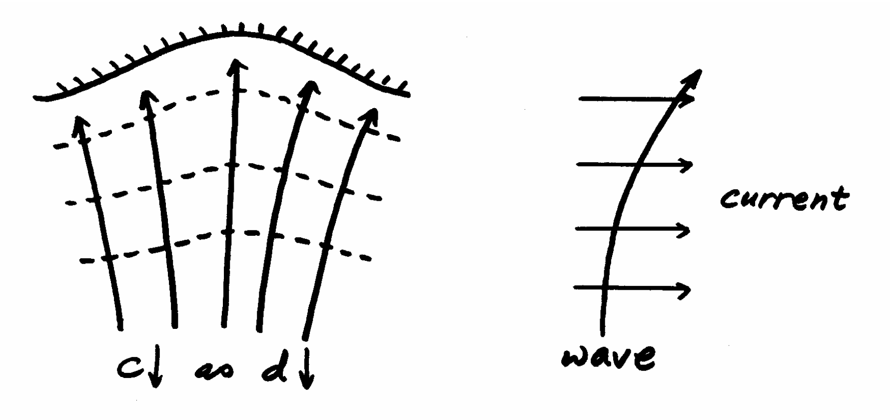
Chapter 4. Wave Refraction, Diffraction, and Reflection

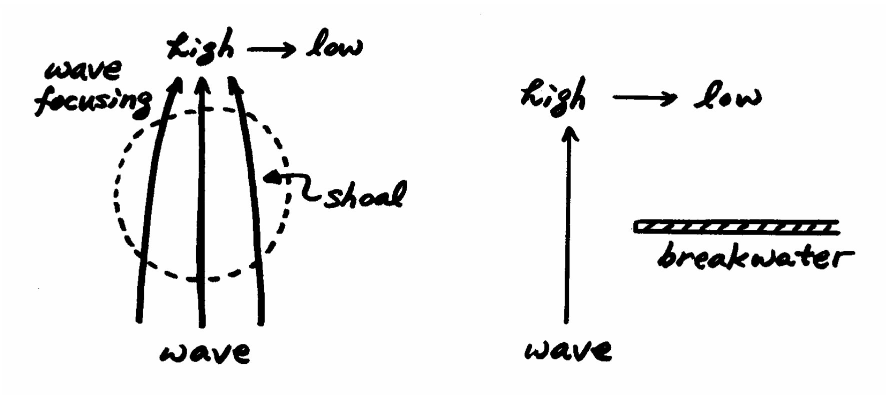
4.1 Three-Dimensional Wave Transformation

Refraction (due to depth and current)



Diffraction (internal or external)

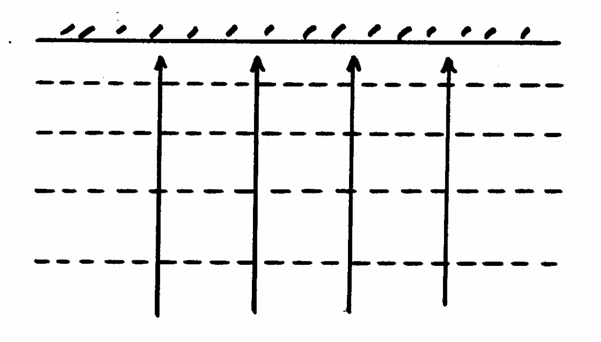
High energy region →(Energy transfer)→ Low energy region



Internal diffraction External diffraction

Refraction, diffraction, shoaling → Change of wave direction and height in nearshore area → Important for longshore sediment transport and wave height distribution → Nearshore bottom change, design of coastal structures

4.2 Wave Refraction

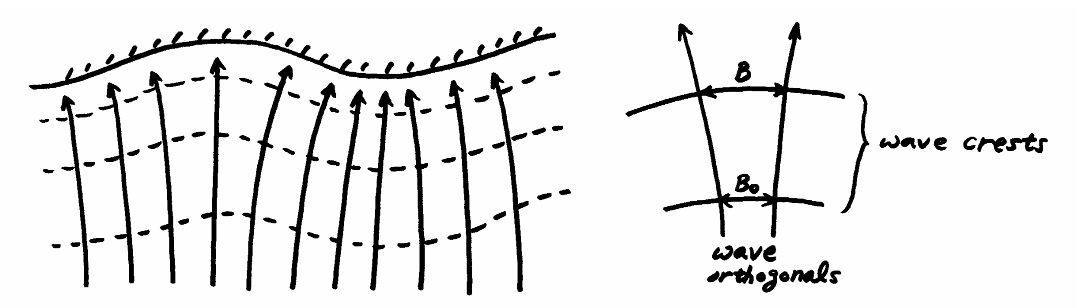


Normal incidence on beaches with straight and parallel bottom contours

↓

No refraction, and only shoaling occurs

$$H = H_0\sqrt{\frac{C_0}{2C_g}} = K_s H_0$$

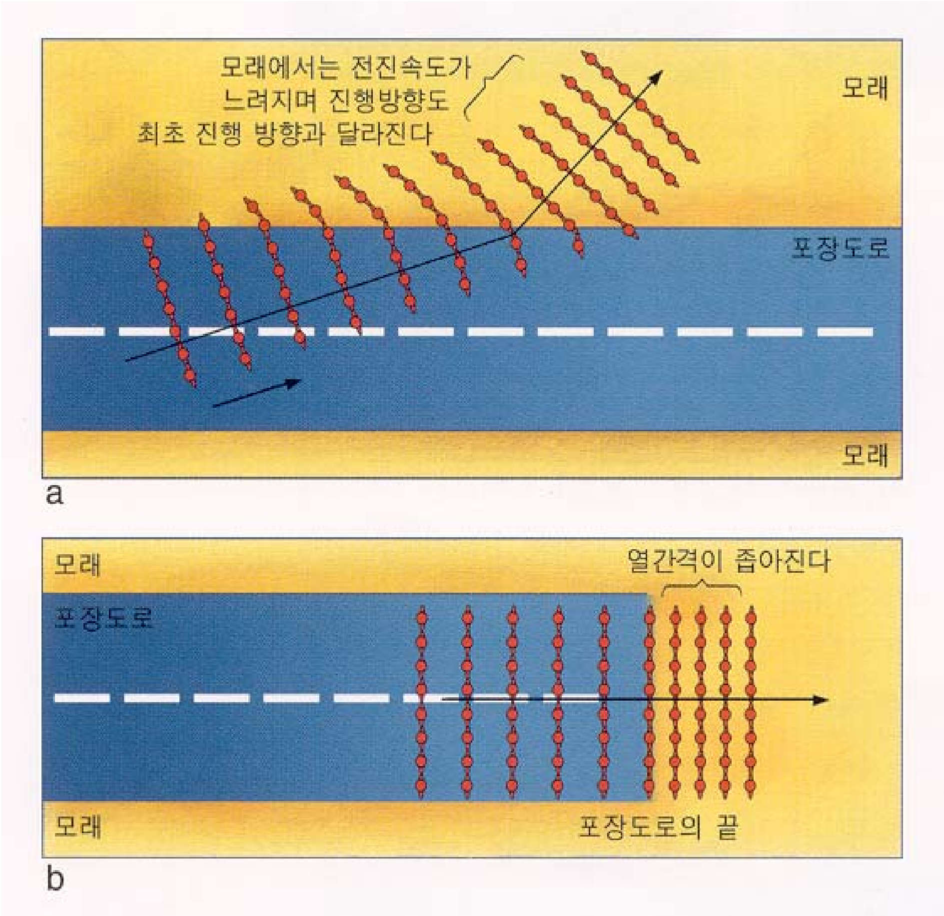


Assuming no energy transfer across wave orthogonals,

$$\bar{E}_0 B_0 = \bar{E}B \quad \Rightarrow \frac{1}{8}\rho g H_0^2 B_0 = \frac{1}{8}\rho g H^2 B \quad \Rightarrow \quad H = H_0\sqrt{\frac{B_0}{B}}$$

For shoaling and refraction,

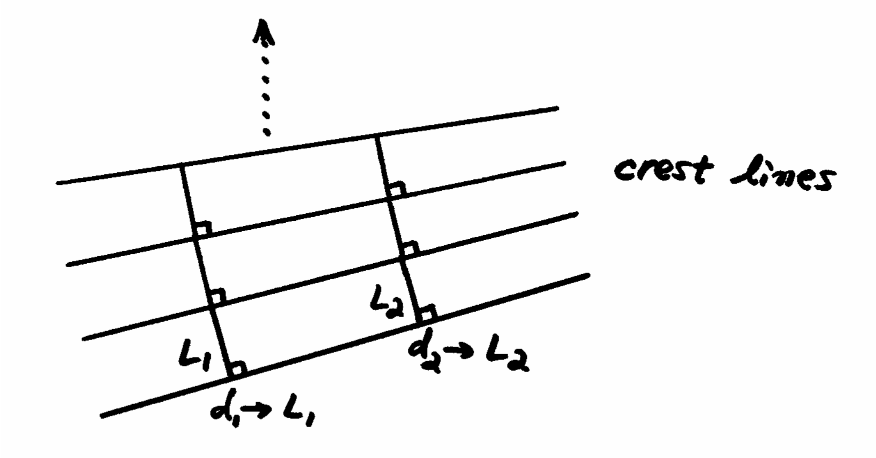
$$H = H_0\sqrt{\frac{C_0}{2C_g}}\sqrt{\frac{B_0}{B}} = H_0 K_s K_r$$



파의 굴절과 천수 현상에 대한 비유적 설명

4.3 Manual Construction of Refraction Diagrams

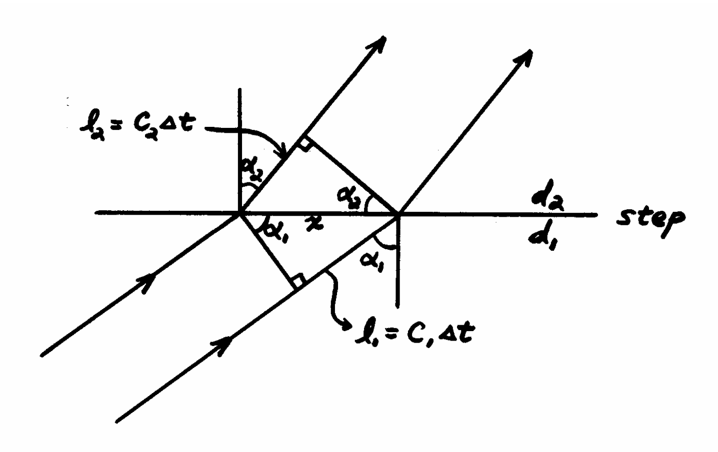
Wave crest method:



Crest lines → Orthogonals (⊥ to crest lines) → Calculate K_r

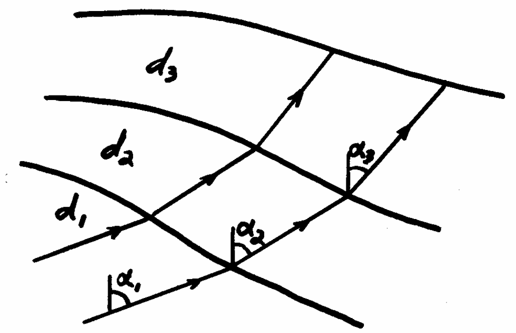
Orthogonal method (Ray tracing method):

- Simpler than wave crests method (K_r is directly calculated from orthogonals)
- Based on Snell's law



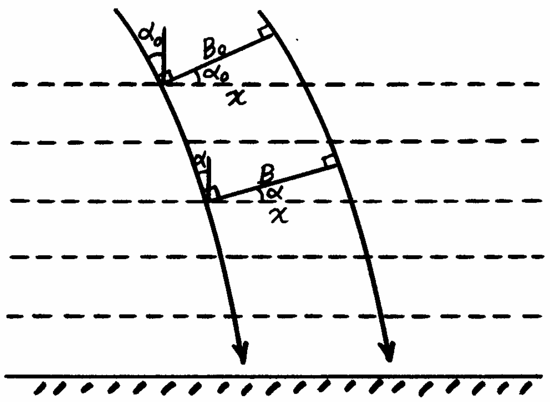
$$\sin\alpha_1 = \frac{l_1}{x} = \frac{C_1 \Delta t}{x}; \quad \sin\alpha_2 = \frac{l_2}{x} = \frac{C_2 \Delta t}{x}$$

$$\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{C_1}{C_2} = \frac{L_1}{L_2} \quad \text{(Snell's law)}$$



$$\frac{\sin\alpha_1}{C_1}=\frac{\sin\alpha_2}{C_2}\quad\rightarrow\quad\alpha_2,\quad\frac{\sin\alpha_2}{C_2}=\frac{\sin\alpha_3}{C_3}\quad\rightarrow\quad\alpha_3,\quad\cdots$$

Straight and parallel bottom contours



$$\cos\alpha_0=\frac{B_0}{x},\quad\cos\alpha=\frac{B}{x}\quad\Rightarrow\quad\frac{B_0}{\cos\alpha_0}=\frac{B}{\cos\alpha}$$

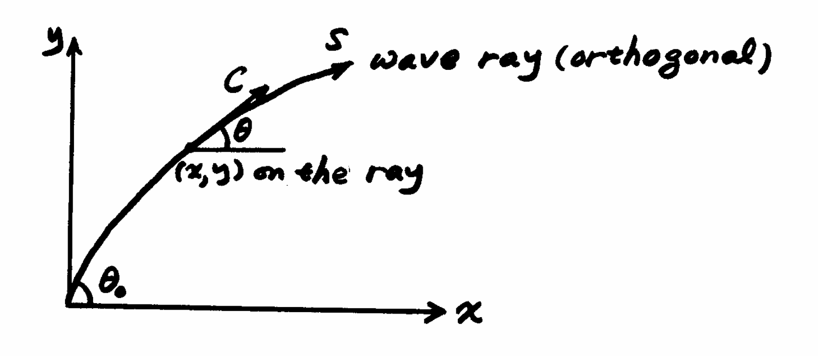
$$K_r=\sqrt{\frac{B_0}{B}}=\sqrt{\frac{\cos\alpha_0}{\cos\alpha}}$$

where

$$\alpha=\sin^{-1}\left(\frac{C}{C_0}\sin\alpha_0\right)\quad\text{by Snell's law}$$

Graphical method is tedious and inaccurate. Moreover, it needs refraction diagrams for different wave periods and directions. Therefore, computational methods have been developed (see Dean, R.G. and Dalrymple, R.A., 1991, *Water Wave Mechanics for Engineers and Scientists,* World Scientific, pp. 109-112)

