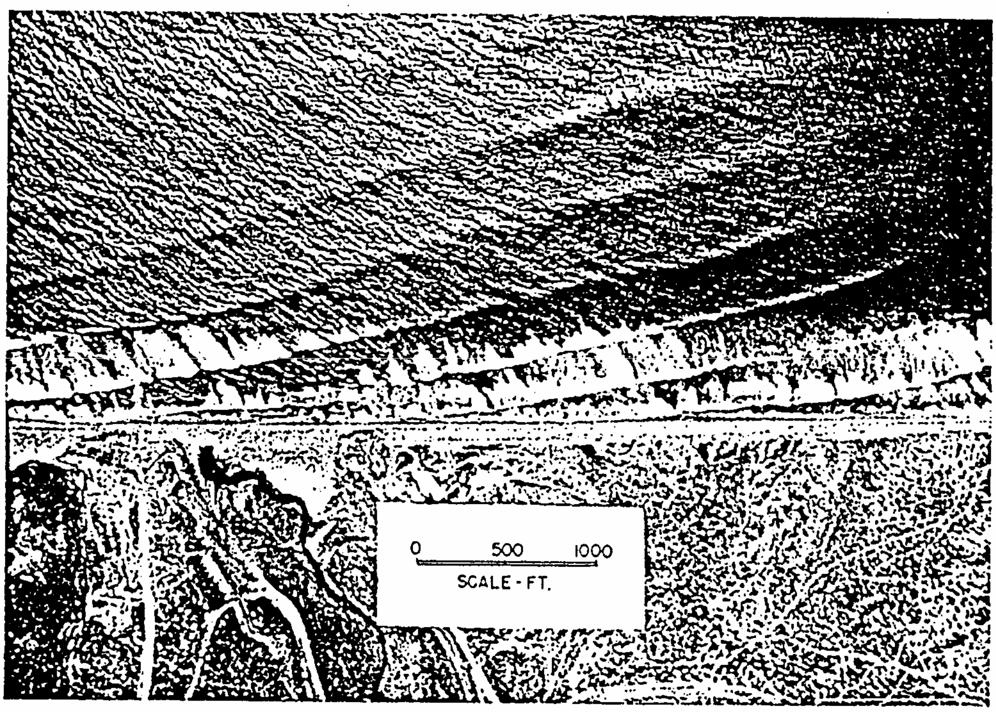
Chapter 6. Wind-Generated waves

6.1 Waves at Sea

local wind wave (short-crested) + swell (long-crested)



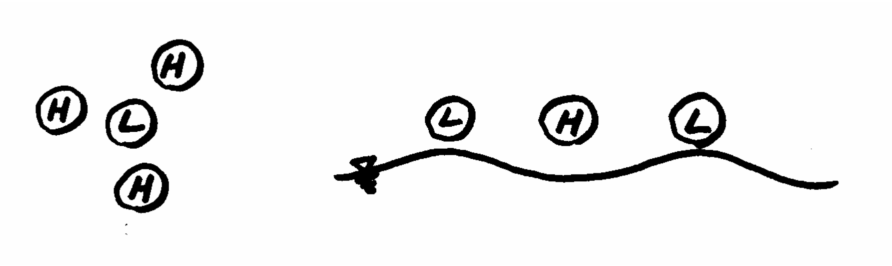
fully developed sea: unlimited fetch and duration of wind; wind energy input is balanced by energy dissipation due to wave breaking.

growing sea: limited fetch and duration of wind (most cases in nature)

6.2 Wind-Wave Generation and Decay

• Phillips (1957): initial stage

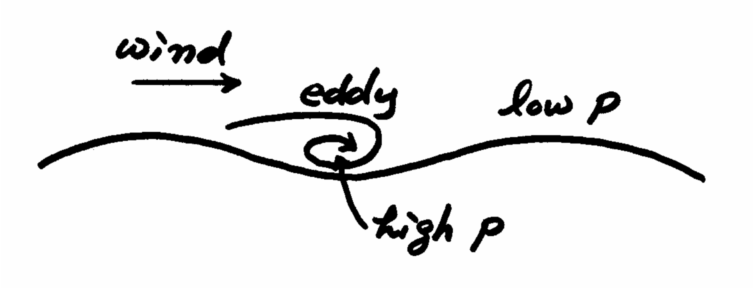
Turbulent wind energy is transferred to water by pressure fluctuation.



$H(t) \propto t$ (linear growth of waves)

• Miles (1957): developing stage

After some waves are developed, eddies are formed at troughs.



