5 STRESS DISTRIBUTION AND SETTLEMENT OF SHALLOW FOUNDATIONS

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5.1 INTRODUCTION

5.1.1 Basic Requirements for a Good Foundation

The basic requirements for a good foundation are that (1) it is safe against complete collapse or failure of the soils upon which it is founded; (2) it experiences no excessive or damaging settlements or movements; (3) environmental and other factors (see below) are properly considered; and (4) the foundation is economically feasible in relation to the function and cost of the overall structure.

Environmental factors and other considerations that may adversely affect the construction and performance of the foundation include:

1. Frost action
2. Shrinking or swelling soils
3. Earthquakes and vibrations
4. Groundwater
5. Underground defects
6. Adjacent structures, excavations, and property lines
7. Scour and wave action

The influence of environmental and other factors has been discussed by Sowers (1962), Perloff and Baron (1976), among others; Sowers (1974) describes some dramatic examples. For a discussion of the effects of frost action in cold regions, see Chapter 19. Shrinking and swelling soils are discussed by Gromko (1974), Chen (1975), Snethen (1979, 1980), and Meyerhof and Fellenius (1985).

A discussion of the effects of foundation vibrations and earthquakes can be found in Chapters 15, 16, and 17. Scour and wave action are discussed in Chapter 18 for offshore structures.

5.1.2 Steps in Ordinary Foundation Design

In order to develop the most economical foundation for a particular structure, the geotechnical engineer ordinarily goes through the following steps (Perloff and Baron, 1976):

1. Establish the scope of the problem.
2. Investigate the conditions at the site.
3. Formulate a trial design.
4. Establish a model of the subsurface to be analyzed.
5. Determine the loads and soil parameters.
6. Perform the analyses.
7. Compare results with other models and experience.
8. Modify the design.
9. Observe the construction.

These steps are followed no matter what is the purpose of the foundation. Analyses are carried out for stability (bearing capacity) and settlement.

The primary requirement is that the foundation must be safe against possible instability; that is, the foundation soils must have adequate bearing capacity to support the loads of the structure. Thus, a bearing capacity analysis is normally done first to insure that the factor of safety is adequate. Bearing capacity calculations are discussed in standard foundation engineering textbooks such as those of Vesić (1975) and Perloff and Baron (1976), and Chapter 4 of this book.

If the bearing capacity is satisfactory, settlement analyses are carried out to estimate both the immediate and long-term settlements. If the bearing capacity is inadequate and/or settlements are too large, the soils are a good candidate for soil improvement or a deep foundation must be used. These points will be discussed later in this chapter.

5.1.3 Shallow Foundations

G. A. Leonards (personal communication, 1973) defines a shallow foundation as one in which the structural loads are transmitted to the soil at an elevation required for the function of the structure itself. Thus, a shallow foundation is not necessarily one that is near the ground surface, but one that is "shallow" in relation to the structure it is supporting. For example, a high-rise building with a five-story underground basement may still be supported on shallow foundations (e.g., spread footings or a mat foundation), even though the foundation elevation might be 20 or 25 m below street level.

Generally, the most economical shallow foundation is isolated spread footings, that is, footings whose area is less than 40% of the total area of the structure. If the footing area is larger than that or combined or strap footings are required, then a mat foundation may be more economical. The various types of shallow foundations and their structural design are discussed by, for example, Peck et al. (1974) and Bowles (1975a, b).

Because most shallow foundations have adequate bearing capacity, or since at least an adequate bearing capacity can relatively easily be achieved, the performance of most shallow foundations is controlled by their settlements. Bearing capacity rarely controls design. With the high factors of safety ordinarily used, especially with building foundations, there is a very low probability of failure. In considering settlements of shallow foundations, three questions must be answered:

1. How do we estimate the movements of the foundation for any given design?
2. What are the tolerable movements?
3. If the estimated movements are greater than the tolerable movements, then what do we do?

In this chapter, only the vertical movements or settlements of shallow foundations are considered. This does not mean that horizontal movements are not important—they are; however, a different approach must be used to make estimates of these movements, as discussed in Chapter 6.

5.2 SETTLEMENT OF SHALLOW FOUNDATIONS

5.2.1 Components of Settlement

The time–settlement history of a shallow foundation or earth structure is illustrated schematically in Figure 5.1. The total settlement s is the sum of the three components as shown in the figure:

$$s = s_i + s_c + s_s \tag{5.1}$$

in which s_i is the *immediate* or *distortion* settlement, s_c is the *consolidation* settlement, and s_s is the *secondary compression* settlement. The immediate component is that portion of the settlement that occurs essentially with load application, primarily as a consequence of distortion (change of shape not change of volume) in the foundation soils. The distortion settlement is generally not elastic, although it is often calculated using elastic theory, especially when cohesive compressible materials are involved.

The other two components of settlement result from the gradual expulsion of water from the voids and the concurrent compression of the soil skeleton. The distinction between consolidation and secondary compression settlements is made on the basis of the physical processes that control the time rate of settlement. Consolidation settlement refers to settlements due to *primary consolidation*, in which the time rate of settlement is controlled by the rate at which water can be expelled from the void spaces in the soil. During *secondary compression*, the rate of settlement is controlled largely by the rate at which the soil skeleton itself yields, compresses, and creeps after the excess hydrostatic pressure is zero; that is, at a constant effective stress. The transition time between these two processes is arbitrarily identified as that time at which excess pore water pressure Δu becomes essentially zero. This time, denoted t_p, is shown in Figure 5.1.

Because the response of soils to applied loads is not linear, the superposition implied by Equation 5.1 is not strictly valid.

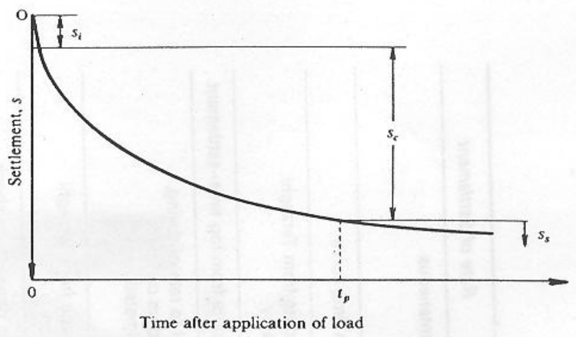


Fig. 5.1 Schematic time–settlement history of the settlement of a shallow foundation. (*Perloff, 1975.*)

However, a consistent and practical alternate approach has not yet been developed, and experience indicates that this approach yields reasonable predictions of settlements for many soil types.

The time–settlement relationship shown in Figure 5.1 is applicable to all soils. However, it should be recognized that the time scale and relative magnitudes of the three components may differ by orders of magnitude for different soil types. For example, water flows so readily through most clean granular soils that the expulsion of water from the pores is, for all practical purposes, instantaneous. Thus, foundations and earth structures on clean granular soils settle essentially simultaneously with the application of load.

The relative importance of each of these components for settlement analyses depends on the soil type, as shown in Table 5.1. As mentioned above, for granular materials, only the immediate settlement is of concern. For clay soils, consolidation settlement is the major concern, but immediate and secondary settlements must also be checked. For organic soils, especially fibrous peats, most of the settlement is secondary compression. Because of their high permeability, consolidation settlement occurs so rapidly as to almost be "immediate", and both components are usually combined for analysis purposes.

5.2.2 Causes of Settlement

It is instructive to look at the causes of settlement that may affect the performance of a structure or foundation. Sowers and Sowers (1970) have listed these as in Table 5.2. It is interesting that only the settlements due to structural loading and groundwater lowering are readily computed. In the cases of environmental loads or load-independent settlements, only the general susceptibility of a particular soil type to such settlements can be stated. Estimates of magnitudes and rates of settlement are virtually impossible in these cases.

5.2.3 Steps in Settlement Analyses

1. Establish the soil profile including the location of the groundwater table. Determine which layer or layers are

TABLE 5.1 RELATIVE IMPORTANCE OF IMMEDIATE, CONSOLIDATION, AND SECONDARY SETTLEMENT FOR DIFFERENT SOIL TYPES.

Soil Type	*Immediate Settlement*	*Consolidation Settlement*	*Secondary Compression*
Sands	Yes	No	No
Clays	Possibly	Yes	Possibly
Organic soils	Possibly (Yes)	Possibly (No)	Yes

TABLE 5.2 CAUSES OF SETTLEMENT[a].

Cause	Form of Mechanism		Amount of Settlement	Rate of Settlement
Structural load	Distortion (change in shape of soil mass)		Compute by elastic theory or empirical rules and procedures	Instantaneous
	Consolidation: change in void ratio under stress	Immediate	Stress–void-ratio curve	From time curve
		Primary consolidation	Stress–void-ratio curve	Compute from Terzaghi theory
		Secondary	Compute from log time–settlement	Compute from log time–settlement
Environmental load	Shrinkage due to drying		Estimate from stress–void ratio or moisture–void ratio and moisture loss limit-shrinkage limit	Equal to rate of drying, seldom can be estimated
	Consolidation due to water table lowering		Compute from stress–void ratio and stress change	Compute from Terzaghi theory
Load-independent (but may be aggravated by load) often environment-related, but not dependent	Reorientation of grains—shock and vibration		Estimate limit from relative density (up to 60–70%)	Erratic, depends on shock, relative density
	Structural collapse—loss of bonding (saturation, thawing, etc.)		Estimate susceptibility and possibly limiting amount	Begins with environmental change, rate erratic
	Raveling, erosion into openings, cavities		Estimate susceptibility but not amount	Erratic, gradual or catastropic, often increasing
	Biochemical decay		Estimate susceptibility	Erratic, often decreases with time
	Chemical attack		Estimate susceptibility	Erratic
	Mass collapse—collapse of sewer, mine, cave		Estimate susceptibility	Likely to be catastrophic
	Mass distortion, shear-creep or landslide in slope		Compute susceptibility from stability analysis	Erratic, catastrophic to slow
	Expansion—frost, clay expansion, chemical attack (resembles settlement)		Estimate susceptibility, sometimes limiting amount	Erratic, increases with wet weather

[a] After Sowers and Sowers (1970). Reproduced with permission of Macmillan Publishing Co.

compressible. Compute the total, neutral, and effective stress profile with depth.

2. Estimate the magnitude and rate of application of the loads applied to the foundation, both during construction and during the estimated economic and service life of the structure. In some structures, the loads applied to the foundation are provided by the structural engineer or architect. In other situations, for example, embankments and tanks, the foundation engineer may estimate the loads.
3. Estimate the change in stress with depth. If the loading is one-dimensional in nature (that is, if the width of the loaded area is significantly greater than the thickness of the compressible layer), then one-dimensional loading and compression may be assumed. In such a case, the change in stress at depth is equal to the stress applied at the surface. If, on the other hand, the width of the loaded area is equal to or less than the thickness of the compressible layer, three-dimensional loading occurs and the applied surface stresses dissipate with depth. Elastic theory or empirical methods are commonly used to estimate the change in stress with depth, but probabilistic methods may also be used.
4. Estimate the preconsolidation pressure. Compare with the effective stress profile computed in (1) above. Determine whether the soil is normally consolidated or overconsolidated.
5. Calculate the consolidation settlements.
6. Estimate the time rate of consolidation settlements.
7. Estimate the rate of secondary compression.
8. See Table 5.1. If necessary, estimate the immediate or distortion settlement. If the foundation soils are cohesive, use elastic theory. If granular, use empirical methods.

