Text: Electrical Engineering Principles and Applications
— A. Hambley, Pearson Education

CH. 1 Introduction

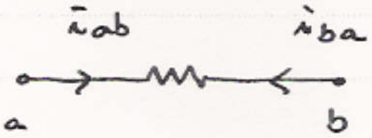
- Electric charge : q (C)

  electron $e = -1.602 \times 10^{-9}$ (C)

- Current : Flow of electric charges

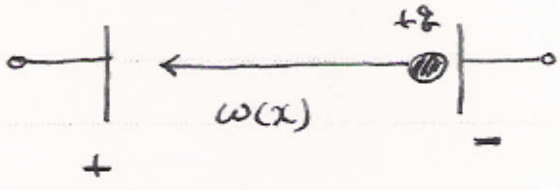
$$i(t) = \frac{dq}{dt} \quad (A)$$

$$q(t) = \int_{t_0}^{t} i(\tau)\, d\tau + q(t_0) \quad (C)$$

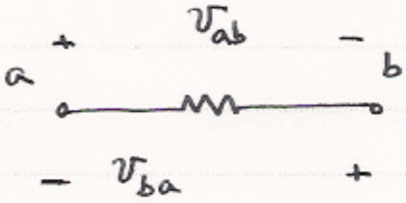


- Electric potential : Voltage

$$v(x) = \frac{dw}{dq} \quad (V)$$



Energy ~~needed~~ required to move unit charge for x.



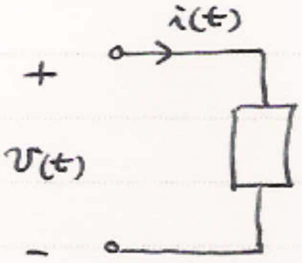
$v_{ab} = -v_{ba}$

v_{ab} : electric potential difference between a and b.

- Energy and power

$$p(t) = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i \quad (W)$$

$p(t) = v(t) \cdot i(t)$ (W) : instantaneous power



if $p(t) = v(t) \cdot i(t) > 0 \Rightarrow$ power consumed. Passive element

if $p(t) = v(t) \cdot i(t) < 0 \Rightarrow$ power generated. Active element.

$$w(t) = \int_{t_1}^{t_2} p(\tau) \cdot d\tau$$

Example 1.3)

§1.4 Kirchhoff's Current Law (KCL)

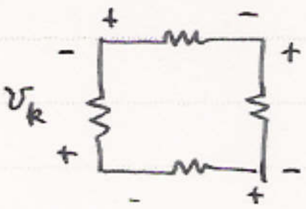
- Net current entering a node is zero.

$$\sum_{node} i_K(t) = 0$$

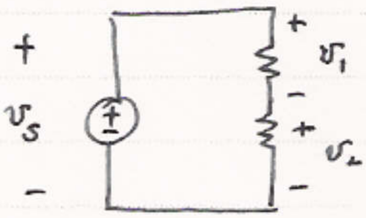
$$\sum_{in} i_K(t) = \sum_{out} i_j(t)$$

§1.5 Kirchhoff's Voltage Law (KVL)

- For any closed path, the sum of voltage drops equals zero.



$$\sum_{loop} v_K(t) = 0$$

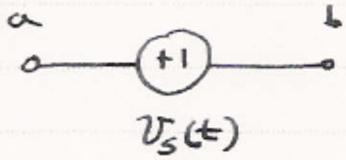


$$\sum_{up} v_K(t) = \sum_{down} v_j(t)$$

- Parallel circuits] See Figures
  Series circuits

§ 1.6 Circuit Elements

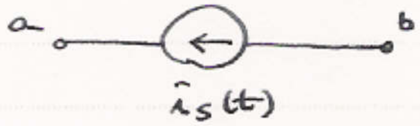
- Independent voltage source



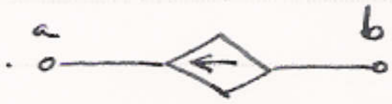
- Dependent voltage source

αv_x — Voltage controlled voltage source
βi_x — current controlled voltage source

- Independent current source



- Dependent current source



αv_x — Voltage controlled current source
βi_x — Current controlled current source

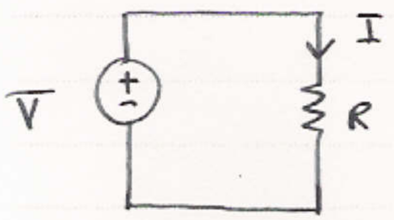
- Ohm's Law

$$i(t) = \frac{v(t)}{R} \rightarrow v(t) = R \cdot i(t)$$

$$R = \frac{v(t)}{i(t)}$$

$R\,(\Omega)$ – resistance

$G\,(\mho, \text{mho}) = \frac{1}{R}$: conductance



$$I = \frac{V}{R}$$

$$P = VI = \frac{V^2}{R} = I^2 R$$

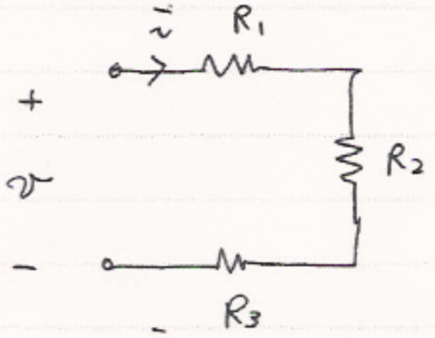
$$R = \frac{\sigma L}{A} \quad (\Omega)$$

L : length
σ : conductivity
A : cross section area

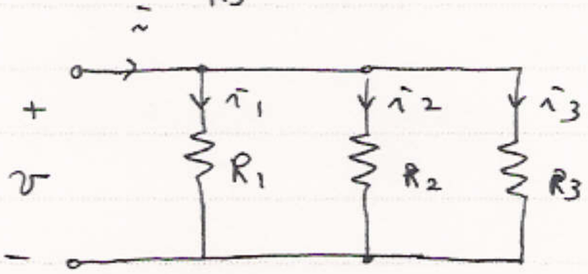
<Example 1.7> Explain

CH. 2 Resistive Circuits

§2.1 Series and parallel Circuit



$$v = iR_1 + iR_2 + iR_3$$
$$= i(R_1 + R_2 + R_3)$$
$$= i\,R_{eq}$$

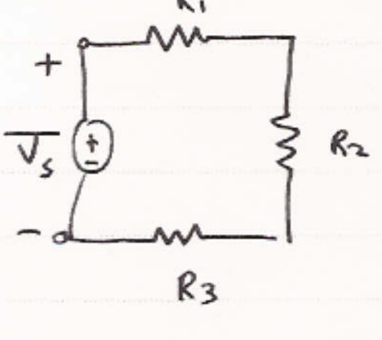


$$i = \cancel{\ } \, i_1 + i_2 + i_3$$
$$= \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3}$$
$$= v \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

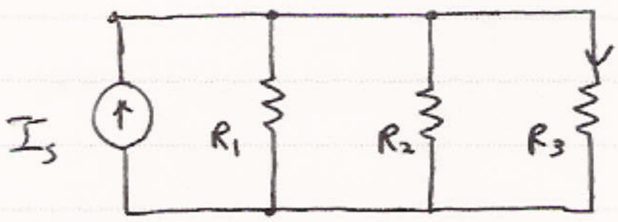
$$\frac{v}{i} = R_{eq} = \frac{1}{(1/R_1 + 1/R_2 + 1/R_3)}$$

<Example 2.2> Explain

§2.3 Voltage ~~division~~ divider and current divider



$$V_2 = \frac{R_2}{R_1 + R_2 + R_3} V_s$$

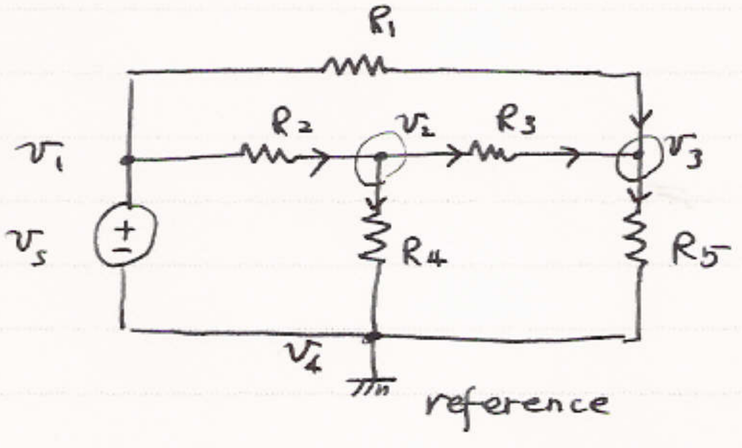


$$I_3 = \frac{\frac{1}{R_3}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)} \cdot I_s$$

§2.4 Node Voltage Analysis

1. Select a reference node ($v = 0$)
2. Assign node voltages as variables.
3. Apply KCL at each node. to get circuit equations
4. Solve for node voltages

Fig. 2.16



$v_1 = v_s$: known (given)

$v_4 = 0$: selected as 0 (reference)

$$\frac{(v_1 - v_2)}{R_2} = \frac{v_2}{R_4} + \frac{(v_2 - v_3)}{R_3} \qquad \text{—— ①}$$

$$\frac{(v_1 - v_3)}{R_1} + \frac{(v_2 - v_3)}{R_3} = \frac{v_3}{R_5} \qquad \text{—— ②}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\mathbb{A} \cdot \mathbb{v} = \mathbb{b}$$

Solve for $\mathbb{v} = \mathbb{A}^{-1} \cdot \mathbb{b}$

Note) 1) Reference node : $v_{ref} = 0$

2) Voltage source $\longrightarrow$ voltage will be known

3) Apply KCL for unknown node voltages.

