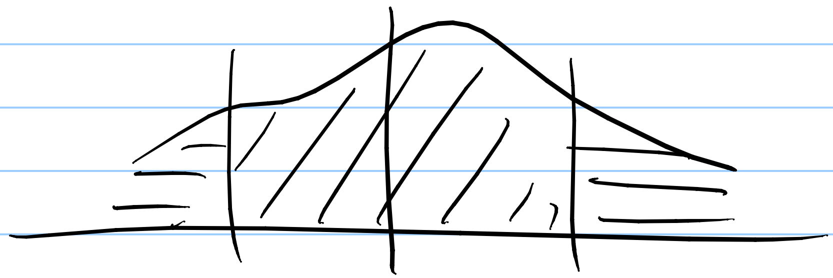
노트 제목 2008-05-22

Markov ineq $P(X \geq c^2) \leq \dfrac{EX}{c^2}$

$$P(X \geq 0) = 1.$$

Chebychev ineq $P(|Y - \mu_Y| \geq c) \leq \dfrac{Var(Y)}{c^2}$



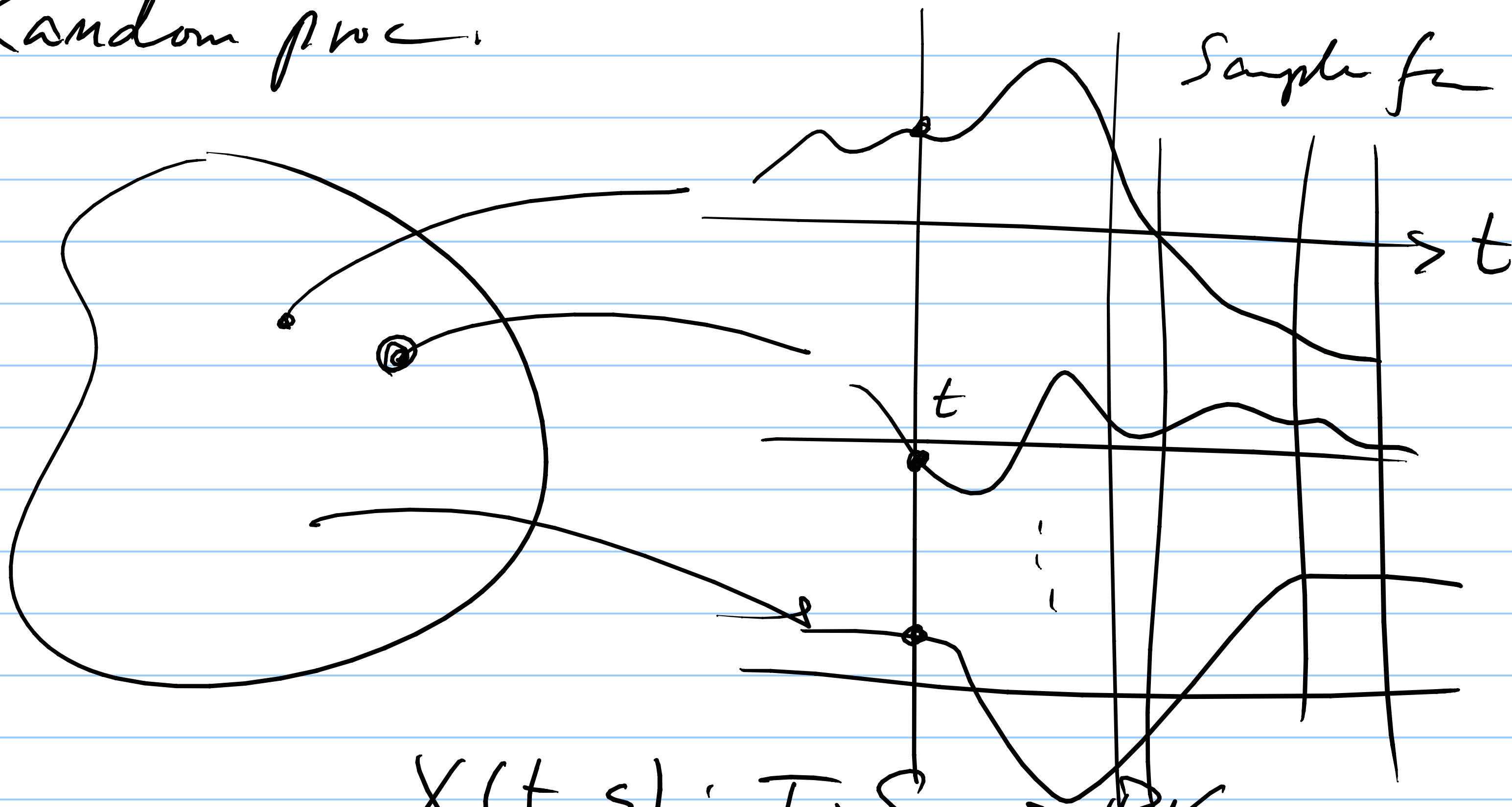