5.2.4 Scope and Organization of Chapter

The chapter is arranged in a sequence approximately corresponding to the computation process for the individual components of settlement described above. Following a discussion of the applicability of the theory of elasticity to the calculation of displacements and stresses in earth masses, methods for estimating the immediate settlements of subsurface materials that are primarily cohesive or cohesionless are described. Next is a discussion of how stresses at depth may be estimated for different loading geometries and boundary conditions. Then the calculation of consolidation settlement including time rate and secondary compression are described. To judge whether the calculated settlements can be tolerated, a discussion of tolerable settlements of buildings and other structures is presented. Finally, the chapter discusses what to do if the estimated settlements are not tolerable. Emphasis is placed on foundation treatment methods not covered elsewhere in this handbook.

Much of this chapter is based on Perloff (1975) and Perloff and Baron (1976). In addition, my colleague G. A. Leonards of Purdue University provided many helpful insights into the fine art of applied foundation engineering.

5.3 APPLICABILITY OF THE THEORY OF ELASTICITY TO CALCULATION OF STRESSES AND DISPLACEMENTS IN EARTH MASSES (After 65 years of soil mechanics, don't we have anything better?)

5.3.1 Rationale for Use of Elastic Theory

The distribution of stresses in earth masses is often estimated using the corresponding distribution in a linear elastic medium with boundary conditions approximating those in the problem of interest. In some cases, elastic theory is also used to estimate displacements as well. Although soils do not behave as linear elastic materials, the rationale for this practice has been the availability of solutions to problems for which the boundary conditions corresponded reasonably well to the boundary conditions for foundation engineering problems, as well as the lack of generally accepted alternatives. Experimental and analytical studies have been carried out to investigate the degree to which the results of elastic theory are applicable to earth masses. Perloff (1975) and Harr (1977) have summarized the conclusions of these investigations.

5.3.2 Applicability for Stress Calculations

Homogeneous Masses When the boundary conditions of the linear elastic analytical model approximate the in-situ boundary conditions, the stress distribution interpreted from the field measurements corresponds reasonably well to that predicted by linear elastic theory, probably because of small deformations and a high factor of safety against collapse. Another reason they probably also work is that specific material constants are not required in vertical stress distributions as predicted by the Boussinesq solutions. Thus, as long as the loading is well below yield with a high factor of safety and only vertical stresses are desired, linear elastic solutions give reasonable results.

Layered Systems Data concerning stress distributions within layered systems is very limited. Only a very few studies are reported in the literature, and those that are do not agree very well with the predictions of multilayer elastic theory. One of the difficulties is the nature of the interlayer shear stress. Solutions are only possible if interlayer shear is assumed to either be zero (frictionless) or with perfect fixity. Neither case is likely to be true in actuality.

In summary, it seems that elastic theory is satisfactory for homogeneous masses but not very good for layered systems. However, it is still used by pavement engineers (e.g., Yoder and Witczak, 1975) for estimating stresses in layered systems, even though it is recognized that the predictions are not very accurate.

5.3.3 Applicability of Elastic Theory to Displacement Calculations

Because displacements depend directly on the nature of the assumed constitutive law, the ability of elastic theory to predict displacements depends more on in-situ nonlinearity and material inhomogeneity than it does on the stress calculations themselves. Settlements due to initial undrained distortion of saturated or nearly saturated cohesive soils, which are subject to moderate increments of stress and where the elastic parameters can be assumed to be approximately constant throughout the mass, may be estimated reasonably well by elastic theory. Again, small strains and a high factor of safety are necessary.

On the other hand, when the in-situ soil conditions are markedly different from the assumptions of linear elastic theory, its use is inappropriate. Examples include the case of cohesionless soil deposits in which the equivalent elastic modulus depends significantly on confinement and where the stress increment due to loading varies significantly throughout the strata; thus, a constant equivalent modulus is not appropriate. It is possible that techniques such as the finite element method (Duncan, 1972; Desai and Christian, 1977) may be appropriate in this situation. Although it may be too cumbersome for routine applications, the finite element method is recommended for major projects, especially when the modulus cannot be assumed to be constant. For routine calculations, semiempirical methods that have been developed are appropriate.

5.3.4 Alternative Approach Using Probabilistic Theory

M. E. Harr (1977) developed an alternate approach to stress distribution problems based on the theory of probability. In particulate media such as soils, the requirements of a continuum theory such as the theory of elasticity are so markedly different from reality that a probabilistic approach has great merit. In contrast to the Boussinesq theory, the properties of the soil are incorporated in the probabilistic stress distribution approach through a parameter ν, which is related to the in-situ coefficient of lateral stress. Harr (1977) shows that the distribution value of the expected normal stress often agrees better with the few published field measurements. Experiments with embedded pressure cells have invariably demonstrated that the theory of elasticity predicts too large vertical stresses and too small lateral stresses in the region of the load. These results more closely correspond to predictions by the probabilistic approach.

In the case of compacted soils, where the coefficient of lateral stress is likely to be rather large, serious disagreements are observed with elastic theory. However, this can be accounted for quite nicely by an increase in the parameter ν, as shown by Harr (1977). Finally, another great advantage of probabilistic theory is that multilayer systems can readily be treated, and no assumptions regarding interlayer stress conditions are required.

Equations for calculating stress distributions using the probabilistic approach will be presented in Section 5.6.5.

5.4 CALCULATION OF INITIAL DISTORTION SETTLEMENTS

5.4.1 Distortion Settlement and Contact Stress

As mentioned earlier, distortion settlement occurs because of a change in shape of the soil mass rather than because of a change in volume. The shape of the deflected soil profile depends on whether the soil is predominantly cohesive or granular and whether the loaded area is rigid or flexible. The possibilities are shown in Figure 5.2. In the case of rigid foundations, the settlements produced are of course uniform, whereas the contact stress distributions under the foundations are very nonuniform. In the case of cohesive soils, at the outer edges of a rigid foundation on a perfectly elastic soil, the stress is infinite. In actuality, as shown in Figure 5.2a, it is limited by the shear strength of the soil. With rigid footings on granular materials, because the confinement is less at the outer edges, the stress is also less. For a very wide footing on granular material (e.g., a stiff mat foundation), settlement would be fairly uniform; near the middle of the mat, the contact stress also would be quite uniform.

As expected, the contact stress distribution for a flexible loaded area is also uniform, but the settlement profiles are quite different, depending on whether the soil is cohesive or granular. These cases are shown in Figure 5.2b. In the case of cohesive soils, which include saturated clays and many rocks, the surface will deform in a shape that is concave upward. The shape of the settlement profile on a granular material is exactly the opposite, concave downward, again because the confining stress near the edges of the footing is lower than in the center. If the sand is confined, it has a higher modulus than at the edges, which means that there is less settlement in the center than at the edges. If the flexible loaded area is very large, then the settlements near the center of the area are relatively uniform

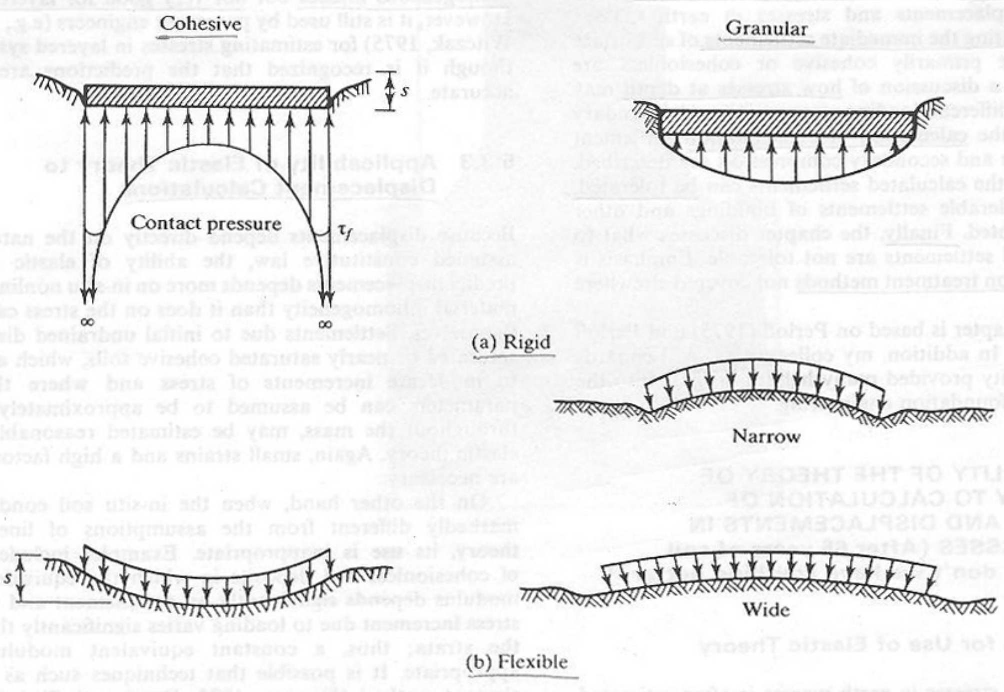


Fig. 5.2 Distribution of settlement and contact stresses for rigid and flexible loaded areas on cohesive and cohesionless soils.

and less than at the edges (Fig. 5.2b). Contact stress distributions are important for the design of foundations and footings (Bowles, 1975a). For the structural design of footings, a linear contact stress distribution is often assumed although this is obviously incorrect from a soil mechanics point of view.

5.4.2 Immediate Settlement of Cohesive Foundations

For soils that are predominately cohesive, linear theory of elasticity is used to estimate the magnitude of initial settlements. Soil profiles are typically simplified, although some solutions involving multiple layer theory are available. Homogeneity and isotropy are implicitly assumed so that only two elastic parameters, the modulus of elasticity E and Poisson's ratio ν are needed. This approach works reasonably well on clay soils if the applied stress level is low; that is, if the factor of safety is large and we do not have plastic yielding in the foundation. If foundation yielding is likely to occur, another approach is recommended, which will be described below.

In many foundations on cohesive soils, the immediate or distortion settlement is a relatively small part of the total vertical movement and, thus, rough estimates are acceptable. A discussion of relative importance of immediate and consolidation settlement will be given later in this section.

