노트 제목 2008-06-03

Stat proc $\Rightarrow$ $\boxed{\begin{aligned} \mu_x(t) &= \mu_x(t+\tau) = \mu_x \\ R_x(t_1, t_2) &= R_x(t_1+\tau, t_2+\tau) \\ &= R_x(t_1 - t_2) \end{aligned}}$

$F_{\underline{X(t_1) \cdots X(t}}$

$\underline{\underline{SSS}} \Rightarrow \underline{\underline{WSS}}$

$\not\Leftarrow$

$\Leftarrow$

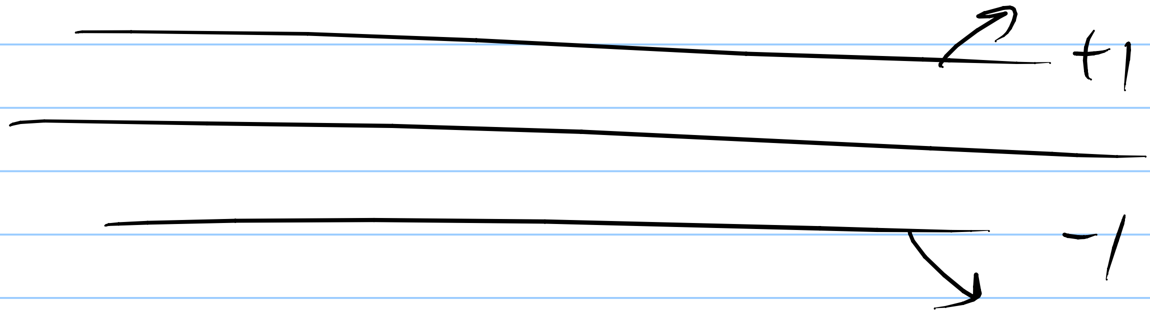
Total avg power: $\underline{\underline{WSS}}$

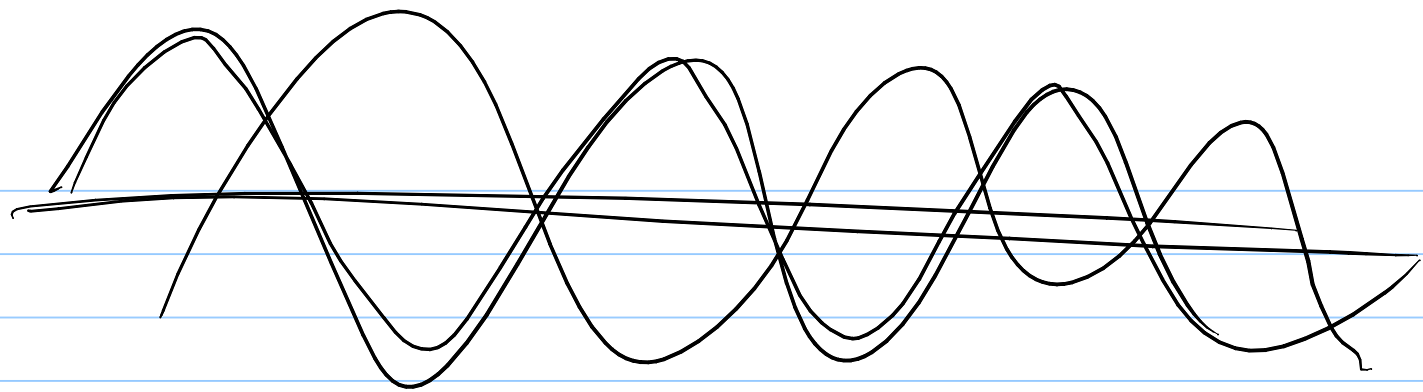
$$R_X(0) = E X^2(t) = E \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} X^2(t)\,dt$$