$H(t) \propto e^{\alpha t}$ (exponential growth of waves)

Assume that wave growth = sum of linear + exponential growth, and find coefficients using field dat

• Hasselmann (1962)

wave interactions → energy transfer to lower frequencies

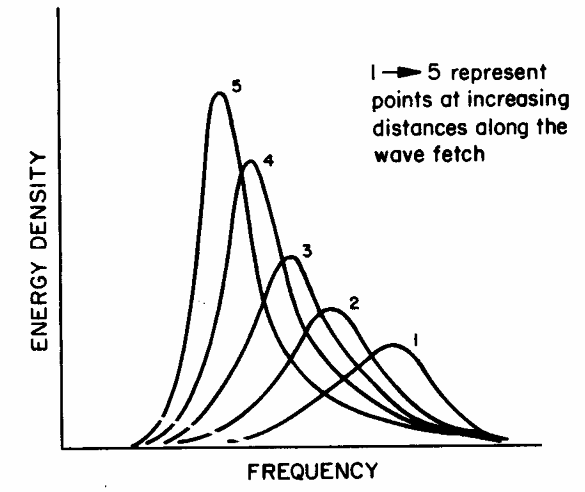
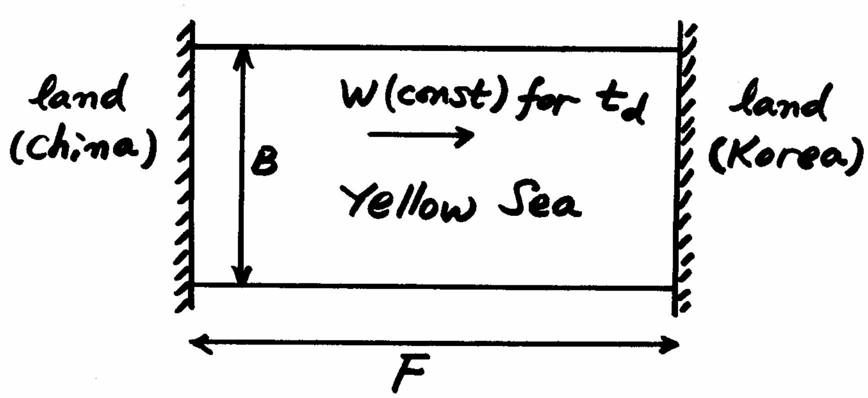


Figure 6.2. Wave spectra growth.

Growth of wind waves depends on

1) fetch length, F
2) wind speed, W
3) duration of wind, t_d
4) fetch width, B
5) water depth, d

⋮



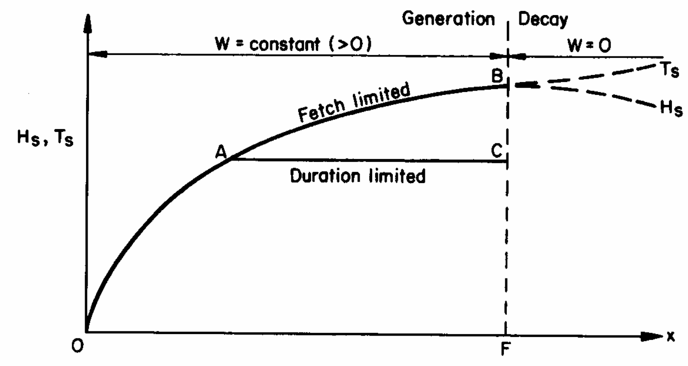


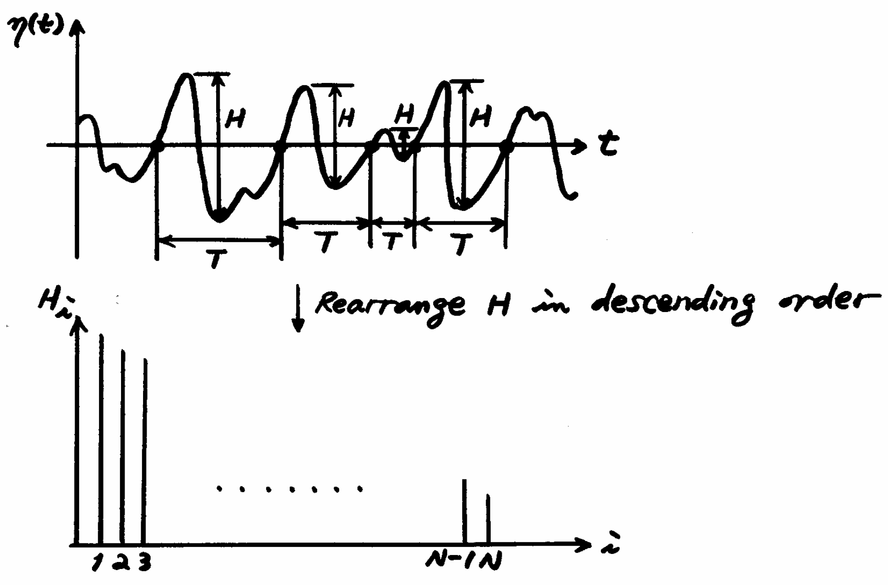
Figure 6.1. Idealized wave growth and decay for a constant wind velocity.