4.4 Numerical Refraction Analysis

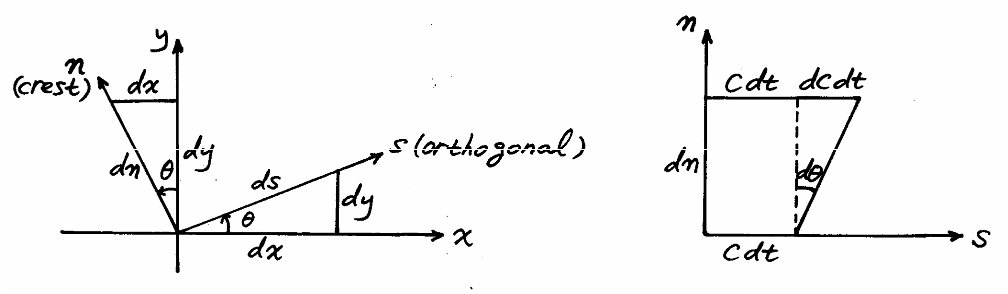


Let t = travel time along a ray of a wave moving with speed $C(x, y)$. Let the wave is located at $[x(t), y(t)]$ at time t. Then

$$\frac{dx}{dt} = C\cos\theta \qquad (1)$$

$$\frac{dy}{dt} = C\sin\theta \qquad (2)$$

$$\theta(x, y) = ?$$



$$dx = ds\cos\theta = -dn\sin\theta$$

$$d\theta \cong \tan d\theta = -\frac{dCdt}{dn} = -\frac{1}{C}\frac{\partial C}{\partial n}ds$$

$$dy = ds\sin\theta = dn\cos\theta$$

$$ds = Cdt$$

$$\frac{\partial\theta}{\partial s} = -\frac{1}{C}\frac{\partial C}{\partial n} = -\frac{1}{C}\left(\frac{\partial C}{\partial x}\frac{\partial x}{\partial n} + \frac{\partial C}{\partial y}\frac{\partial y}{\partial n}\right) = -\frac{1}{C}\left(-\sin\theta\frac{\partial C}{\partial x} + \cos\theta\frac{\partial C}{\partial y}\right)$$

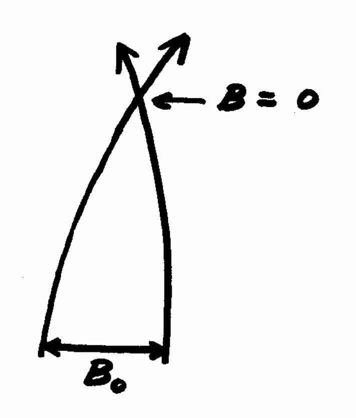
$$= \frac{1}{C}\left(\sin\theta\frac{\partial C}{\partial x} - \cos\theta\frac{\partial C}{\partial y}\right) \qquad (4.5)$$

$$\frac{\partial\theta}{\partial t} = \frac{\partial\theta}{\partial s}\frac{\partial s}{\partial t} = C\frac{\partial\theta}{\partial s} = \sin\theta\frac{\partial C}{\partial x} - \cos\theta\frac{\partial C}{\partial y} \qquad (3)$$

Solve equations (1), (2), (3) for $x(t)$, $y(t)$, and $\theta(x,y)$.

Problems of ray tracing method:

(1) Wave ray crossing



$$K_r = \sqrt{\frac{B_0}{B}} \to \infty \text{ at ray-crossing point}$$
$$H = H_0 K_s K_r \to \infty \text{ at ray-crossing point}$$

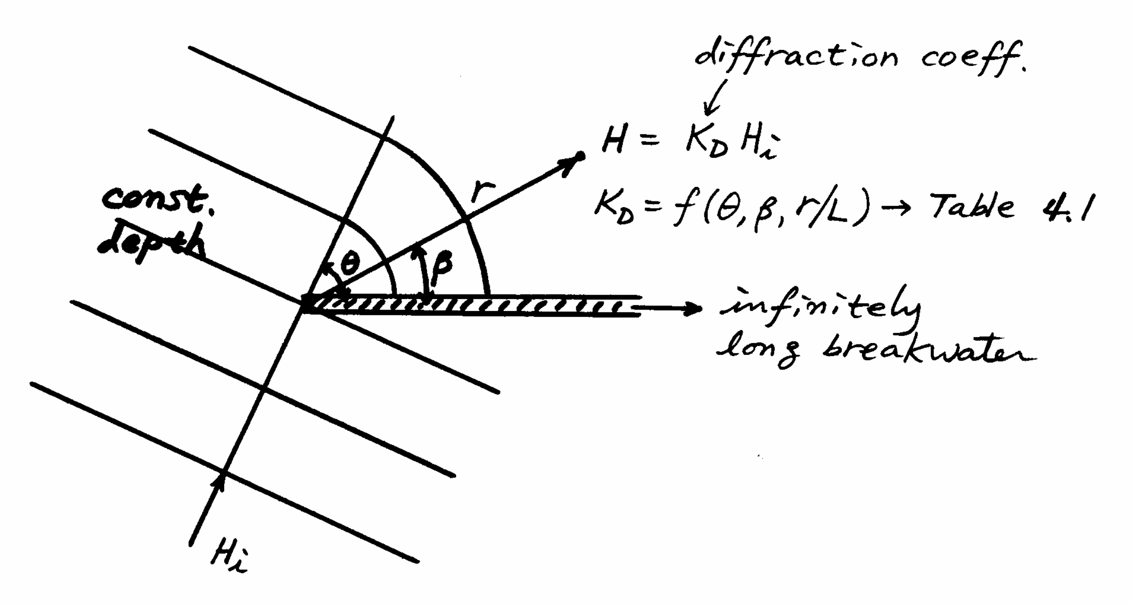
(2) Interpolation is needed in the region of sparse orthogonals

To resolve these problems, finite-difference method is used, which solves Eq. (4.6) for θ on (x,y) grid. Wave height is also calculated at each grid point using conservation of energy equations.

4.5 Refraction by Currents (read text)

4.6 Wave Diffraction

Based on Penney and Price (1952) solution



Use diffraction diagrams in Shore Protection Manual (SPM, Figs. 2.28-2.39), in which θ varies from $15°$ to $180°$ with increment of $15°$. In the figures, x and y are given in units of wavelength. Therefore, we need to calculate L for given T and d.

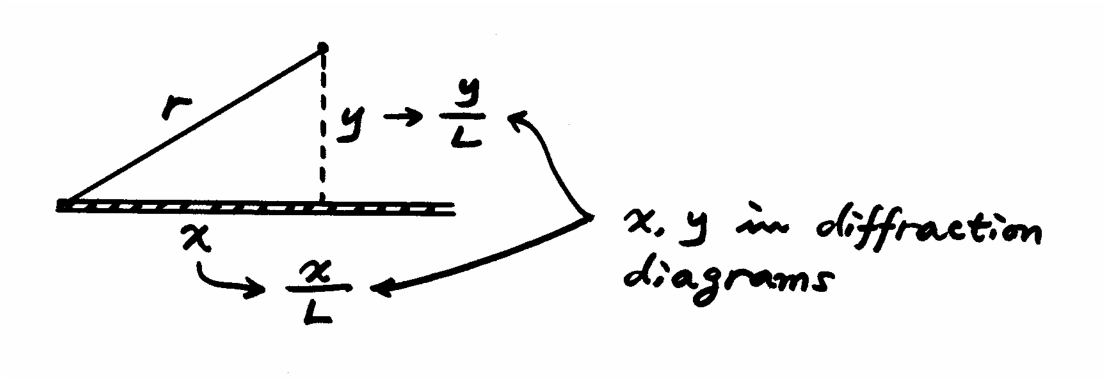


Figure 2-27. Wave diffraction at Channel Islands Harbor breakwater, California.

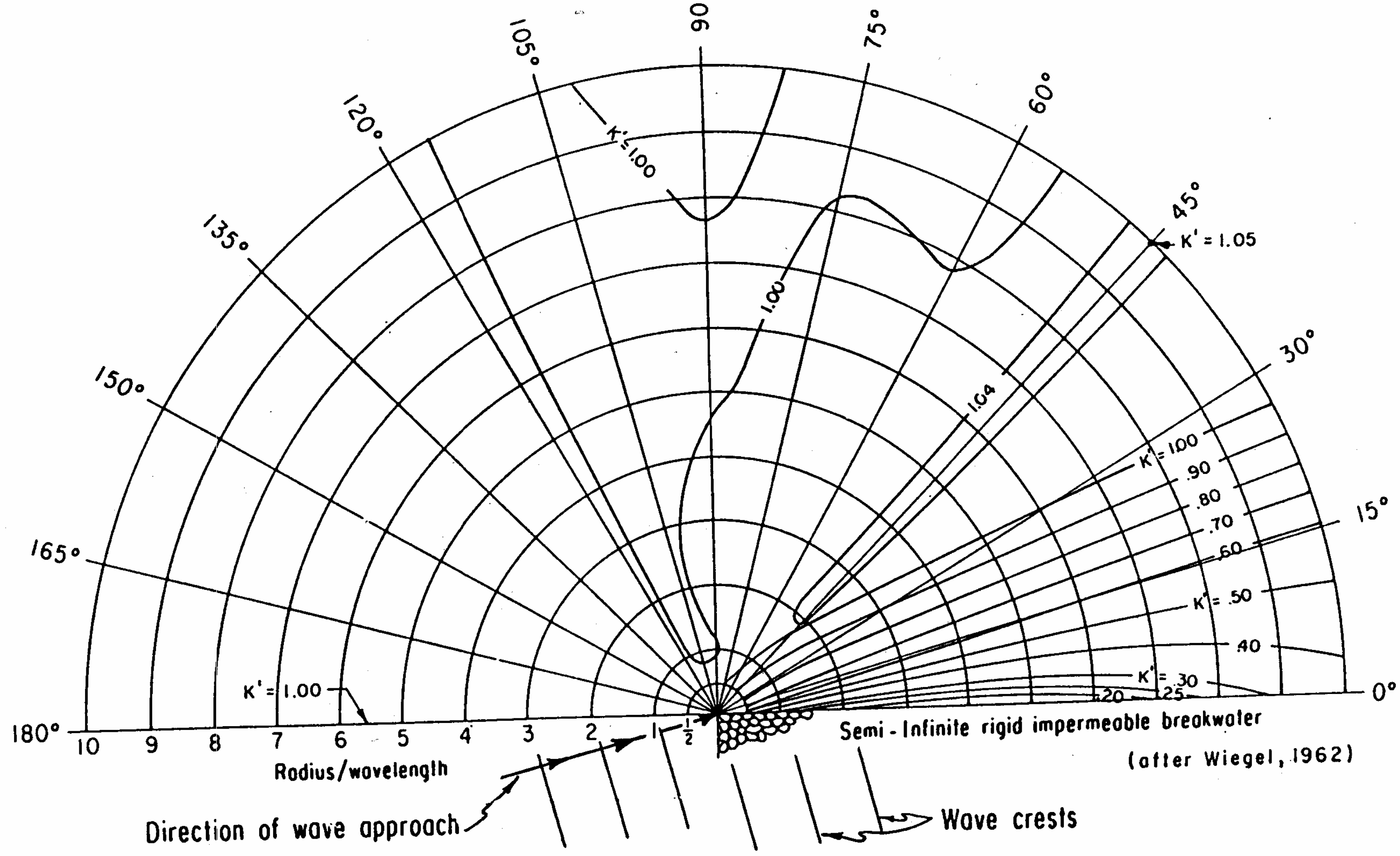


Figure 2-28. Wave diffraction diagram--15° wave angle.

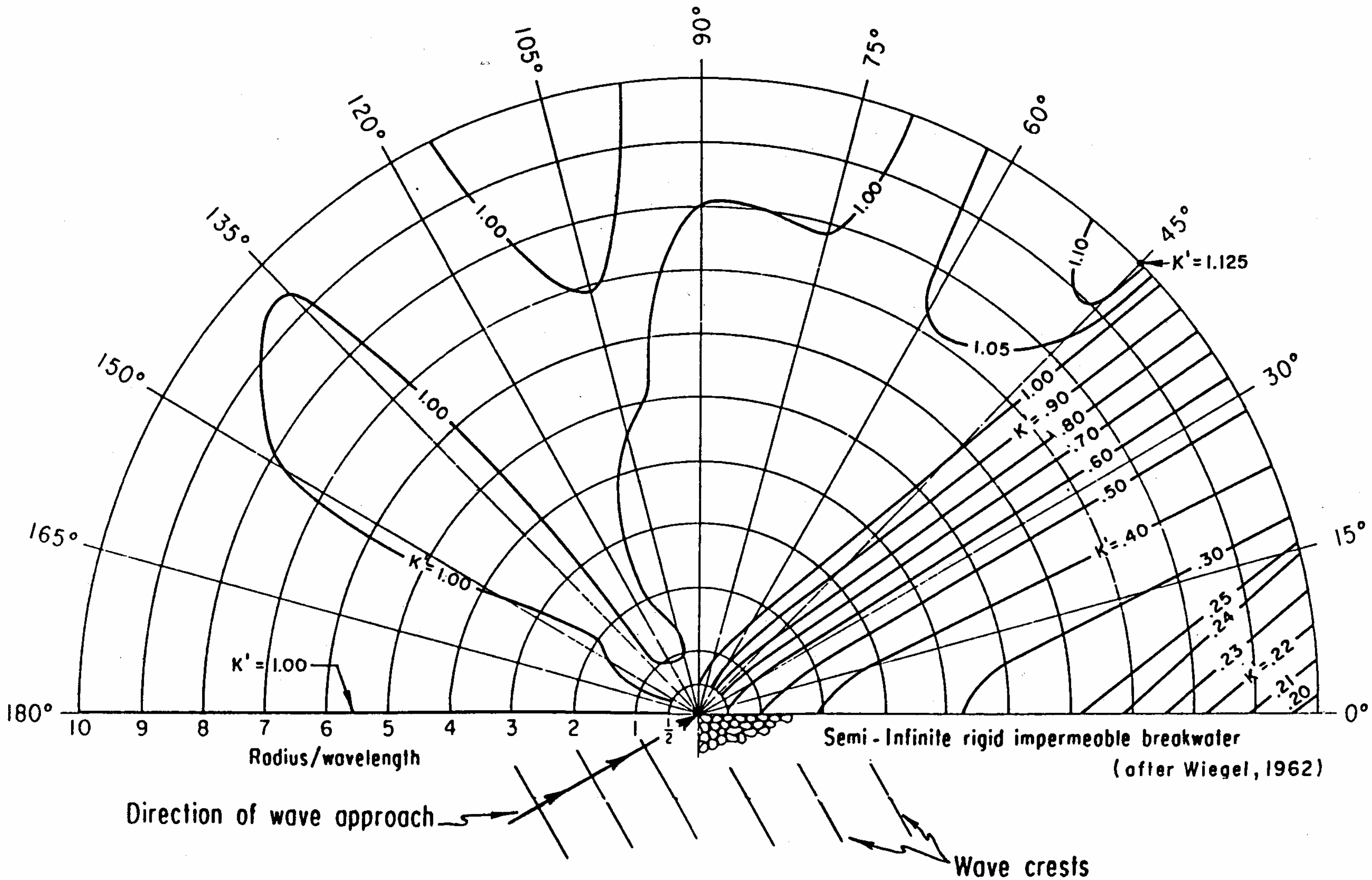


Figure 2-29. Wave diffraction diagram--30° wave angle.