4) # of unknown voltages = # of equations

Fig. 2.18 : Explain

Fig. 2.19 : Explain

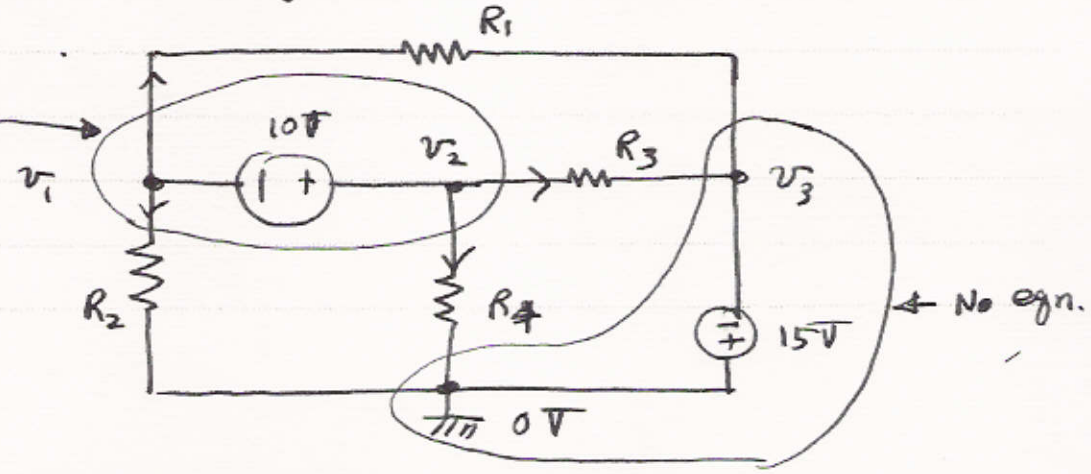
Fig. 2.20 : Explain

Fig. 2-23 : Explain

- Super node : Treat voltage source as a node
  Apply KCL at the supernode (area)

Fig. 2.24

KCL



$v_3 = -15$ V : Known

$v_2 = v_1 + 10$: constraint

KCL at Supernode :

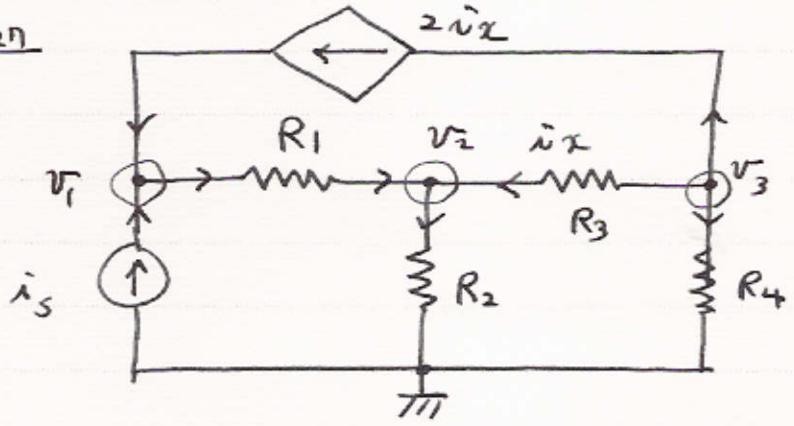
$$\frac{v_1}{R_2} + \frac{v_1 - (-15)}{R_1} + \frac{v_1 + 10}{R_4} = 0$$

Solve for v_1.

Note) (# of equations) = (# of nodes) − 1 − (# of VS)

− Circuit with dependent source

Fig. 2.27



$$i_s + 2i_x = \frac{v_1 - v_2}{R_1} \quad \text{— ①}$$

$$\frac{v_1 - v_2}{R_1} + \frac{v_3 - v_2}{R_3} = \frac{v_2}{R_2} \quad \text{— ②}$$

$$2i_x + \frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} = 0 \quad \text{— ③}$$

where, $i_x = \dfrac{v_3 - v_2}{R_3}$ — ④

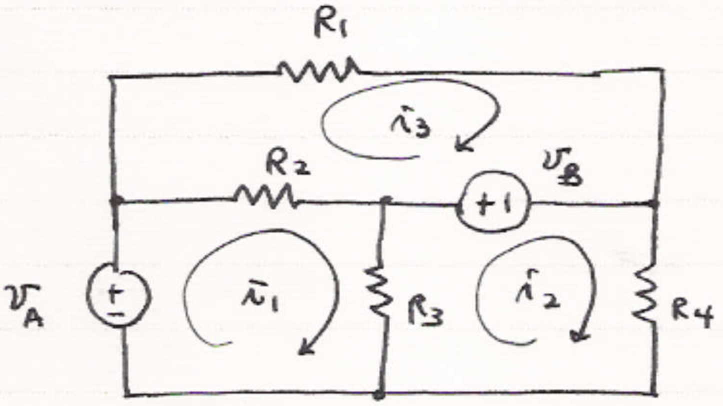
§2.5 Mesh Current Analysis

- Mesh : Elementary Loop.

- 1. Select meshes (loops)
  2. Assign mesh currents as variables
  3. Apply KVL for each mesh to get circuit eqns.
  4. Solve for the mesh current.

- (# of equations) = (# of meshes) − (# of CS).

- Fig. 2.32



$$v_A = R_2 (i_1 - i_3) + R_3 (i_1 - i_2)$$

$$0 = R_3 \cdot (i_2 - i_1) + v_B + R_4 \cdot i_2$$

$$0 = R_1 \cdot i_3 + v_B + R_2 (i_3 - i_1)$$

→ Solve for $\{i_1, i_2, i_3\}$

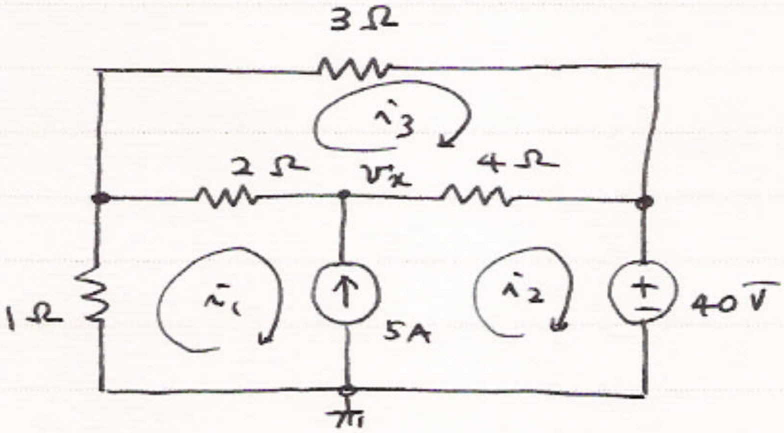
(f) - Compare to Node Analysis

- Note) Loop current and Branch current

Fig. 2.32 b) Explain.

- Super Mesh : Mesh by eliminating current source

Fig 2.36



1. $i_1 + 2(i_1 - i_3) + v_x = 0$ — ①

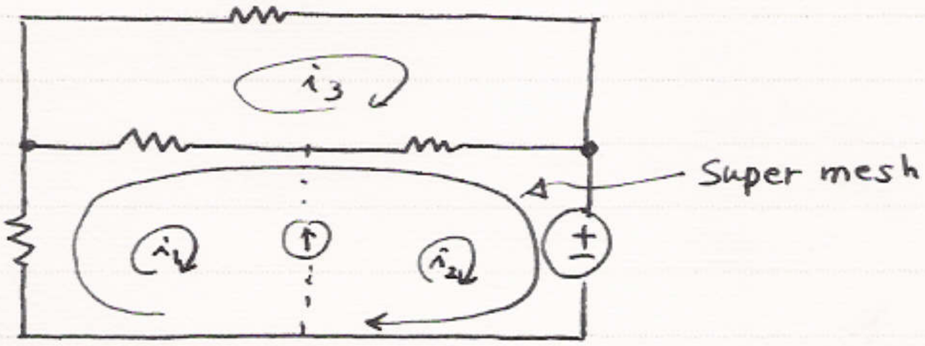
$-v_x + 4(i_2 - i_3) + 40 = 0$ — ②

$3 \cdot i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$ — ③

$i_2 - i_1 = 5$ — ④

Note) Dummy variable v_x is introduced.

After ~~By~~ Eliminating current source,



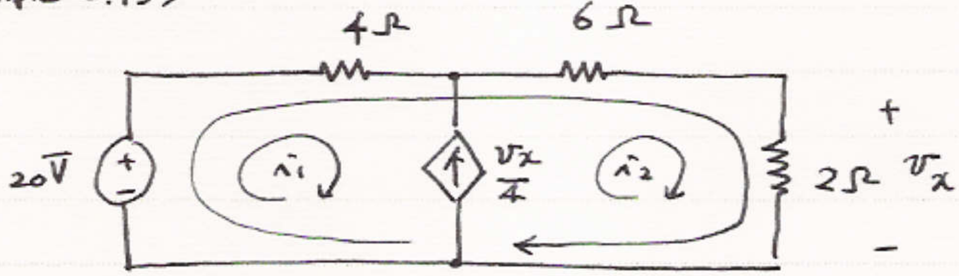
apply KVL along the super mesh.