μ_Y

Chernoff bd-

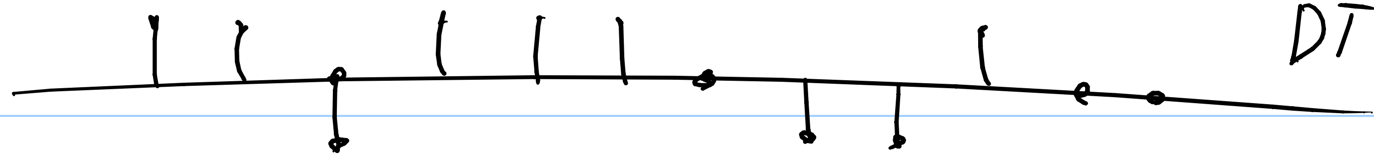
$$P(X \geq c) \leq \min_{s \geq 0} e^{-sc}\phi_X(s) \leq e^{-sc}\phi_X(s)$$

mgf

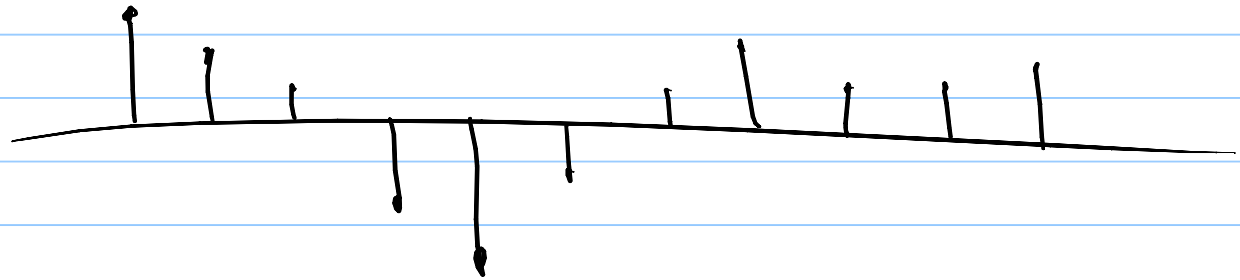
Random proc.