If $t_d > F / C_g$, fetch-limited, $H, T = f(W, F)$

If $t_d = x_A / C_g < F / C_g$, duration-limited, $H, T = f(W, t_d)$

6.3 Wave Record Analysis for Height and Period

Zero-crossing method:



H_n = average of the highest $n\%$ of the wave heights e.g. $H_{10} = H_{1/10} = \frac{1}{N/10}\sum_{i=1}^{N/10} H_i$

$\uparrow$ in text $\quad$ $\uparrow$ more common expression

$$H_{33} = H_{1/3} = \frac{1}{N/3}\sum_{i=1}^{N/3} H_i \equiv H_s \quad \text{(significant wave height)}$$

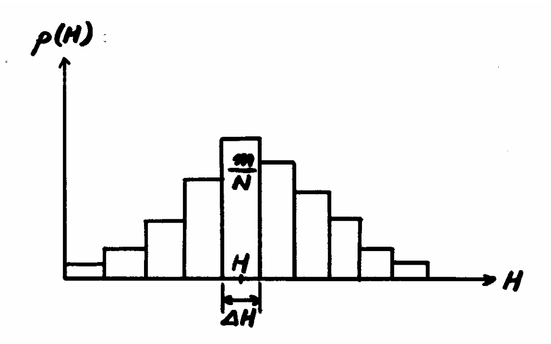
$$H_{100} = \overline{H} = \frac{1}{N}\sum_{i=1}^{N} H_i$$

$$H_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} H_i^2} \quad \text{(root-mean-squared wave height)}$$

$$T_s = T_{H_{1/3}} = \frac{1}{N/3}\sum_{i=1}^{N/3} T_{H_i}\ ; \quad T_{H_i} = \text{period of the wave of height of } H_i$$

<u>Wave height distribution</u>

Find probability density function, $p(H)$, by plotting histogram of wave height:



m waves out of N have

$$H - \frac{\Delta H}{2} < H_i < H + \frac{\Delta H}{2}$$

$$p(H)\Delta H = \frac{m}{N}$$

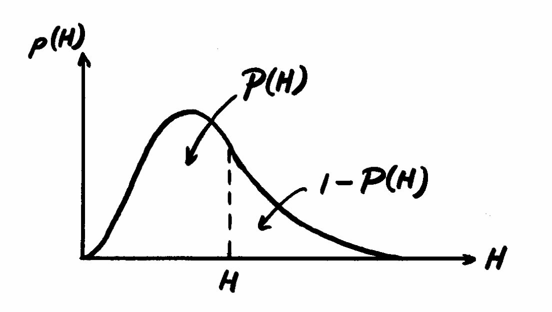
$\int_0^\infty p(H)dH = 1$ for probability density function

Longuet-Higgins: $p(H)$ is given by a Rayleigh distribution

$$p(H) = \frac{2H}{H_{rms}^2} e^{-(H/H_{rms})^2}$$

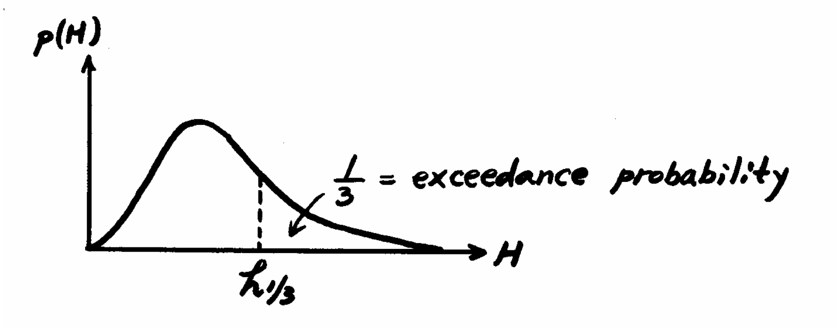
cumulative probability, $P(H) = \text{probability}(H' \leq H) = \int_0^H p(H)dH = 1 - e^{-(H/H_{rms})^2}$

exceedance probability = $\text{probability}(H' > H) = 1 - P(H) = e^{-(H/H_{rms})^2}$



$$\overline{H} = \int_0^\infty Hp(H)dH = 0.886 H_{rms}$$

$$H_s = ?$$



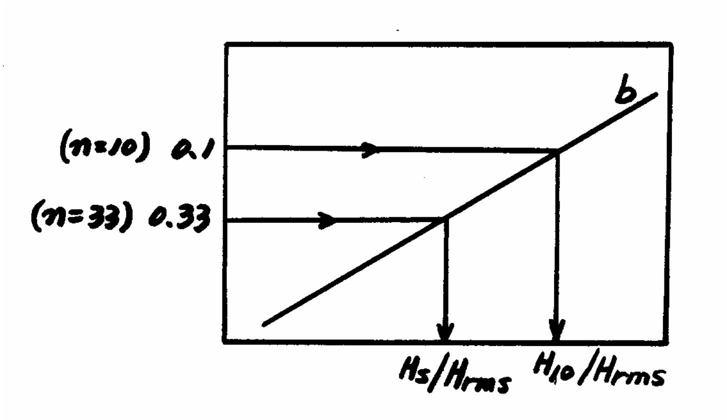
$$\exp\left[-\left(h_{1/3}/H_{rms}\right)^2\right] = \frac{1}{3} \rightarrow h_{1/3}$$

$$H_s = \frac{\int_{h_{1/3}}^\infty Hp(H)dH}{\int_{h_{1/3}}^\infty p(H)dH} = \frac{\int_{h_{1/3}}^\infty Hp(H)dH}{1/3} = 1.416 H_{rms} \cong \sqrt{2} H_{rms}$$