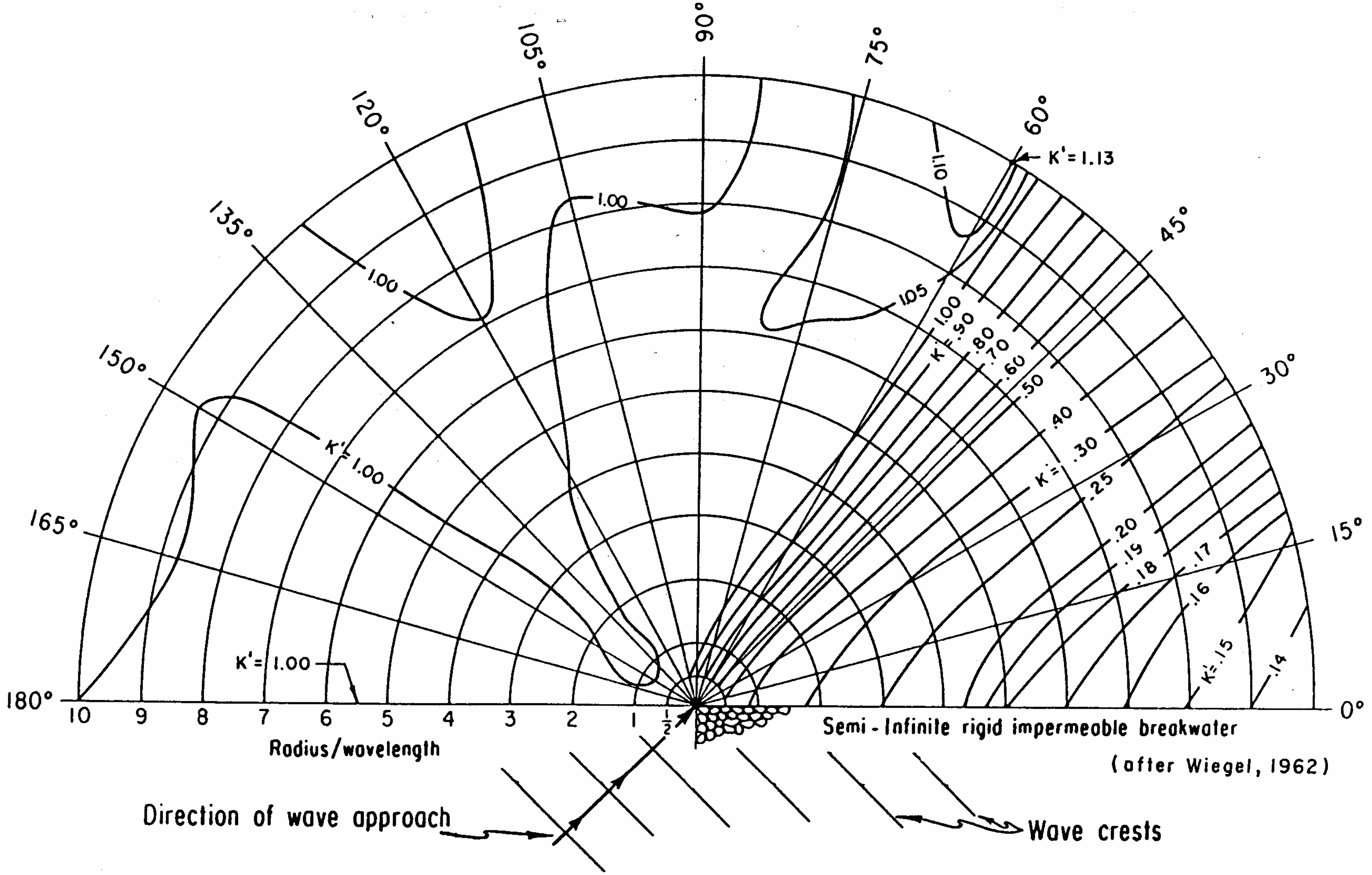


Figure 2-30. Wave diffraction diagram--45° wave angle.

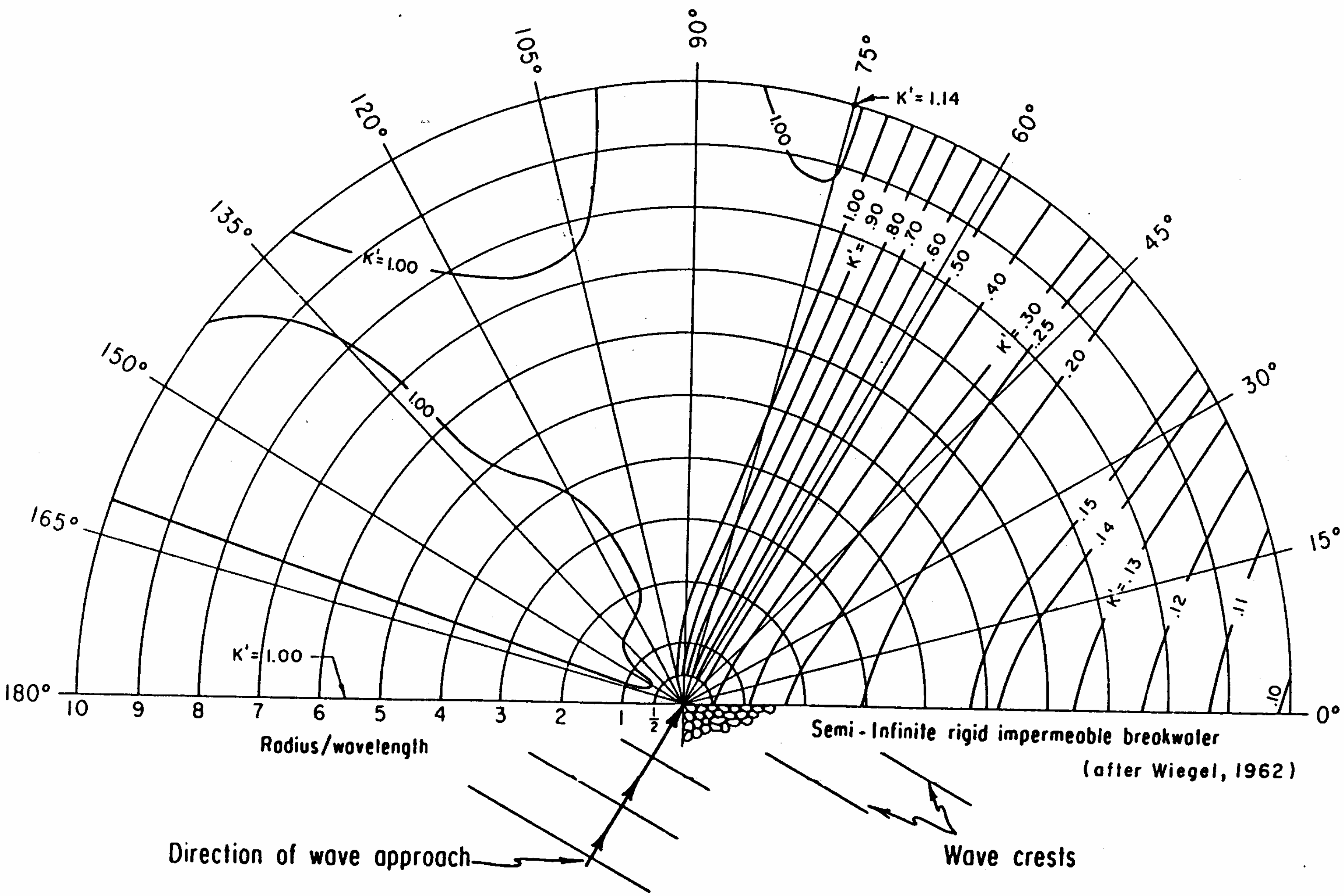


Figure 2-31. Wave diffraction diagram--60° wave angle.

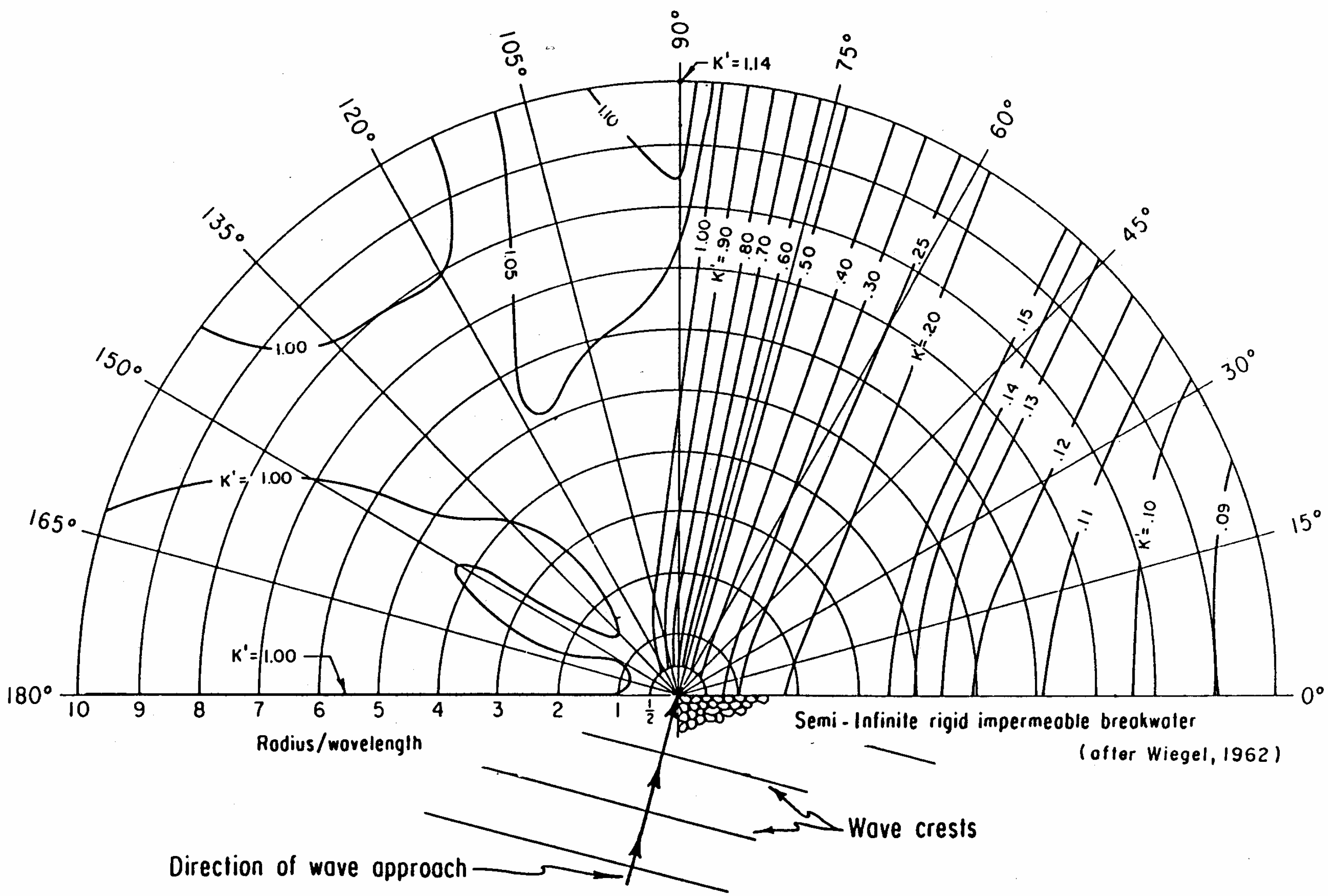


Figure 2-32. Wave diffraction diagram--75° wave angle.

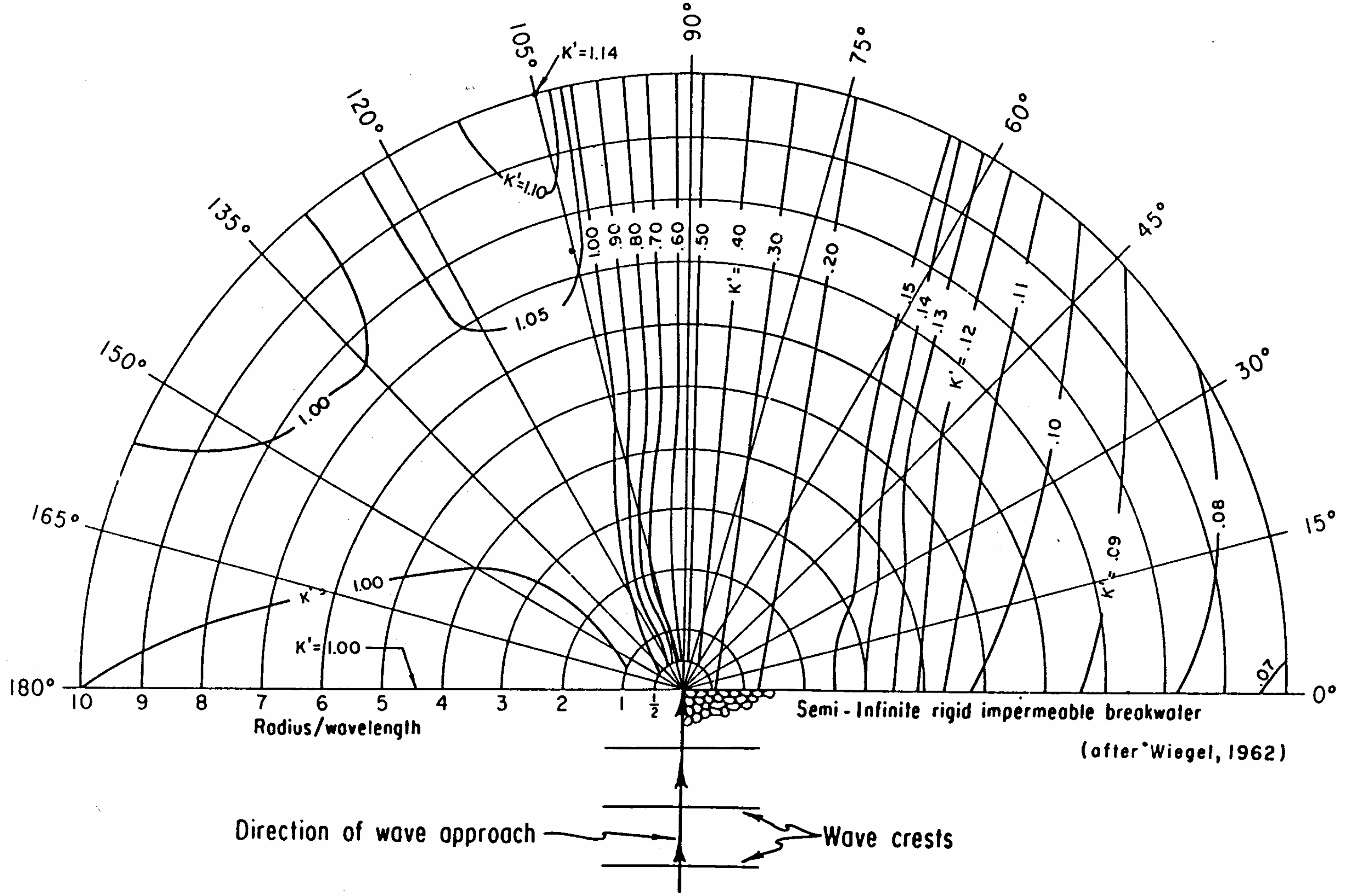


Figure 2-33. Wave diffraction diagram--90° wave angle.