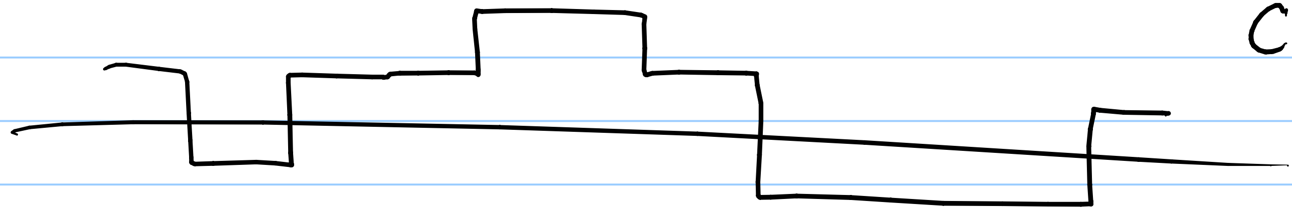
$X(t, s) : T \times S \to \mathbb{R}$



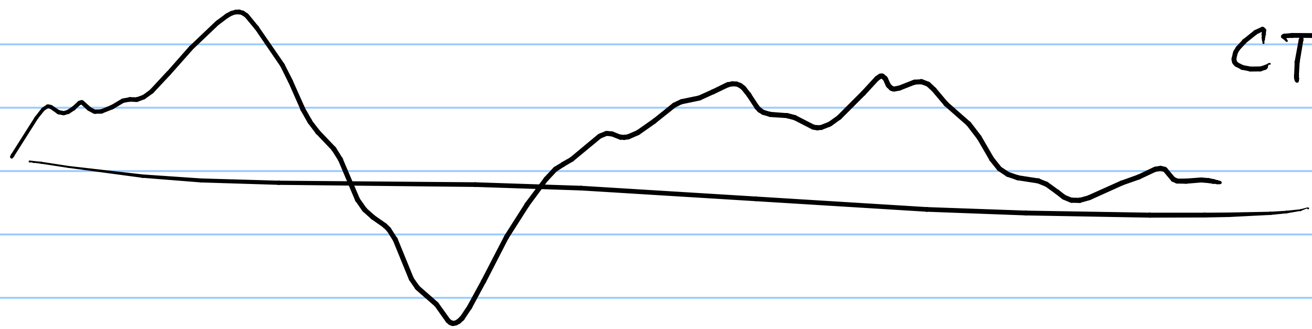
DTDV



DTCV

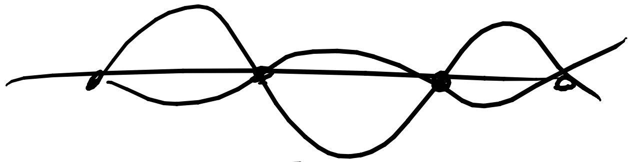


CTDV

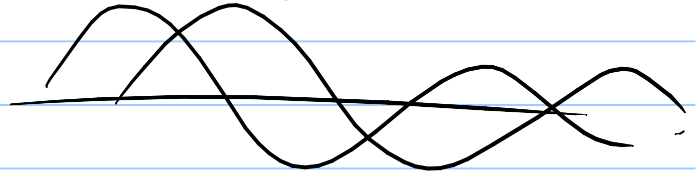


CTCV

$X(t) = A \cos 2\pi t$

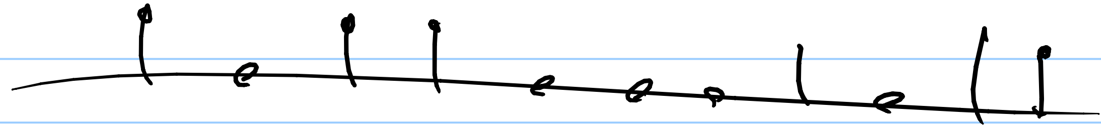


$X(t) = \cos 2\pi (t + \theta)$

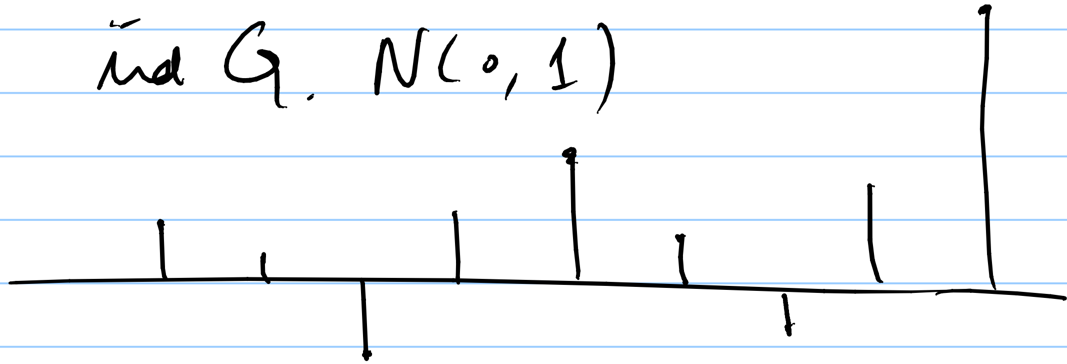


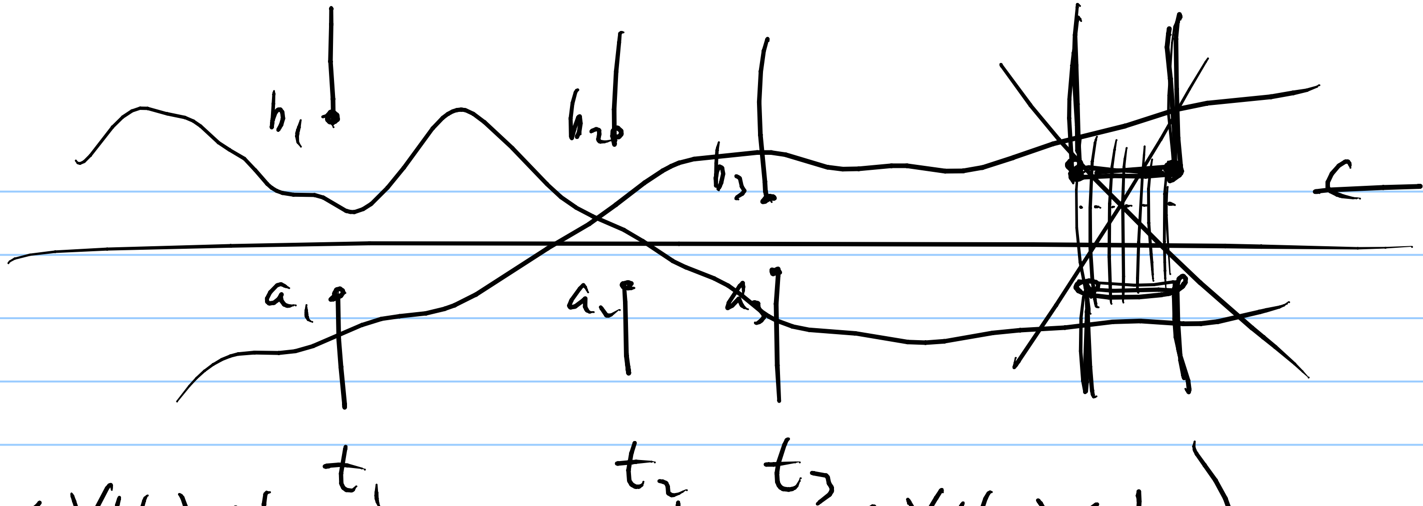
unif rv $(-\pi, \pi]$

$X(t) = X_t$ iid Ber rvs = Ber rp.



iid G. $N(0,1)$





$$P\left(a_1 \le X(t_1) \le b_1,\ a_2 \le X(t_2) \le b_2,\ a_3 \le X(t_3) \le b_3\right)$$

$$f_{X(t_1)X(t_2)X(t_3)}(x_1, x_2, x_3)$$

Poisson Ctg proc. $N(t)$

(1) $N(t_1) - N(t_0)$ is Poi w/ mean $\lambda(t_1 - t_0)$

(2) nonoverlapping $[t_0, t_1]$ & $[t_0', t_1']$

$N(t_1) - N(t_0)$ & $N(t_1') - N(t_0')$ indep

$$P(X(t)=n) = \overset{m}{\binom{m}{n}} p^n (1-p)^{m-n}$$

$$\Delta \to 0 \qquad \underset{0}{\lambda \underset{\downarrow}{\Delta}} = \underset{\downarrow \atop 0}{p} \qquad \lambda = \frac{p}{\Delta}$$