Ergodicity

$$E X(t) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} X(t)\,dt$$

$X(t) = Z$ ± 1 equiprob





$$\overline{E}X(t+\tau)X(t) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} X(t+\tau)X(t)\,dt$$

$$R_{XY}(t_1, t_2) = E X(t_1) Y(t)$$

$X(t)$ $Y(t)$ joint stationarity

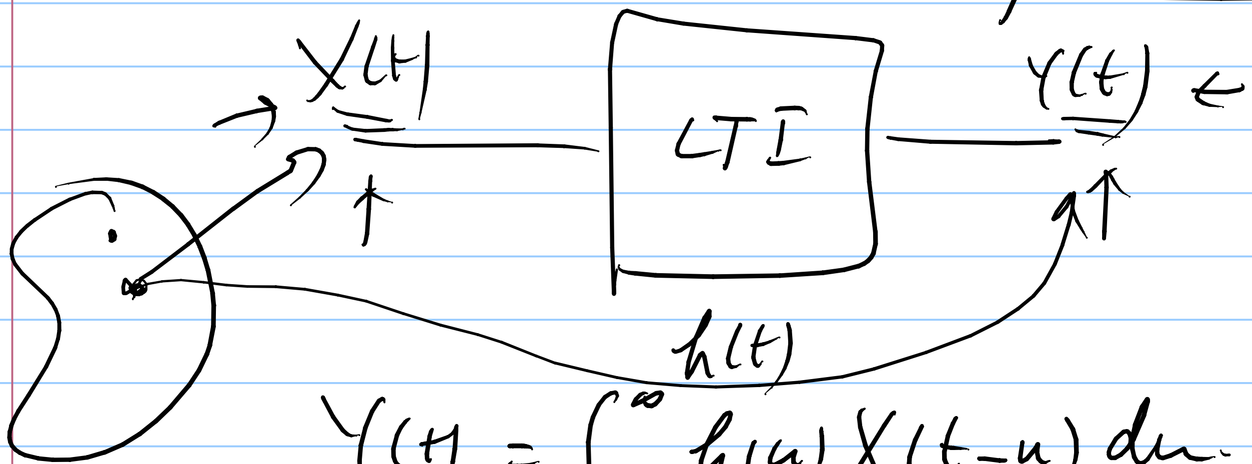
jwss (1) $X(t)$ wss

(2) $Y(t)$ WSS

(3) $R_{XY}(t_1, t_2) = R_{XY}(t_1+\tau, t_2+\tau)$

$= R_{XY}(t_1 - t_2)$

G rp $\underline{X} = \begin{pmatrix} X(t_1) \\ \vdots \\ X(t_n) \end{pmatrix}$



$$Y(t) = \int_{-\infty}^{\infty} h(u) X(t-u)\, du = X(t) * h(t)$$

$$\underline{\underline{\mu_Y}}(t) = EY(t) = \int_{\infty}^{\infty} h(u)\, EX(t-u)\, du.$$

$$= \mu_X(t) * h(t)$$

$$\text{WSS} \quad \mu_Y = \underline{\underline{\mu_X}} \int_{\infty}^{\infty} h(u)\, du.$$

$$\underline{\underline{R_Y}}(t_1, t_2) = E \int h(u) X(t_1-u)\, du \int h(v) X(t_2 - v)\, dv$$

$$= \iint h(u) h(v) \underbrace{E X(t_1-u) X(t_2-v)}_{R_X(t_1-u,\; t_2-v)} du\, dv$$

$$t_1 - t_2 - u + v.$$

$$R_Y(\tau) \overset{\text{WSS}}{=} \underline{R_X(\tau) * h(\tau) * h(-\tau)} \leftarrow$$

$$R_{XY}(t_1, t_2) = E\,X(t_1)\,Y(t_2)$$

$$= E\,\underline{X(t_1)} \int h(u)\, X(\underline{t_2 - u})\, du$$

If WSS $= \int h(u)\, R_X(t_1 - t_2 + u)\, du$

$$\underline{R_{XY}(\tau) = R_X(\tau) * h(-\tau)}$$

$$\underline{R_{YX}(\tau) = R_X(\tau) * h(\tau)}$$

$$R_Y(\tau) = \sum_k \sum_l h(k)\, h(l)\, R_X(\tau - k + l) \leftarrow$$

$$\underline{R_X(0)} = E\,X^2(t) = \lim_{K \to \infty} \frac{1}{2K+1} \sum_{-K}^{K} X^2(k)$$