A Distributed Load at or Near the Surface of a Deep Layer When the foundation problem can be approximated as one or more uniformly distributed loads acting on circular or rectangular areas near the surface of a relatively deep stratum, the vertical settlement can be estimated by

$$s_i = C_s q B \left(\frac{1-\nu^2}{E_u} \right) \tag{5.2}$$

where

s_i = settlement of a point on the surface
C_s = shape and rigidity factor
q = magnitude of the uniformly distributed load
B = characteristic dimension of the loaded area as shown in Figure 5.3
E_u = Young's modulus (undrained)
ν = Poisson's ratio

The coefficient C_s accounts for the shape and rigidity of the loaded area and for the position of the point for which the settlement is being calculated. Values of C_s are given in Table 5.3.

EXAMPLE 5.1

A structure is to be supported on a stiff reinforced concrete mat foundation whose dimensions are 20 m by 50 m. The load on the mat is to be uniformly distributed; its magnitude is 65 kPa. The mat rests on a deep saturated deposit of saturated clay for which the average undrained Young's modulus is approximately 40 MPa. Estimate the immediate settlement at the center and corner of the mat.

Solution

Since the mat is stiff, use the rigid factors from Table 5.3a. With $L/B = 50/20 = 2.5$, the shape factors for both the center and corner are determined by interpolation to be $C_s = 1.20$. Thus from Equation 5.2 the immediate surface settlement at both the center and corner of the mat is $s_i = 1.20(65)(20)\ [(1 - 0.5^2)]$ divided by $[(40 \times 10^3)] = 0.029$ m = 29 mm.

For comparison, the shape factors for a flexible mat would be determined by interpolation to be

At the center $C_s = 1.63$
At the corner $C_s = 0.81$

Thus, the immediate surface settlements are

At the center $s_i = 40$ mm
At the corner $s_i = 20$ mm

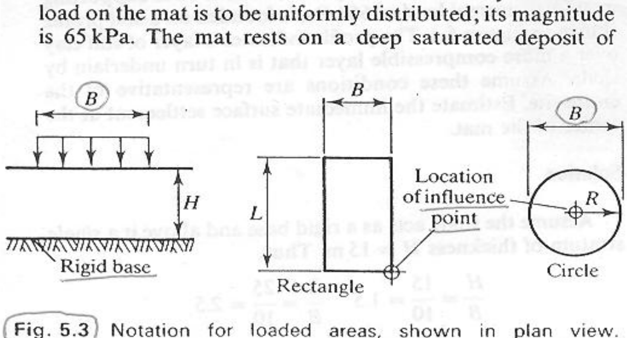


Fig. 5.3 Notation for loaded areas, shown in plan view. (*U.S. Navy, 1982.*)

A mat foundation is usually neither completely flexible nor completely rigid, depending on its size and thickness and how heavily reinforced it is. If it is large, the distribution of contact pressure may be nearly uniform over its center portion. At the corners and edges, however, the rigidity of the mat may be significant (owing to its thickness and the amount of reinforcing), and settlements are likely to be less than predicted. In a saturated clay, because of settlement in the middle portion of the foundation, some heave may occur in the outer portions because of undrained (constant volume) loading and shear.

Effect of Layered Systems In actuality, most soil profiles are not homogeneous and deep. If the thickness of the top layer is large relative to the dimensions of the loaded area, immediate surface settlement may be calculated as if the soil were a homogeneous layer of infinite depth. However, if the upper stratum is relatively thin, the effect of layering must be taken into consideration. This is likely to be especially important when a soft compressible stratum is underlain by rock or very hard or dense soils. This special case may be approximated by a layer of elastic material of finite thickness underlain by a rigid base. Settlements for this case may be determined by Equation 5.2, but using a shape factor C_s that accounts for the presence of the rigid base. Values for these shape factors C_s are tabulated in Table 5.3b for the settlement under the center of a rigid circular area and under the corner of flexible rectangular areas. These shape factors depend upon both the shape of the loaded area and the thickness of the compressible stratum relative to the width of the loaded area, as illustrated in Figure 5.3.

Examination of Table 5.3 indicates the importance of the presence of a rigid boundary. When $H/B = 0.5$, the reduction in surface displacements of the center of the loaded area relative to that for the halfspace is greater than 50 percent.

EXAMPLE 5.2

Compute the immediate settlement at the center of the uniformly loaded (flexible) area measuring 6 m × 6 m. The applied surface stress is 200 kPa and the depth to firm bottom is 3 m. Assume the undrained elastic modulus is 10 000 kPa and $\nu = 0.5$.

Solution

Use the C_s values for the corners of four equally sided rectangles 3 m × 3 m. In this case, $H/B = 1$, $L/B = 1$, and,

TABLE 5.3 SHAPE AND RIGIDITY FACTORS, C_s, FOR CALCULATING SETTLEMENTS OF POINTS ON LOADED AREAS AT THE SURFACE OF AN ELASTIC HALFSPACE[a].

a. Infinite Depth

Shape and Rigidity	*Center*	*Corner*	*Edge/Middle of Long Side*	*Average*
Circle (flexible)	1.00		0.64	0.85
Circle (rigid)	0.79		0.79	0.79
Square (flexible)	1.12	0.56	0.76	0.95
Square (rigid)	0.82	0.82	0.82	0.82
Rectangle: (flexible) length/width				
2	1.53	0.76	1.12	1.30
5	2.10	1.05	1.68	1.82
10	2.56	1.28	2.10	2.24
Rectangle: (rigid) length/width				
2	1.12	1.12	1.12	1.12
5	1.6	1.6	1.6	1.6
10	2.0	2.0	2.0	2.0

b. Limited Depth Over a Rigid Base

H/B	*Center of Rigid Circular Area Diameter = B*	*Corner of Flexible Rectangular Area L/B = 1*	*L/B = 2*	*L/B = 5*	*L/B = 10*	*L/B = ∞ (strip)*
		ν = 0.50				
0	0.00	0.00	0.00	0.00	0.00	0.00
0.5	0.14	0.05	0.04	0.04	0.04	0.04
1.0	0.35	0.15	0.12	0.10	0.10	0.10
1.5	0.48	0.23	0.22	0.18	0.18	0.18
2.0	0.54	0.29	0.29	0.27	0.26	0.26
3.0	0.62	0.36	0.40	0.39	0.38	0.37
5.0	0.69	0.44	0.52	0.55	0.54	0.52
10.0	0.74	0.48	0.64	0.76	0.77	0.73
		ν = 0.33				
0	0.00	0.00	0.00	0.00	0.00	0.00
0.5	0.20	0.09	0.08	0.08	0.08	0.08
1.0	0.40	0.19	0.18	0.16	0.16	0.16
1.5	0.51	0.27	0.28	0.25	0.25	0.25
2.0	0.57	0.32	0.34	0.34	0.34	0.34
3.0	0.64	0.38	0.44	0.46	0.45	0.45
5.0	0.70	0.46	0.56	0.60	0.61	0.61
10.0	0.74 (vs. 0.79)	0.49 (vs. 0.56)	0.66 (vs. 0.76)	0.80 (vs. 1.05)	0.82 (vs. 2.10)	0.81 (vs.)

[a] After U.S. Navy (1982).

from Table 5.3, $C_s = 0.15$:

$$S_i = 0.15(200)(3)\left(\frac{1-0.5^2}{10\,000}\right) \times 4 = 27\ \text{mm}$$

If the soil profile consists of a relatively thin stiffer layer underlain by a less stiff layer of greater depth, then the stresses from the surface load must be distributed to the top of the less compressible layer. Use the stress distribution techniques discussed in Section 5.6.

Analytical and/or numerical methods for the determination of displacements in multilayered systems are available for cases other than those in Table 5.3 (see Poulos and Davis, 1974). A number of multilayer solutions are now available in computer codes. Except for pavements and special foundations, however, the use of multilayered computer analyses is generally not justified, because the material parameters are not accurately determined, the boundary conditions and interface conditions between the strata are not that well known, and, finally, approximations may be required to fit the geometry of the real problem to that for which the solution is available. In many situations an approximate analysis of the intermediate settlement is sufficient, as illustrated in the following two examples.

EXAMPLE 5.3

The mat foundation of Example 5.1, 20 m × 50 m supporting a uniform normal load of 65 kPa, is founded on a soil profile shown in Figure 5.4. The profile indicates a layer of stiff clay over a more compressible layer that is in turn underlain by shale. Assume these conditions are representative of the entire site. Estimate the immediate surface settlement at the center of the mat.

Solution

Assume the shale acts as a rigid base and above it a *single* stratum of thickness $H = 15$ m. Thus,

$$\frac{H}{B} = \frac{15}{10} = 1.5 \qquad \frac{L}{B} = \frac{25}{10} = 2.5$$

The shape factor C_s obtained from Table 5.3b by linear

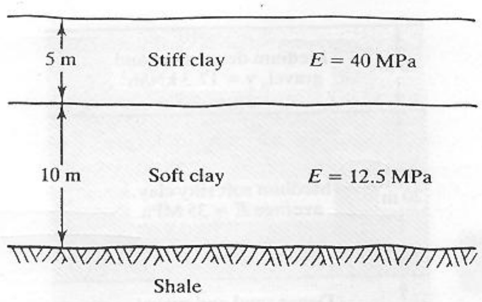


Fig. 5.4 Soil profile for Example 5.3.

interpolation is 0.21. Substituting this value into Equation 5.2 leads to a calculated surface settlement (assuming $v = 0.5$) of

$$s_i = (0.21)(65)(10)\left(\frac{1-0.5^2}{E}\right) \times 4 = 410 \times \frac{10^3}{E} \quad (\text{kPa}\cdot\text{m})$$

The potential immediate surface displacement can be bounded by estimating the settlement using the moduli of the two compressible strata, or $10\,\text{mm} < s_i < 33\,\text{mm}$. A better estimate can be obtained by using an *equivalent* Young's modulus weighted by the relative thicknesses of the two strata, or

$$E_{eq} = \frac{5(40 \times 10^3) + 10(12.5 \times 10^3)}{15} = 21.7 \times 10^3\,\text{kPa}$$

Thus, the immediate settlement is 19 mm.

The settlement predicted by this approach could exceed that determined by multilayer solutions because the load distribution effect of the upper stiffer layer is not accounted for in the weighted average E_{eq}.

Another way to obtain an estimate of s_i, especially when the stiffness of the upper layer is much greater than that of the lower layer, is to assume that the immediate settlement results primarily from distortions within the less stiff layer. This layer is extended all the way from the ground surface to the top of the underlying shale and the elastic settlement is calculated as before using Table 5.3 and Equation 5.2. To account for the actual depth of the less-stiff material, the settlement that would occur in the thinner layer of the material overlying the rigid base is subtracted from the settlement determined assuming the entire depth was soft. See Perloff (1975) for an example.

Correction for Low Factor of Safety and Large Undrained Shear Deformations If the factor of safety against bearing capacity failure (see Chapter 4) is less than about 3, the immediate settlement should be modified to take into account undrained plastic yielding that occurs in the foundation. A semiempirical procedure that takes this into account was developed by D'Appolonia et al. (1971). See also U.S. Navy (1982) and Foott and Ladd (1981).