Fig. 6.5: line a $\rightarrow$ exceedance prob. $\left(\frac{H'}{H_{rms}} > \frac{H}{H_{rms}}\right)$

Note: P in the y-axis of Fig. 6.5 must be $1-P$

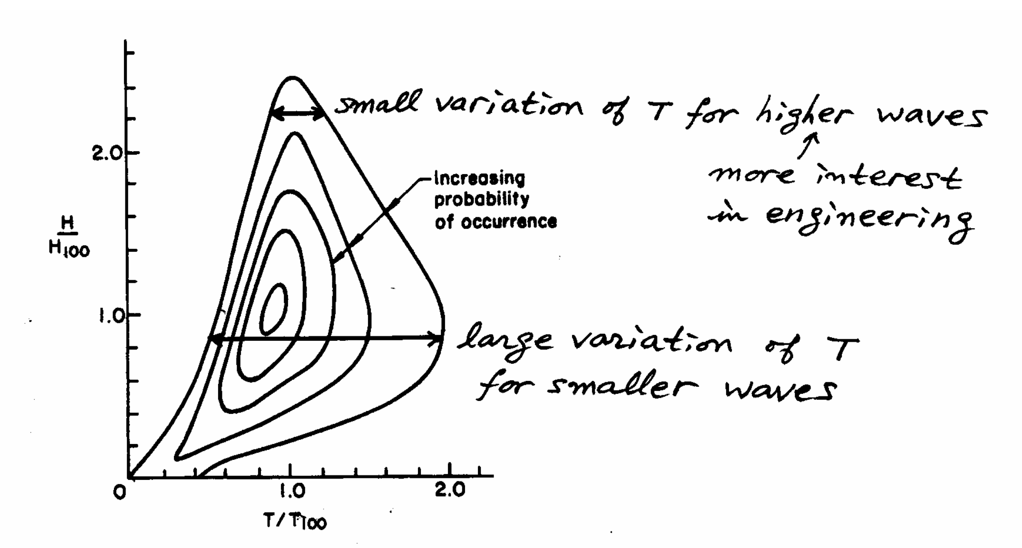
line b $\rightarrow$ average of the highest n% waves

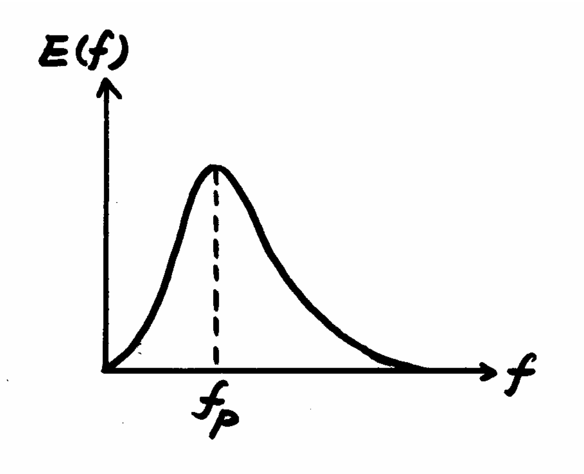


$H_{\max} = 0.707 H_s \sqrt{\ln N} \cong 2.0 H_s$: used for design of offshore structures

Wave period distribution

Joint probability of H and T





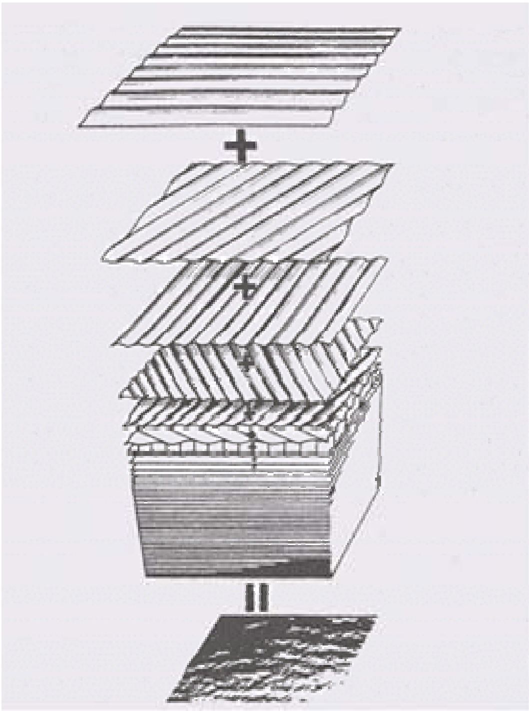
$$T_p = \frac{1}{f_p}\ ; \quad T_s \cong 0.95 T_p$$

where T_s = significant wave period from zero-crossing method

T_p = peak period from spectral analysis.