Super Mesh : $1 \cdot i_1 + 2(i_1 - i_3) + 4(i_2 - i_3) + 40 = 0$ — ①

$3 \cdot i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$ — ②

$i_2 - i_1 = 5$: constraint

- <Example 2.13>

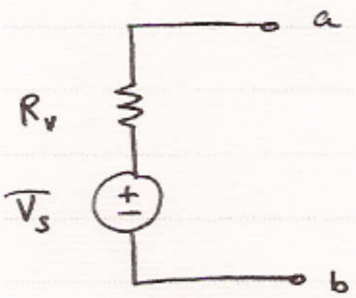


$20 = 4 \cdot i_1 + 6 \cdot i_2 + 2 \cdot i_2$ — ①

$i_2 - i_1 = \frac{1}{4} \cdot v_x = \frac{1}{4} \cdot (2 \cdot i_2)$ — ②

§2-6 Thevenin and Norton Equivalent Circuits

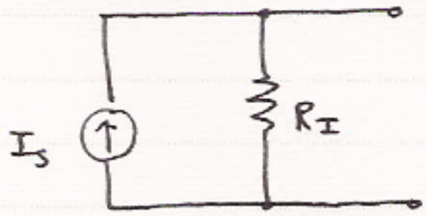
- Source Transformation



$V_{oc} = V_s$ — open circuit voltage

$I_{sc} = \dfrac{V_s}{R_s}$ — short circuit current

$\Updownarrow$



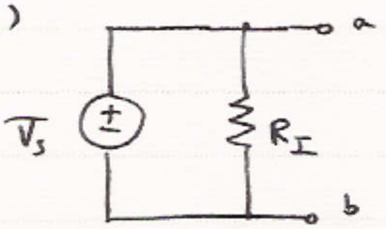
$V_{oc} = R_I \cdot I_s$

$I_{sc} = I_s$

If $I_s = \dfrac{V_{oc}}{R_V}$ and $R_V = R_I$,

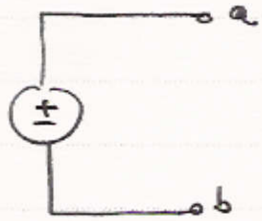
then two circuits are equivalent.

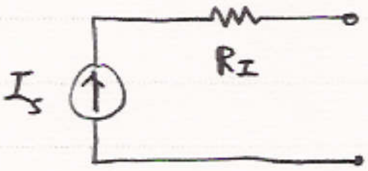
Note)



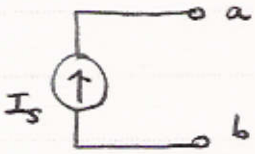
~~R_I in parallel~~

$\Leftrightarrow$

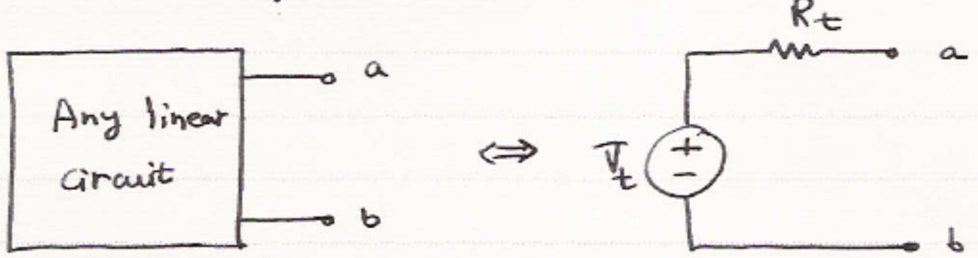




$\Leftrightarrow$



- Thevenin Equivalent Circuit



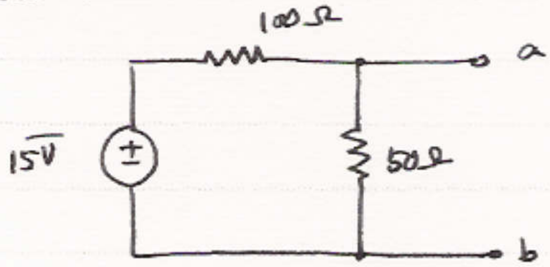
$V_t = V_{oc}$: Thevenin source voltage

R_t : internal resistance

$I_{sc} = \dfrac{V_t}{R_t}$: short circuit current

$\rightarrow\ R_t = \dfrac{V_{oc}}{I_{sc}} = \dfrac{V_t}{I_{sc}}$

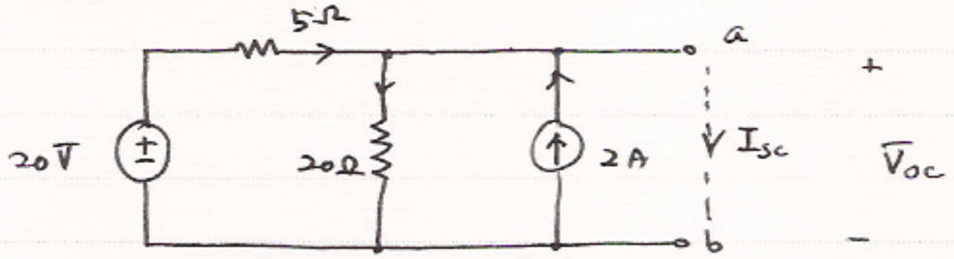
Example 2.14>



$V_{oc} = 5V$, $I_{sc} = 0.15\ A$

$R_t = \dfrac{5}{0.15} = 33.3\ \Omega$

<Example 2.15>



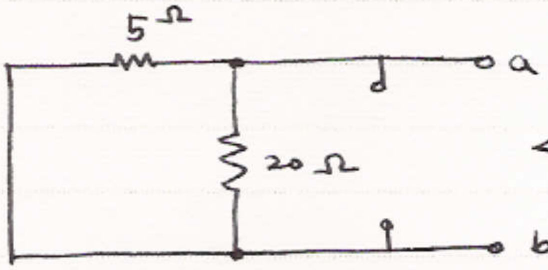
$$\frac{20 - V_{oc}}{5} + 2 = \frac{V_{oc}}{20} \longrightarrow V_{oc} = 24\ V$$

$$I_{sc} = \frac{20}{5} + 2 - \frac{0}{20} \longrightarrow I_{sc} = 6\ A$$

$$R_t = \frac{24}{6} = 4\ \Omega$$

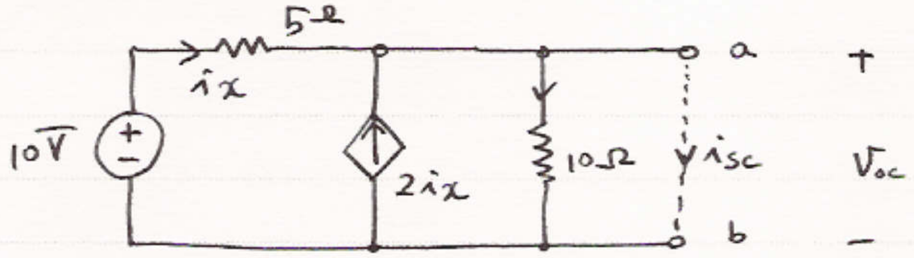
Note) Source elimination

- Short independent VS → super node
- Open independent CS → super mesh



$\leftarrow R_t = 5 \| 20 = 4\ \Omega$

<Example 2.16>

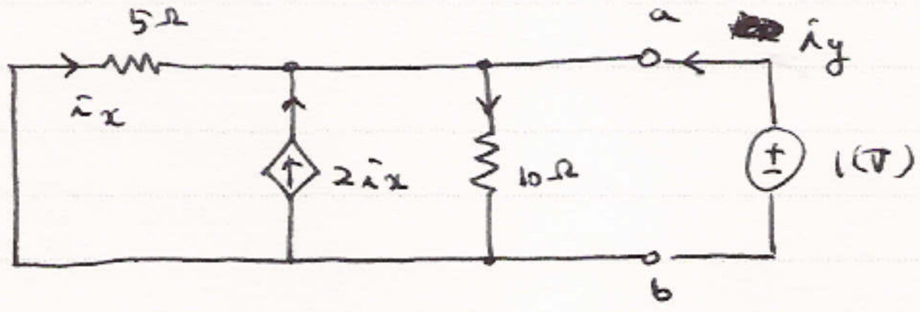


$$\frac{10 - V_{oc}}{5} + 2 \times \frac{10 - V_{oc}}{5} = \frac{V_{oc}}{10} \longrightarrow V_{oc} = 8.57\ (V)$$

$$\frac{10 - 0}{5} + 2 \times \frac{10}{5} = i_{sc} \longrightarrow i_{sc} = 6\ (A)$$

$$R_t = \frac{V_{oc}}{i_{sc}} = 1.43\ (\Omega)$$

By eliminating the voltage source, we get

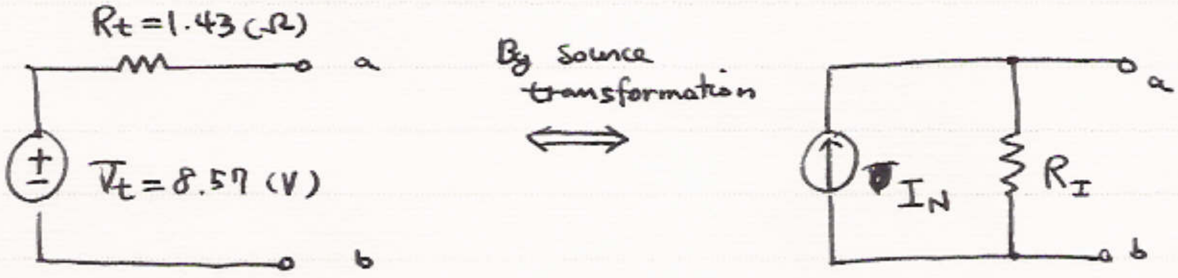


And assume a dummy VS is applied on a-b.