Heave of Excavations It may occur that a significant portion of the heave of excavations above compressible strata arises from undrained distortions within the strata. This heave, and the subsequent settlement resulting when the structural load within the excavation is applied to the foundation within the excavation, may be significant, particularly in the case of a partially or fully compensated mat foundation. For major projects, use of the finite element method is recommended. See Clough and Schmidt (1977) for discussions and examples.

For preliminary estimation purposes, a common approach is to use the loading approximation in Figure 5.5a, in which the upper boundary of the elastic medium is presumed to be at the base of the excavation, subjected to an upward uniform strip load of magnitude $-\gamma D^2$, where D is the depth of the excavation. Such an analysis fails to account for the influence of the material surrounding the excavation on the distribution of stresses and therefore displacements within the soil below this excavation.

An alternate approach has been developed by Perloff (1975) based on the work of Baladi (1968), who obtained solutions for the heave at the base of a strip excavation in a linear elastic medium. By determining the heave at various depths within the elastic medium, an approximation of the expected heave when only a limited depth of deformable material overlies a rigid boundary (Fig. 5.5b) can be made. Such an analysis may be useful for preliminary estimation purposes in lieu of a finite element analysis. The magnitude of heave or rebound at the base of a strip excavation of rectangular cross section is determined by this method from

$$r_d(\text{strip}) = \Delta_{\text{strip}} \frac{\gamma D^2}{E} \tag{5.3}$$

in which Δ_{strip} is a dimensionless heave factor whose magnitude is determined by the geometry of the excavation and the position for which the rebound r_d is being calculated. The heave factor for the centerline and the edge of the excavation are given in Figure 5.6, for a variety of excavation geometries and depths to a rigid boundary.

When the length-to-width ratio of the excavation (L/B) is less than about 5, the limited length of the excavation must be accounted for. This is done by assuming that the relative effect of excavation shape on heave of a rectangular excavation will be similar to that for the settlement of a uniformly loaded area. Thus, the heave of a rectangular excavation can be calculated by

$$r_d = C_r'' \Delta_{\text{strip}} \frac{\gamma D^2}{E} \tag{5.4}$$

in which C_r'' is determined from shape and rigidity factors given by Perloff (1975) and plotted in Figure 5.7 for base heave at the center and midpoint of the long side of the excavation. The use of these figures is illustrated in the following example.

EXAMPLE 5.4

A foundation excavation, 20 m × 30 m in plan, is to be carried out to a depth of 10 m in the soil profile shown in Figure 5.8. Estimate the heave at the center of the excavation due to undrained distortion of the silty clay layer.

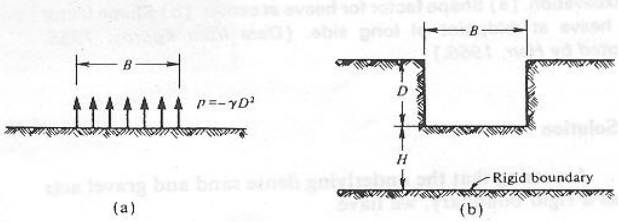


Fig. 5.5 (a) Common loading approximation for heave. (b) Linear elastic excavation analysis. (*Perloff, 1975.*)

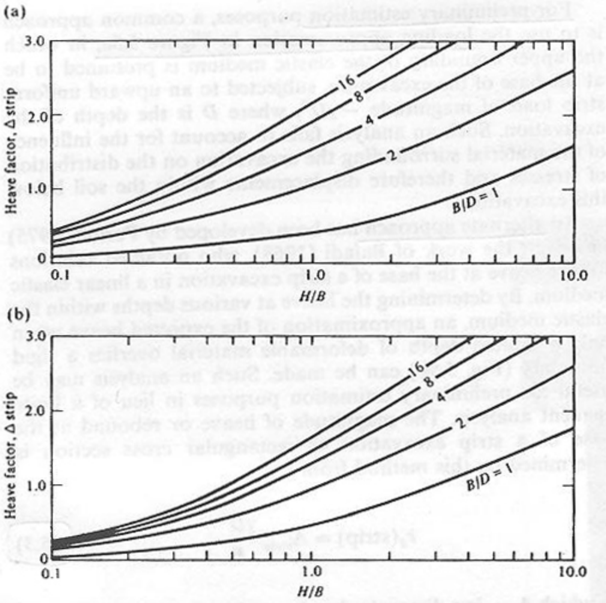


Fig. 5.6 Heave at base of strip excavation in linearly elastic medium of limited thickness. (a) Heave at center line. (b) Heave at edge. (*Based on analysis of Baladi, 1968.*)

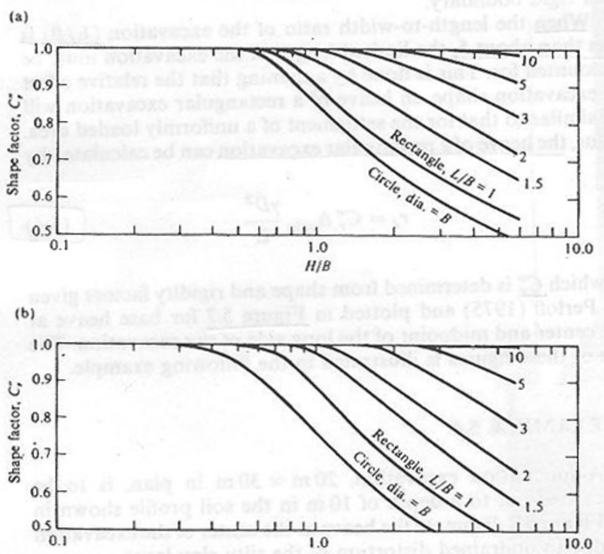


Fig. 5.7 Factors for correcting heave of strip excavation for shape of excavation. (a) Shape factor for heave at center. (b) Shape factor for heave at midpoint of long side. (*Data from Egorov, 1958, as cited by Harr, 1966.*)

Solution

Assuming that the underlying dense sand and gravel acts as a rigid boundary, we have

$$\frac{B}{D} = \frac{20}{10} = 2 \quad \frac{H}{B} = \frac{20}{20} = 1$$

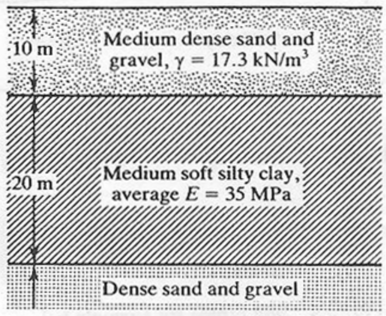


Fig. 5.8 Soil profile for Example 5.4.

and from Figure 5.6a, $\Delta_{strip} = 1.08$. For

$$\frac{L}{B} = \frac{30}{20} = 1.5$$

the shape factor is, from Figure 5.7a, $C_r'' = 0.97$. Thus, the heave can be calculated from Equation 5.4 as

$$r_d = (0.97)(1.08)\frac{(17.3\ \text{kN/m}^2)(10)^2}{35 \times 10^3\ \text{kPa}}$$
$$= 0.05\ \text{m} = 52\ \text{mm}$$

In the *Canadian Foundation Engineering Manual* (Meyerhof and Fellenius, 1985), a method is shown for calculating settlement in compressible soils using the net stress at a characteristic point. The net stress includes the decrease in applied foundation stress due to the excavated soil. Lambe and Whitman (1969) used the stress path method (Lambe, 1967) to predict heave of excavations.

5.4.3 Evaluation of Elastic Parameters

The magnitude of the calculated immediate distortion settlements (and heave) depends directly on the values of the elastic parameters (Young's modulus and Poisson's ratio) used in the calculations. Because cohesive soils are not linear elastic materials, these "elastic parameters" must be properly evaluated so that when they are substituted into the appropriate equations, correct estimates of the initial distortion settlement will be obtained.

For saturated clay soils, which deform at constant volume during the limited time required to develop the elastic distortion settlement, a Poisson's ratio of $\nu = 0.5$, corresponding to an incompressible medium, is usually assumed. Although this assumption may not be strictly correct, the magnitude of the computed settlement is not especially sensitive to small changes in Poisson's ratio.

Laboratory Tests Determination of the appropriate value of the equivalent Young's modulus E_u, an undrained modulus, is much more difficult. The ideal way would be to use the initial slope or tangent modulus of the stress–strain curve, as determined from triaxial compression or unconfined compression tests (Fig. 5.9). Alternatively, a secant modulus could be used, determined for the stress level estimated to occur in the field, for example, 25 or 50 percent of σ_{max}. There is, however, ample laboratory and field evidence to indicate that the values so obtained are too low, often only a small percentage of the field value. There are two primary reasons for this discrepancy. (1) Sample disturbance during sampling and preparation of the

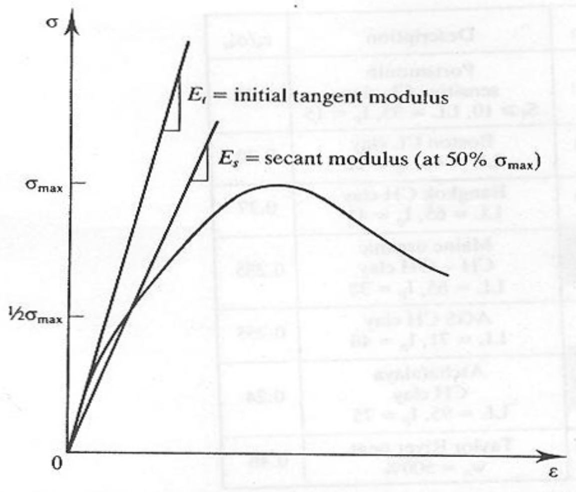


Fig. 5.9 Definitions of the initial tangent and secant moduli, for example, at 50 percent σ_{max}.

specimen for laboratory testing leads to a reduced undrained stiffness; the modulus is one of the properties most sensitive to disturbance effects. (2) Defects such as fissures are common in a great many sedimentary soil deposits. These inhomogeneities are usually unimportant to the settlement of a structure because they are small relative to the dimensions of the foundation. However, such defects may strongly influence a small laboratory test specimen and produce spuriously low measured modulus values in a laboratory test, especially unconfined compression tests. Because these two factors reduce the E_u, there is a tendency to overpredict the immediate settlements in the field.

On the basis of limited laboratory evidence, Perloff (1975) proposes a procedure for obtaining a suitable value of an equivalent field modulus. Ladd et al. (1977) and Foott and Ladd (1981) recommend the use of the direct sample shear test on high-quality undisturbed samples.