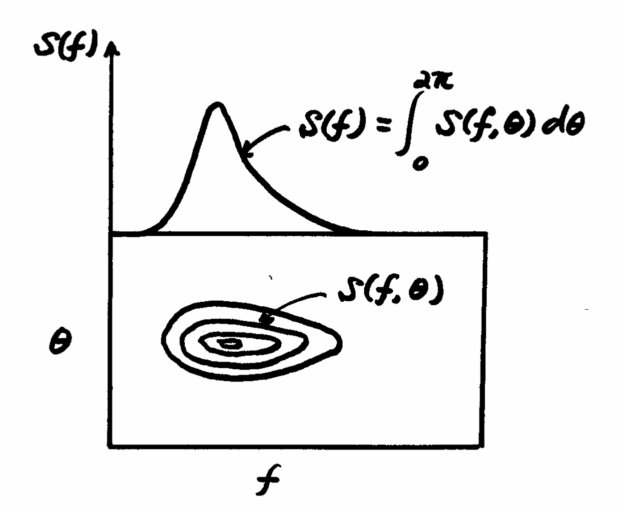
6.4 Wave Spectral Characteristics

Irregular waves = superposition of many sinusoidal waves of different frequency, amplitude, phase, and direction



Using directional spectrum analysis, we obtain directional spectrum, $S(f,\theta)$.

Assuming single wave direction, we obtain frequency spectrum, $S(f)$.



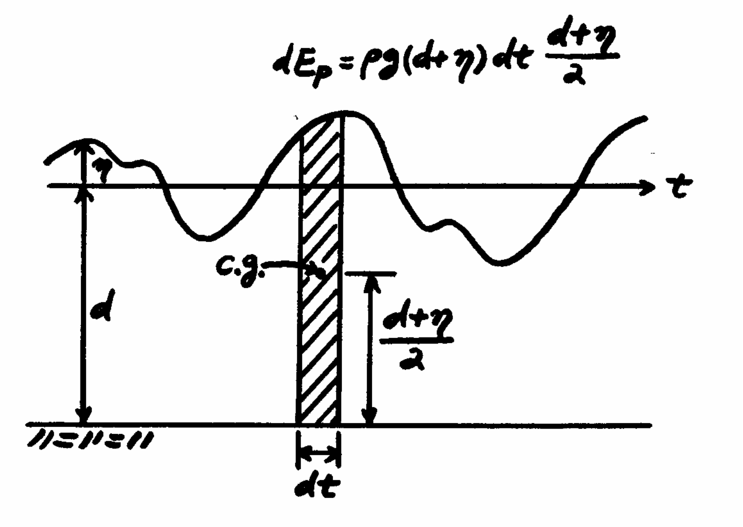
For a sinusoidal wave, energy per unit surface area or energy density is

$$\overline{E} = \frac{1}{8}\rho g H^2$$

Since ρg = constant, we can write

$$S(f,\theta)dfd\theta = \sum_{f}^{f+df}\sum_{\theta}^{\theta+d\theta}\frac{H^2}{8}; \quad H = H(f,\theta)$$

$$S(f)df = \sum_{f}^{f+df}\frac{H^2}{8}; \quad H = H(f)$$



$$\overline{E}_p = \frac{1}{T_*}\int_0^{T_*} dE_p$$

$$= \frac{1}{T_*}\int_0^{T_*} \frac{\rho g}{2}(d+\eta)^2\, dt$$

$$= \frac{\rho g}{2T_*}\int_0^{T_*}(d^2 + 2d\eta + \eta^2)dt$$

$$= \frac{\rho g}{2}d^2 + \frac{\rho g}{2T_*}\int_0^{T_*}\eta^2 dt$$

where T_* = length of wave measurement. Considering the potential energy due to waves,

$$\overline{E}_p = \frac{\rho g}{2T_*}\int_0^{T_*}\eta^2 dt$$

Since $\overline{E}_p = \overline{E}_k$, the total energy density is

$$\overline{E} = 2\overline{E}_p = \frac{\rho g}{T_*}\int_0^{T_*}\eta^2 dt = \rho g\overline{\eta^2}$$

where the over-bar of $\overline{\eta^2}$ denotes time-average. Note that $\overline{\eta^2}$ is the variance of η. The definition of variance is $Var(x) = E\left[(x-\mu)^2\right]$. In our case, the mean, μ is zero.

For a discretely sampled η,

$$\overline{E} = \frac{\rho g}{N}\sum_{i=1}^{N}\eta_i^2$$

where N = number of samples in T_*.