$$i_y = \frac{1}{10} - 3 \cdot i_x = \frac{1}{10} - 3 \times \left(\frac{-1}{5}\right) = \frac{7}{10}\ (A)$$

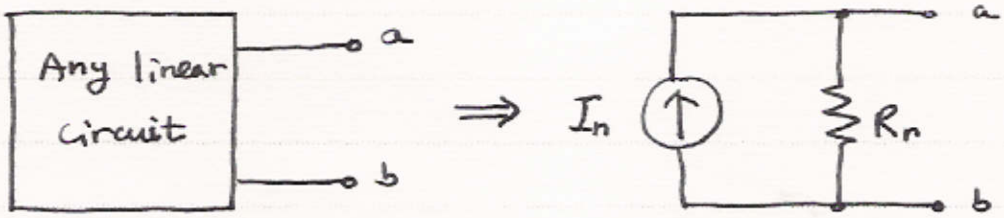
$$\rightarrow R_t = \frac{1\ (V)}{i_y\ (A)} = \frac{10}{7}\ (\Omega)$$

We get the Thevenin's circuit as



where $I_N = \dfrac{V_t}{R_t}$ and $R_I = R_t$.

- Norton's Equivalent Circuit

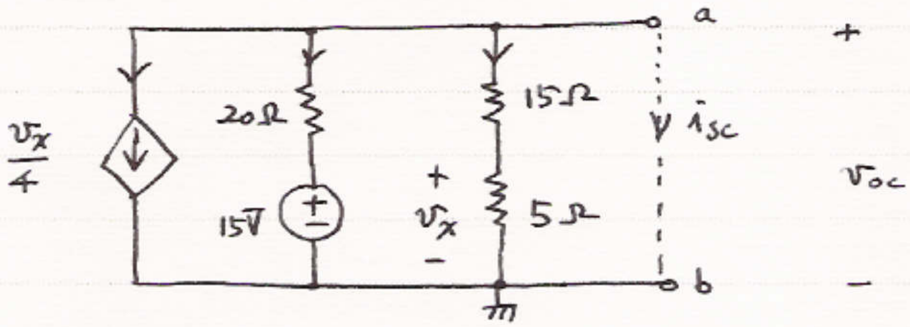


$$I_{sc} = I_n$$

$$V_{oc} = R_n \cdot I_n$$

$$R_n = \frac{V_{oc}}{I_{sc}}$$

<Example 2.17>



OPEN: $\frac{v_x}{4} + \frac{v_{oc}-15}{20} + \frac{v_{oc}}{20} = 0$ —— ①

where $v_x = \frac{1}{4} v_{oc}$ —— ②

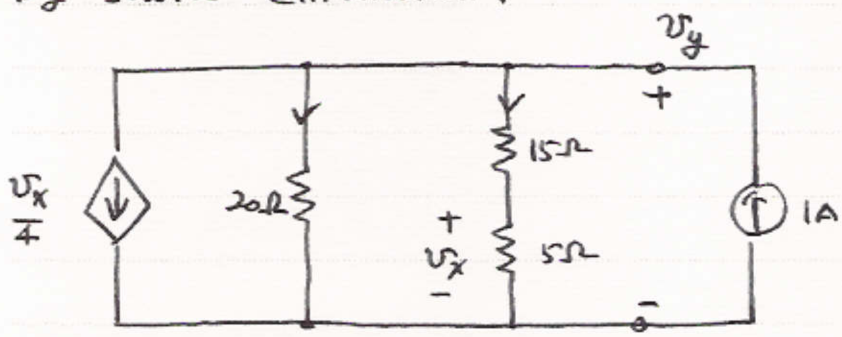
$\rightarrow v_{oc} = 4.62 \ (V)$

SHORT: $\frac{v_x}{4} + \frac{0-15}{20} + \frac{0}{20} + i_{sc} = 0$ —— ①

where $v_x = 0$ —— ②

$\rightarrow i_{sc} = 0.75 \ (A)$

By source elimination;

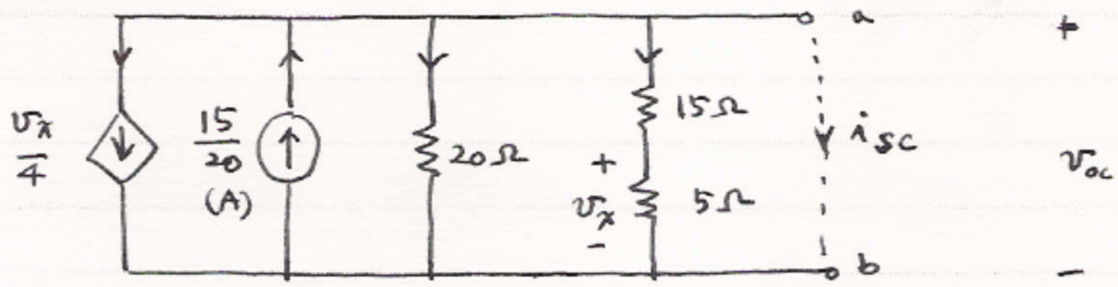


$$\frac{v_x}{4} + \frac{v_y}{20} + \frac{v_y}{20} = 1 \qquad \text{— ①}$$

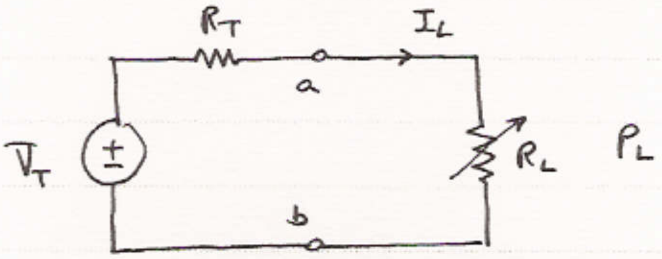
where $\quad v_x = \frac{1}{4} v_y \qquad$ — ②

$\rightarrow \quad v_y = 6.15 \quad (V)$

By source transformation :



– Maximum Power Transfer



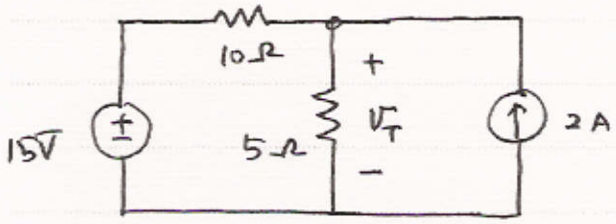
$$P_L = \left(\frac{V_T}{R_T + R_L} \right)^2 \cdot R_L$$

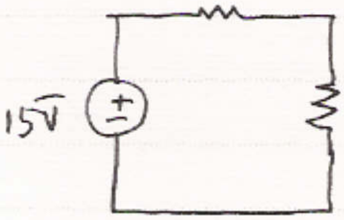
$$\frac{dP_L}{dR_L} = 0 \longrightarrow \underline{R_T = R_L}$$

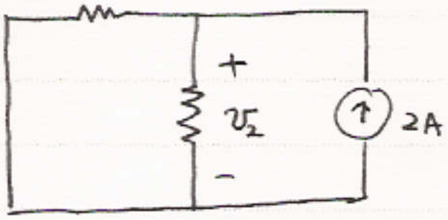
$$P_{L\,max} = \frac{V_T^2}{4R_T}$$

§2.11 Superposition Principle

<Example 2.20>

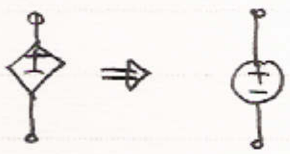
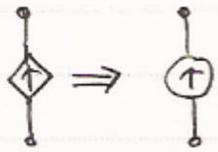






→ $v_T = v_1 + v_2$ by superposition (linearity)

– With dependent source :