FT $$\int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt = X(f)$$

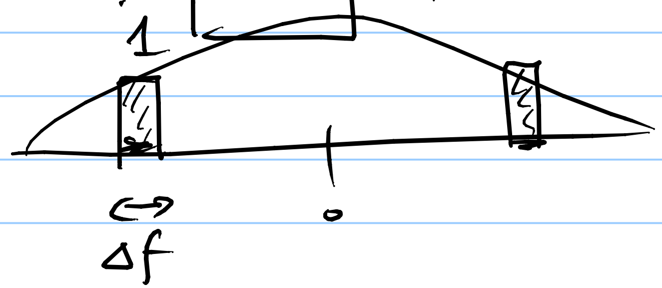
deterministic

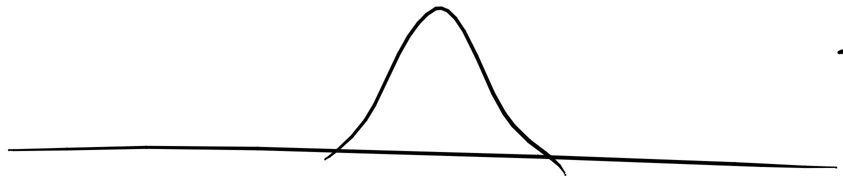
$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi ft}\, df$$

Energy spectrum

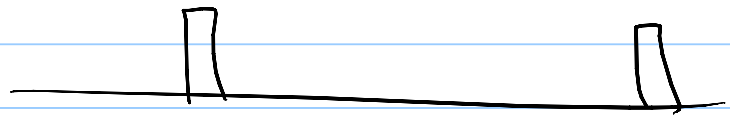
$$\int |X(f)|^2\, df = \int x^2(t)\, dt = \text{total energy}$$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) \longrightarrow \int \Big|\overbrace{X(f)\,H(f)}^{Y(f)}\Big|^2 df = \int_{\text{passbands}} |X(f)|^2\, df$$



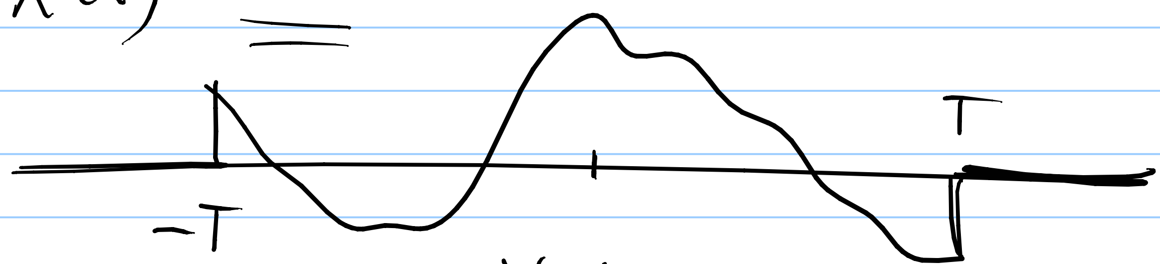


$$= \int_{f_0-\frac{\Delta f}{2}}^{f_0+\frac{\Delta f}{2}} |X(f)|^2 \, df$$



$$+ \int_{-f_0-}^{-f_0+}$$

$X(t)$ $\underline{\underline{\text{WSS}}}$.



$X_T(t)$

$$E \lim_{T\to\infty} \boxed{\frac{1}{2T} \left|\tilde{X}_T(f)\right|^2} = \lim_{T\to\infty} E \, \frac{1}{2T} \left| \int_{-T}^{T} x(t) \, e^{-j2\pi f t} \, dt \right|^2$$

total power

Periodogram.

$$\underbrace{S_x(f)}_{} = \underbrace{\int_{-\infty}^{\infty} R_x(\tau)\, e^{-j2\pi f\tau}\, d\tau}_{}$$

Power Spectral Density

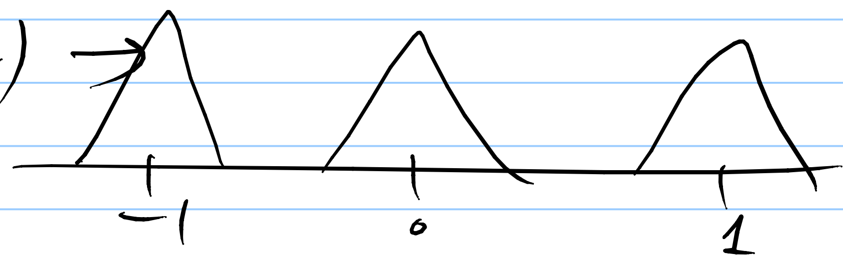
Wiener-Kinchin Thm.