$$\to \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$

Indep increment condition

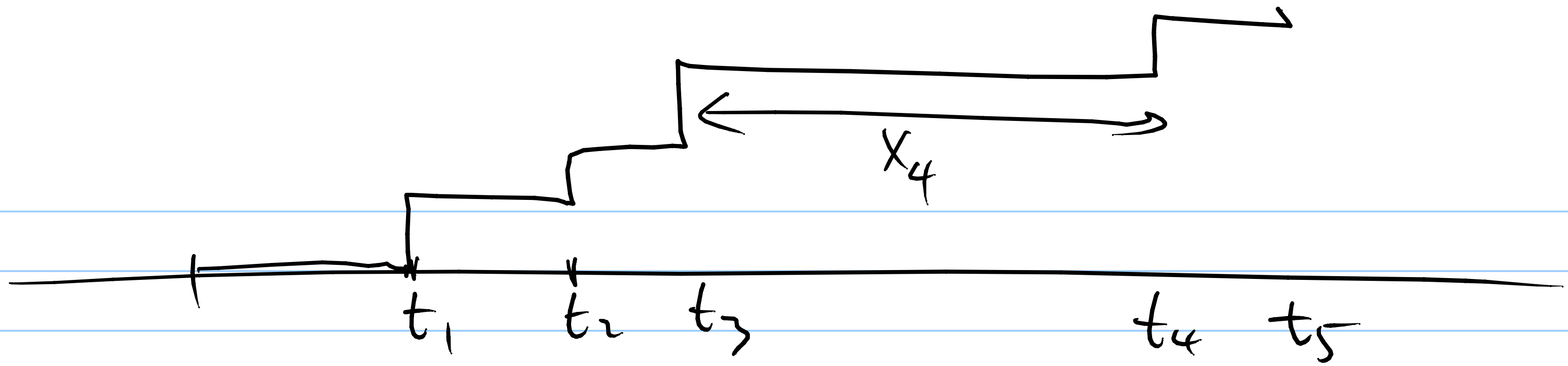
<u>indep time proc</u>

$$p_{N(t_1)\cdots N(t_k)}(n_1, \dots, n_k) \qquad N(0)=0$$

$$= p_{N(t_1)-N(0), \dots, N(t_k)-N(t_{k-1})}(n_1-0, \dots, n_k-n_{k-1})$$

$$\left(\begin{aligned} &P(X+Y=a, X=b) \\ &= P(Y=a-b, X-0=b-0) \end{aligned} \right)$$

$$= \frac{(\lambda(t_1-0))^{(n_1-0)}}{(n_1-0)!} e^{-\lambda(t_1-0)} \cdots \frac{(\lambda(t_k-t_{k-1}))^{n_k-n_{k-1}}}{(n_k-n_{k-1})!} e^{-\lambda(t_k-t_{k-1})}$$



Point process: $T_1 < T_2 < T_3 < T_4 < T_5$

Inter-arrival time proc: $X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_5$

$T_1 - T_0 \quad T_2 - T_1 \quad T_3 - T_2 \quad \dots \quad T_5 - T_4$

$T_0 = 0$

X_i's are iid Expon.