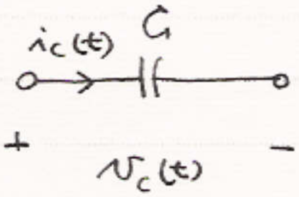
Replace  , 

then apply the superposition principle

<Fig. 2-62> Explain

CH.3 Inductance and Capacitance

§3-1 Capacitor



C (F)

$$q(t) = C \cdot v_c(t) \qquad : \text{charge}$$

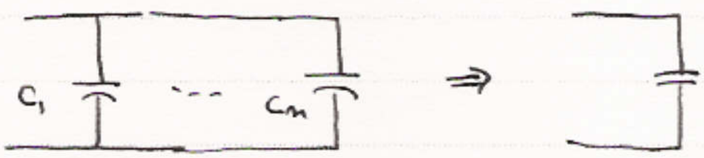
$$i_c(t) = \frac{dq(t)}{dt} = C \cdot \frac{dv_c(t)}{dt}$$

$$v_c(t) = v_c(0) + \frac{1}{C}\int_0^t i_c(\tau)\cdot d\tau$$

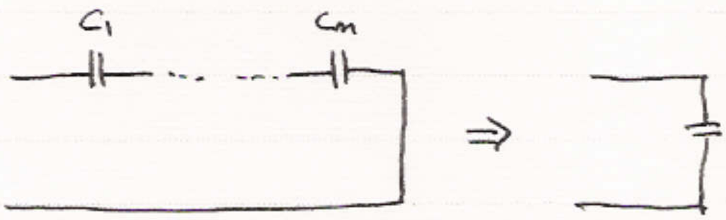
$$p_c(t) = v_c(t)\cdot i_c(t) = C\cdot v_c(t)\cdot \frac{dv_c(t)}{dt} \qquad (W)$$

$$w_c(t) = \int_{t_0}^{t} p(\tau)\cdot d\tau = \int_0^{v_c(t)} C\cdot v_c\, dv_c \qquad : \text{Stored energy}$$

$$= \frac{1}{2} C\cdot v_c^2(t) = \frac{1}{2} v_c(t)\cdot q(t) = \frac{1}{2}\frac{q^2(t)}{C} \qquad (J)$$

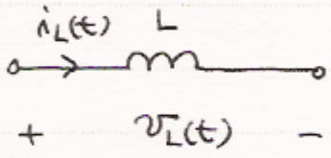


$$C_{parallel} = \Sigma C_k$$



$$C_{series} = \frac{1}{\frac{1}{C_1} + \cdots + \frac{1}{C_n}}$$

§3-4 Inductor



$$\lambda(t) = L \cdot i_L(t) \qquad : \text{Flux linkage}$$

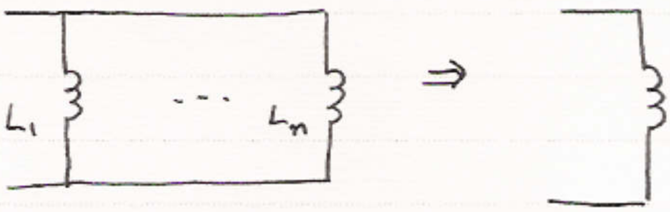
$$v_L(t) = \frac{d\lambda(t)}{dt} = L \cdot \frac{di_L(t)}{dt}$$

$$i_L(t) = i_L(0) + \frac{1}{L}\int_0^t v_L(\tau)\cdot d\tau$$

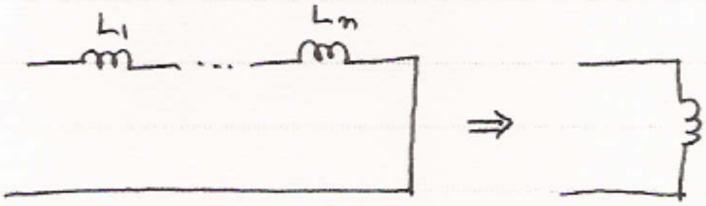
$$P_L(t) = v_L(t)\cdot i_L(t) = L \cdot i_L(t) \cdot \frac{di_L}{dt}$$

$$\omega_L(t) = \int_0^t P_L(\tau)\cdot d\tau = \int_0^{i_L(t)} L \cdot i_L \cdot di_L \qquad : \text{stored energy}$$

$$= \frac{1}{2} L \cdot i_L^2(t) = \frac{1}{2} i_L(t)\cdot\lambda(t) = \frac{1}{2}\frac{\lambda^2(t)}{L} \quad (J)$$

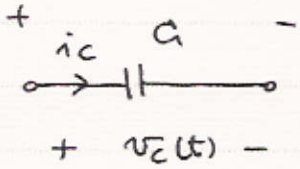


$$L_{parallel} = \frac{1}{\frac{1}{L_1} + \cdots + \frac{1}{L_n}}$$



$$L_{series} = L_1 + \cdots + L_n$$

Date : 22-1



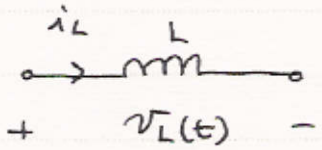
$$i_C(t) = C\,\frac{dv_C}{dt}$$

$v_C(t)$: continuous

$$v_C(0^-) = v_C(0^+)$$

open for DC

Take $v_C(t)$ as variable



$$v_L(t) = L \cdot \frac{di_L}{dt}$$

$i_L(t)$: continuous

$$i_L(0^-) = i_L(0^+)$$

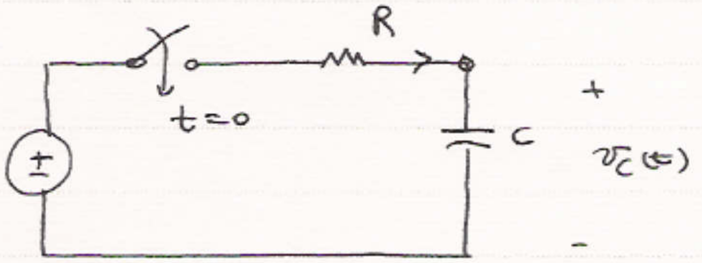
Short for DC

Take $i_L(t)$ as variable

CH. 4 Transients

§4-1 R–C Circuits

$V_s = v_s$



$t = 0^- : \quad v_c(0^-) = 0 = v_c(0^+)$

$t \geq 0^+ :$

$$v_s(t) = R \cdot i(t) + v_c(t)$$

$$i(t) = C \frac{dv_c}{dt}$$

$$v_s(t) = RC \cdot \frac{dv_c}{dt} + v_c(t)$$

$$\frac{dv_c}{dt} + \frac{1}{RC} \cdot v_c(t) = \frac{V_c}{RC}$$

$$s + \frac{1}{RC} = 0 \longrightarrow s = -\frac{1}{RC}$$

$$v_c(t) = A \cdot e^{-\frac{t}{RC}} + B$$

$t \to \infty$; steady state

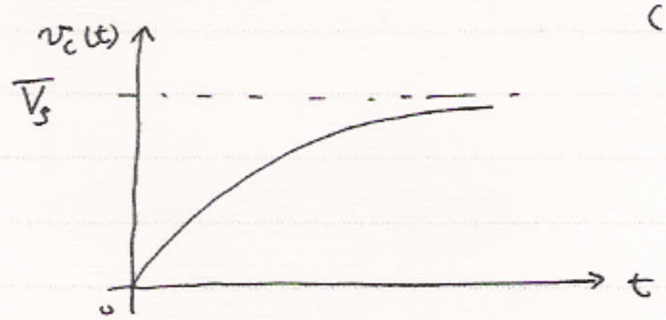
$$v_c(\infty) = B = V_s$$

$t = 0$: initial condition

$$v_c(0) = A + V_s = 0 \quad , \quad A = -V_s$$

$$\therefore \ v_c(t) = V_s \left[1 - e^{-\frac{1}{RC} t} \right] , \quad \text{for } t \geq 0$$

Where, $\tau = RC$: time constant
(speed of change)



cf) Using KCL,

$$\frac{v_s - v_c(t)}{R} = C \frac{dv_c}{dt}$$

$$v_s(t) = RC \frac{dv_c}{dt} + v_c(t)$$

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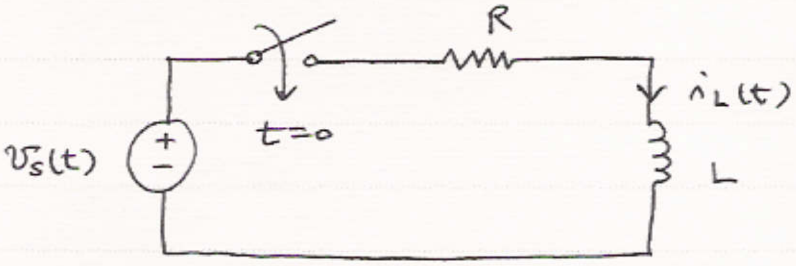
§4-3 R-L Circuits



$$t=0^- : \quad i_L(0^-) = 0 \quad = i_L(0^+)$$

$$t \geq 0^+ : \quad v_s(t) = R \cdot i_L(t) + L \cdot \frac{di_L}{dt}$$

$$\frac{di_L}{dt} + \frac{R}{L} \cdot i_L(t) = \frac{1}{L} V_s$$

$$s + \frac{R}{L} = 0 \longrightarrow s = -\frac{R}{L}$$

$$i_L(t) = A \cdot e^{-\frac{R}{L}t} + B$$

$$t \to \infty : \quad i_L(\infty) = B = \frac{V_s}{R}$$

$$t = 0^+ : \quad i_L(0) = A + \frac{V_s}{R} = 0$$

$$\therefore i_L(t) = \frac{V_s}{R}\left[1 - e^{-\frac{t}{(L/R)}}\right], \quad (t \geq 0)$$

where $\tau = \frac{L}{R}$: time constant

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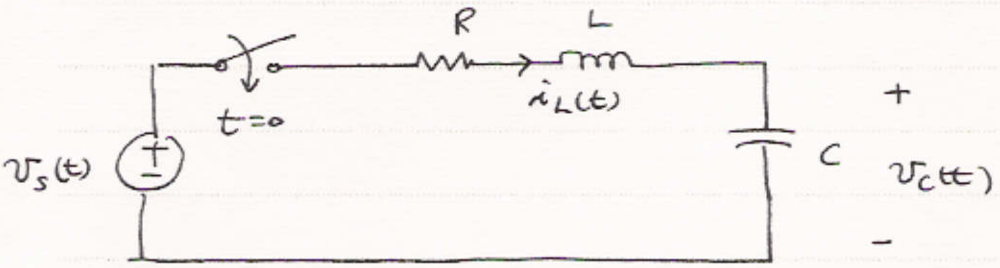
§4-5 Second Order Circuits



