노트 제목 2008-05-20

mgf: $\phi_X(s) = E e^{sX}$

$$\left.\frac{d^n \phi_X(s)}{ds^n}\right|_{s=0} = EX^n$$

$Y = aX + b$ $\qquad \phi_Y(s) = e^{bs}\phi_X(as)$

$W = X + Y$ $\qquad \phi_W(s) = \phi_X(s)\,\phi_Y(s)$

indep

$W = X_1 + \cdots + X_n$

indep Poiss.

$$\phi_W(s) = e^{(\alpha_1 + \cdots + \alpha_n)(e^s - 1)}$$

$$\phi_W(s) = e^{(\mu_1 + \cdots + \mu_n)s + \frac{(\sigma_1^2 + \cdots + \sigma_n^2)s^2}{2}}$$

$$\phi_X(j\omega) = E\, e^{j\omega X} = \int f_X(x)\, e^{j\omega x}\, dx$$

Char fn of X

$$\underline{\underline{R}} = \sum_{i=1}^{N} X_i \quad N \text{ rv independent of } X_i\text{'s}$$

$$ER = \underbrace{EE(R|\underline{\underline{N}})} = \mu_X \mu_N$$

$$Var(R) = \mu_N Var(X) + \underline{Var(N)\mu_X^2}$$

$$\underline{\underline{\phi_R(s)}} = \underline{\underline{\phi_N}}(\ln \underline{\underline{\phi_X}}(s))$$

Central limit thm

$$Var\left(\sum^{n} X_i\right) = n\sigma_X^2 \longrightarrow \infty$$

$$\mathrm{Var}\left(\frac{1}{n}\sum^{n} X_i\right) = \frac{\sigma_X^2}{n} \to 0$$

$$\frac{1}{n}\sum^{n} X_i \xrightarrow{\text{WLLN}} EX = \mu_X$$

$$\mathrm{Var}\left(\frac{1}{\sqrt{n}}\sum^{n} X_i\right) = \sigma_X^2$$

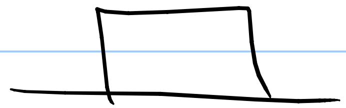
$$\frac{1}{\sqrt{n}}\sum^{n} X_i \to G \text{ rv w/ mean } \sqrt{n}\,\mu_X$$

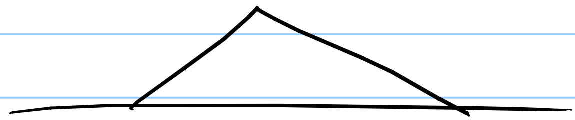
$$\text{Var } \sigma_X^2$$

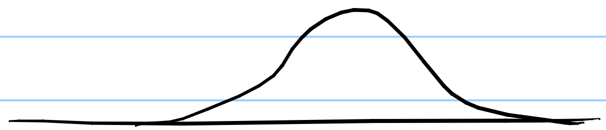
$$\frac{1}{\sqrt{n}}\sum^{n} \frac{X_i - \mu_X}{\sigma_X} \longrightarrow N(0,1)$$

Moment $\quad EX^n$ nth moment

$E(X-\mu_X)^n$ nth central moment







5, 6

$X_1 \neq X_2 = \dots\dots$

Example: Service time Unif $\underline{[0,1]}$ hour.

50 customers. X_i

$$P(30 \le \underline{W} \le 40)$$

$$EW = 50 \cdot \frac{1}{2} = 25 \text{ hrs.}$$

$$\mathrm{Var}(W) = 50 \cdot \frac{1}{12} = \frac{25}{6}$$

$$P(30 \le W \le 40) = \Phi\left(\frac{40-25}{\sqrt{\frac{25}{6}}}\right) - \Phi\left(\frac{30-25}{\sqrt{\frac{26}{6}}}\right)$$

Cdf of G.