Recalling

$$H_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}H_i^2}$$

we get

$$H_{rms}^2 = \frac{1}{N}\sum_{i=1}^{N}H_i^2$$

$$\frac{\rho g}{8}H_{rms}^2 = \frac{1}{N}\sum_{i=1}^{N}\frac{\rho g}{8}H_i^2 = \frac{1}{N}\sum_{i=1}^{N}E_i = \overline{E}$$

$$\therefore\ \overline{E} = \frac{\rho g}{8}H_{rms}^2 = \frac{\rho g}{16}H_s^2$$

From Eq. (6.8)

$$S(f)df = \sum_{f}^{f+df} \frac{H^2}{8}$$

$$\overline{E} = \sum_{\text{all } f} \frac{\rho g}{8} H^2 = \rho g \int_0^\infty S(f)df \equiv \rho g m_0$$

where m_0 = zeroth moment of spectrum. The nth moment is given by

$$m_n = \int_0^\infty f^n S(f)df$$

Since $\overline{E} = \rho g \overline{\eta^2}$, $\overline{\eta^2} = m_0$. Also, since $\overline{E} = \frac{\rho g}{16} H_s^2$, $H_s = 4\sqrt{m_0} \equiv H_{m0}$

In deep water, $H_s \cong H_{m0}$. As kd decreases, $H_s > H_{m0}$ (see Fig. 6.7)

6.5 Wave Spectral Models

General form of frequency spectrum: $S(f) = \frac{A}{f^5} e^{-B/f^4}$

where A, B = empirical constants

<u>Bretschneider spectrum</u>

$$S(T) = \frac{3.44 T^3 \overline{H}^2}{\overline{T}^4} e^{-0.675(T/\overline{T})^4}$$

Using $S(f) = S(T)T^2$ and $T = 1/f$,

$$S(f) = \frac{3.44 T^5 \overline{H}^2}{\overline{T}^4} e^{-0.675(T/\overline{T})^4}$$

$$= \frac{3.44 \overline{H}^2}{\overline{T}^4 f^5} e^{-0.675 \overline{T}^{-4}/f^4}$$

Bretschneider-Mitsuyasu spectrum (applicable to finite depth)

$$S(f) = 0.205 H_s^2 T_s (T_s f)^{-5} \exp\left[-0.75 (T_s f)^{-4}\right]$$

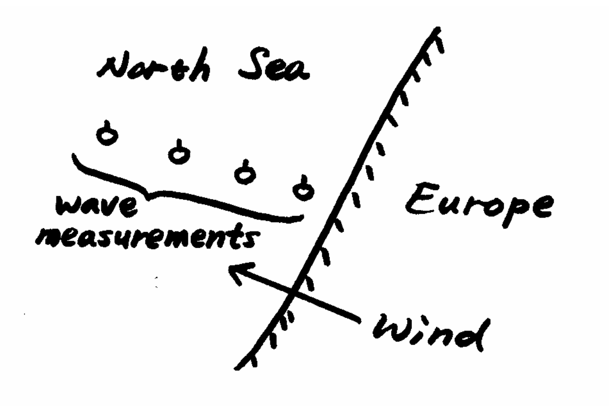
Pierson-Moskowitz spectrum (for fully developed seas)

$$S(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} e^{-0.74(g/2\pi W f)^4}$$

where $\alpha = 8.1 \times 10^{-3}$ and W = wind speed at 19.5 m above SWL $\cong (1.05 \sim 1.1) W_{10}$

JONSWAP spectrum (growing seas in deep water)

Based on data of JOint North Sea WAve Project

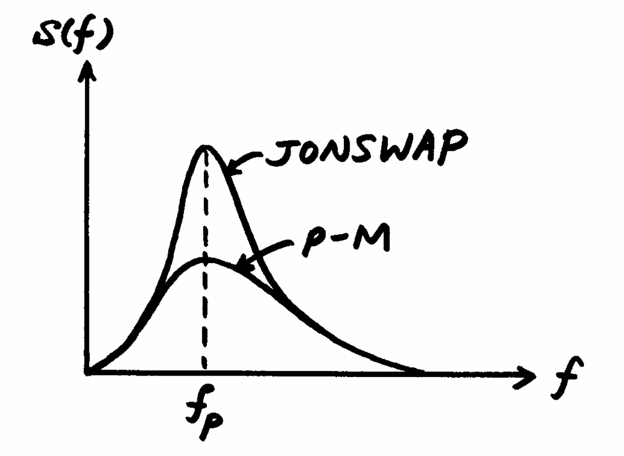


$$S(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} e^{-1.25(f_p/f)^4} \gamma^{\exp\left[-(f-f_p)^2/2\sigma^2 f_p^2\right]}$$

where γ = peak enhancement factor (typical value = 3.3), and

$$\sigma = \begin{cases} 0.07 & \text{for } f < f_p \\ 0.09 & \text{for } f \geq f_p \end{cases}$$