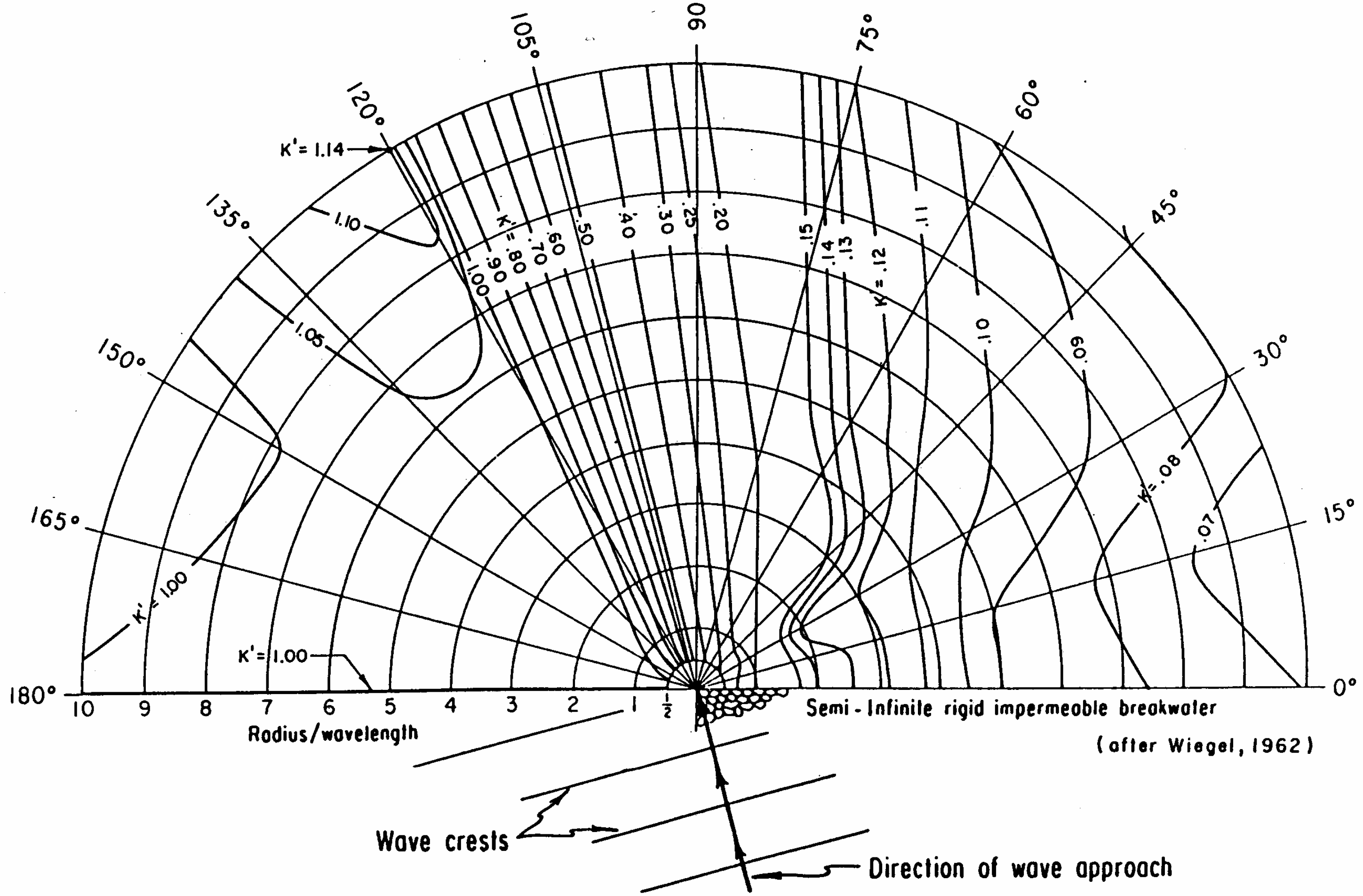


Figure 2-34. Wave diffraction diagram--105° wave angle.

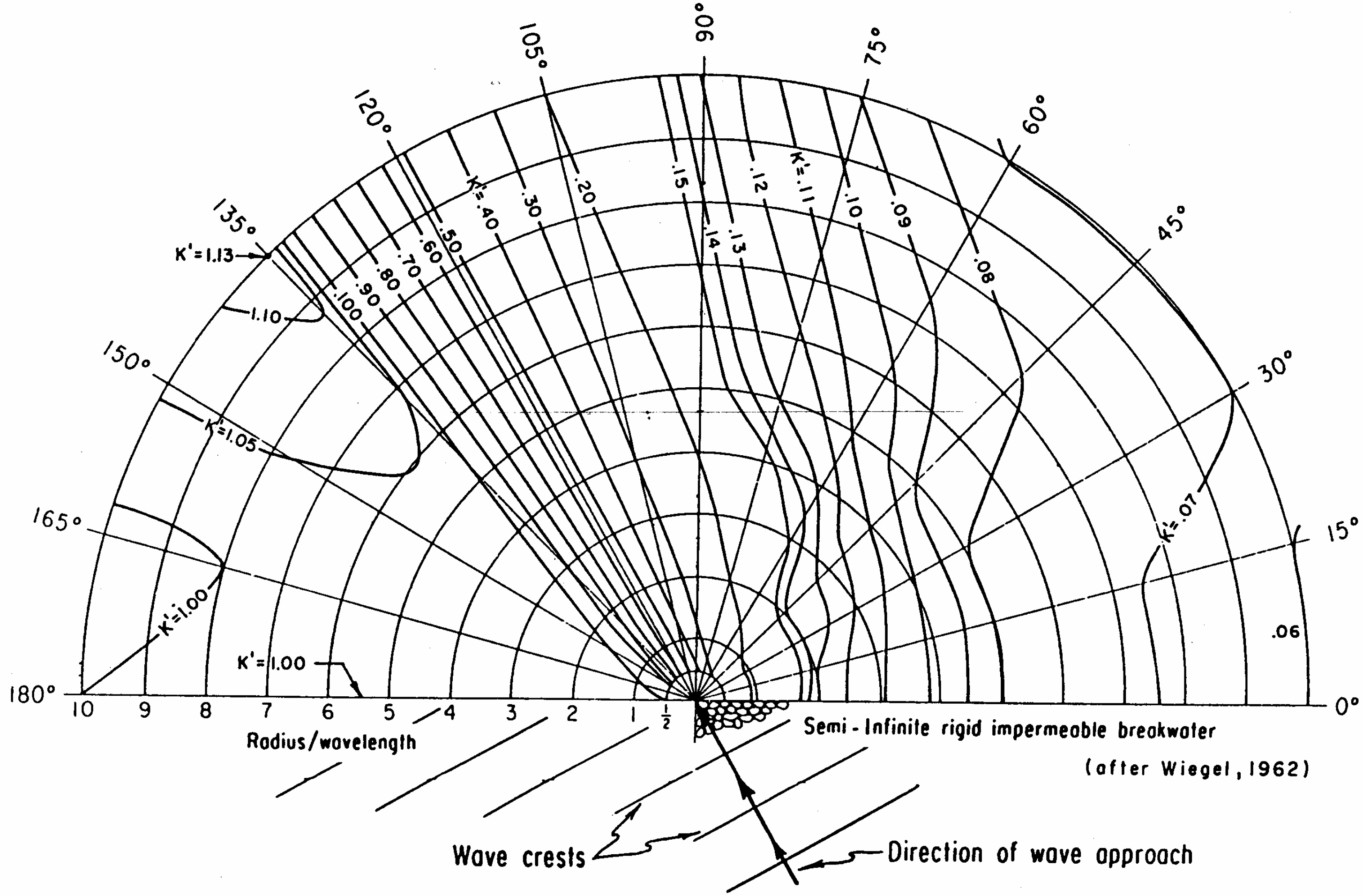


Figure 2-35. Wave diffraction diagram--120° wave angle.

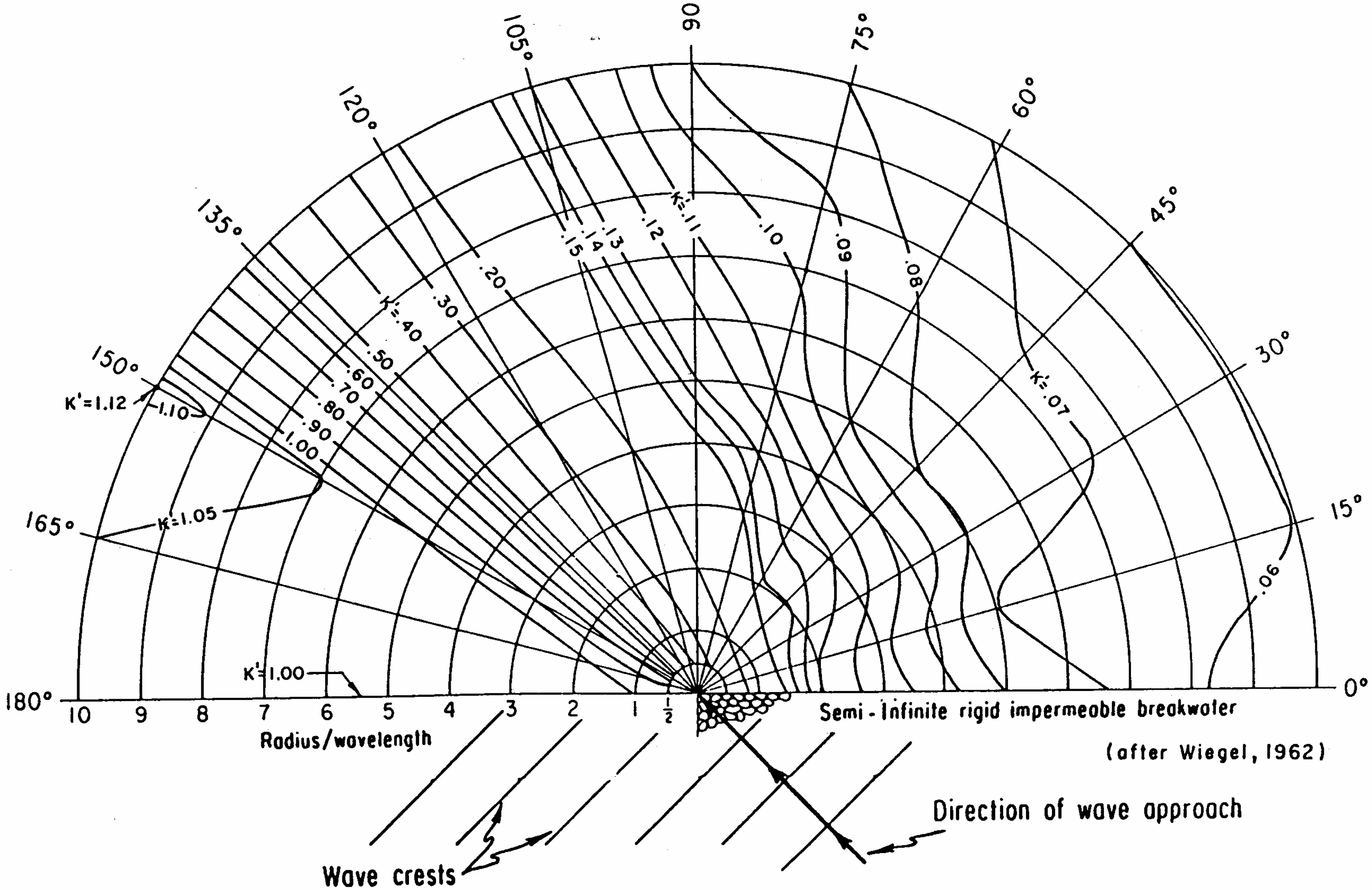


Figure 2-36. Wave diffraction diagram--135° wave angle.

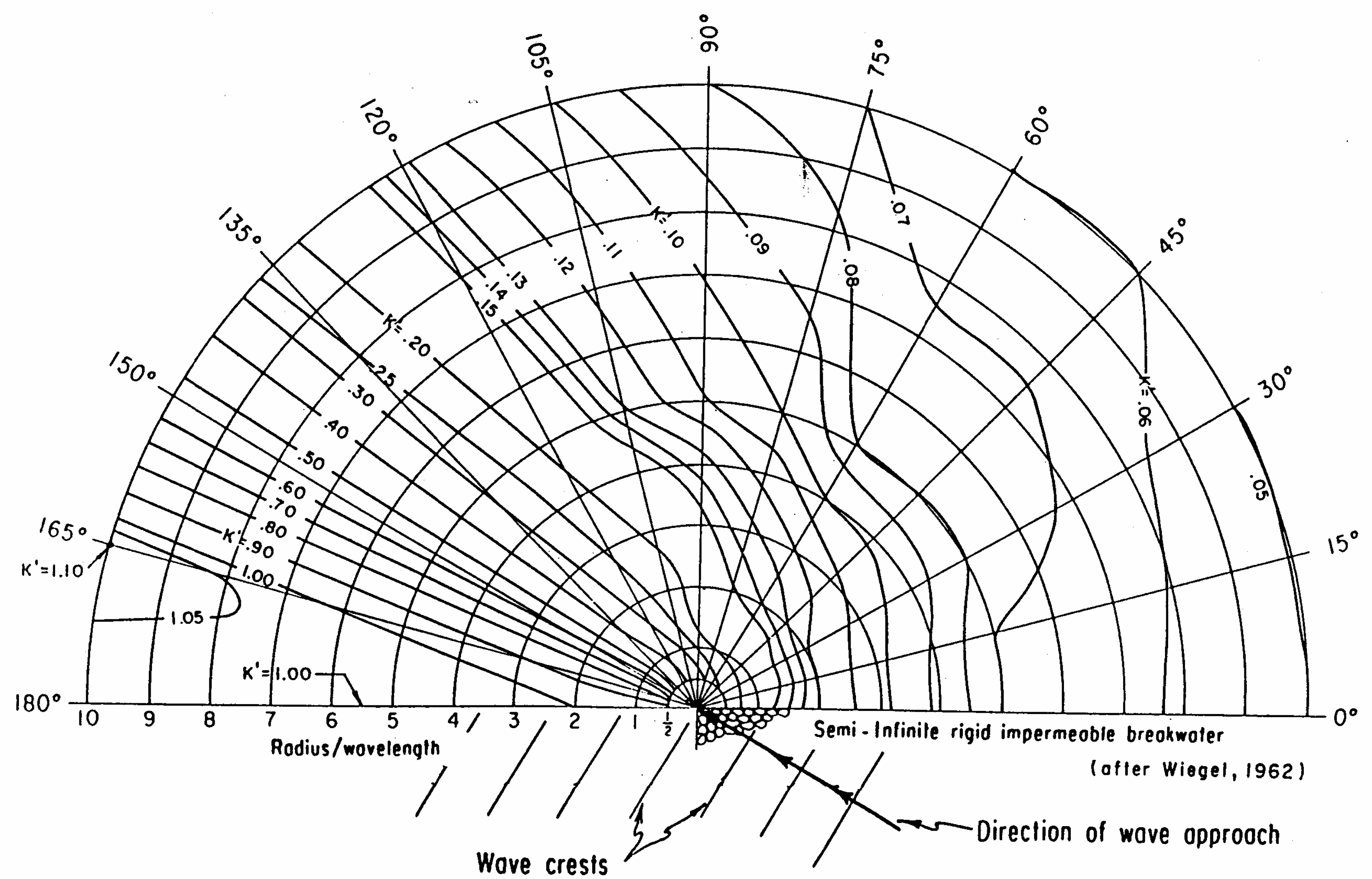


Figure 2-37. Wave diffraction diagram--150° wave angle.