Markov ineq. $\underline{X \geq 0}$

$$\underline{P(X \geq c^2)} \leq \frac{EX}{c^2}$$

pf: $EX = \underbrace{\int_0^{c^2} x f_X(x)\,dx}_{} + \int_{c^2}^{\infty} \underline{x} f_X(x)\,dx.$

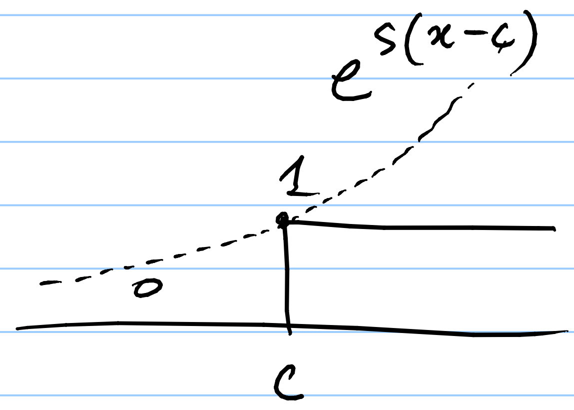
$$\geq \int_{c^2}^{\infty} c^2 f_X(x)\,dx = c^2 P(X \geq c^2)$$

Chebychev. ineq

$$\underline{P(|Y-\mu_Y| \geq c)}$$

$$= P\left((Y-\mu_Y)^2 \geq c^2\right) \leq \frac{E(Y-\mu_Y)^2}{c^2} = \underbrace{\frac{Var(Y)}{c^2}}$$

Chernoff bound



$$P(X \geq c) = \int_c^{\infty} f_X(x)\,dx$$

$$= \int_{-\infty}^{\infty} u(x-c) f_X(x)\,dx$$

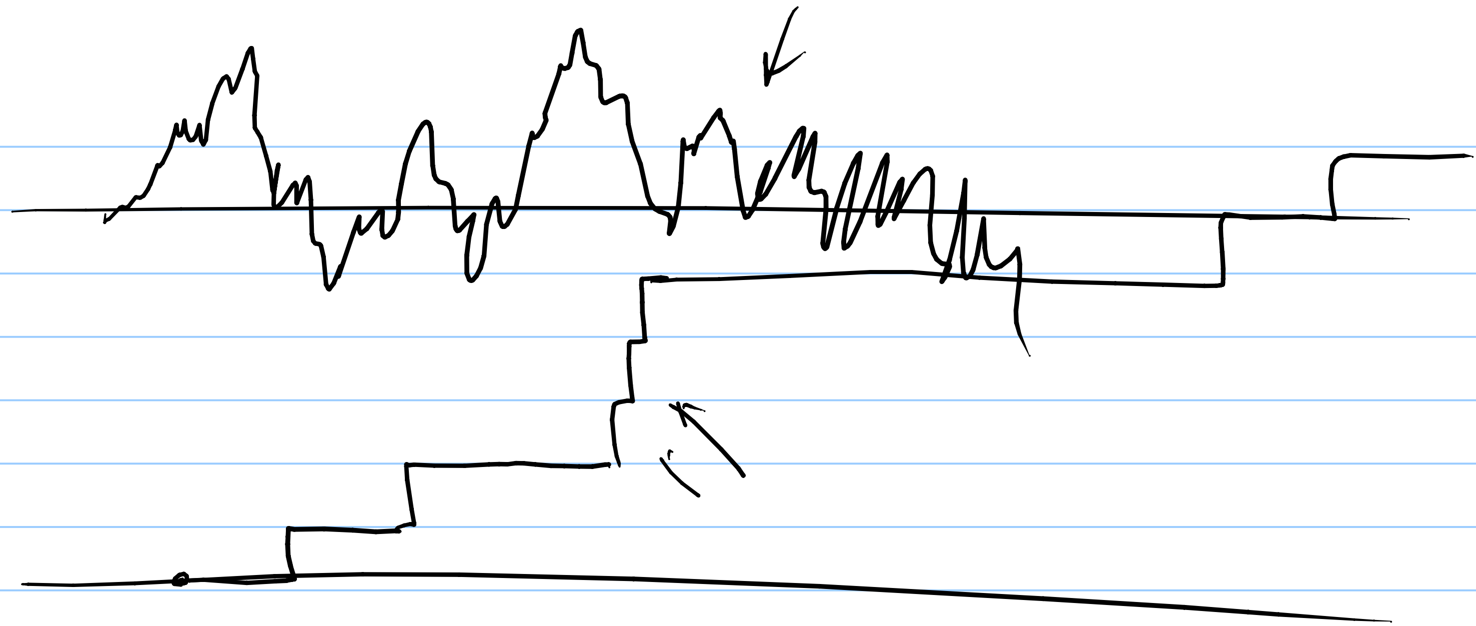
$$\leq \int_{-\infty}^{\infty} e^{s(x-c)} f_X(x)\,dx = e^{-sc} \underbrace{\int_{-\infty}^{\infty} e^{sx} f_X(x)\,dx}_{\phi_X(s)}$$

