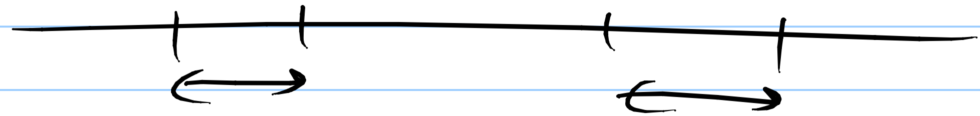
노트 제목 2008-05-29

Poisson

1. $N(t_1) - N(t_0)$ is Poisson rv
2. indep incre.



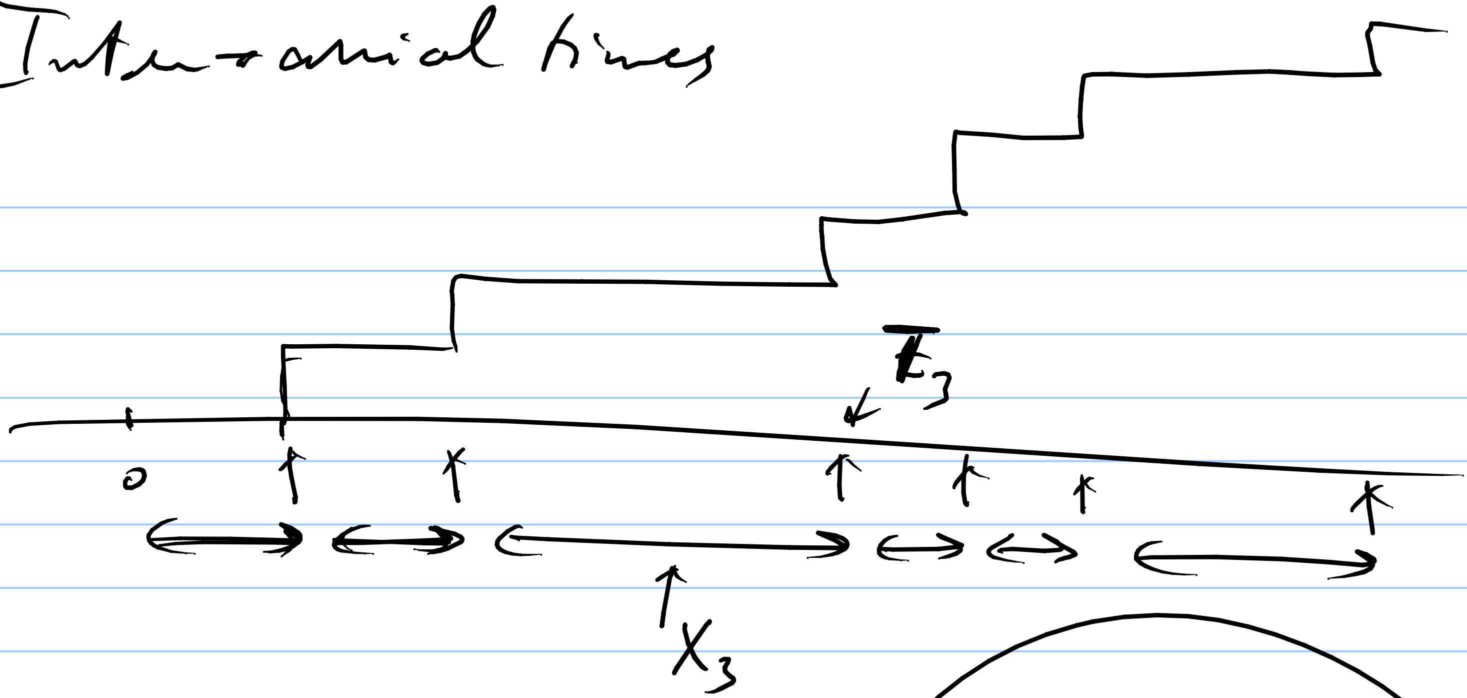
λ : arrival rate.