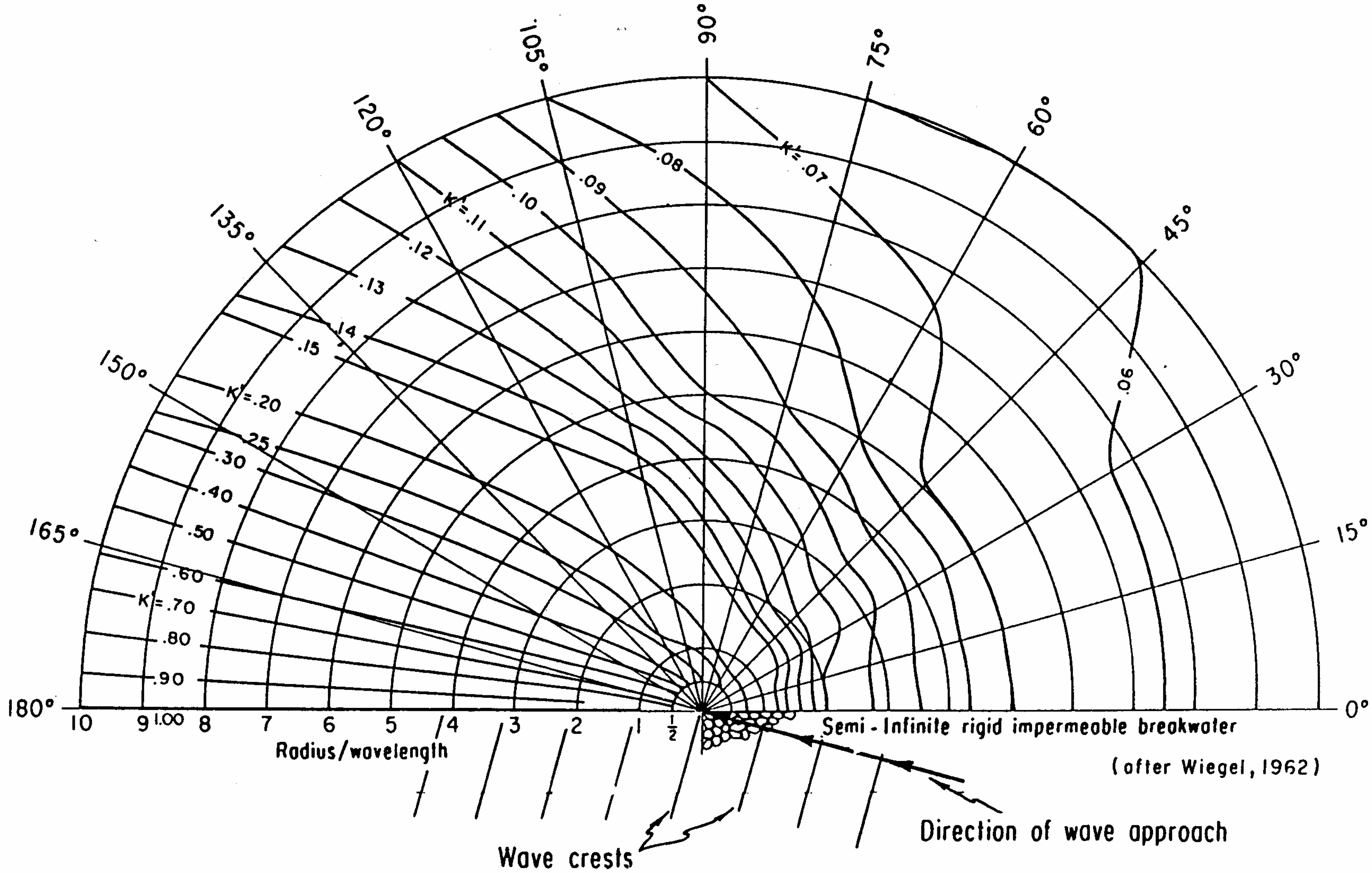


Figure 2-38. Wave diffraction diagram--165° wave angle.

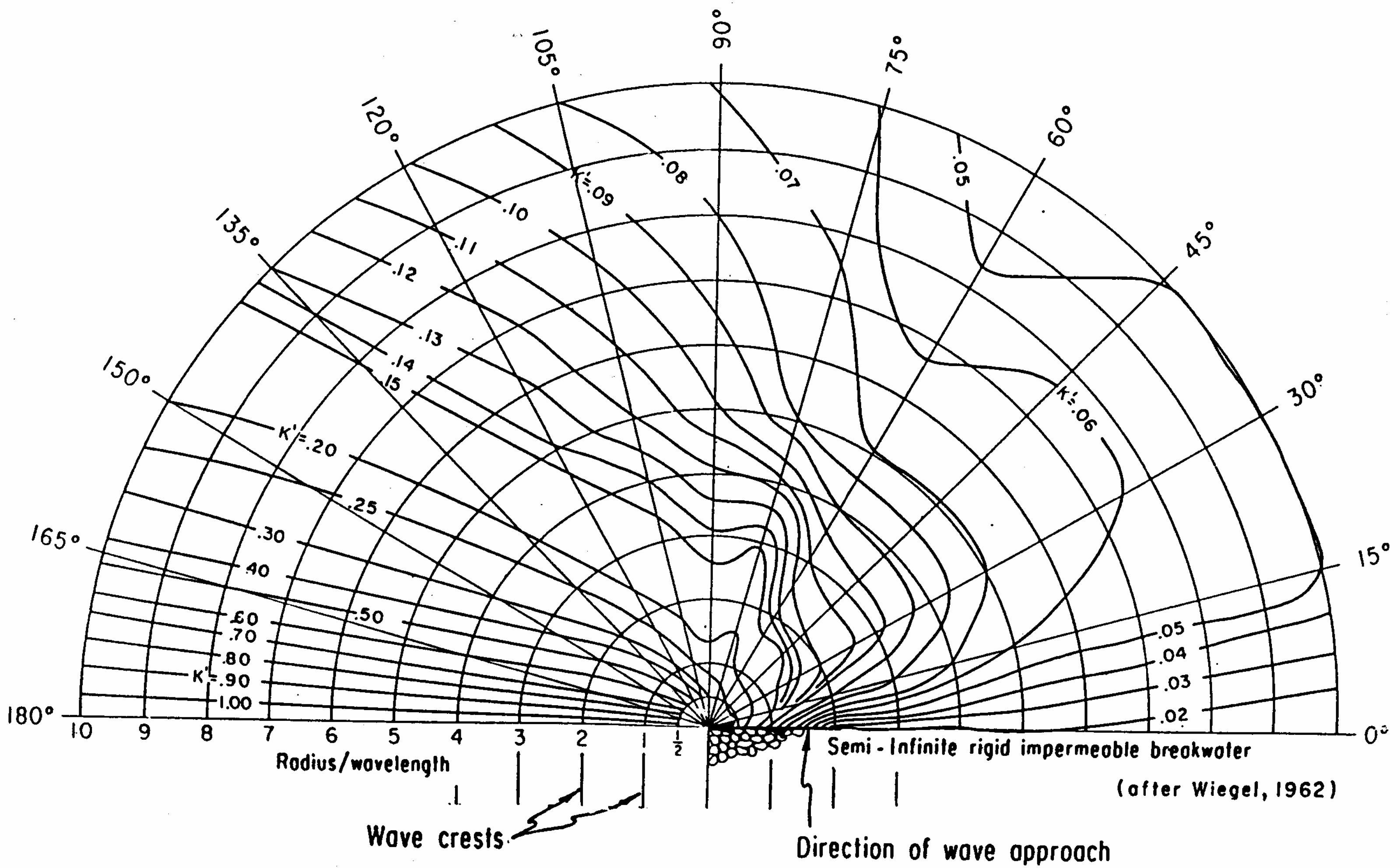
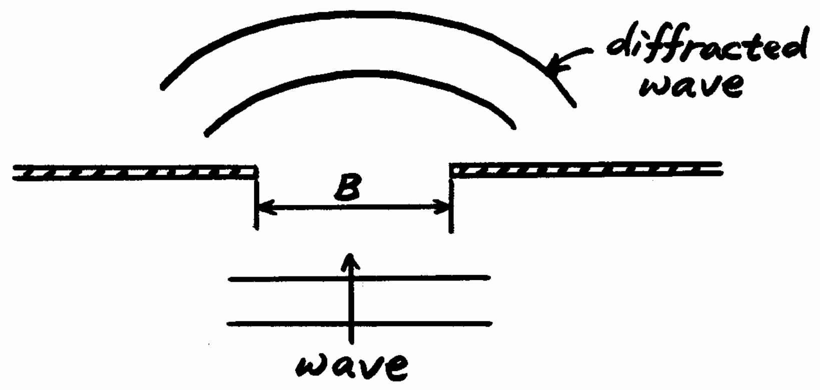


Figure 2-39. Wave diffraction diagram--180° wave angle.

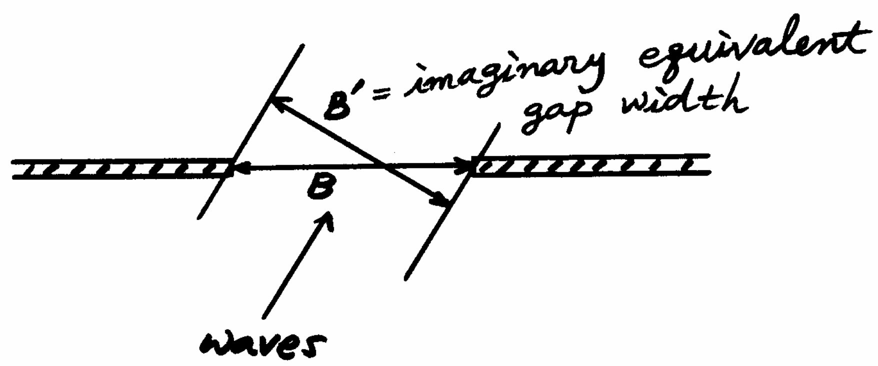
Waves passing through a gap ($B \leq 5L$)



Normal incidence: $B/L = 0.5, 1.0, 1.41, 1.64, \cdots, 5.0$ (SPM Figs. 2.43-2.52)

Oblique incidence: $B/L = 1.0$; $\theta = 0°, 15°, 30°, 45°, 60°, 75°$ (SPM Figs. 2.55-2.57)

Approximate method for oblique incidence: Use diffraction diagrams for normal incidence by replacing B by B'. See SPM Fig. 2.58 for comparison with exact solutions.



If $B > 5L$ (wide gap), use diffraction diagram for each breakwater separately, by assuming no interaction between breakwaters.

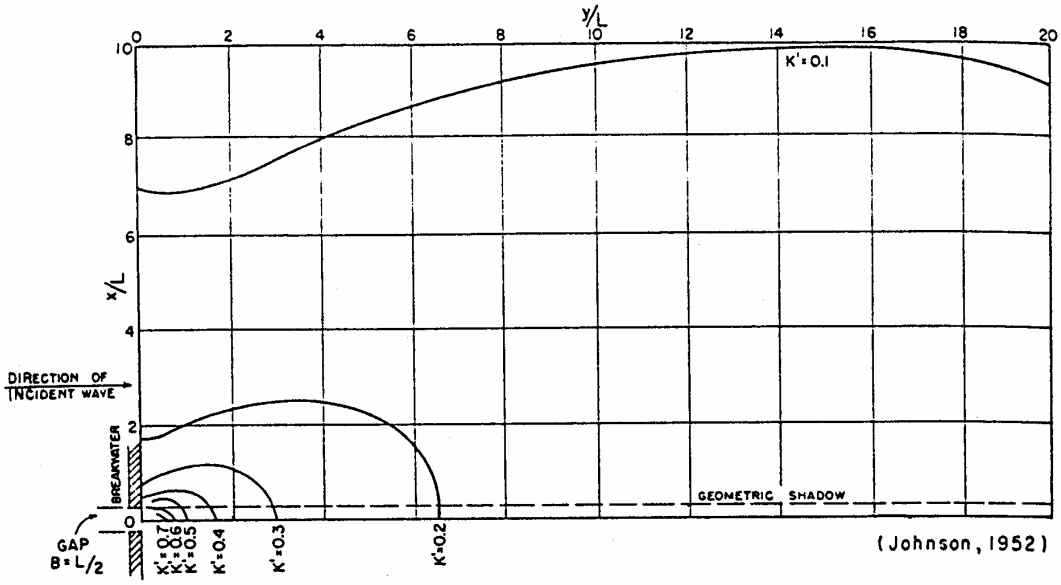


Figure 2-43. Contours of equal diffraction coefficient gap width = 0.5 wavelength (B/L = 0.5).