$$\left(N(t),\ 0 \le t < \infty \right) \sim \left(T_1, T_2, \dots \right)$$

$$\sim (X_1, X_2, \cdots\cdots)$$

$$T_1 = X_1, \quad T_2 = X_1 + X_2, \quad T_3 = X_1 + X_2 + X_3 \cdots$$

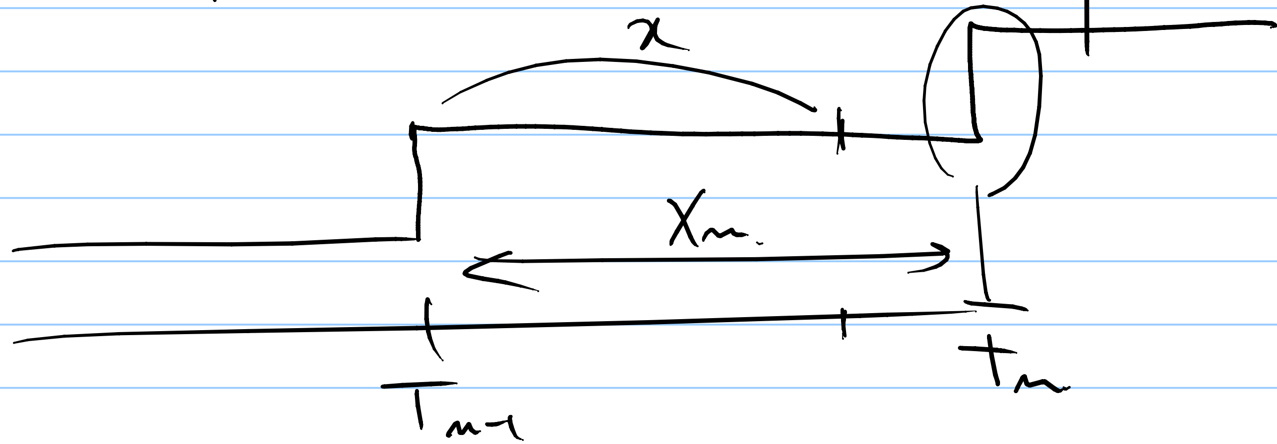
$$T_m - T_{m-1}$$

$$\parallel$$

$$P(X_m \le x) = 1 - P(X_m > x)$$

$$= 1 - P(X_m > x \mid X_1 = x_1, \cdots, X_{m-1} = x_{m-1})$$

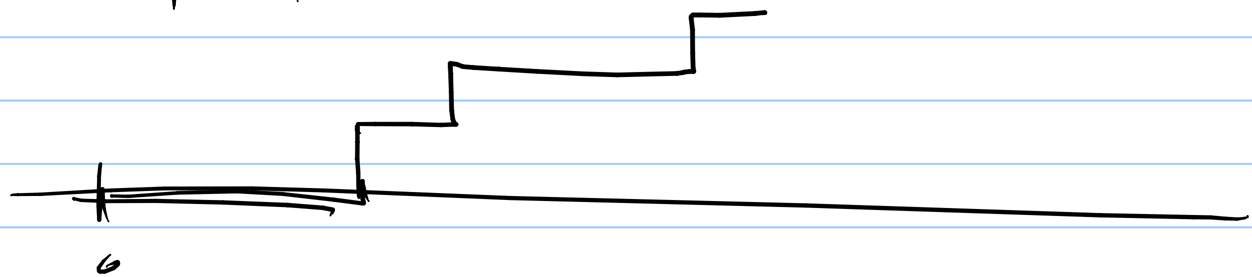
$$= 1 - P(N(t_{m-1} + x) - N(t_{m-1}) = 0 \mid X_1 = x_1 \cdots X_{m-1} = x_{m-1})$$