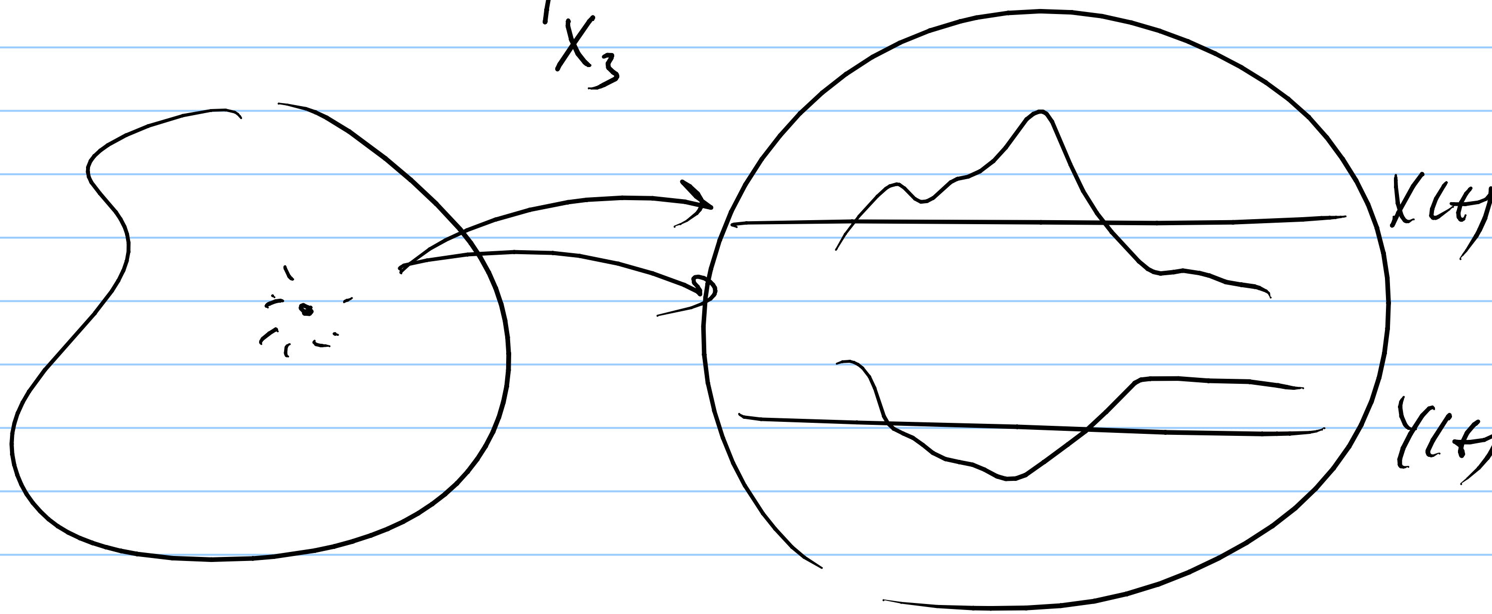
$$p_{N(t_1) \cdots N(t_k)}(n_1, \cdots, n_k)$$

$$= p_{N(t_1)-N(t_0), \cdots, N(t_k)-N(t_{k-1})}(n_1 - n_0, \cdots, n_k - n_{k-1})$$