JONSWAP spectrum with $\gamma = 1.0$ = Pierson-Moskowitz spectrum



<u>TMA spectrum</u> (includes effect of finite water depth)

$$S_{TMA} = S_J \Phi(f,d)$$

where $\Phi(f,d)$ = Kitaigordskii shape function for finite depth effect:

$$\Phi(f,d) = \begin{cases} 0.5\omega_d^2 & \text{for } \omega_d < 1 \\ 1 - 0.5(2-\omega_d)^2 & \text{for } 1 \le \omega_d \le 2 \\ 1 & \text{for } \omega_d > 2 \end{cases}$$

where $\omega_d = 2\pi f (d/g)^{1/2}$

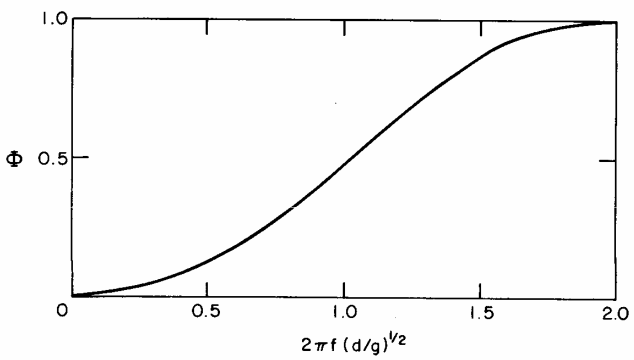


Figure 6.9. Correction factor for TMA spectrum.

Directional wave spectra

Frequency spectrum assumes waves with many different frequencies but a single direction. The real waves consist of many component waves with different frequencies and directions. Therefore, we need directional wave spectra.

$$S(f,\theta) = S(f)G(f;\theta)$$

where $G(f;\theta)$ = directional spreading function, which represents directional distribution of wave energy. In general, $G(f;\theta)$ varies with frequency, f :

small f $\rightarrow$ long-period waves $\rightarrow$ narrow spreading
large f $\rightarrow$ short-period waves $\rightarrow$ wide spreading

We take

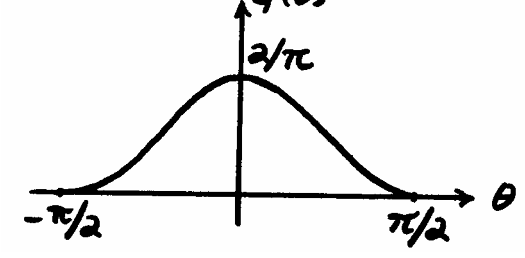
$$\int_{-\pi}^{\pi} G(f;\theta)d\theta = 1$$

so that $G(f;\theta)$ represents relative magnitude of directional spreading of wave energy.

$$\therefore \text{ total energy} = \int_0^\infty \int_{-\pi}^{\pi} S(f,\theta)d\theta df$$
$$= \int_0^\infty \int_{-\pi}^{\pi} S(f)G(f;\theta)d\theta df$$
$$= \int_0^\infty S(f)df$$

Cosine square function:

$$G(f;\theta) = G(\theta) = \frac{2}{\pi}\cos^2\theta \quad \leftarrow \text{independent of } f$$



Mitsuyasu-type function:

$$G(f;\theta) = G(s)\cos^{2s}\left(\frac{\theta}{2}\right); \quad G(s) = \frac{2^{2s-1}}{\pi}\frac{\Gamma^2(s+1)}{\Gamma(2s+1)}$$

$$s = \begin{cases} s_{\max}(f/f_p)^5 & \text{for } f < f_p \\ s_{\max}(f/f_p)^{-2.5} & \text{for } f > f_p \end{cases}$$

$s = s_{\max}$ at $f = f_p$, and s decreases as $|f - f_p|$ increases.

Swell $\rightarrow$ larger $s_{\max}$ $\rightarrow$ narrow spreading

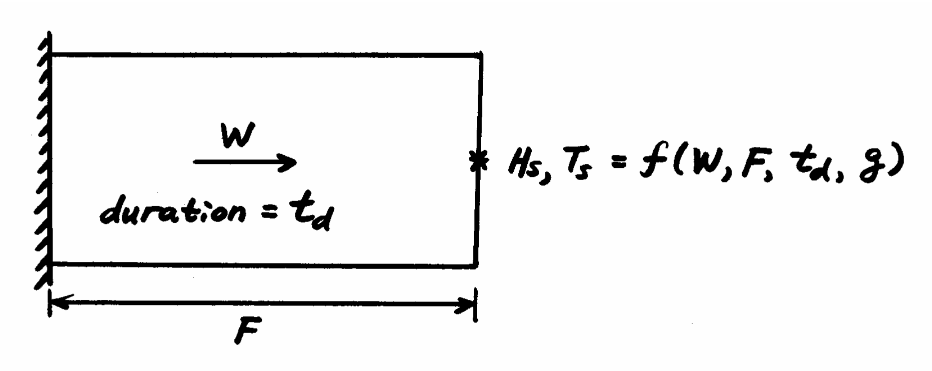
Wind wave $\rightarrow$ smaller $s_{\max}$ $\rightarrow$ wide spreading

6.6 Wave Prediction – SMB Method

↑

Sverdrup, Munk, Bretschneider

Consider a box storm:



By dimensional analysis,

$$\frac{gH_s}{W^2}, \frac{gT_s}{2\pi W} = f\left(\frac{gF}{W^2}, \frac{gt_d}{W}\right)$$

Using Fig. 6.10,

$$\frac{gF}{W^2} \rightarrow H_s, T_s \qquad (1)$$

$$\frac{gt_d}{W} \rightarrow H_s, T_s \qquad (2)$$

Choose the smaller values of (1) and (2). If (1) is smaller, it is fetch-limited condition. If (2) is smaller, duration-limited condition.