$t=0^-$: $i_L(0^-) = 0 \quad (A)$, $v_C(0^-) = V_{C0} \quad (V)$

$t \geq 0^+$: $v_s(t) = R \cdot i_L(t) + L \cdot \dfrac{di_L}{dt} + v_C(t)$ — ①

$i_L(t) = C \cdot \dfrac{dv_C}{dt}$ — ②

② → ① :

$$v_s(t) = RC \cdot \frac{dv_C}{dt} + LC \cdot \frac{d^2 v_c}{dt^2} + v_c(t)$$

$$\frac{d^2 v_c}{dt^2} + \frac{R}{L} \cdot \frac{dv_C}{dt} + \frac{1}{LC} \cdot v_C(t) = \frac{1}{LC} \cdot v_s(t) \quad — ③$$

② : $v_C'(0^-) = \dfrac{1}{C} i_L(0^-) = 0$

$$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0 \quad — ④$$

$s = s_1 , s_2$

$$v_C(t) = A \cdot e^{s_1 t} + B \cdot e^{s_2 t} + C_1 \quad — ⑤$$

- Solutions of 2nd order equation

$$\frac{d^2x(t)}{dt^2} + 2\alpha \frac{dx(t)}{dt} + \omega_o^2 \cdot x(t) = f(t)$$

$$x(t) = x_p(t) + x_c(t)$$

- Forced response :

$$\frac{d^2x_p}{dt^2} + 2\alpha \cdot \frac{dx_p}{dt} + \omega_o^2 \cdot x_p(t) = f(t)$$

- Complementary solution :

$$s^2 + 2\alpha s + \omega_o^2 = 0$$

Let $\zeta = \dfrac{\alpha}{\omega_o}$: damping ratio

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

1) $\zeta > 1$: $x_c(t) = K_1 \cdot e^{s_1 t} + K_2 \cdot e^{s_2 t}$ (over damped)

2) $\zeta = 1$: $x_c(t) = K_1 \cdot e^{s_1 t} + K_2 \cdot t \cdot e^{s_1 t}$ (critically damped)

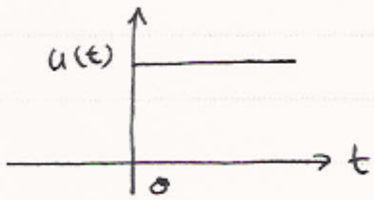
3) $\zeta < 1$: $x_c(t) = K_1 \cdot e^{-\alpha t} \cdot \cos(\omega_n t) + K_2 \cdot e^{-\alpha t} \cdot \sin(\omega_n t)$ (under damped)

where $S_1, S_2 = -\alpha \pm j\omega_n$

$$= -\alpha \pm j\sqrt{\omega_o^2 - \alpha^2}$$

– Unit step function:

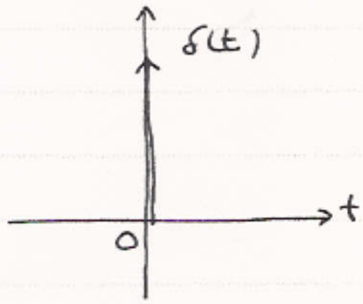
$$u(t) = \begin{cases} 1 \;, & (t \geq 0) \\ 0 \;, & (t < 0) \end{cases}$$



– Impulse function (delta function)

$$\frac{du(t)}{dt} = \delta(t)$$

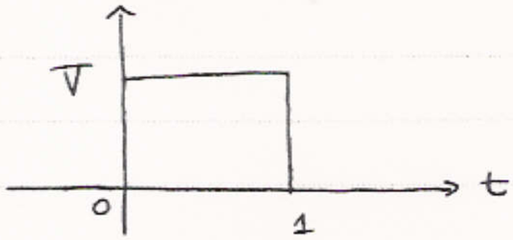
$$\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1$$



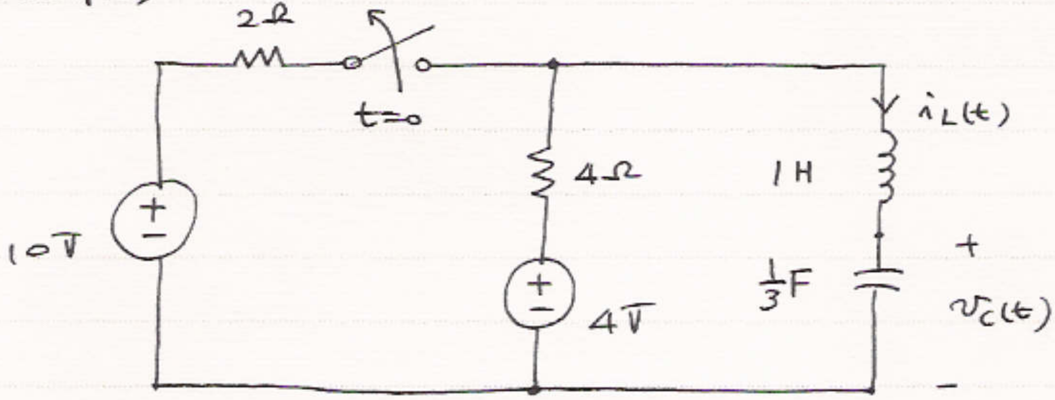
$$\int_{-\infty}^{\infty} f(t) \cdot \delta(t - t_0) \cdot dt = f(t_0)$$

– Rectangular function

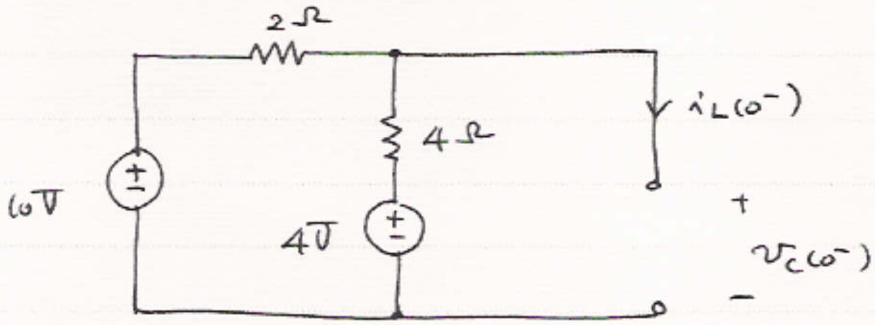
$$V \left[u(t) - u(t-1) \right]$$



Example)



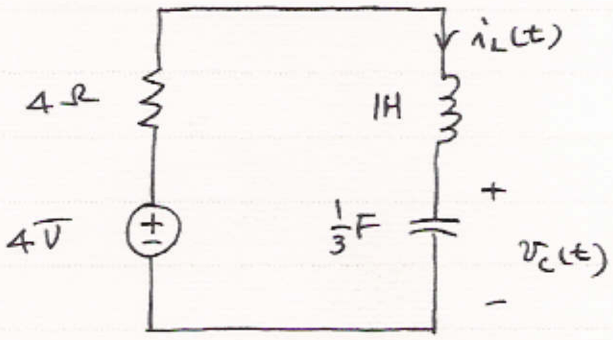
a) $t=0^-$: Initial condition



$i_L(0^-) = 0 \ (A) \quad = i_L(0^+)$

$v_C(0^-) = 8 \ (V) \quad = v_C(0^+)$

b) $t \geq 0^+$:



$$\begin{cases} 4 = 4 \cdot i_L(t) + 1 \cdot \dfrac{di_L}{dt} + v_C(t) & \text{— ①} \\ i_L(t) = \dfrac{1}{3} \dfrac{dv_C}{dt} & \text{— ②} \end{cases}$$