$N(t_0) = 0$, $n_0 = 0$

Interarrival times

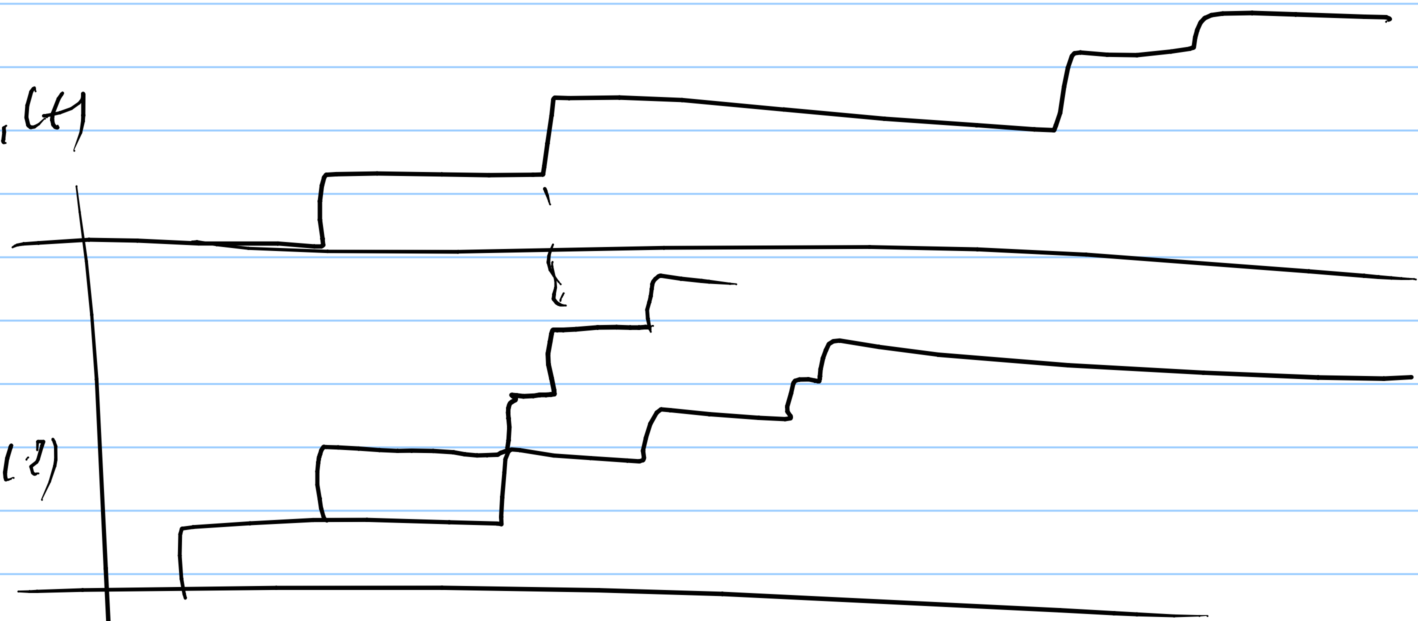




$(X(t_1), \dots, X(t_n)) \quad (Y(u_1) \cdots Y(u_m))$

$N_1(t)$

$N_2(t)$



$N_1(t) + N_2(t)$ $\qquad \lambda_1 + \lambda_2$

$$N(t) \begin{array}{l} \nearrow N_1(t) \\ \searrow N_2(t) \end{array}$$

$\mu_X(t) = EX(t)$

$R_X(t_1, t_2) = EX(t_1)X(t_2)$ $\quad$ $R_{XY}(t_1, t_2) = EX(t_1)Y(t_2)$

$C_X(t_1, t_2) = E(X(t_1) - \mu_X(t_1))(X(t_2) - \mu_X(t_2))$

$C_{XY}(t_1, t_2) =$

$Y \qquad Y$

Example: $X(t) = \cos 2\pi(t + \Theta)$ $\quad [-1/2, 1/2]$

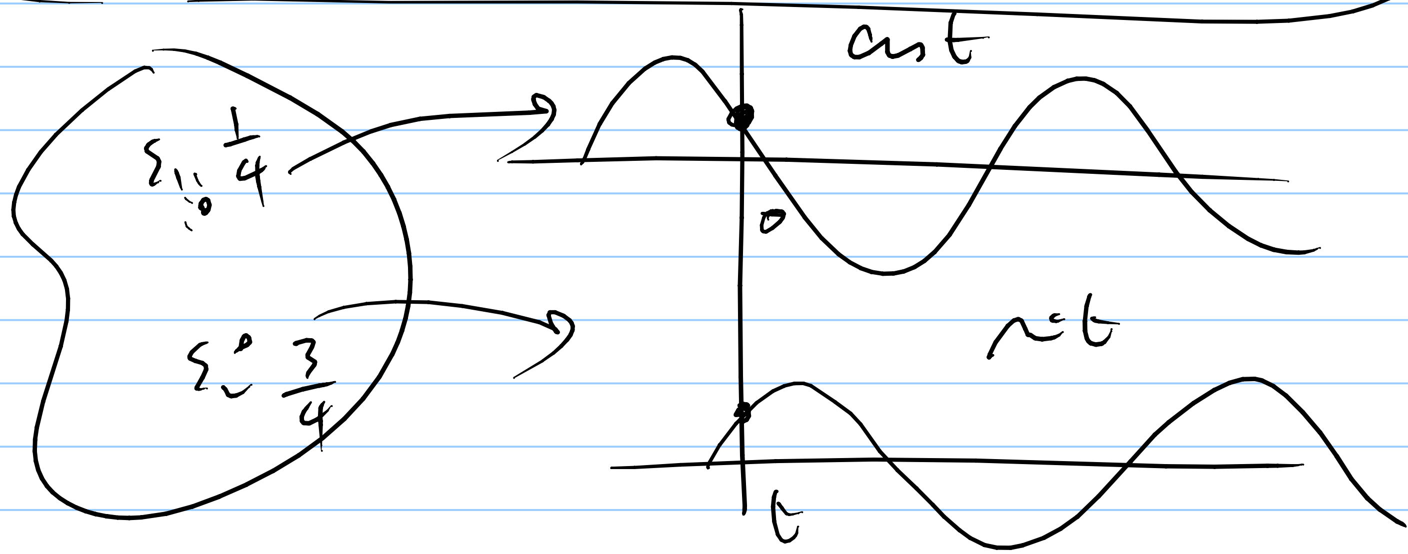
$\mu_X(t) = 0$

$R_X(t_1, t_2) = \frac{1}{2}\cos(t_1 - t_2) = R_X(t_1 + \tau, t_2 + \tau)$

Stationarity

$X(t)$

$$\boxed{f_{X(t_1)\cdots X(t_m)}(\underline{x}_1, \ldots, \underline{x}_m) = f_{X(t_1+\tau)\cdots X(t_m+\tau)}(\underline{x}_1, \ldots, \underline{x}_m)}$$



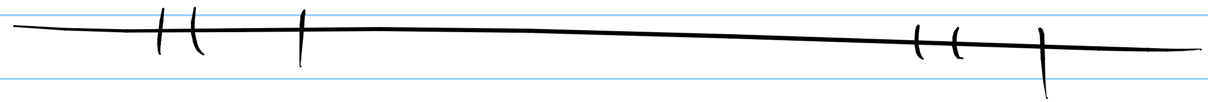
$$EX(t) = \cos t \cdot \frac{1}{4} + \sin t \cdot \frac{3}{4}$$