For a typhoon, use Eqs. (6.37) and (6.38):

R = radius to maximum wind speed (W_R)
$\Delta p = p_a - p_e$ = strength of typhoon
V_F = forward speed of typhoon
$\alpha \cong 1$ for slow moving typhoon

6.7 Wave Prediction – Spectral Models

$$S(f) = \frac{A}{f^5} e^{-B/f^4} \quad \leftarrow \text{general form}$$

A, B = empirical constants = $f(W, F, t_d)$

Given $W, F, t_d \rightarrow S(f) \rightarrow H_s \cong H_{m0} = 4\sqrt{m_0}; \quad m_0 = \int_0^\infty S(f)df$

$$\frac{\partial S}{\partial f} = 0 \text{ at } f = f_p \rightarrow \text{find } f_p \rightarrow T_p \rightarrow T_s = 0.95T_p$$

SPM: W, F, t_d →(w/o calculation of $S(f)$)→ H_{m0}, T_p for JONSWAP spectrum

$$W_A = 0.71W^{1.23}$$

$$\frac{gH_{m0}}{W_A^2} = 0.0016\left(\frac{gF}{W_A^2}\right)^{0.5} \qquad (6.40)$$

$$\frac{gT_p}{W_A} = 0.286\left(\frac{gF}{W_A^2}\right)^{0.33} \qquad (6.41)$$

$$\frac{gt_d'}{W_A} = 68.8\left(\frac{gF}{W_A^2}\right)^{0.66} \qquad (6.42)$$

where t_d' = minimum duration for fetch-limited condition, whereas t_d = actual duration.

If $t_d \geq t_d'$, fetch-limited → Use Eqs. (6.40), (6.41)

If $t_d < t_d'$, duration-limited → Calculate F using $t_d' = t_d$ with Eq. (6.42)

→ Use Eqs. (6.40), (6.41) with new F.

6.8 Numerical Wave Prediction Models (read text)

6.9 Extreme Wave Analysis

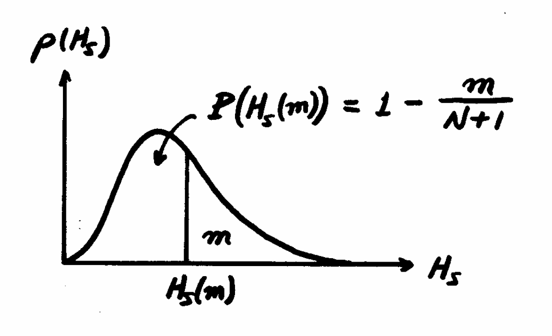
Return period (再現期間)?

Ex) H_s of 50 year return period = significant wave height which can occur once in every 50 years on the average.

How can we estimate H_s of 50 year return period with limited data (e.g. 2 year data)?

<u>Return period analysis using Gumbel distribution</u>

1) H_s was measured every hour for 2 years.
2) Select daily maximum H_s.
   number of data, $N = 365 \times 2 = 730$,
   r = time interval in years = $\frac{1}{365} = 0.00274$
3) Rearrange H_s in descending order from $H_s(1)$ to $H_s(N)$.
4) Compute cumulative probability, $P(H_s)$ for each H_s.



5) Plot H_s vs $-\ln\{-\ln[P(H_s)]\}$ $\rightarrow$ Find β, γ for best fit.

Gumbel distribution: $P(H) = \exp\left\{-\exp\left[-\left(\frac{H-\gamma}{\beta}\right)\right]\right\}$

$$H = -\beta \ln\{-\ln[P(H)]\} + \gamma$$

6) Calculate $P(H_s)\big|_{T_r}$ by using

$$\frac{r}{T_r} = 1 - P(H_s) \;\rightarrow\; P(H_s)\big|_{T_r=50} = 1 - \frac{r}{T_r} = 1 - \frac{0.00274}{50} = 0.9999452$$

7) Calculate H_s (T_r = 50 yr) by

$$H_s\big|_{T_r=50} = -\beta \ln\{-\ln(0.9999452)\} + \gamma$$

Use similar procedures for other distributions (see Table 6.1)

Encounter probability:

$E = 1 - e^{-T/T_r}$ ← 재현기간 T_r 인 사건이 기간 T 동안에 발생할 확률