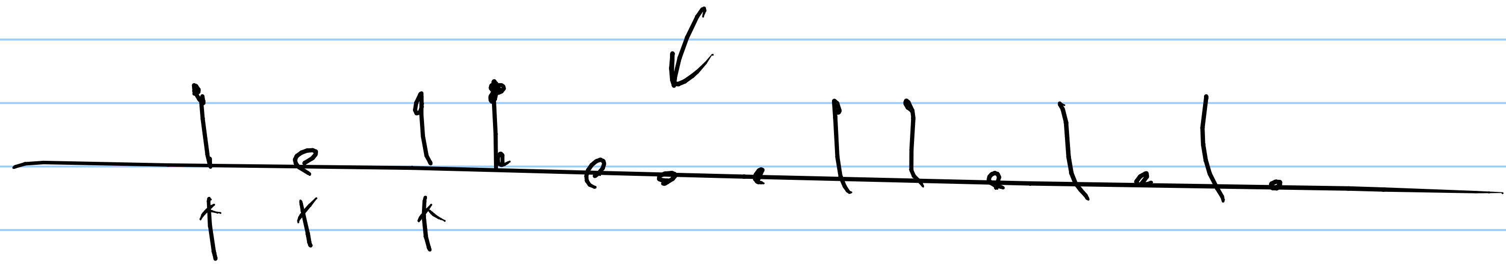
$$P(X \geq c) \leq \min_{s \geq 0} e^{-sc} \phi_X(s)$$