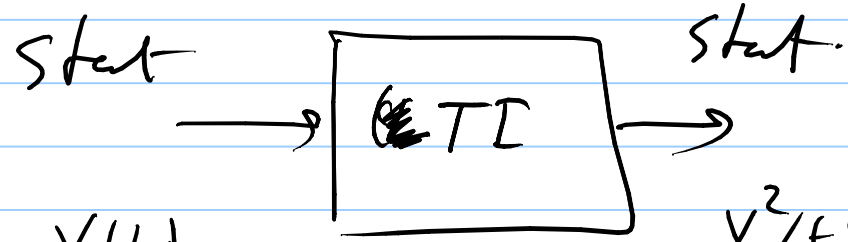
$$\underbrace{E\,X(t_1)X(t_2)}_{\underline{\underline{Y}}} = \cos t_1 \cos t_2 \cdot \underline{\underline{\tfrac{1}{4}}} + \sin t_1 \sin t_2 \cdot \underline{\underline{\tfrac{3}{4}}}$$

$$\underline{\underline{\cos t_1 \sin t_2}}$$

$$= \sum_{\underline{\underline{y}}} y\, p_Y(y)$$

Example: iid





$X(t)$ $\qquad$ $X^2(t)$, $X(t)-X(t-\tau)$, $e^{X(t)}$

$$\begin{cases} \underline{\underline{\mu_X(t)}} = \int_{-\infty}^{\infty} x \, \underline{\underline{f_{X(t)}}}(x)\, dx = \mu_X = \underline{\underline{\mu_X(t+\tau)}} \\ \underline{\underline{R_X(t_1,t_2)}} = \iint_{-\infty}^{\infty} x_1 x_2 \, \underline{f_{X(t_1)X(t_2)}(x_1,x_2)}\, dx_1 dx_2 \end{cases}$$

$$= \iint x_1 x_2 \, f_{X(t_1+\tau)X(t_2+\tau)}(x_1,x_2)\, dx_1 dx_2$$

$$= \underline{\underline{R_X(t_1+\tau, t_2+\tau)}}$$

<u>Wide-sense stationarity</u>

μ_X $\qquad$ $R_X(\tau) = E X(t+\tau) X(t)$

SSS $\Rightarrow$ WSS $\Leftarrow$

$\not\Leftarrow$

$\Leftarrow$ Gauss.