② → ① :

$$4 = \frac{4}{3} \frac{dv_C}{dt} + \frac{1}{3} \cdot \frac{d^2 v_C}{dt^2} + v_C(t)$$

$$\frac{d^2 v_C}{dt^2} + 4 \cdot \frac{dv_C}{dt} + 3 \cdot v_C(t) = 12 \quad \text{— ③}$$

$$s^2 + 4s + 3 = (s+3)(s+1) = 0 \quad \text{— ④}$$

$$\rightarrow v_C(t) = \left[A \cdot e^{-3t} + B \cdot e^{-t}\right] + C$$

$$0 + 4 \cdot 0 + 3 \cdot C = 12 \longrightarrow C = 4$$

$$v_C(t) = A \cdot e^{-3t} + B \cdot e^{-t} + 4$$

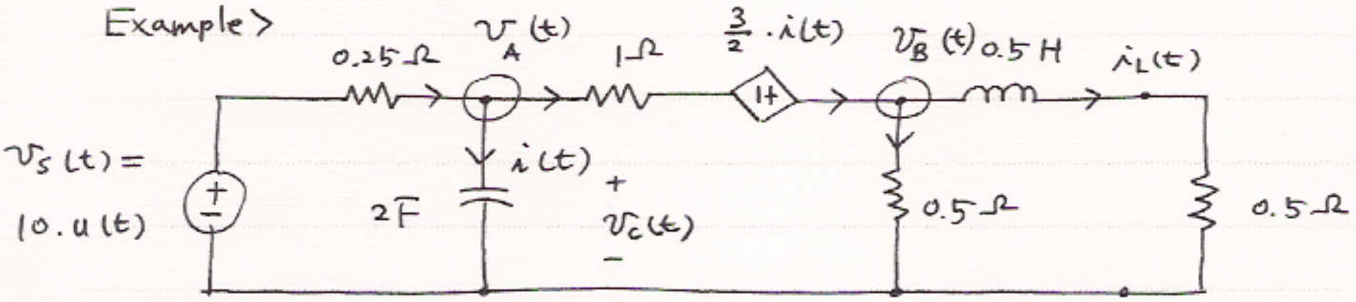
$$② : \ v_C'(0^-) = 3 \cdot i_L(0^-) = 0$$

$$v_C(0^+) = A + B + 4 = 8$$

$$v_C'(0^+) = -3A - B = 0$$

$$\therefore v_C(t) = -2 \cdot e^{-3t} + 6 \cdot e^{-t} + 4, \quad (t \geq 0)$$

Example>



a) $t = 0^-$; No active source

$$v_C(0^-) = 0 \quad (V) \quad = v_C(0^+)$$

$$i_L(0^-) = 0 \quad (A) \quad = i_L(0^+)$$

b) $t \geq 0^+$: Nodal Analysis

$$\begin{cases} v_A(t) = v_C(t) \\ v_B(t) = 0.5\,\frac{di_L}{dt} + 0.5\,i_L(t) \end{cases} \quad \text{and} \quad i(t) = 2 \cdot \frac{dv_C}{dt}$$

A: $$\frac{10 - v_C}{0.25} = 2 \cdot \frac{dv_C}{dt} + \frac{v_C - \left[\left\{0.5\,\frac{di_L}{dt} + 0.5\,i_L(t)\right\} - \frac{3}{2} \cdot 2\,\frac{dv_C}{dt}\right]}{1} \quad \text{—①}$$

B: $$\frac{v_C - \left[\left\{0.5\,\frac{di_L}{dt} + 0.5 \cdot i_L(t)\right\} - \frac{3}{2} \cdot 2\,\frac{dv_C}{dt}\right]}{1}$$

$$= \frac{0.5 \cdot \frac{di_L}{dt} + 0.5 \cdot i_L(t)}{0.5} + i_L(t) \quad \text{—②}$$

Arrange ① and ② as the state equation :

$$\begin{bmatrix} \dfrac{dv_c(t)}{dt} \\ \\ \dfrac{di_L(t)}{dt} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_c(t) \\ \\ i_L(t) \end{bmatrix} + \begin{bmatrix} u_1(t) \\ \\ u_2(t) \end{bmatrix}$$

Initial Conditions are obtained as

$$v_c'(0^+) = a_{11} \cdot v_c(0^+) + a_{12} \cdot i_L(0^+) + u_1(0^+)$$

$$i_L'(0^+) = a_{21} \cdot v_c(0^+) + a_{22} \cdot i_L(0^+) + u_2(0^+)$$

In general , the second order ODE is expressed as

$$\begin{cases} p \cdot x_1 = a_{11} \cdot x_1 + a_{12} \cdot x_2 + u_1(t) \\ \\ p \cdot x_2 = a_{21} \cdot x_1 + a_{22} \cdot x_2 + u_2(t) \end{cases}$$

where $p = \frac{d}{dt}$ operator.

To get a single variable differential equation ,

$$\begin{cases} (p - a_{11}) \cdot x_1 - a_{12} \cdot x_2 = u_1 \\ -a_{21} \cdot x_1 + (p - a_{22}) \cdot x_2 = u_2 \end{cases}$$

And eliminate x_2 and get

$$(p^2 - a_{11} \cdot p - a_{22} \cdot p + a_{11} \cdot a_{22}) \cdot x_1 - a_{12} \cdot a_{21} \cdot x_1$$
$$= (p - a_{22}) \cdot u_1 + a_{12} \cdot u_2$$

$$\frac{d^2 x_1}{dt^2} - (a_{11} + a_{22}) \cdot \frac{dx_1}{dt} + (a_{11} \cdot a_{22} - a_{12} \cdot a_{21}) \cdot x_1(t)$$
$$= \frac{du_1}{dt} - a_{22} \cdot u_1 + a_{12} \cdot u_2 \qquad \text{— ⓐ}$$

Similarly,

$$\frac{d^2 x_2}{dt^2} - (a_{11} + a_{22}) \cdot \frac{dx_2}{dt} + (a_{11} \cdot a_{22} - a_{12} \cdot a_{21}) \cdot x_2(t)$$
$$= \frac{du_2}{dt} - a_{11} \cdot u_1 + a_{21} \cdot u_2 \qquad \text{— ⓑ}$$

CH. 10 Diodes

§ Semiconductors

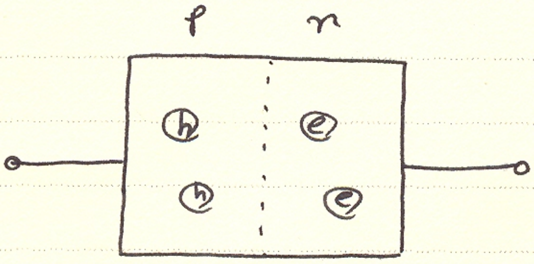
- Silicon : covalent bond
- Carriers { electrons (−) / holes (+)
- p-type { majority carriers — holes / minority carriers — electrons

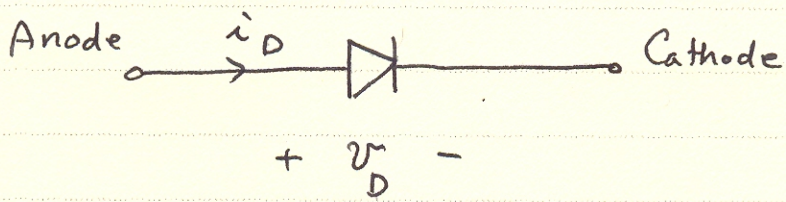
  n-type { majority carriers — electrons / minority carriers — holes
- Doner — Phosphorus : donates electrons : n-type