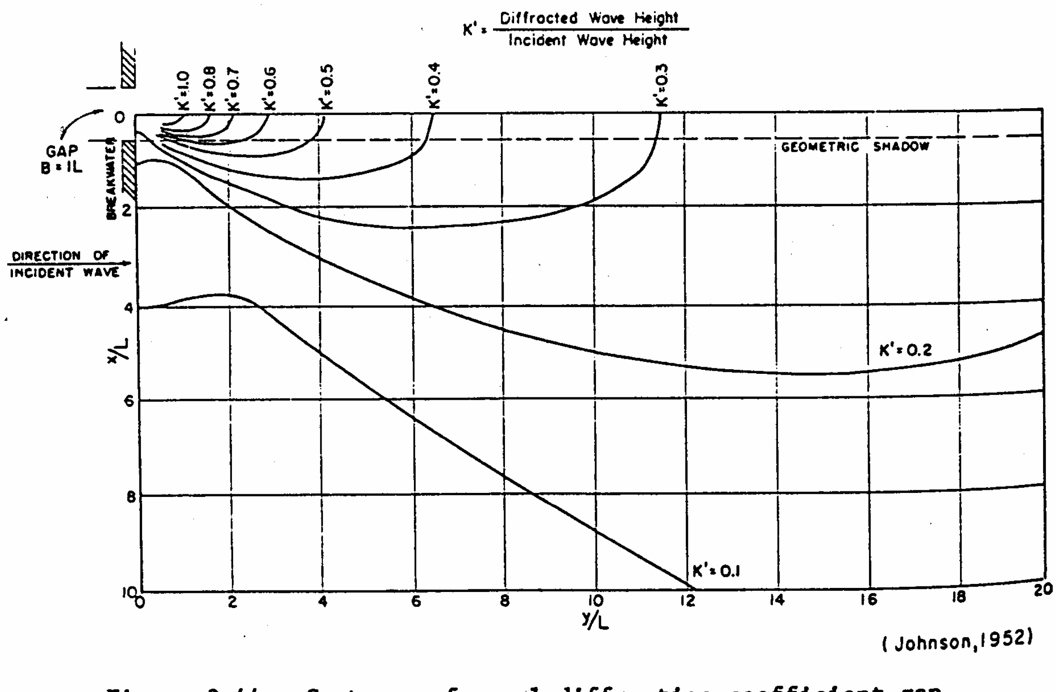


Figure 2-44. Contours of equal diffraction coefficient gap width = 1 wavelength (B/L = 1).

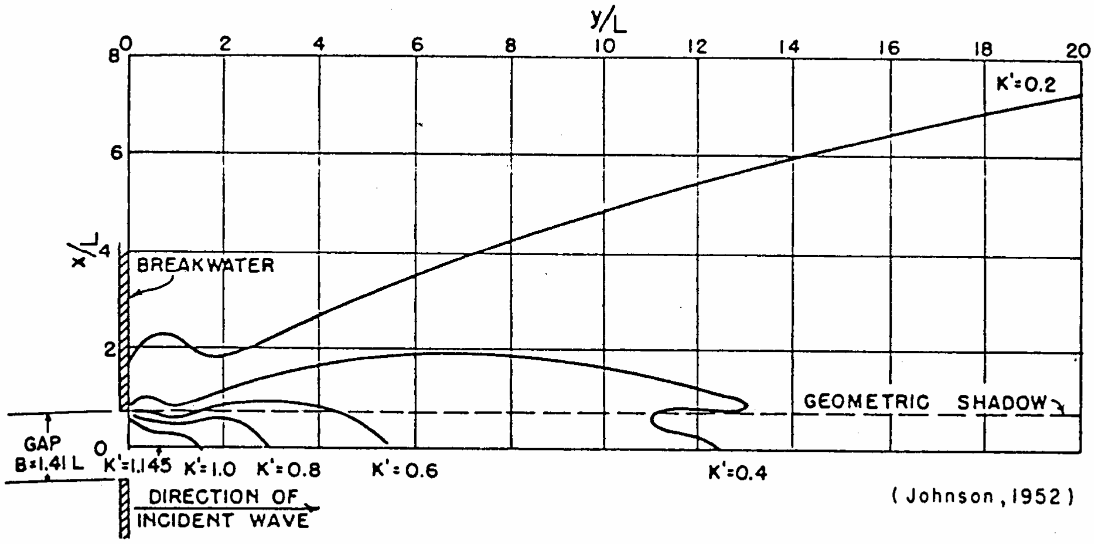


Figure 2-45. Contours of equal diffraction coefficient gap width = 1.41 wavelengths (B/L = 1.41).

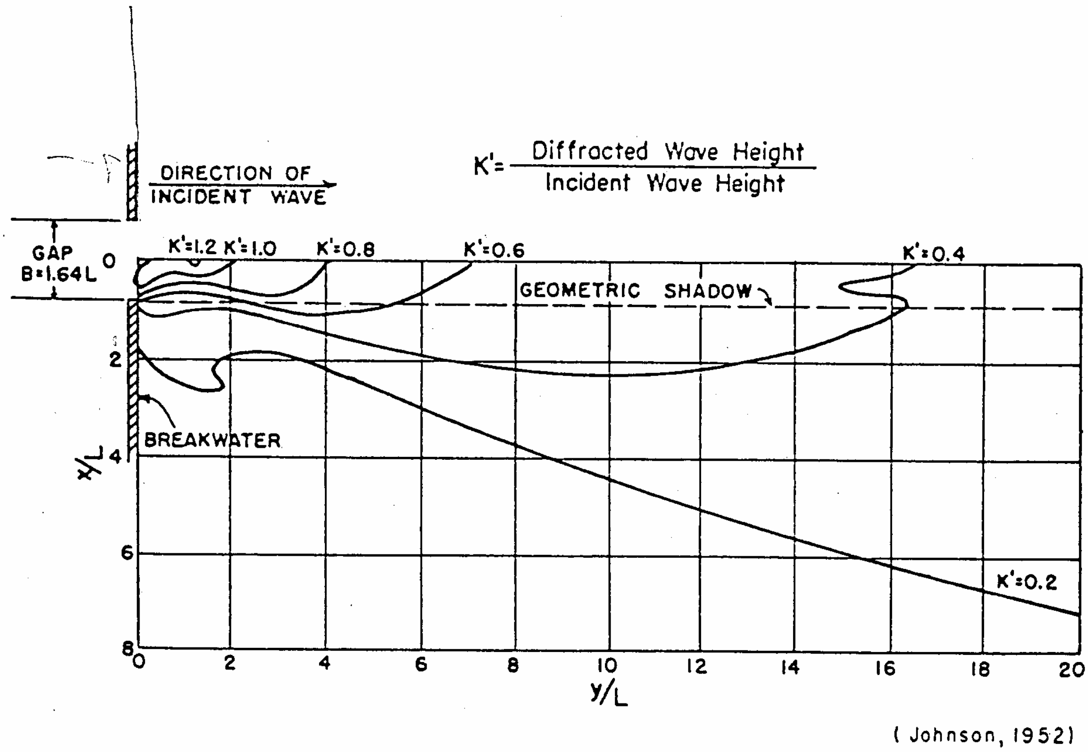


Figure 2-46. Contours of equal diffraction coefficient gap width = 1.64 wavelengths (B/L = 1.64).

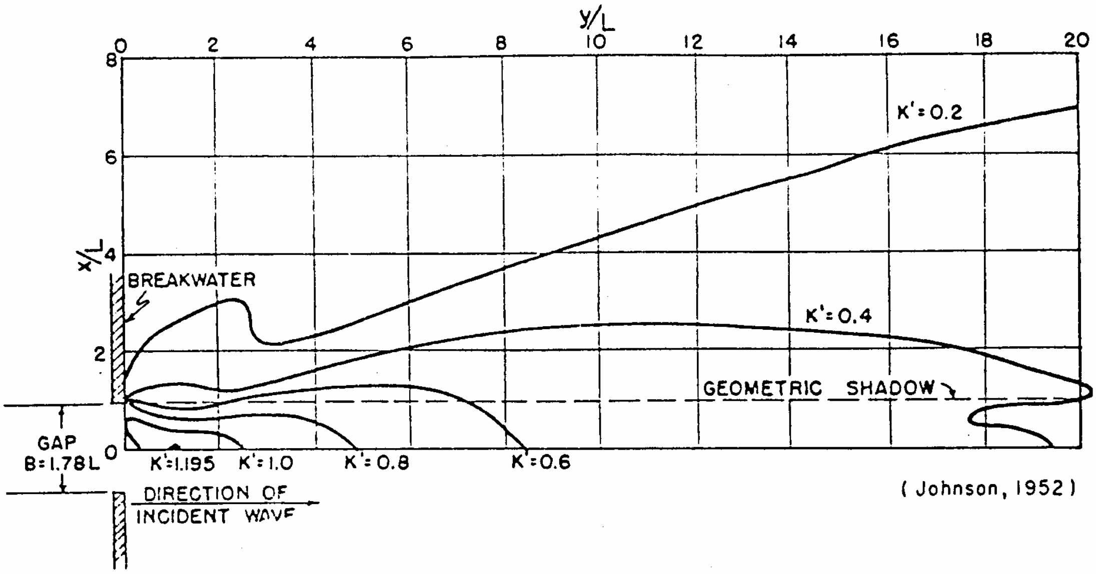


Figure 2-47. Contours of equal diffraction coefficient gap width = 1.78 wavelengths (B/L = 1.78).

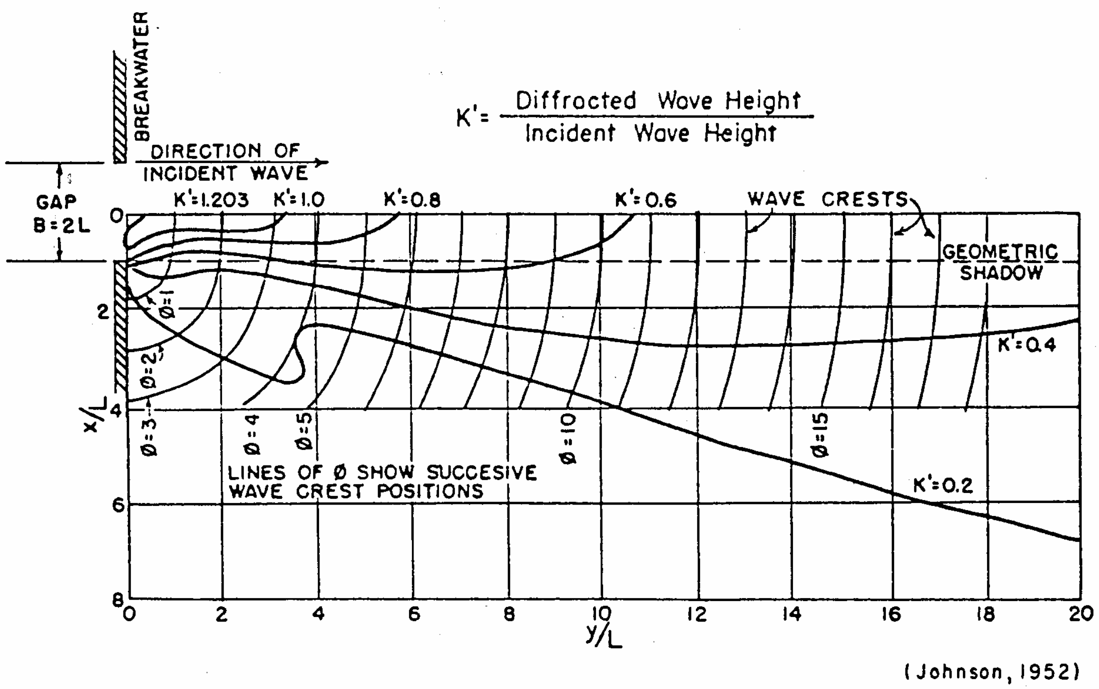


Figure 2-48. Contours of equal diffraction coefficient gap width = 2 wavelengths (B/L = 2).

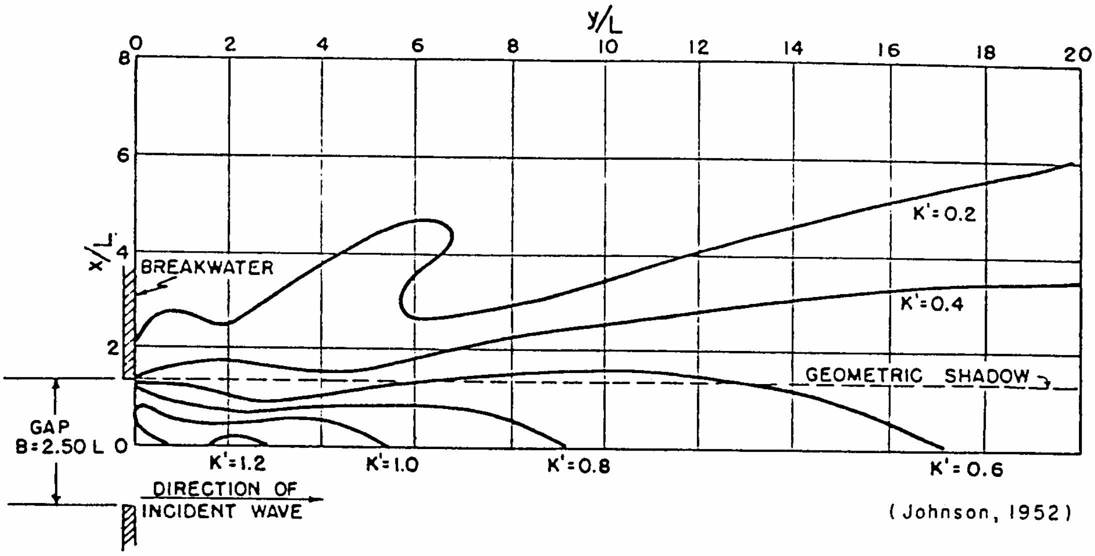


Figure 2-49. Contours of equal diffraction coefficient gap width = 2.50 wavelengths (B/L = 2.50).

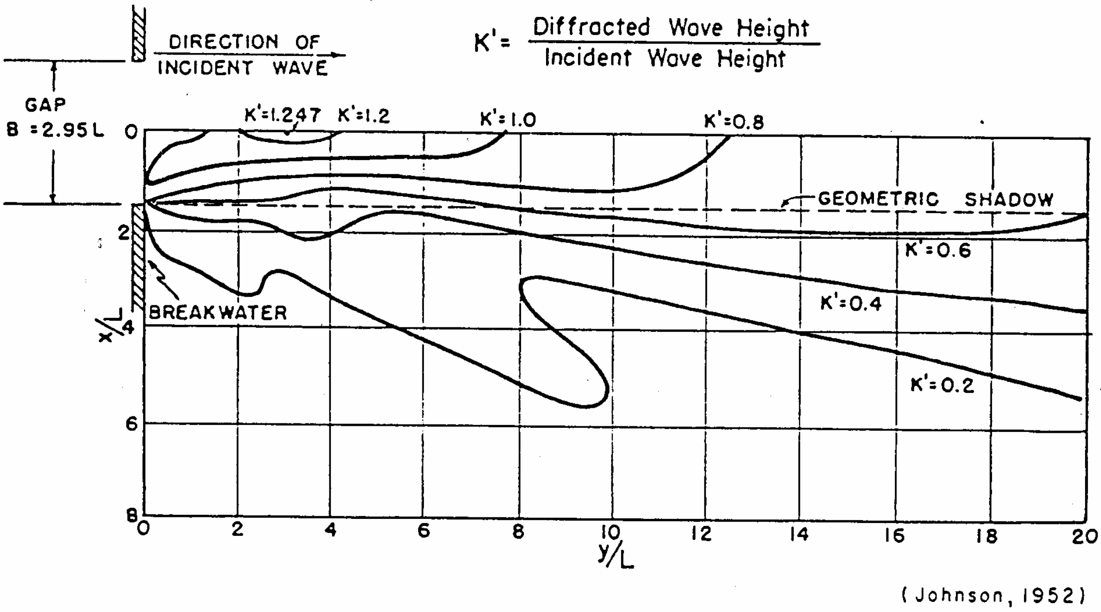


Figure 2-50. Contours of equal diffraction coefficient gap width = 2.95 wavelengths (B/L = 2.95).