unif $[0,1]$

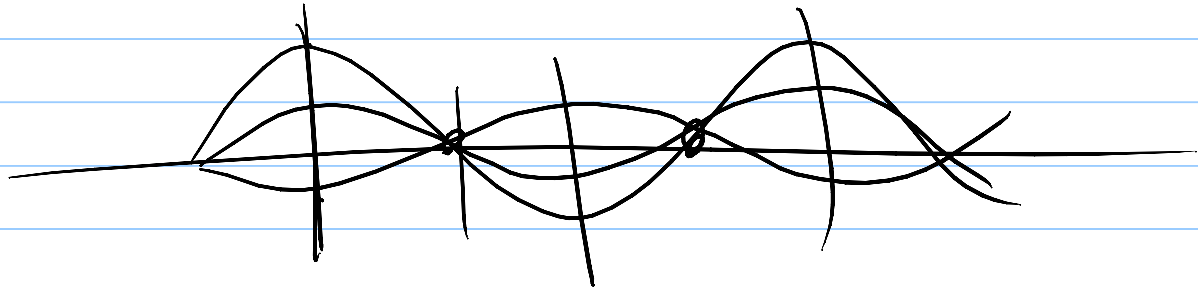
Example: $X(t) = \cos 2\pi(t + \Theta)$

$\mu_X = 0$

unif $[-1,1]$

$R_X(\tau) = \frac{1}{2} \cos 2\pi\tau$

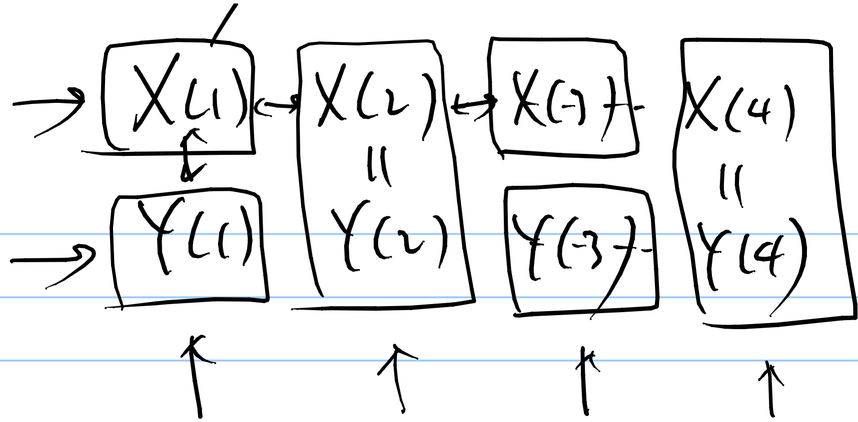
$X(t) = A \cos 2\pi t$



$X(t), Y(t),$ $R_{XY}(t_1, t_2) = E X(t_1) Y(t_2)$

Ber(p)

Joint stat



$$f_{X(t_1)\cdots X(t_m)Y(u_1)\cdots Y(u_l)}(x_1,\cdots,x_m,y_1,\cdots,y_l)$$

$$= f_{X(t_1+\tau)\cdots X(t_m+\tau)Y(u_1+\tau)\cdots Y(u_l+\tau)}(x_1,\cdots,x_m,y_1,\cdots,y_l)$$

jwss

(1) $X(t)$ wss

(2) $Y(t)$ wss

(3) $R_{XY}(t_1,t_2)$ shift inv $= R_{XY}(t_1+\tau, t_2+\tau)$

$X(t)$ stat, $Y(t)$ stat, indep

$$\Rightarrow \underbrace{f_{X(t_1)\cdots X(t_m)}(x_1,\cdots,x_m)}_{\parallel} \underbrace{f_{Y(u_1)\cdots Y(u_\ell)}(y_1,\cdots,y_\ell)}_{\parallel}$$

$$= f_{X(t_1+\tau)\cdots X(t_m+\tau)}(x_1,\cdots,x_m)\, f_{Y(u_1+\tau)\cdots Y(u_\ell+\tau)}(y_1,\cdots,y_\ell)$$