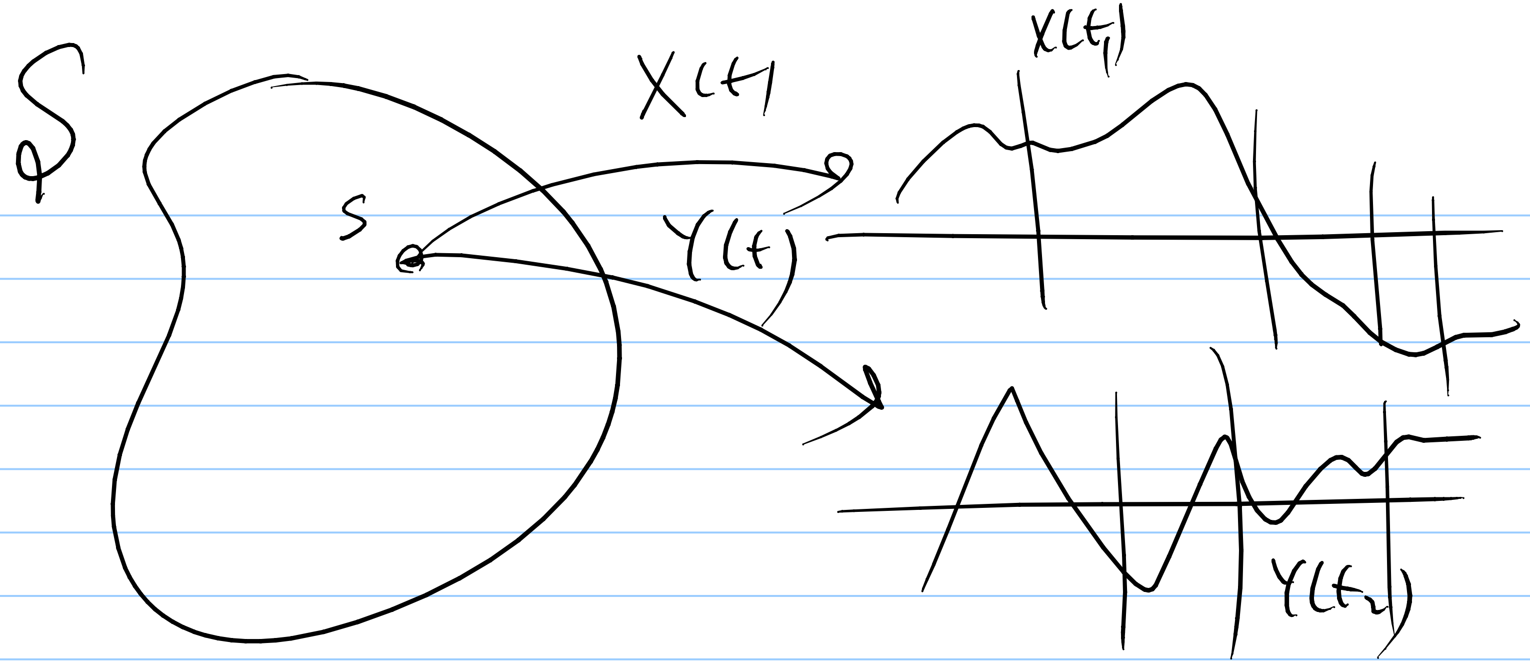


$$= 1 - P\Big(\underbrace{N(t_{n-1}+x) - N(t_{n-1}) = 0}_{\begin{cases} \dfrac{(\lambda x)^{\overset{=1}{0}}}{0!_{=1}}\, e^{\lambda x}, & x > 0 \\ 0, & x \le 0 \end{cases}}\Big)$$

$$= \begin{cases} 1 - e^{\lambda x}, & x > 0 \\ 0, & \text{else.} \end{cases} \quad \text{expon cdf}$$

$X_1, X_2, \ldots$





$$\left(\underline{X(t_1)\, X(t_2)\, Y(u_1)\, Y(u_2)\, Y(u_3)} \right)$$

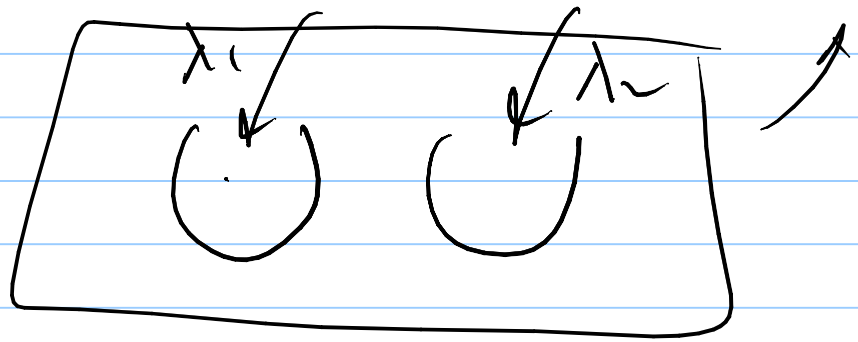
$$f_{X(t_1)\cdots X(t_n)\, Y(u_1)\cdots Y(u_m)}^{\underline{X(t),\ Y(t)}}(x_1, \cdots, x_n, y_1, \cdots, y_m)$$

$$= f_{X(t_1),\cdots X(t_n)}(x_1,\cdots,x_n)\, f_{Y(u_1)\cdots Y(u_m)}(y_1,\cdots,y_m)$$

<u>$X(t)$ & $Y(t)$ are indep.</u>

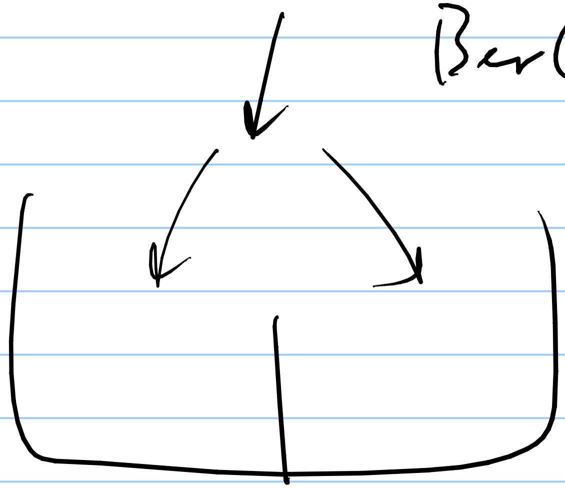
$$\left.\begin{array}{l} N_1(t) \sim \mathrm{Poi}(\lambda_1) \\ N_2(t) \sim \mathrm{Poi}(\lambda_2) \end{array}\right\} \text{indep.}$$

$$N_1(t) + N_2(t) \sim \mathrm{Poi}(\lambda_1 + \lambda_2)$$



$$N(t) \sim \text{Poi}(\lambda) \quad \begin{matrix} \nearrow N_1(t) \sim \text{Poi}(\lambda p) \\ \searrow N_2(t) \sim \text{Poi}(\lambda(1-p)) \end{matrix}$$