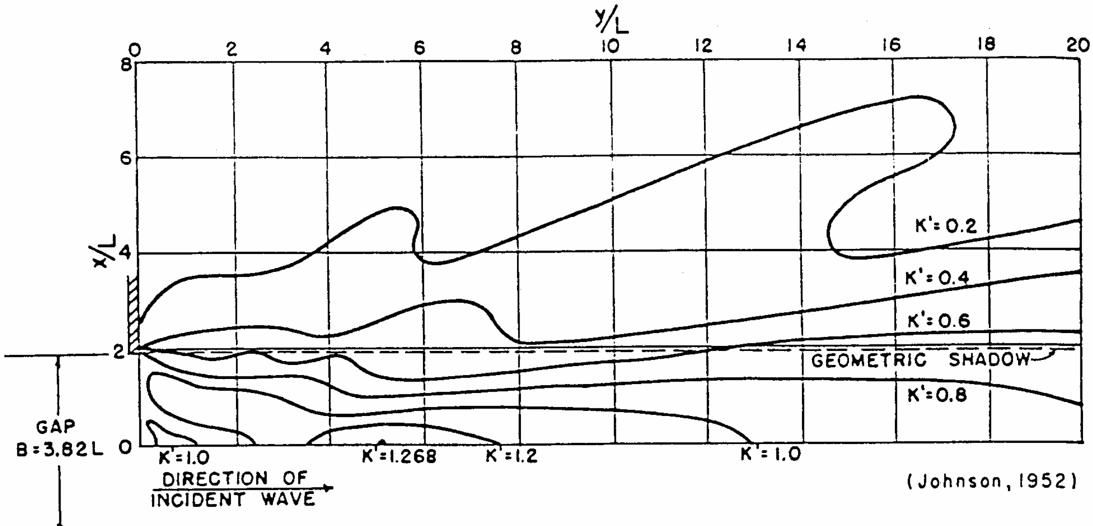


Figure 2-51. Contours of equal diffraction coefficient gap width = 3.82 wavelengths (B/L = 3.82).

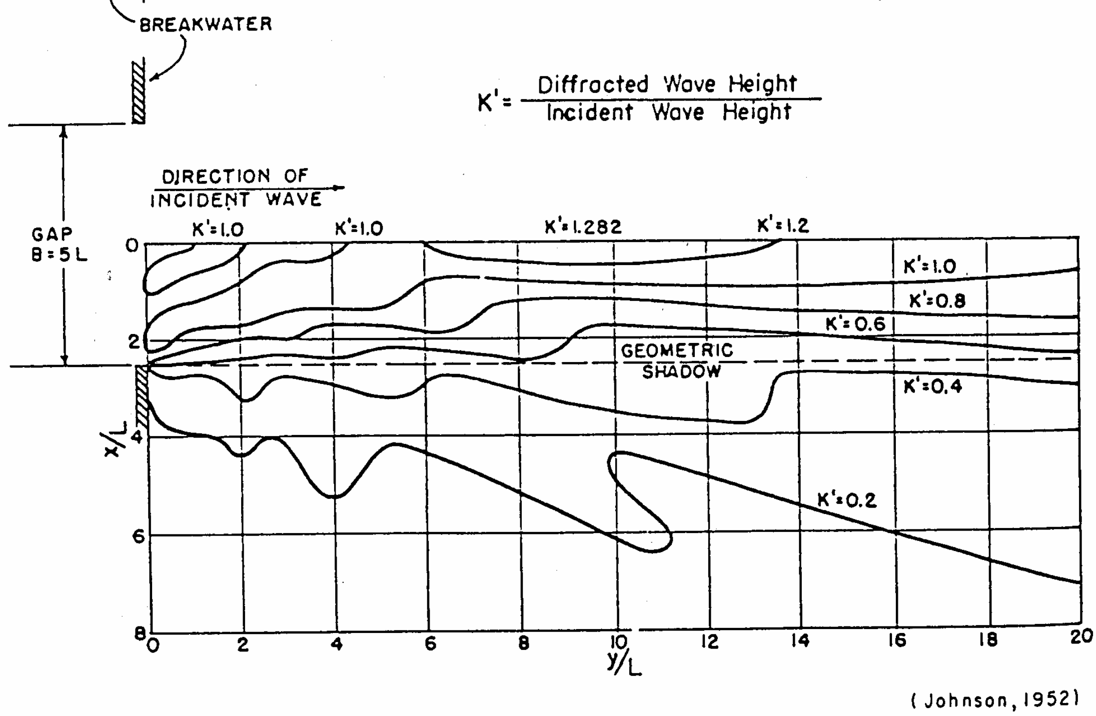


Figure 2-52. Contours of equal diffraction coefficient gap width = 5 wavelengths (B/L = 5).

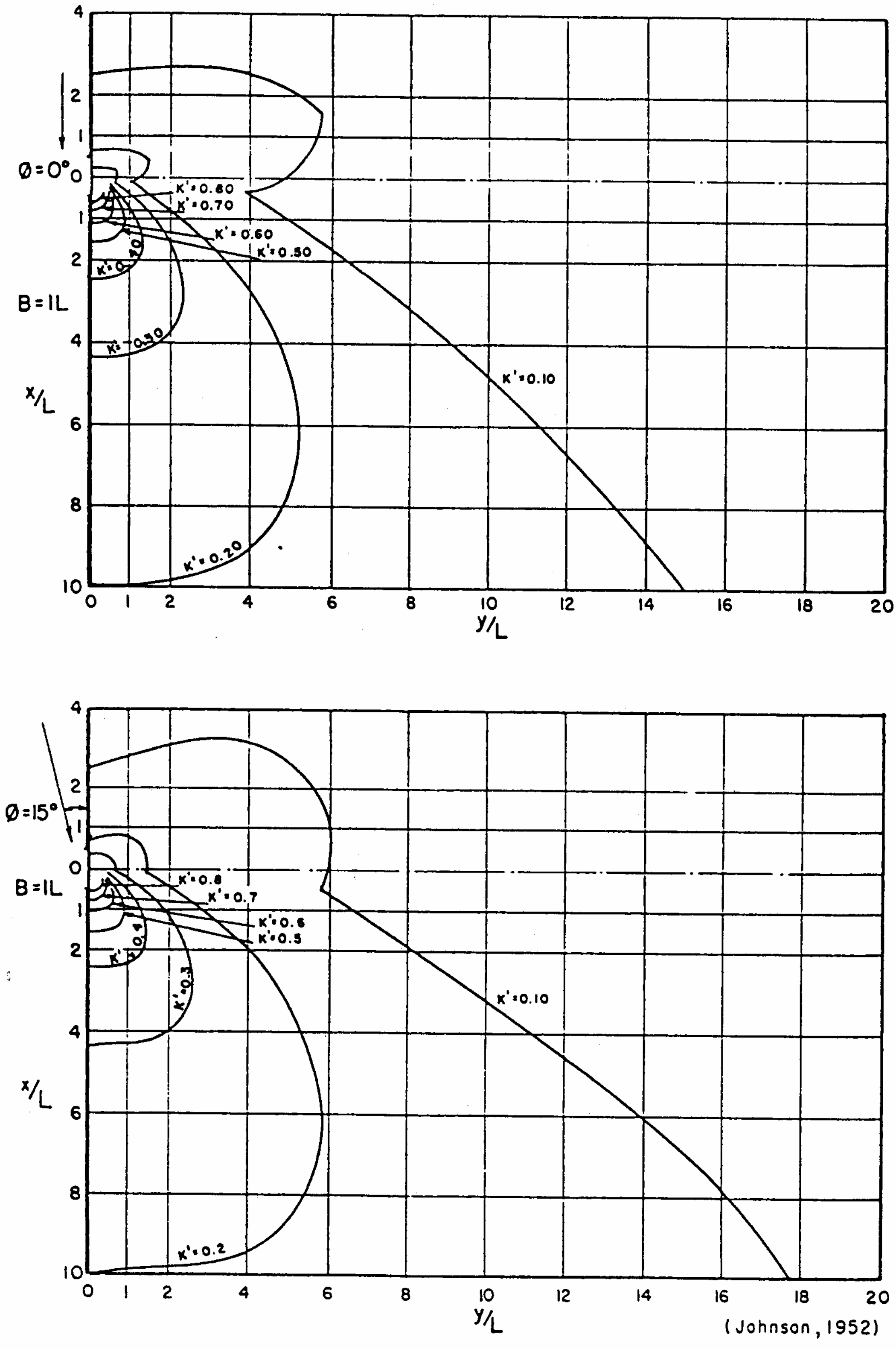


Figure 2-55. Diffraction for a breakwater gap of one wave-length width where $\phi = 0^{\circ}$ and 15°.

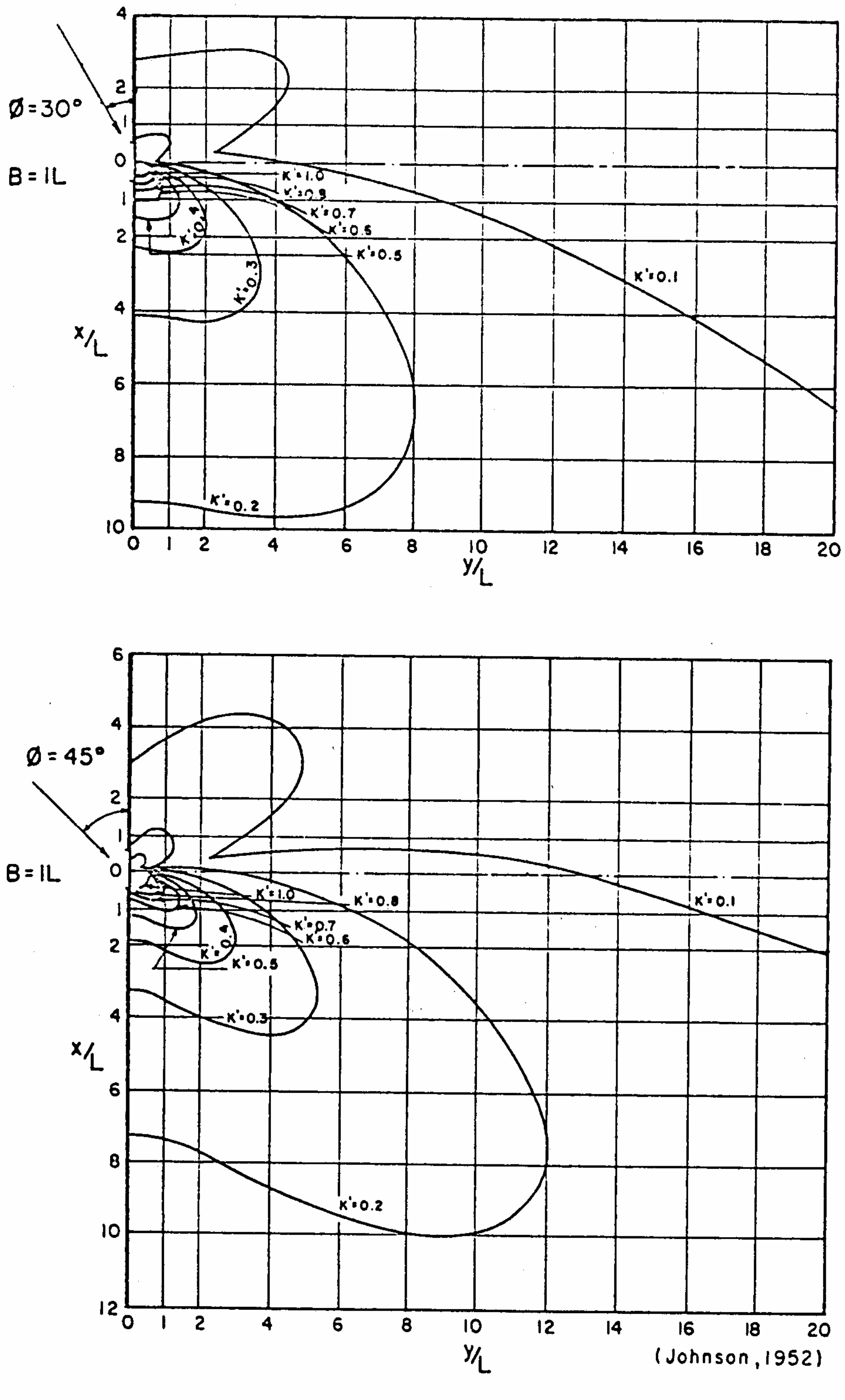


Figure 2-56. **Diffraction for a breakwater gap of one wavelength width where ϕ = 30° and 45°.**