Field Plate Load Tests Because of the problems that affect the determination of the undrained modulus in laboratory tests, field plate bearing tests (ASTM D1194) are sometimes conducted for important projects. In these tests, all the parameters in Equation 5.2 are known except the factor $(1 - \nu^2)/E_u$, which can then be determined by back calculation. However, it should be kept in mind that if the seat of the settlement of the plate is different from that of the foundation, load tests on small plates, typically 30 cm to 75 cm in diameter, cannot be simply extrapolated to predict the settlements of prototype foundations; that is, the settlement of the foundation may be influenced by the presence of compressible strata that are far below the zone of influence of a small test plate. The best approach is to have access to the compressible layers during the exploration program to conduct the plate load tests. Disturbance of the surface can be partially overcome by cycling the load at least five times. This test should be carried out at the expected foundation elevation. Because of the relatively shallow influence of the loaded plates, it may be advisable to use two different size plates and test at two different depths, and scale up the modulus to the prototype foundation. A further complicating factor in the plate tests is that once the load on the plate exceeds about half the ultimate or failure load, settlements start to accelerate as the load is increased. Thus, back-calculated E_u values are very dependent upon the level of shear stress imposed by the plate.

Empirical Relations Because of all the problems with laboratory determination of E_u and because large scale field loading tests are expensive, it is common to assume that E_u is somehow related to the undrained shear strength. A common approximation (Bjerrum, 1963, 1972) is to use the ratio E_u/τ_f ranging between 500 and 1500, with τ_f determined either by the field vane shear or the undrained triaxial compression test. The lowest value is for highly plastic clays where the applied load is large compared to the value of $\sigma'_p - \sigma'_{vo}$; (that is, the stress added to the foundation is relatively large). The higher value is for clays of low plasticity, where the added load is relatively small. D'Appolonia et al. (1971) reported an average E_u/τ_f of 1200 for load tests at ten sites, but for the clays of higher plasticity the range was 80–400. Values have been found ranging from 40 to 3000 (Simons, 1974). These cases plus a few others taken from the literature are plotted versus PI in Figure 5.10. Stiff fissured soils and glacial tills are not included. There is much scatter for $PI < 50$ and not much data for $PI > 50$. It seems reasonable to simply use Bjerrum's recommendation (E_u/τ_f of 500 and 1500), modified as required by procedures developed by D'Appolonia et al. (1971) for estimating immediate settlements of soft clays.

Another factor that strongly affects the undrained shear strength of clays is stress history, and stress history also affects Young's modulus. Information from Ladd et al. (1977) is shown in Figure 5.11. The relationship is not so simple because E_u/τ_f depends strongly on the level of shear stress. In general, however, it decreases with increasing overconsolidation ratio for a given stress level, as shown in Figure 5.11. Duncan and Buchignani (1976) also present a relationship between undrained modulus and OCR (Table 5.4) that may be used.

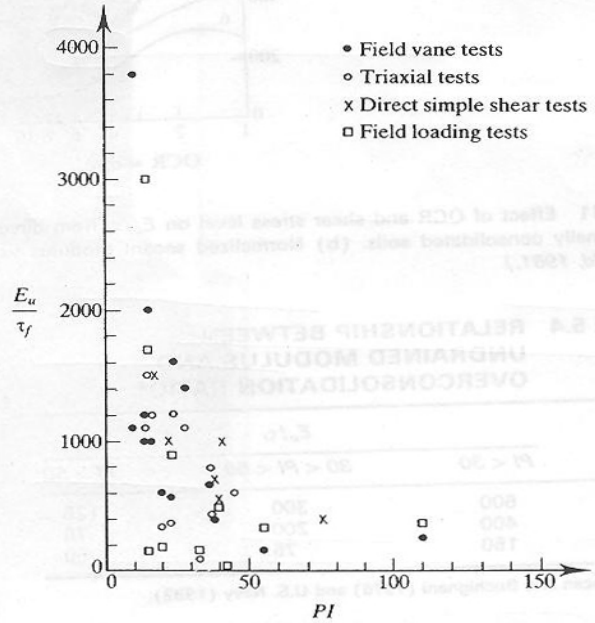
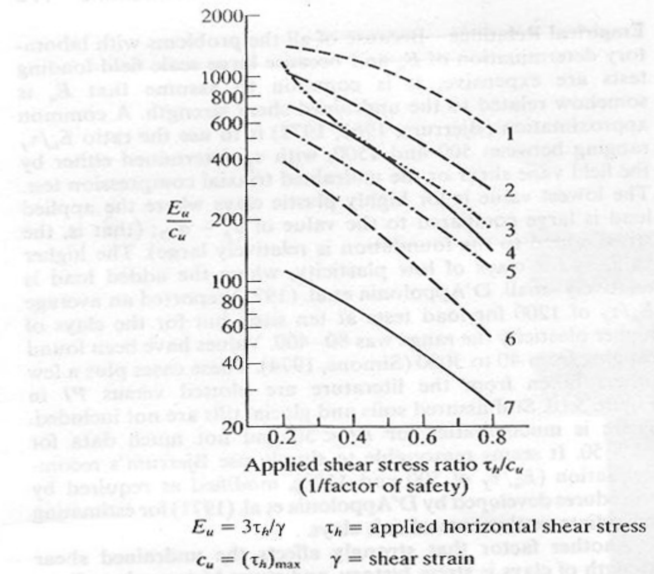


Fig. 5.10 The ratio E_u/τ_f versus PI as reported by various authors (see Holtz and Kovaćs, 1981, for references cited). Sources: Bozozuk (1963); Bozozuk and Leonards (1972); D'Appolonia et al. (1971); DiBaglio and Stenhamar (1975); Hansbo (1960); Holtz and Holm (1979); Ladd and Edgers (1972); LaRochelle and Lefebvre (1971); Raymond et al. (1971); Simons (1974); Tavenas et al. (1974).



No.	Description	c_u/σ'_{vc}
1[1]	Portsmouth sensitive CL clay $S_t \geq 10$, LL = 35, I_p = 15	0.20
2[1]	Boston CL clay LL = 41, I_p = 22	0.20
3[1]	Bangkok CH clay LL = 65, I_p = 41	0.27
4[1]	Maine organic CH – OH clay LL = 65, I_p = 38	0.285
5[2]	AGS CH clay LL = 71, I_p = 40	0.255
6[1]	Atchafalaya CH clay LL = 95, I_p = 75	0.24
7[3]	Taylor River peat w_n = 500%	0.46

(1) From Ladd and Edgers (1972)
(2) MIT for Dames and Moore
(3) MIT for Haley and Aldrich

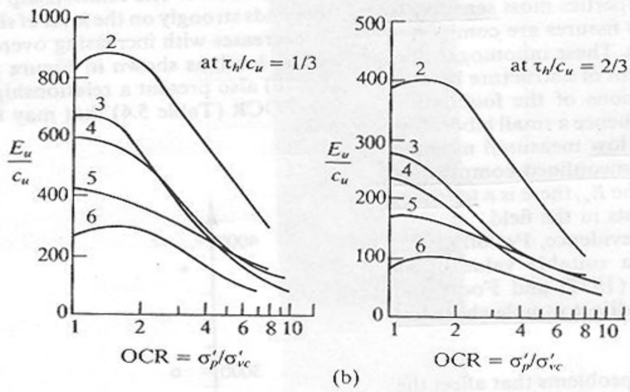


Fig. 5.11 Effect of OCR and shear stress level on E_u/τ_f from direct simple shear tests. (a) Normalized secant modulus vs. stress level for normally consolidated soils. (b) Normalized secant modulus vs. overconsolidation ratio. (*Data from Ladd et al., 1977, and Foott and Ladd, 1981.*)

TABLE 5.4 RELATIONSHIP BETWEEN UNDRAINED MODULUS AND OVERCONSOLIDATION RATIO[a].

	E_u/τ_f		
OCR	*PI* < 30	30 < *PI* < 50	*PI* > 50
< 3	600	300	125
3–5	400	200	75
> 5	150	75	50

[a] After Duncan and Buchignani (1976) and U.S. Navy (1982).

In-Situ Tests Besides using a plate bearing test to determine the in-situ elastic modulus, it would be possible to use it directly for estimating the immediate settlements. As before, the primary requirement is that the soil volume under the foundation stressed by the plate should have some relation to the volume stressed by the proposed foundation. Included in this possibility are large-scale loading tests utilizing, for example, an embankment fill or a large tank of water. In these latter cases, geotechnical instrumentation is required for measurement of settlements and excess pore pressures.

Another approach is to use the pressuremeter test (Baguelin et al., 1978; Meyerhof and Fellenius, 1985). Through selection of appropriate empirical coefficients, the immediate settlement can be determined from the pressuremeter modulus. Other candidate in-situ tests for this purpose are the screw plate compressometer and the Marchetti dilatometer.

5.4.4 Importance of Immediate Settlement Calculations

As mentioned earlier, the immediate or distortion settlement is often not a significant portion of the total foundation settlement, so only rough estimates are ordinarily required for estimating this component of the total settlement. However, if as occurs in certain circumstances, the immediate settlement is an important part of the total settlement, then it is worth the effort to obtain a good estimate of the undrained elastic modulus. Recommendations in Section 5.4.3 above are appropriate in this case.

From Equation 5.2, immediate settlement is directly proportional to the applied load. Sometimes, to provide an adequate factor of safety in bearing capacity, the footing size is increased or a large mat foundation is chosen. Designers should realize that the immediate settlement also increases as the foundation size increases.

Large footings and heavy loads cause large immediate settlements, especially in weaker soils, and a correction for low factor of safety and large undrained shear deformations has already been mentioned. As noted by Foott and Ladd (1981), these occur especially in soils that have a high plasticity index and also contain organic matter. They recommend estimating the soil moduli using K_0 consolidated direct simple shear tests and using the empirical observations by D'Appolonia et al. (1971) for lateral yield and low factors of safety. With appropriate consideration of soil type and available consolidation time, their method should indicate conditions where initial and creep-induced settlements could be a significant design problem.

Burland et al. (1977) make some empirical observations to help designers decide when immediate settlements are likely to be an important part of the total settlement of a foundation. For soft soils in which the applied stress will exceed the preconsolidation pressure, typically the immediate settlement component is about 10 percent of the consolidation component. Therefore, it is relatively unimportant, although even this magnitude may be a problem if the structure is sensitive to rapid settlements. On the other hand, for stiff soils in which the applied stress does not exceed the preconsolidation pressure, then the immediate settlement may be as much as 50 to 60 percent of the total settlement. It also decreases as the depth of the compressible layer decreases. Even for deep layers of overconsolidated or stiff clay, the immediate settlement is unlikely to exceed 70 percent of the total settlement, and it may in an extreme case be as low as 25 percent for nonhomogeneous and anisotropic soils. Average values of the ratio s_i/s appear to range from 0.5 to 0.6. The total settlement s, because the secondary component is so small in stiff soils, can be estimated quite adequately from oedometer tests.

Effect of Footing Size on Immediate Settlement Perloff and Baron (1976) give three instructive examples on what happens to immediate settlements as the size of footings is changed. Increasing the size of a square footing to support a given total load leads to a reduction in elastic settlement in proportion to the increased footing dimension. In other words, with the same modulus, if the applied stress q is reduced, one should obtain less settlement.