$\text{Ber}(p)$



Moments of a r.p.

$$EX(t) = \mu_X(t) \quad \text{mean function.}$$

$$R_X(\underline{t_1}, \underline{t_2}) = EX(t_1)X(t_2) \quad \underline{\text{auto}}\text{-correlation}$$

$$C_X(t_1, t_2) = E\big(X(t_1) - \mu_X(t_1)\big)\big(X(t_2) - \mu_X(t_2)\big)$$

auto-covariance

$$\underline{\underline{t_1 - t_2}} \qquad \underline{(t_1+\tau) - (t_2+\tau)}$$

$$\tau$$

Example: $X(t) = A \cos 2\pi t$

$$\mu_X(t) = \mu_A \cos 2\pi t$$

$$R_X(t_1, t_2) = E\, A \cos 2\pi t_1\, A \cos 2\pi t_2$$

$$= (EA^2)\, \frac{1}{2}\Big(\cos 2\pi(\underline{\underline{t_1 - t_2}}) + \cos 2\pi(\underline{\underline{t_1 + t_2}})\Big)$$

$$X(t) = \cos 2\pi(t+\Theta) \quad \Theta \ \text{unif} \ \left(-\tfrac{1}{2}, \tfrac{1}{2}\right]$$

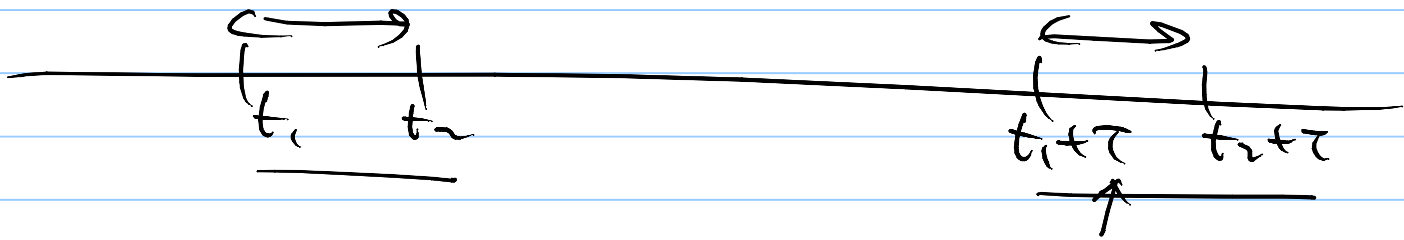
$$\mu_X(t) = E \cos 2\pi(t+\Theta)$$

$$= \int_{-1/2}^{1/2} \cos 2\pi(t+\theta) \cdot 1 \, d\theta = 0$$

$$R_X(t_1,t_2) = \int_{-1/2}^{1/2} \underbrace{\cos 2\pi(t_1+\theta) \cos 2\pi(t_2+\theta)}_{\frac{1}{2}\left(\cos 2\pi(t_1-t_2) + \cos 2\pi(t_1+t_2+2\theta)\right)} 1 \, d\theta$$

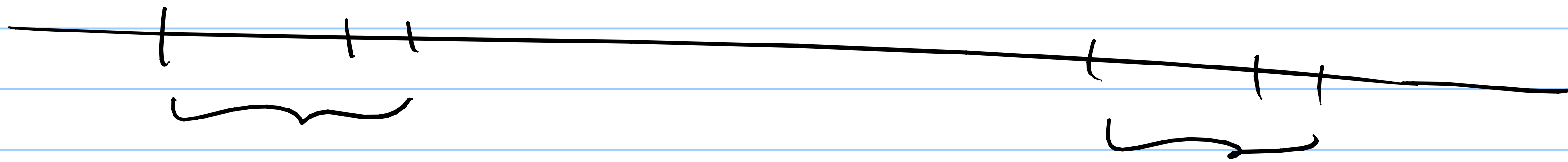
$$= \frac{1}{2} \cos 2\pi(t_1 - t_2)$$

shift invariant



Stationary Process.

$$f_{X(t_1),\dots,X(t_m)}(x_1,\dots,x_m) = f_{X(t_1+\tau),\dots,X(t_m+\tau)}(x_1,\dots,x_m)$$



iid rps.