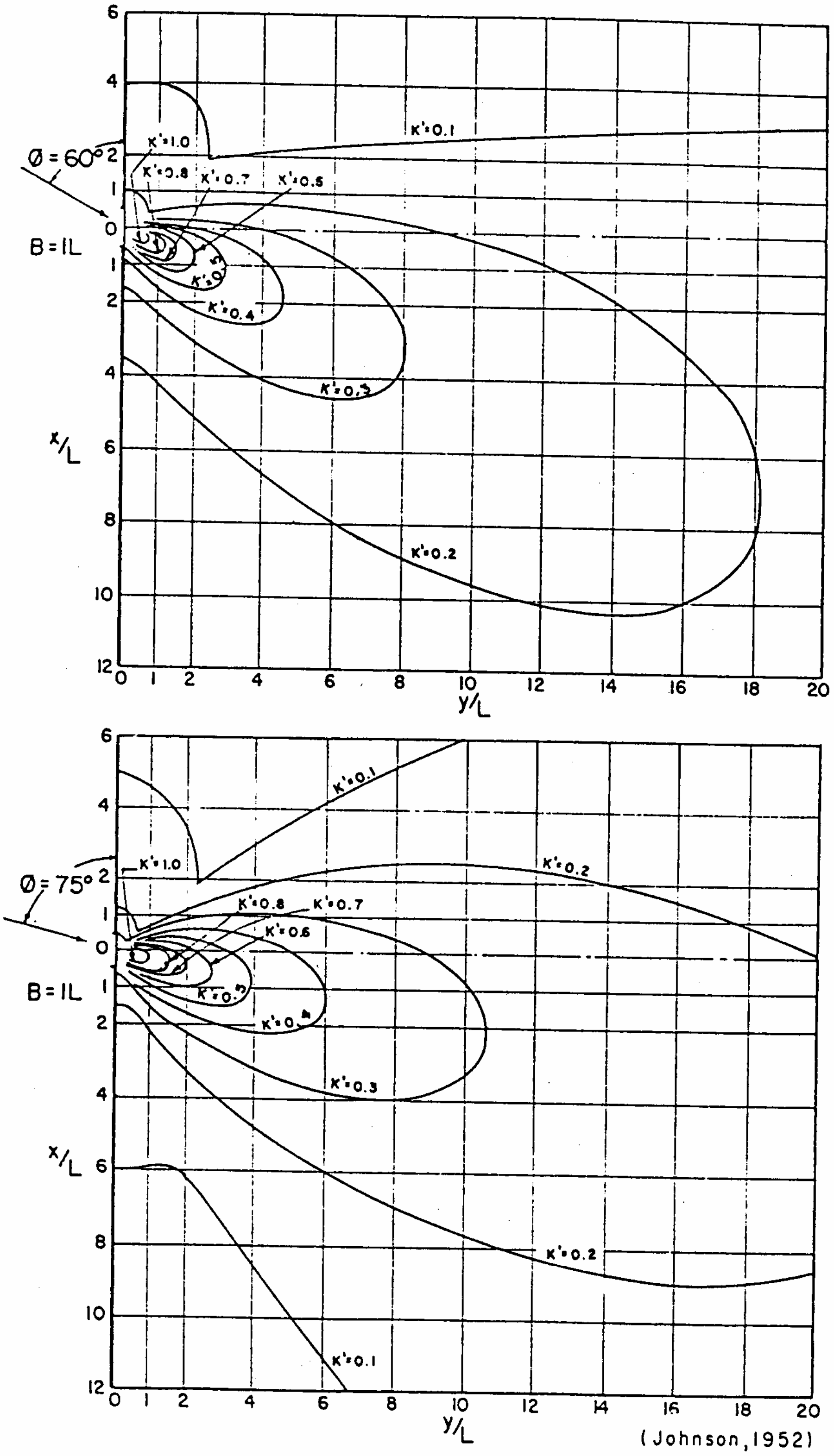
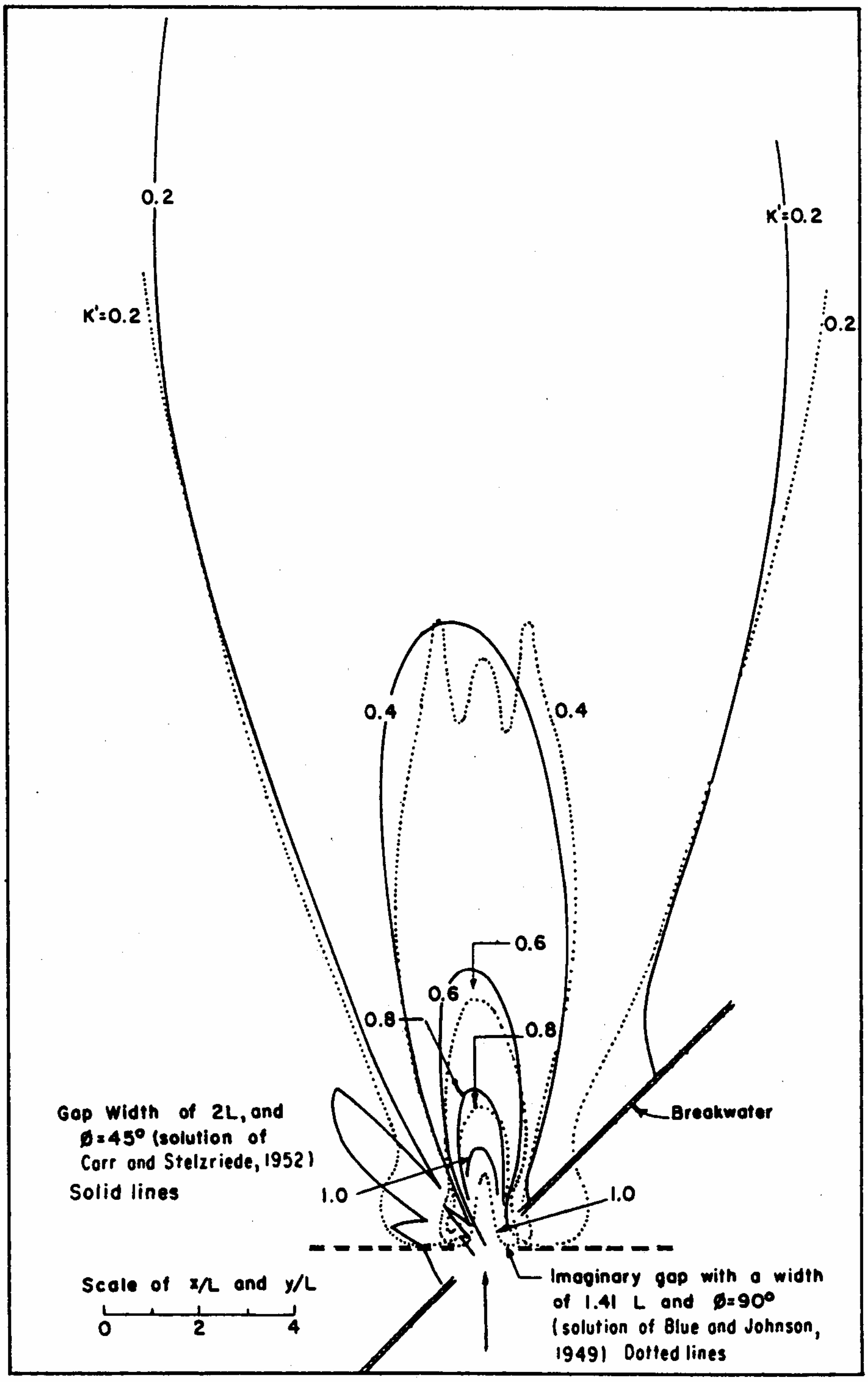


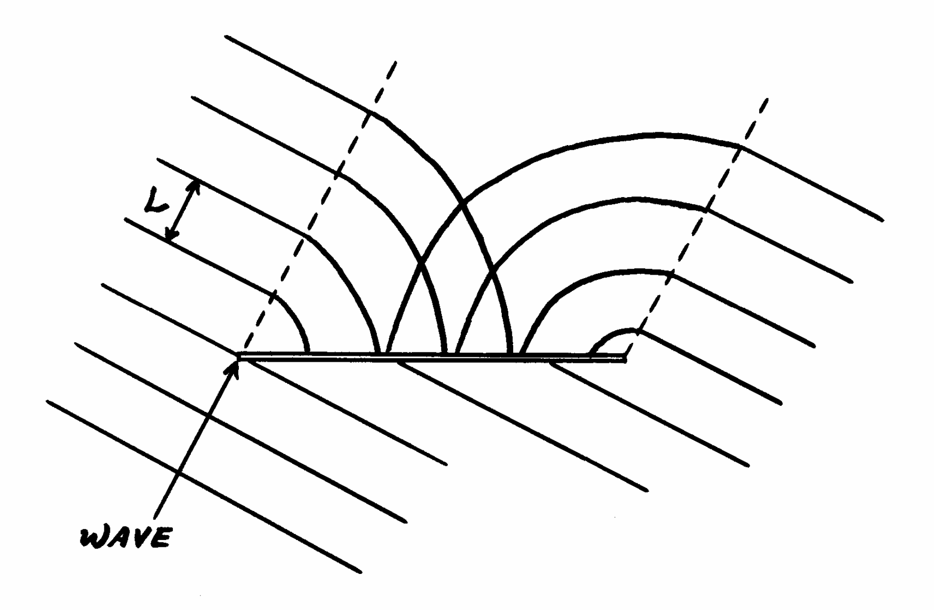
Figure 2-57. Diffraction for a breakwater gap of one wave-length width where $\phi = 60^{\circ}$ and 75°.



(Johnson, 1952)

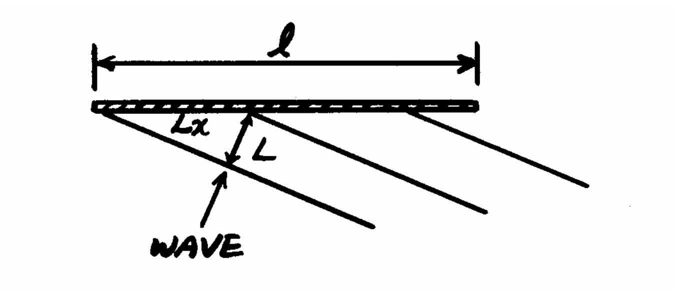
Figure 2-58. Diffraction diagram for a gap of two wavelengths and a 45° approach compared with that for a gap width $\sqrt{2}$ wavelengths with a 90° approach.

Waves behind an offshore breakwater



$$K_D^2 = K_{D_L}^2 + K_{D_R}^2 + 2K_{D_L}K_{D_R}\cos\theta$$

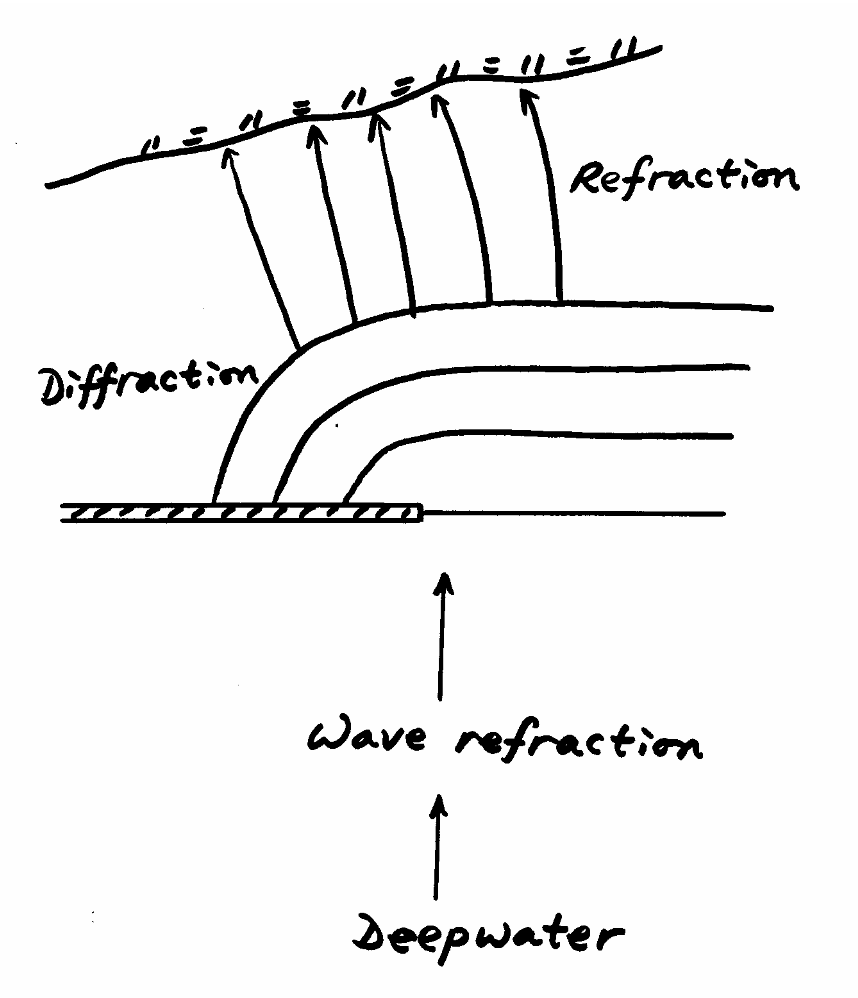
where $\cos\theta$ = phase difference between two diffracted waves:



$$\theta = \left\{\frac{l}{L_x} - \mathrm{INT}\left(\frac{l}{L_x}\right)\right\} \times 2\pi$$

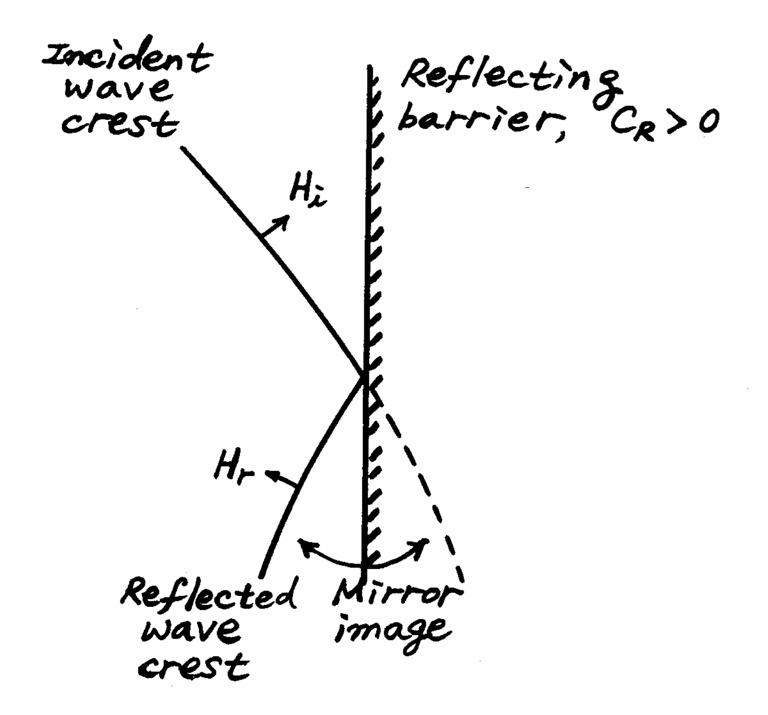
For normal incidence, $\theta = 0°$, $\cos\theta = 1.0$

4.7 Combined Refraction and Diffraction



More general approach: Use mild-slope equation (Berkhoff, 1972, Computation of combined refraction-diffraction, Proceedings of 13th International Conference on Coastal Engineering, ASCE, pp. 471-490).

4.8 Wave Reflection



$$H_r = C_R H_i$$

where C_R = reflection coefficient ≤ 1.0

$$C_R \cong \begin{cases} 0.1 \sim 0.2 & \text{sand beach} \\ 0.4 & \text{Tetrapod on breakwater} \\ 0.7 & \text{rock beach} \\ 0.9 & \text{vertical wall} \end{cases}$$