Is this true for a strip footing with a constant load per unit length? Increasing the width to reduce the unit pressure does *not* reduce the elastic settlement, as long as a constant total load per unit length is applied.

Finally, they consider two square footings of width B and nB, respectively, which are subjected to the same unit pressure. So the total load carried by each footing is not the same. Their analysis shows that the larger footing will settle n times as much as the smaller one. Thus, different sized footings subjected to the same unit pressure will not have equal immediate settlements.

5.5 DISTORTION SETTLEMENT OF GRANULAR SOILS

Virtually all settlement of granular soils can be considered to be immediate (Table 5.1). This is because even if the sands are below the groundwater table, and completely saturated, excess pore pressures dissipate rapidly during loading. Although the magnitude of settlement may be significantly less than might be obtained with similar foundations on cohesive soils, the settlements of structures on sand must be considered and accurately estimated because most structures are more sensitive to distortion settlement with rapid loadings than they are if the distortion occurs slowly. Further, granular soils are more likely to be heterogeneous than many sedimentary clay deposits. Because we have no rational theory for prediction of the settlement of shallow foundations on granular soils, we use empirical procedures in engineering practice, and these procedures will be the subject of this section.

We begin by a discussion of the sources of settlement and the factors that cause shear strains in granular soils when they are loaded. Then we describe a number of methods that have been developed for estimating immediate settlements on cohesionless soils, concentrating especially on the Schmertmann (1970) method and other procedures using in-situ tests.

The sources of settlement in granular soils are: (1) shear strains, which result in changes in shape after loading, and (2) changes in volume, which can be either positive or negative (dilation or compression). Both factors are functions of the initial void ratio and confining pressure, and both occur in granular soils and result in surface settlements.

5.5.1 Factors Affecting Sand Compressibility (and How to Determine Them)

Although granular materials are not elastic, we can consider the elastic constants in Equation 5.2 to be equivalent approximations. If so, then the magnitude of load (the contact pressure and size of the footing) directly affects the settlement of a granular layer. The thickness of the granular layer must also influence the settlement. Thickness is determined by the site exploration program and the geology of the site.

The magnitude of the load, contact pressure, and size of footing are determined from the preliminary design. Sometimes presumptive bearing values are used to size the foundation elements or, if a bearing capacity analysis has been carried out previously, estimated working footing stresses may be used.

G. A. Leonards (1987, personal communication) has indicated that the primary factors that influence the compressibility of granular materials are:

1. Soil characteristics
2. State of stress in the ground
3. State of compaction
4. Stress history

Soil Characteristics Characteristics of granular soils such as gradation, grain size, angularity, roughness, and mineral hardness affect compressibility. For the same packing, relative density, stress history, and stress level, a better gradation decreases compressibility, while increasing angularity results in an increase in compressibility. On the other hand, increasing the roughness of the grains and their size decreases the compressibility. The propensity for grain crushing depends on the type of mineral, particle shape, and stress level. Some minerals are more susceptible to crushing than others. Everything else being equal, the compressibility increases as hardness decreases. A number of other crushing factors are discussed by Hardin (1985). At ordinary footing loads, crushing of mineral grains does not significantly contribute to settlement. However, grain crushing could be important in micaceous sands and silts.

Soil characteristics are determined through a program of site investigation and laboratory testing.

State of Stress The second important factor is the state of stress in the ground. If the sand is subjected to a large horizontal stress, there is less tendency towards volume change and less settlement. Large horizontal stresses result from prestressing, discussed below, either due to geologic factors or to construction loadings. Even if some of the vertical overburden stress is removed, a certain percentage of the horizontal stress remains. K_0, the coefficient of lateral earth pressure at rest, may increase from approximately 0.5 up to 1.0 or greater. Therefore, if the soil is again loaded, less settlement will occur. Other factors related to the state of stress include the location of groundwater table, depth of foundation, void ratio, and possible prestressing and prestraining. These last two factors are discussed below.

How is the state of stress determined? A quantitative determination is not easy. From the geology and site investigation, certain inferences can be made about the possibility of prestraining and therefore the presence of large residual lateral stresses in the ground. Plate load tests may be useful in this regard as are other in-situ measurements such as those from earth pressure cells and pressuremeter and dilatometer tests.

State of Compaction State of compaction includes the packing, density, and orientation of the sand particles. It is very important to know the initial state of compaction and its variability. For example, if the deposit is loose and variable, large and possibly detrimental settlements will occur in the foundation. If the deposit is dense, then the sand tends to expand when sheared and small, often negligible, settlements result.

How is the state of compaction determined? Sands are very difficult to sample undisturbed; therefore, in-situ tests, the SPT and Dutch cone penetrometer, for example, are used to correlate with in-situ density.

Most compressibility obviously will occur with looser deposits. In terms of relative density or density index, it has been estimated that greater than 90 percent of the compression occurs between $D_r = 0.10$ and 0.70. Very little settlement occurs as the density index exceeds 0.70. In terms of packing at a given relative density, the compressibility of the least favorable packing is probably 2 to 4 times the compressibility of most favorable packing. It has a relatively minor importance for footing settlements.

Stress History The most important factor influencing the compressibility of granular soils is the stress history, or more precisely, the strain history of the deposit. If the sand deposit has been previously loaded or strained, a large decrease in compressibility (increase in equivalent modulus), and therefore a large decrease in settlement, results. The normally consolidated compressibility is at least five times greater, with typical values between 8 and 16 times, and it may even approach 30 times greater than the overconsolidated or prestrained compressibility. This occurs because if a sand is loaded in compression, for example in a triaxial test, and somewhere before failure the sample is unloaded and then reloaded, the unload/reload modulus E_r is much steeper than the initial tangent modulus E_t (Fig. 5.12). This effect is greater in a loose sand. The unload/reload modulus in very loose sand can easily be 5 to 30 times less than the initial tangent modulus. If the designer overestimates settlements by a factor of 5 to 30, then expensive and unnecessary foundation treatment or deep foundations may be selected. Therefore, it is very important to determine if possible whether a sand deposit has previously been loaded.

Prestraining and its effects have been discussed by Dahlberg (1975), Lambrechts and Leonards (1978), and Jamiolkowski et al. (1985).

Because sands are very easily disturbed during sampling, it is almost impossible to determine in a laboratory test whether

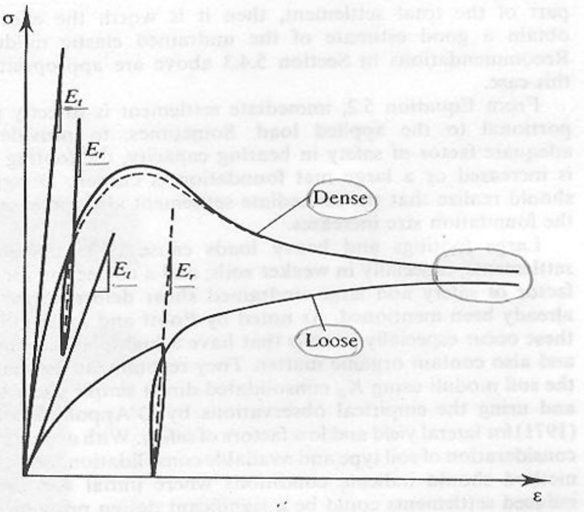


Fig. 5.12 Stress–strain curve for a typical sand in loose and dense states.

a sample has been prestrained. Therefore, indirect means are used. Examination of the geology and geologic history of the deposit can be very instructive. For example, if construction excavation and replacement has taken place, or if sand dunes have moved over the area, as often happens in lake shore and sea coastal regions, then the sand has almost certainly been preloaded. In-situ tests used to measure soil variability such as the SPT or the Dutch cone penetrometer (CPT) are quite insensitive to changes in modulus due to prestraining because they measure the ultimate or failure strength of the material. Penetration resistance is increased very slightly, probably less than 10 percent, owing to prestraining, whereas an increase in compressibility from 5 to 30 times can occur. Therefore, correlations between, for example, SPT and/or CPT and modulus can be in error by a factor of 5 to 30.

Plate load tests and other in-situ tests also may be used to determine whether a sand deposit has been prestressed. Good references on prestraining and in-situ tests include Dahlberg (1975) and Jamiolkowski et al. (1985). The prerequisites for a successsful plate load test on sand are discussed in some detail by Terzaghi and Peck (1967).

Plate load tests may be somewhat difficult to interpret. A small prestress effect may occur if an excavation is made to enable the plate test to be performed near the proposed foundation elevation. The location of the groundwater table may also affect the results, as will soil variability. If the deposit has a highly variable in-situ density, the plate load test will give inappropriate results. The most important point is to use a correct factor to scale from the plate load test up to the footing size, and the Terzaghi and Peck (1967) chart and scaling factor have been slightly modified by G. A. Leonards (personal communication, 1987). The Terzaghi and Peck charts were developed for normally consolidated sands above the groundwater table and relative density measured by the SPT. If the results of a plate load test indicate a significant difference from the charts, the deposit is most likely prestressed.

Use of correlations developed for normally consolidated sands may yield very misleading results if they are used for sands that have been previously prestrained. Therefore, it is best to try to measure in-situ compressibility directly by the use of plate load and other in-situ tests such as the pressuremeter, dilatometer, or screwplate, as discussed near the end of this section.

5.5.2 Procedures to Estimate the Settlements of Foundations on Granular Materials

Elastic Theory and Recent Modifications According to Harr (1966), elastic theory is utilized for estimating the settlements of shallow foundations on granular materials in the U.S.S.R. This process must involve some empirical corrections, because of the great influence confining pressure has on the modulus and compressibility. A related development is the Oweis (1979) method for predicting settlements based on an equivalent linear model using the deformation that would be caused by plate load tests at depth. The model was calibrated by use of actual plate load test data. Recently Bowles (1987) proposed that settlements of shallow foundations on sands could be estimated using Steinbrenner's (1936) modification of the Boussinesq equations by adjusting appropriately the strain influence factor. For the cases he investigated, good results were obtained. In another recent development, Hardin (1987) proposed a model for one-dimensional strain that appears to represent very adequately the shear stress–strain behavior for normally consolidated cohesionless soils over a wide range of stresses. The model is potentially useful for estimating settlements of structures that can be approximated by one-dimensional strain.

Empirical Procedures In view of the difficulties with (1) elastic theory and (2) plate load and other in-situ tests, a number of empirical procedures have been developed. These include the procedures by Terzaghi and Peck (1967) and by Peck et al. (1974) based on the results of the standard penetration test (SPT). Schmertmann (1970) developed a procedure using cone penetration tests to determine in-situ compressibility, and this method and its later modifications will be described in some detail in this chapter. Other empirical in-situ procedures will also be described briefly. Finally, Burland and Burbidge (1985) have conducted an extensive study of some 200 case histories of settlements of shallow foundations on granular materials and expanded on a similar study reported by Burland et al. (1977). We will use the Burland procedures to check results from the other empirical procedures.