Properties (a) $S_x(f) \geq 0$

(b) $R_X(0) = E X^2(t) = \int_{-\infty}^{\infty} S_x(f)\, df$

(c) $S_x(f) = S_x(-f)$

Discrete-time r.p. $R_x(k)$



(b)' $R_x(0) = EX^2(k) = \int_{-1/2}^{1/2} S_x(f)\,df$

$$X(t) \longrightarrow \boxed{\underline{\underline{h(t)}}} \longrightarrow Y(t)$$

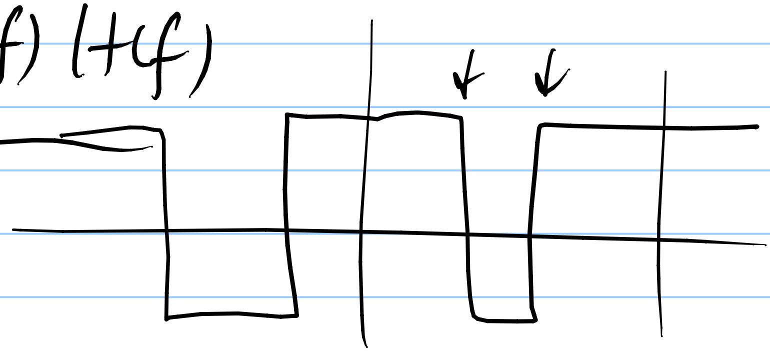
$$R_Y(\tau) = R_X(\tau) * \overset{H(f)}{h(\tau)} * \overset{H^*(f)}{h(-\tau)}$$

$$\underline{S_Y(f) = S_X(f)\,|H(f)|^2} \qquad |\underline{Y(f)}|^2 = |\underline{X(f)}|^2\,|H(f)|^2$$

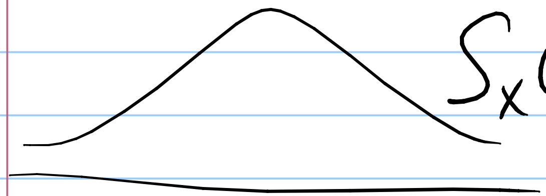
$$\underline{S_{XY}(f) \triangleq FT\big(R_{XY}(\tau)\big) = S_X(f)\,H^*(f)}$$