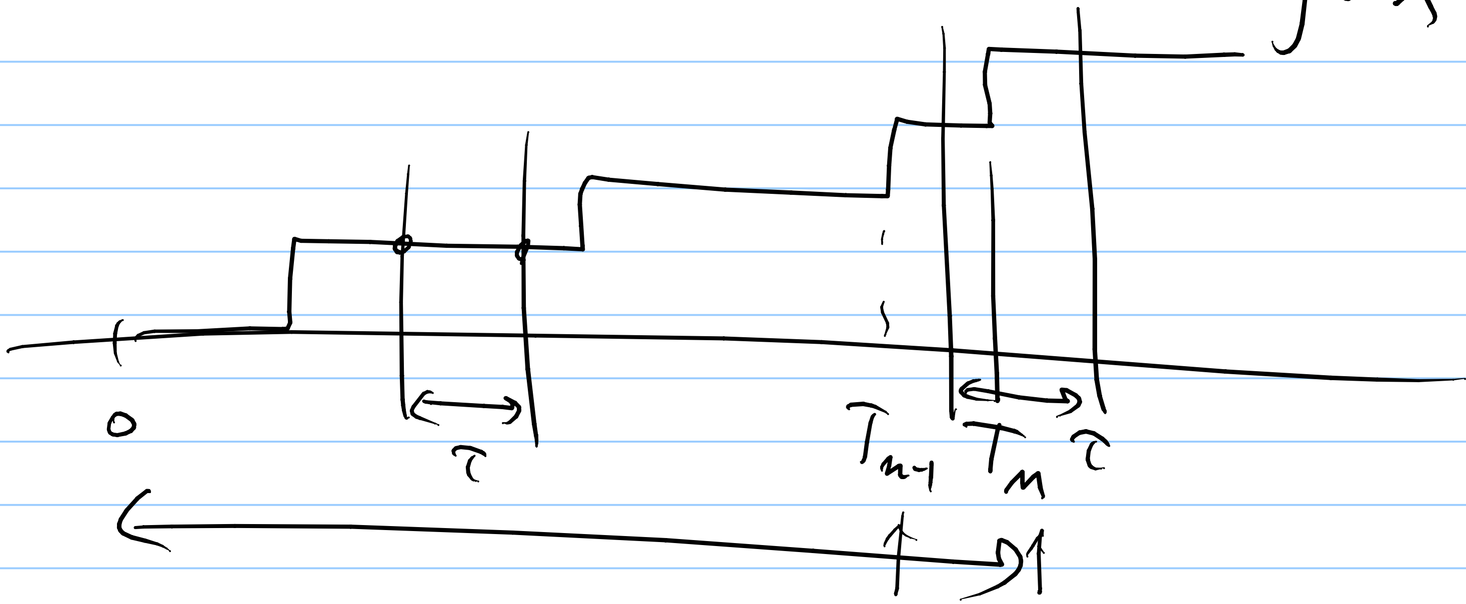
indep

$$\underline{\underline{R_{XY}(t_1,t_2)}} = E\,X(t_1)\,Y(t_2) = \iint xy\, \underbrace{f_{X(t_1)Y(t_2)}(x,y)}_{f_{X(t_1)}(x)\, f_{Y(t_2)}(y)}\, dx\,dy$$

$$= \int x f_{X(t_1)}(x)\,dx \int y f_{Y(t_2)}(y)\,dy$$

$$= \underline{\underline{\mu_X}} \cdot \underline{\underline{\mu_Y}}$$

$X(t)$ wss, $Y(t)$ wss, indep $\Rightarrow$ $X(t), Y(t)$ jwss



$\cancel{M(t)} \Rightarrow \underline{N(t_1) - N(t_1 - \tau)}$ $\qquad$ $\underline{N(t_2) - N(t_2 - \tau)}$

$\underline{\underline{M(t)}} = \underline{N(t) - N(t - \tau)}$

<u>Gauss. rp</u>

$$X(t) \qquad \underline{\underline{\left(X(t_1), \cdots, X(t_n) \right)}}^t = \underline{X}$$

$$\|$$

$$WSS \Longleftrightarrow \underline{\underline{SSS}}$$

$$\underline{\mu_{\underline{X}}} = \begin{pmatrix} E X(t_1) \\ E X(t_2) \\ \vdots \\ E X(t_n) \end{pmatrix} = \begin{pmatrix} \mu_X(t_1) \\ \mu_X(t_2) \\ \vdots \\ \mu_X(t_n) \end{pmatrix} \overset{WSS}{=} \mu_X \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$\underline{C_{\underline{X}}} = \begin{pmatrix} Var(X_1) & Cov(X_1, X_2) & \cdots & Cov(X_1, X_n) \\ Cov(X_2, X_1) & Var(X_2) & \cdots & Cov(X_2, X_n) \\ \vdots & & \ddots & \vdots \\ Cov(X_n, X_1) & Cov(X_n, X_2) & \cdots & Var(X_n) \end{pmatrix}$$

$$= \begin{pmatrix} C_X(t_1+\tau, t_1+\tau) & C_X(t_1+\tau, t_2) & \cdots & C_X(t_1, t_m) \\ C_X(t_2, t_1) & C_X(t_2, t_2) & \cdots & C_X(t_2, t_m) \\ \vdots & \vdots & \ddots & \vdots \\ C_X(t_m, t_1) & C_X(t_m, t_2) & \cdots & C_X(t_m, t_m+\tau) \end{pmatrix}$$

$$X(t) \longrightarrow \boxed{LTI} \longrightarrow Y(t)$$

$\parallel$

filter.

$h(t)$

$=$

$H(f)$