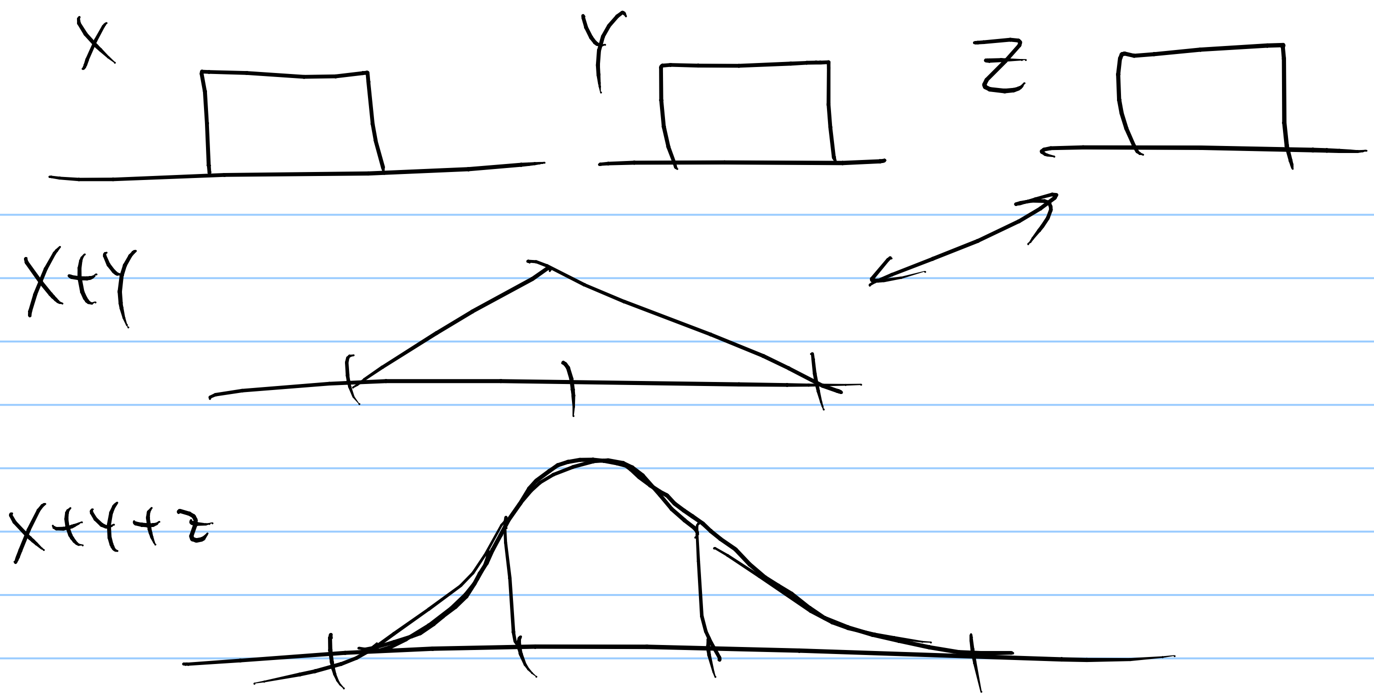
$$\left(\frac{d}{dw}\int_{a}^{\underline{g(w)}} f_{XY}(x,y)\,dx = f_{XY}(\underline{\underline{g(w)}}, y)\cdot\frac{dg(w)}{dw}\right)$$

$$f_W(w) = \begin{cases} \int_{-\infty}^{\infty} f_{XY}(w-y, y)\, dy \\ \int_{-\infty}^{\infty} f_{XY}(x, w-x)\, dx. \end{cases}$$

X, Y indep Convolution.

$$f_W(w) = \begin{cases} \int_{-\infty}^{\infty} f_X(w-y) f_Y(y)\, dy. \\ \int_{-\infty}^{\infty} f_X(x) f_Y(w-x)\, dx \end{cases}$$

$$p_W(w) = \begin{cases} \sum p_X(w-y)\, p_Y(y) \\ \sum p_X(x)\, p_Y(w-x) \end{cases}$$



Moment generating fn. (mgf)

$$\phi_X(s) = E e^{sX}$$

$$= \int_{-\infty}^{\infty} e^{sx} \underline{\underline{f_X(x)}}\, dx$$

$$\left(\sum_{x} e^{sx} p_X(x) \right)$$