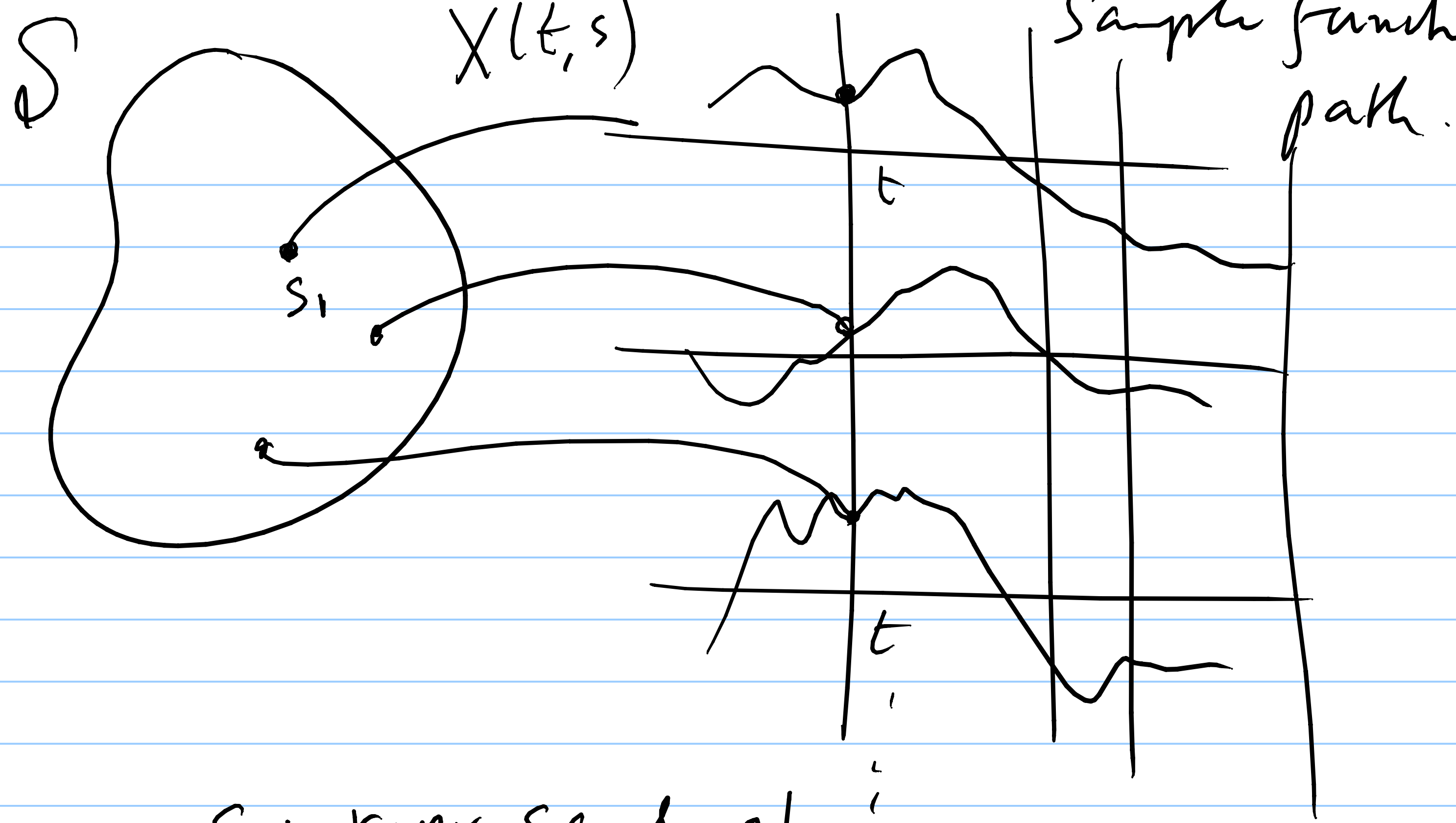
Ch 10 Stochastic Process

Random sequence function

$X \qquad \underline{X} = \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix} \qquad X(t)$