$$S_{YX}(f) = S_X(f)\,H(f)$$

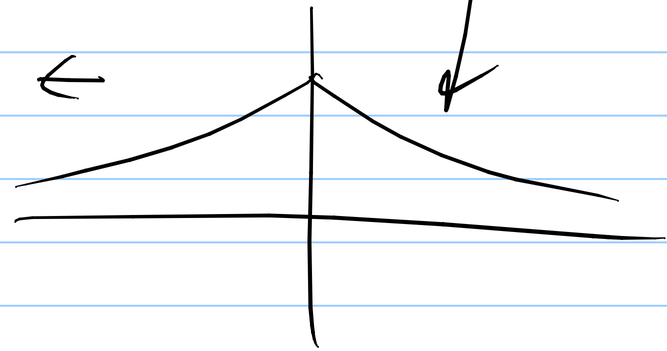
$\boxed{S_X(f)}$



Example: $X(t)$ wss. $R_X(\tau) = A e^{-b|\tau|}$

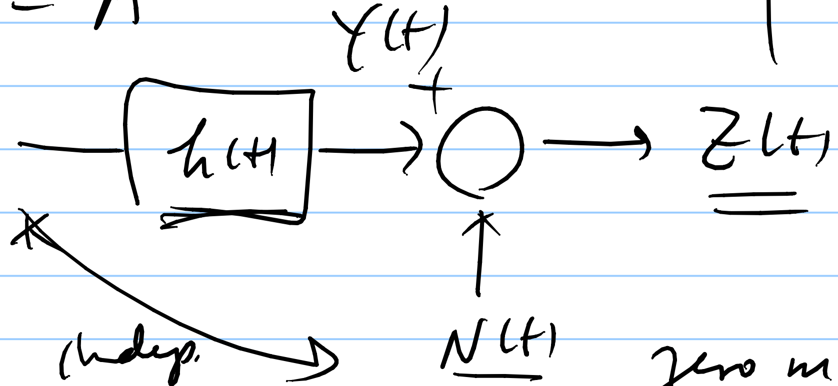


$$S_X(f) = \frac{2Ab}{(2\pi f)^2 + b^2} \leftarrow$$



$$E X^2(t) = A$$

Example: $X(t)$ wss



(AWGN) zero mean

$$Y(t) = X(t) * h(t)$$

$$S_Y(f) = S_X(f)\,|H(f)|^2$$

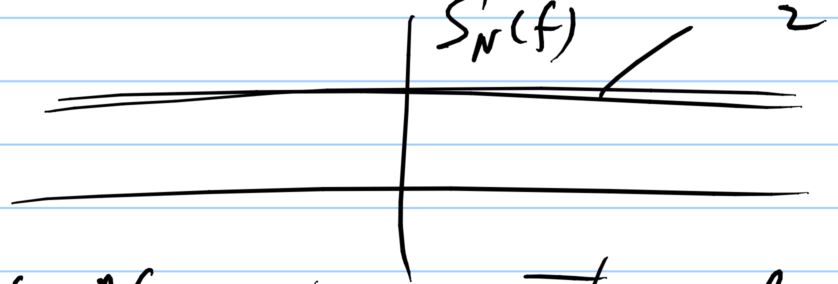
$$\underline{R_Z(\tau)} = \underline{R_Y(\tau)} + \underline{R_N(\tau)} + \cancel{\underline{R_{YN}(\tau)}}^{\,0} + \cancel{\underline{R_{NY}(\tau)}}^{\,0}$$

$$= E\big(Y(t+\tau) + N(t+\tau)\big)\big(Y(t) + N(t)\big)$$

$$S_Z(f) = S_Y(f) + S_N(f) + \cancel{S_{YN}(f)}^{\,0} + \cancel{S_{NY}(f)}^{\,0}$$

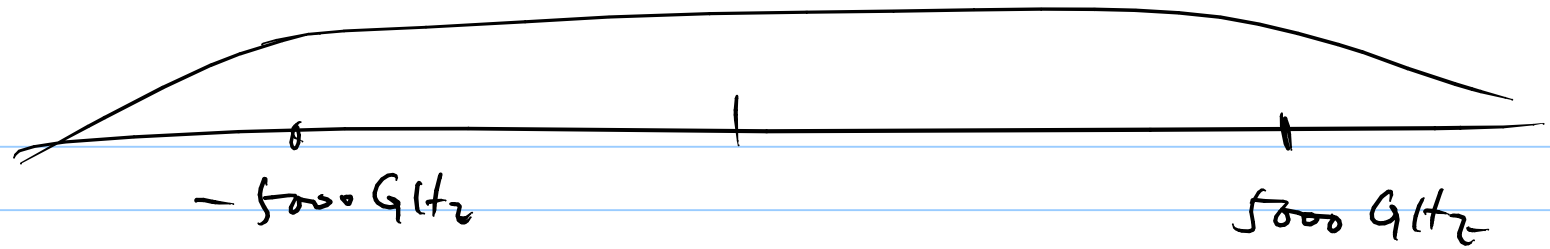
$$= S_X(f)\,|H(f)|^2 + \underline{\underline{S_N(f)}}$$

$\frac{N_0}{2}$



$$R_N(0) = \int_{-\infty}^{\infty} \underline{\underline{S_N(f)}}\, df = \infty \qquad \text{Thermal noise.}$$

$$\frac{N_0}{2}$$



−5000 GHz
5000 GHz