5.5.3 Schmertmann (1970) Procedure

In 1970, J. H. Schmertmann proposed a new procedure for estimating the settlement of shallow foundations on granular soils. Although empirical, the procedure has a rational basis in the theory of elasticity, finite element analyses, and observations from field measurements and laboratory model studies. From the theory of elasticity, the distribution of vertical strain ε_z within the linear elastic halfspace subjected to a uniformly distributed load over an area at the surface can be determined by

$$\varepsilon_z = \frac{\Delta q}{E} I_z \qquad (5.5)$$

where

- Δq = the intensity of the uniformly distributed load
- E = Young's modulus of the elastic medium
- I_z = a strain influence factor, which depends only upon the Poisson's ratio and the location of the point for which the strain is being evaluated

Based on the results of displacement measurements within sand masses loaded by model footings, as well as finite element analyses and deformations of materials with nonlinear stress–strain behavior, the distribution of strain within loaded granular masses is very similar in form to that for a linear elastic medium.

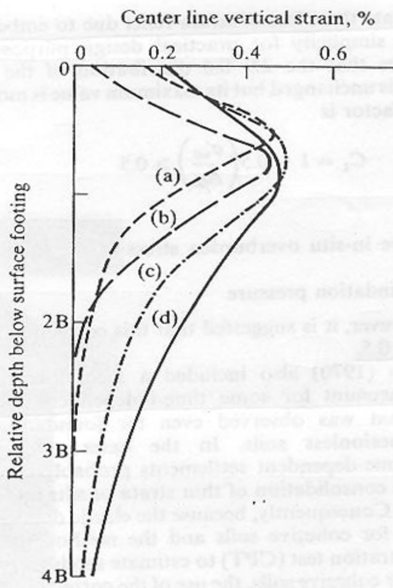


Fig. 5.13 Comparisons of vertical strain distributions from FEM studies and from rigid model tests. (a) Hartman FEM axisymmetric, $B = 6.1$ m, $\Delta p = 287$ kN/m^2. (b) Brown model, $B = 150$ mm, $L/B = 1$, $\Delta p = 10$ kN/m^2. (c) Hartman FEM plane strain, $B = 6.1$ m, $\Delta p = 383$ kN/m^2. (d) Brown model, $B = 150$ mm, $L/B = 4$, $\Delta p = 10$ kN/m^2. Both (a) and (c) are with Duncan and Chang $K = 60\,000$, $\phi = 42°$, $\nu = 0.4$, $K_0 = 0.50$. Both (b) and (d) are averages of 3 tests. (*Schmertmann et al., 1978.*)

Some typical results of model tests and finite element analyses reported by Schmertmann (1978) are shown in Figure 5.13. Other results are shown by Schmertmann (1970) and Perloff (1975). On the basis of these observations, Schmertmann (1970) suggested that for practical purposes the distribution of vertical strain within a granular mass could be expressed by Equation 5.5 in which the Young's modulus might vary from point to point. The strain influence factor could be approximated by a triangle with a maximum value of 0.6 at $z/B = 0.5$ and $I_z = 0$ at a depth of $z/B = 2$. Schmertmann (1970) refers to this as a "$2B$–0.6 distribution."

The surface settlement s_i is the integration of the strains:

$$s_i = \int_{z=0}^{\infty} \varepsilon_z dz \qquad (5.6a)$$

which can be approximated by

$$s_i = \Delta q \int_0^{2B} \frac{I_z}{E} dz \qquad (5.6b)$$

This relationship can be approximated further as a summation of settlements of convenient approximately homogeneous layers, or

$$s_i = C_1 C_2 \Delta q \sum_{i=1}^{n} \left(\frac{I_z}{E}\right)_i \Delta z_i \qquad (5.6c)$$

in which

- Δq = net load intensity at the foundation depth
- I_z = strain influence factor from the $2B$–0.6 distribution
- E = appropriate Young's modulus at the middle of the ith layer of thickness Δz_i
- C_1, C_2 = correction factors as described below

To incorporate the effect of strain relief due to embedment and yet retain simplicity for practical design purposes, the method assumes that the $2B$–0.6 distribution of the strain influence factor is unchanged but its maximum value is modified. The suggested factor is

$$C_1 = 1 - 0.5\left(\frac{\sigma'_{v0}}{\Delta q}\right) \geqslant 0.5 \qquad (5.7)$$

where

σ'_{v0} = effective in-situ overburden stress at the foundation depth
Δq = net foundation pressure

In all cases, however, it is suggested that this correction factor not be less than 0.5.

Schmertmann (1970) also included a second correction factor, C_2, to account for some time-independent increase in settlement that was observed even for foundations on presumably cohesionless soils. In the cases studied by Schmertmann, time-dependent settlements probably occurred as a result of the consolidation of thin strata of silts and clays within the sands. Consequently, because the elastic distribution is inappropriate for cohesive soils and the method uses the Dutch cone penetration test (CPT) to estimate modulus, which is questionable for cohesive soils, the use of the correction factor C_2 is not recommended; therefore, use C_2 equal to 1.0 in Equation 5.6c.

No account was taken in the original procedure of the influence of foundation shape on the strain distribution, because as a foundation shape changes from approximately axisymmetric to approximately plane strain conditions, the angle of shearing resistance increases and the stresses at a given depth also increase. These two effects were thought to cancel each other, giving a strain distribution that is, perhaps, not very different for a wide range of length-to-width ratios.

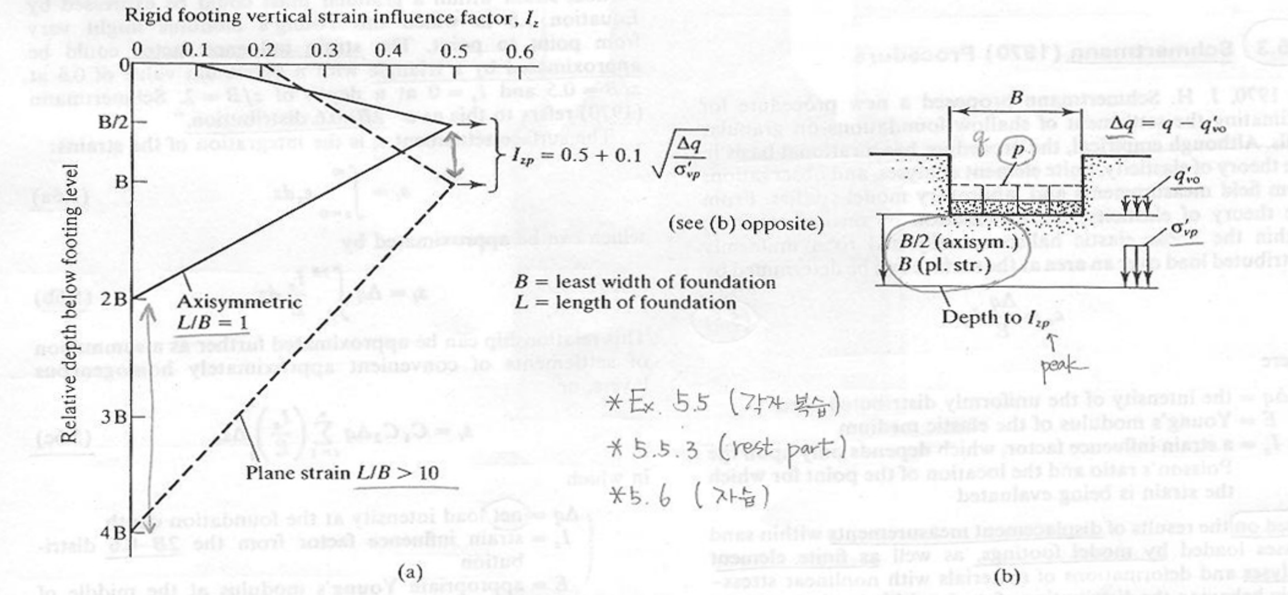
Model test results suggest that when a rigid boundary lies within the $2B$–0.6 distribution, the distribution of the strain influence factor will be simply truncated at the depth, with the slopes of the distribution remaining as for the homogeneous case.

Modifications of 1978 A number of modifications have been made by Schmertmann et al. (1978) and Schmertmann (1978). The strain influence diagram was modified slightly on the basis of extensive analytical studies, and axisymmetric and plane strain loadings are now considered separately. The modified strain influence diagram is shown in Figure 5.14. Note that the depth of the strain influence factor goes to $2B$ for the axisymmetric case and to $4B$ for plane strain conditions. The maximum value of the influence factor is at least 0.5 plus an incremental increase relative to the effective vertical overburden pressure at the depth of the maximum value. An explanation of the pressure terms in I_{zp} is shown in Figure 5.14b. Schmertmann (1978) recommends that if L/B is greater than 1 and less than 10, both the axisymmetric and plane strain cases can be calculated and interpolated for the actual L/B ratio.

As before, this method is only appropriate for normally loaded sands where the bearing capacity of the sand is adequate. If the sand has been prestrained by previous loading, then the real settlements will, as explained earlier, be greatly overpredicted by this method. Schmertmann (1978) recommends that a tentative reduction in settlement after preloading or other means of compaction of half the predicted settlement be used, and this is probably still conservative. There may also be some additional settlement effect due to dynamic, cyclic, or vibratory loads. This of course is a very serious potential problem for loose sands below the water table. Some type of densification or prestressing is an easy and effective way of reducing the potential for liquefaction or other undesirable behavior.

The correction factors C_1 (Eq. 5.7) and C_2 are unchanged. Also, as before, the correction factor C_2 is subject to question.

The use of this method to estimate the settlement of a shallow foundation on sand is illustrated by an example later in this section.



*Ex. 5.5 (각자 복습)
*5.5.3 (rest part)
*5.6 (자습)

Fig. 5.14 Modified strain influence factor diagrams for use in Schmertmann method for estimating settlement over sand. (a) Simplified strain influence factor distributions. (b) Explanation of pressure terms in equation for I_{zp}. (*Schmertmann, 1978.*)

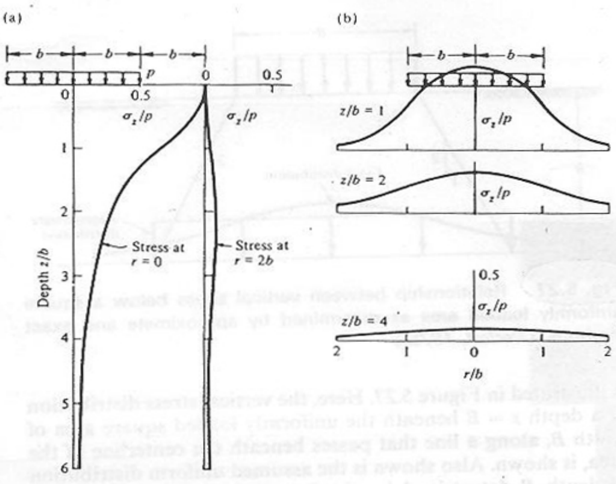


Fig. 5.23 Distribution of vertical stress due to a loaded circular area on a linear elastic halfspace (a) along vertical lines, (b) along horizontal lines. (*Perloff, 1975.*)

equal to twice the width of the loaded area ($z/b = 4$ in Figure 5.23a). Beneath the point outside the loaded area, the stress is zero at the ground surface and increases to a maximum at a depth of approximately 1.2 times the diameter of the loaded area ($z/b = 2.4$). However, the magnitude of this stress changes only slightly with depth below this point.