s : repr sample pt

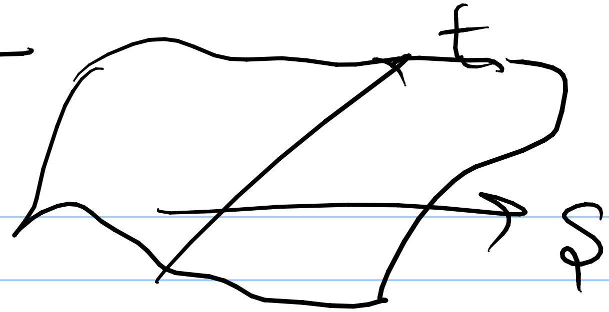
t : index (time) $X(t,s)$

$X(t,s) : S \to F$ function space ←

$X(t,s)$: indexed family of rvs ←

$$X(t,s): T \times S \to \mathbb{R}$$

$$(t,s) \longrightarrow X(t,s)$$

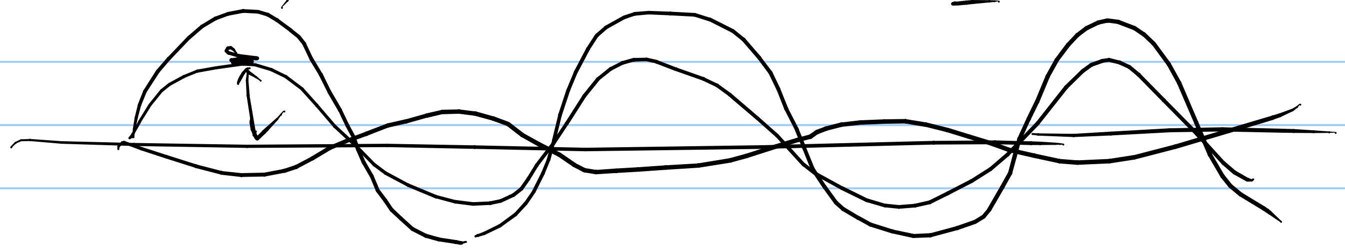


$$\{a,b\} \times \{c,d,e\}$$

$$= \{(a,c),\ (a,d),\ (a,e),\ (b,c),\ (b,d),\ (b,e)\}$$

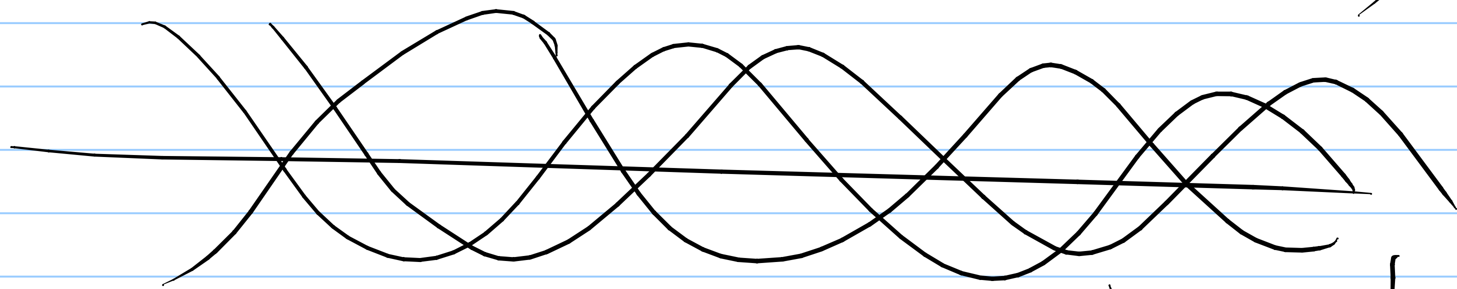
$$F_{X_1 X_2 \cdots X_n \cdots}(x_1, \dots, x_n)$$

Examples: $X(t) = A \cos 2\pi t$ $\quad \underline{A}$: unif$[-1,1]$



$X(0) = \underline{\underline{A}}$ $\quad X(\frac{1}{8}) = \underline{\underline{\frac{\sqrt{2}}{2}A}}$ $\quad , X(\frac{1}{4}) = 0$

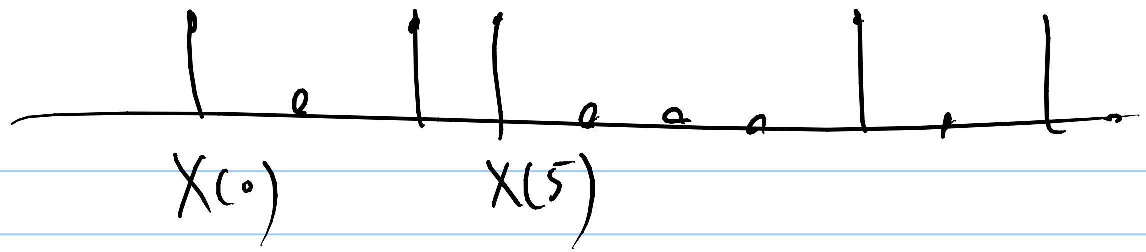
$X(t) = \cos 2\pi(t + \theta)$ $\quad$ unif $[0,1)$

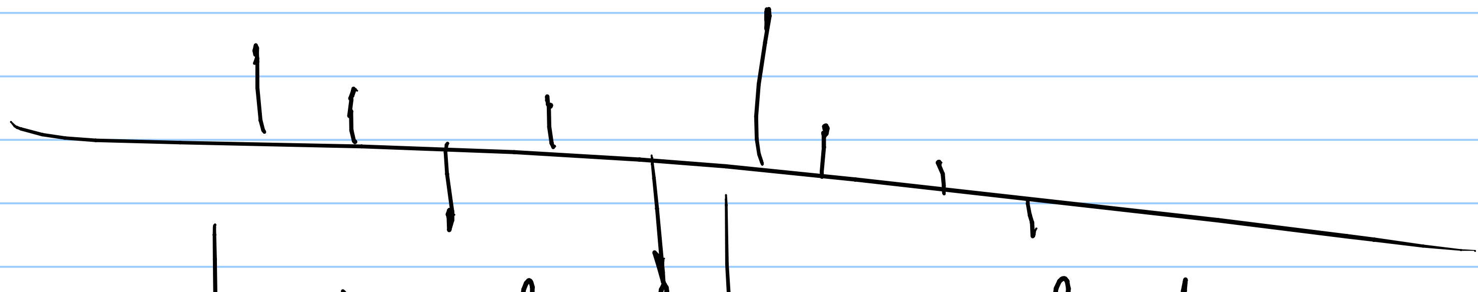


$\underline{\underline{X(0)}} = \cos 2\pi\theta$

$X(\frac{1}{8}) = \cos(2\pi\theta + \frac{\pi}{4})$

$X(\frac{1}{4}) = \cos(2\pi\theta + \frac{\pi}{2}) = \sin 2\pi\theta$



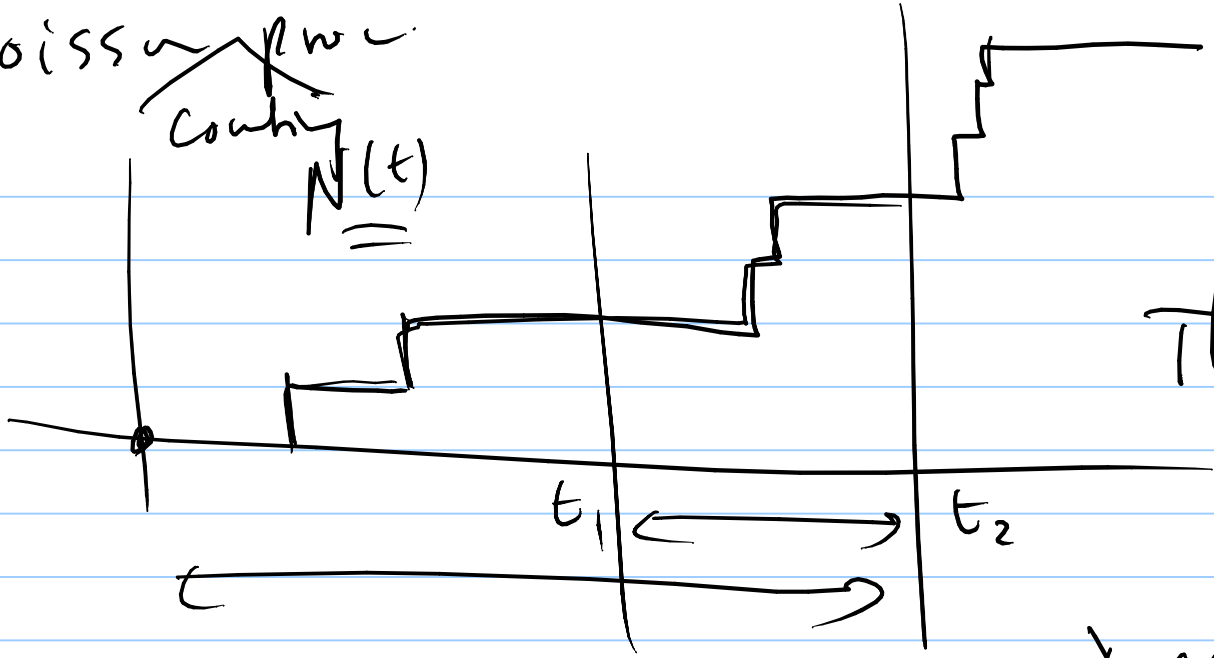


	cts valued	disc valued
cts tm	cvct	dvct
disc tm	cvdt	dvdt

Poisson proc.

counting

$\underline{N}(t)$

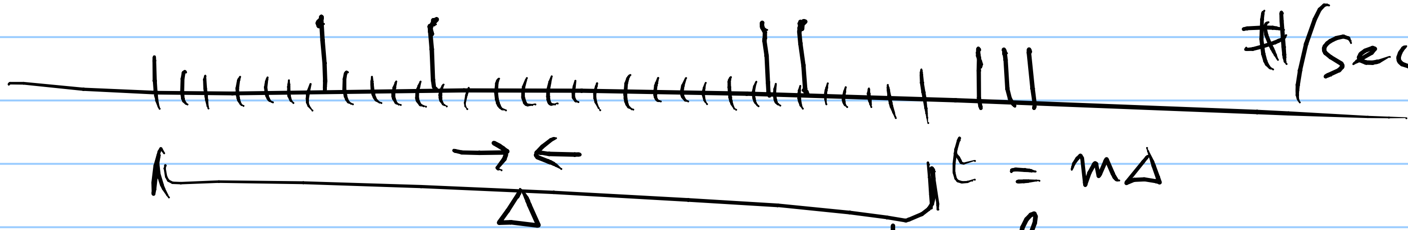


$\lambda(\underline{t_2 - t_1})$

Thm 2.8.

λ arrival rate

#/sec.



$t = m\Delta$

Δ

$p = \lambda\Delta$

m intervals

n arrivals

$$p_{\underline{N_m}}(n) = \binom{m}{n} (\lambda\Delta)^n (1-\lambda\Delta)^{m-n}.$$

$$\to \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$

$$N_m : B_{in} \xrightarrow[\delta \to 0]{} N(t) \text{ Poiss.}$$