The distribution of vertical stress along selected horizontal lines beneath the center of the loaded area is shown in Figure 5.23b. The stress is more concentrated beneath the area at shallow depths, but tends to spread more at increasing depths.

Another useful way to view the distribution of stresses is as contours of equal vertical stress called isobars (or arctic taverns), as shown in Figure 5.24 for a circular loaded area. The area contained within a given stress contour experiences stresses larger than the stress level indicated by that contour. For example, the zone within the stress contour for which $\sigma_z = 0.05p$ contains all of the material subjected to stresses (resulting from the loaded area) of that magnitude or greater. Because of the shape of this zone, it is often referred to as the bulb of pressure. Note that for a loaded area of a given shape on the surface of a linear elastic halfspace, the size of the pressure bulb is proportional to the size of the loaded area. Thus, when considering the settlement potential of a large structure, it should be remembered that the stresses increase with depth in direct proportion to the size of the loaded area.

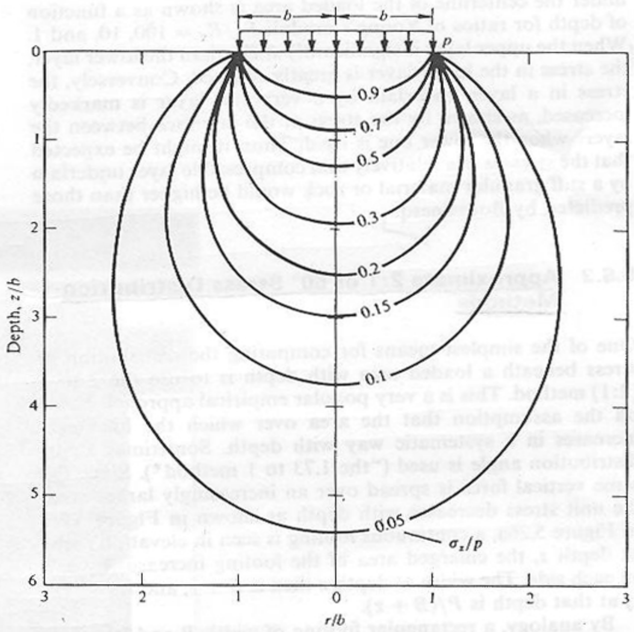


Fig. 5.24 Contours of constant vertical stress (isobars) beneath a uniformly loaded circular area on a linear elastic halfspace. (*Perloff, 1975.*)

5.6.2 Effects of Layered Systems

It is frequently necessary to determine the stresses in a compressible layer that underlies one or more layers of different mechanical properties. This problem has been analyzed by considering layered systems consisting of different elastic properties. Solutions are discussed by Poulos and Davis (1974), and Perloff (1975) gives a number of references. The most extensive use of layered elastic theory has been by pavement engineers.

Typical results of one such analysis are given in Figure 5.25, in which a uniformly distributed load is shown acting on a circular area on the surface of a two-layer elastic system. In this case, the thickness of the upper layer has been chosen equal to the radius of the loaded area. The vertical stress distribution

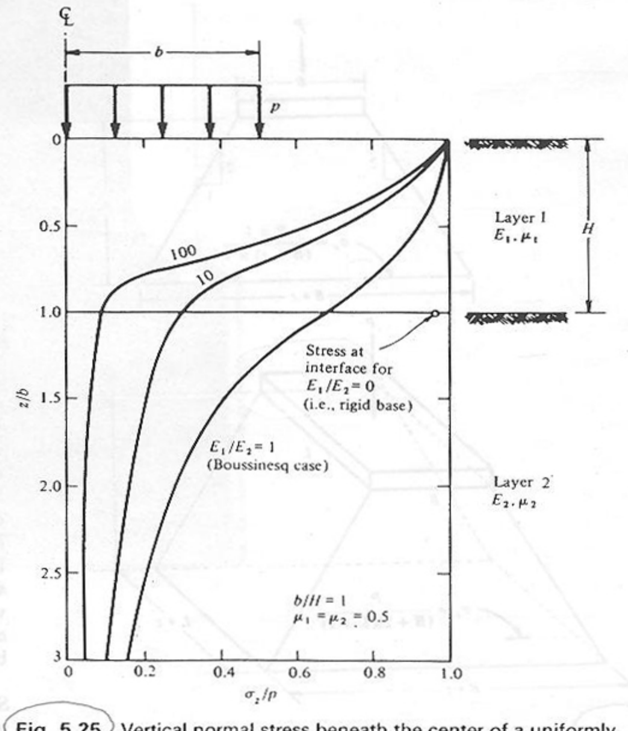


Fig. 5.25 Vertical normal stress beneath the center of a uniformly loaded circular area at the surface of a two-layer elastic system. (*Modified from Burmister, 1958, by Perloff, 1975.*)

under the centerline of the loaded area is shown as a function of depth for ratios of Young's moduli $E_1/E_2 = 100$, 10, and 1. When the upper layer is significantly stiffer than the lower layer, the stress in the lower layer is greatly reduced. Conversely, the stress in a layer underlain by a very stiff layer is markedly increased, as shown by the stress at the interface between the layers when the lower one is rigid. Thus, it might be expected that the stresses in a relatively thin compressible layer underlain by a stiff granular material or rock would be higher than those predicted by Boussinesq.

5.6.3 Approximate 2:1 or 60° Stress Distribution Methods

One of the simplest means for computing the distribution of stress beneath a loaded area with depth is to use the 2 to 1 (2:1) method. This is a very popular empirical approach based on the assumption that the area over which the load acts increases in a systematic way with depth. Sometimes a 60° distribution angle is used ("the 1.73 to 1 method"). Since the same vertical force is spread over an increasingly larger area, the unit stress decreases with depth as shown in Figure 5.26. In Figure 5.26a, a continuous footing is seen in elevation view. At depth z, the enlarged area of the footing increases by $z/2$ on each side. The width at depth z then is $B + z$, and the stress σ_z at that depth is $P/(B + z)$.

By analogy, a rectangular footing of width B and length L would have an area of $(B + z)(L + z)$ at a depth z, as shown in Figure 5.26b. The corresponding stress at depth z is also shown in the figure.

The relationship between the approximate distribution of stress determined by the 2:1 method and the exact distribution

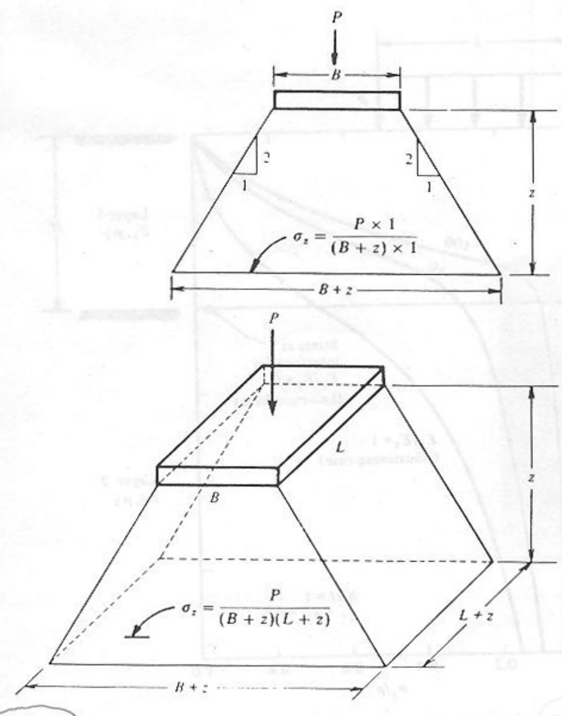


Fig. 5.26 Approximate method for distribution of vertical stress due to surface load.

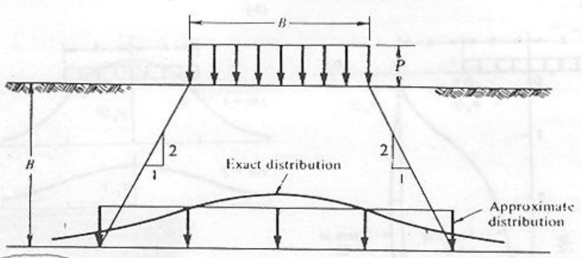


Fig. 5.27 Relationship between vertical stress below a square uniformly loaded area as determined by approximate and exact methods. (*Perloff, 1975.*)

is illustrated in Figure 5.27. Here, the vertical stress distribution at a depth $z = B$ beneath the uniformly loaded square area of width B, along a line that passes beneath the centerline of the area, is shown. Also shown is the assumed uniform distribution at depth B determined by the 2:1 method. The discrepancy between these two methods decreases as the ratio of depth to the size of the loaded area increases.

5.6.4 Effects of Gravity Structures

Embankments In most situations in foundation engineering, the loads imposed on a soil by the foundation can be simply represented by a system of boundary stresses without significant error in the calculated stress distributions. However, for stresses due to earth embankments, such an approximation may lead to important differences in the calculated stresses. Perloff (1975) suggests that a more reasonable approach is to consider the embankment and foundation as a single body loaded only by its own weight. As noted by Poulos and Davis (1974), there may be some inaccuracy in the results given by Perloff (1975) and Perloff and Baron (1976). The results are based on original studies by Baladi (1968), who used conformal mapping to solve the differential equations. The way self-weight is considered causes some inconsistencies, as the width to height ratio of the embankment becomes large. Thus, in using the Perloff (1975) charts, care should be exercised for cases where $L/H > 1$.

For important projects in which an accurate estimation of stresses is important, a finite element analysis (e.g., Duncan, 1972) is recommended.

Effect of Soft Foundations As shown by finite element analyses of a stiff elastic embankment overlying a less-stiff elastic foundation of limited depth (Perloff, 1975), the distribution of stresses arising from the weight of an embankment is in general affected by the magnitude of the relative stiffness of the embankment and its foundation. As the relative stiffness of the embankment increases, the vertical stress beneath the center decreases, especially as the ratio of the moduli becomes large. Shear stress at the base of the embankment is significantly decreased, and only a minor increase in horizontal stresses is observed. However, for modular ratios greater than 10, the lower half of the embankment exhibits rather large tensile stresses. Whether these actually occurred in an embankment would depend on the stress–strain characteristics of the embankment material and possible cracking and stress redistribution.

Stress Relief Due to Excavations Another circumstance in which stresses due to gravity forces are frequently of interest is that of unloading due to an excavation. Evaluation of the heave (rebound) that occurs undrained and could be analyzed by