  Acceptor — Boron : accepts electrons : p-type

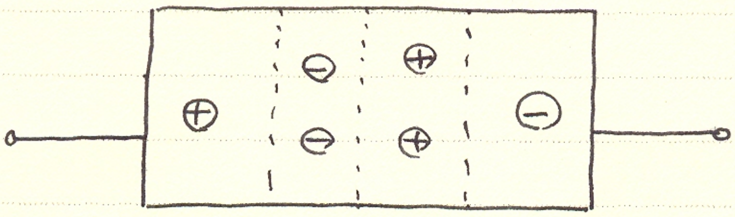
§ Diodes

- p–n junction



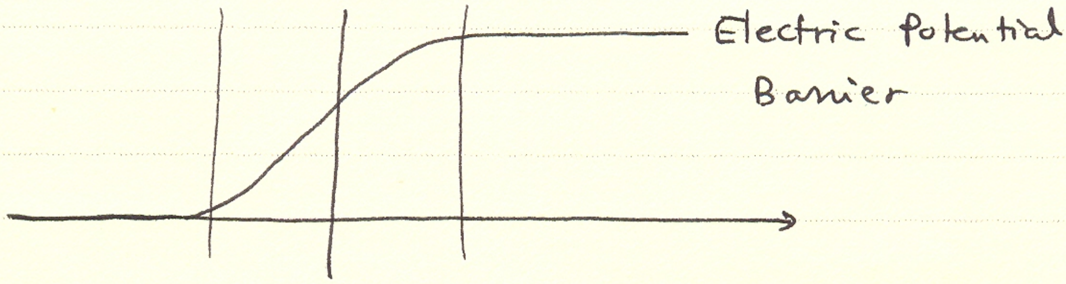


- Unbiased p–n junction

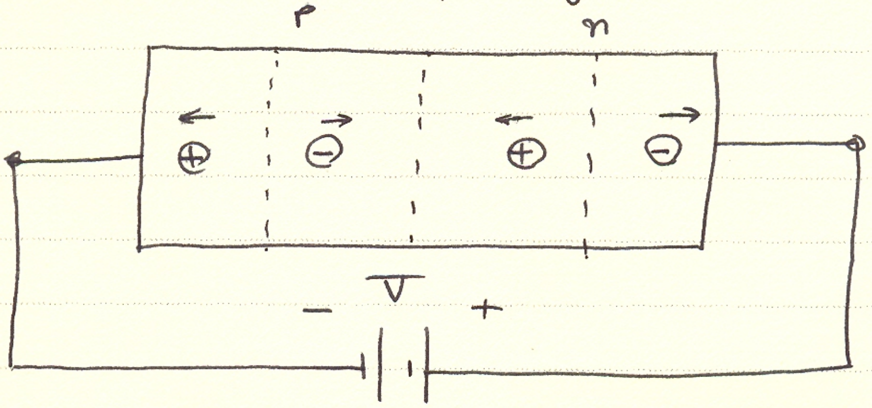


$\rightarrow$ holes
$\leftarrow$ electrons
} by diffusion

depletion region

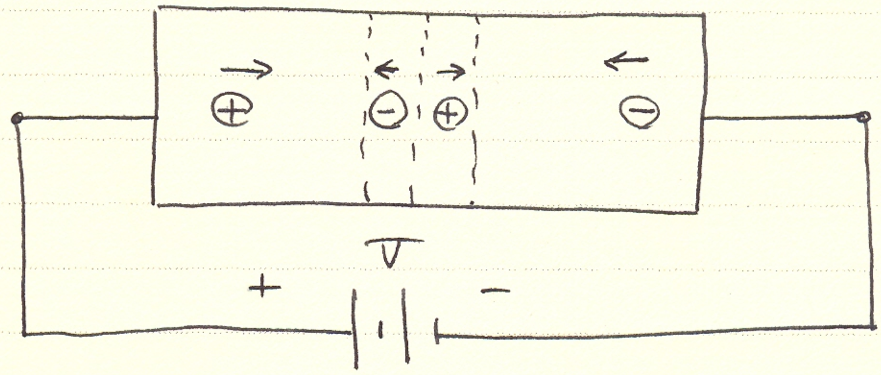


- Reverse biased pn - junction



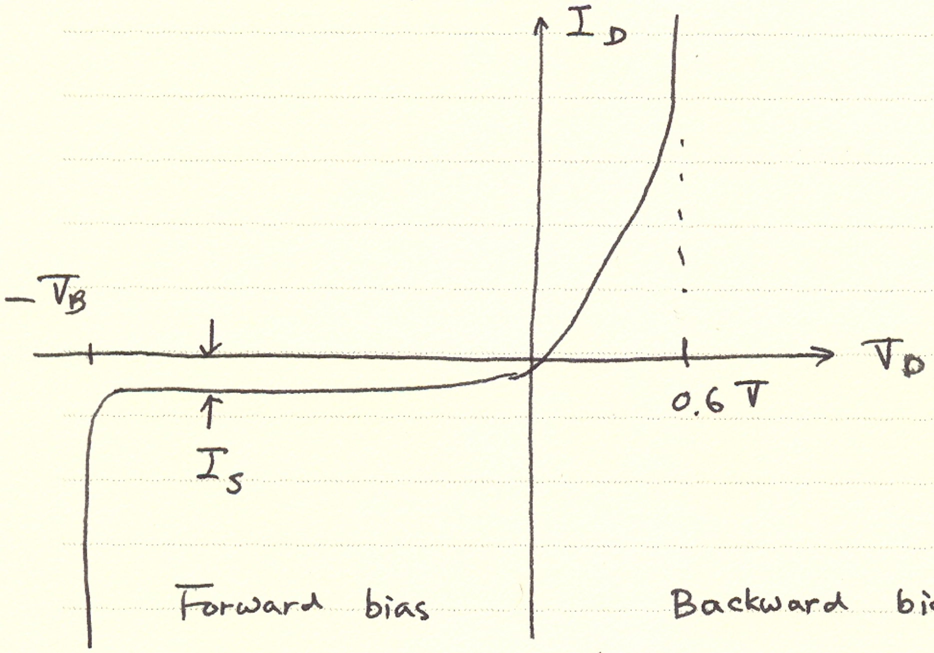
{ depletion region increases
potential barrier increase by V — no more current

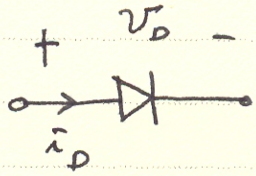
- Forward biased p-n junction



{ depletion region ~~decreases~~ neutralized
Potential barrier decreases → Current flows

• shockley Equation





$$\boxed{i_D = I_S \cdot \left[\exp\left(\frac{v_D}{n V_T} \right) - 1 \right]}$$ Remember!

I_S : saturation current ($\sim 10^{-7}$ A)

n : emission coefficient (~ 1 for Si)

$V_T = \frac{kT}{q} = \frac{T}{11600}$: Thermal voltage

$k = 1.38 \times 10^{-23}$: Boltzmann constant

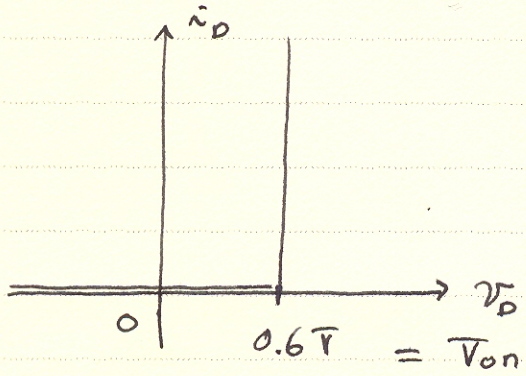
$q = 1.60 \times 10^{-9}$: electron change

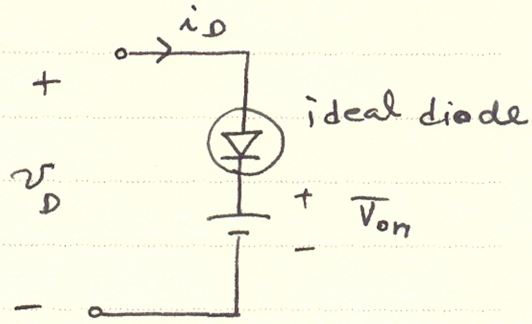
V_B : Breakdown voltage

- At room temperature : $T = 293°K$

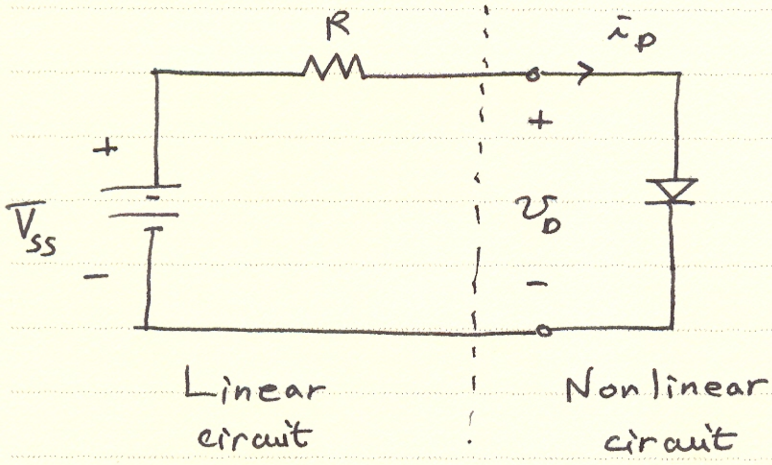
$$i_D \cong \begin{cases} I_s \cdot e^{40 v_D} & , (v_D > 0.1\,V) \\ -I_s & , (v_D < -0.1\,V) \end{cases}$$

- By approximation,





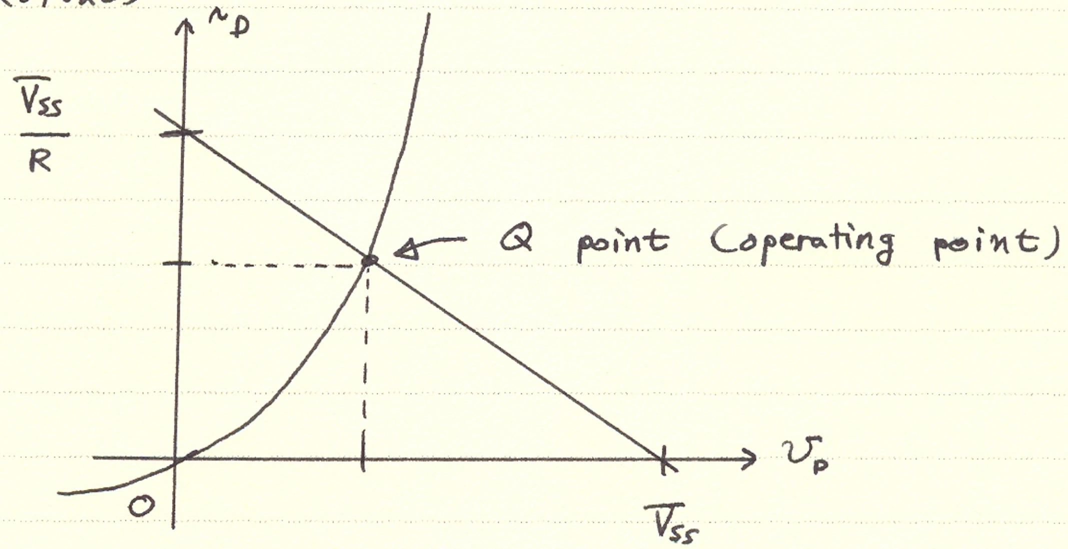
§ 10-2. Load Line Analysis : Graphical Analysis



Linear : $v_D = V_{SS} - R \cdot i_D$

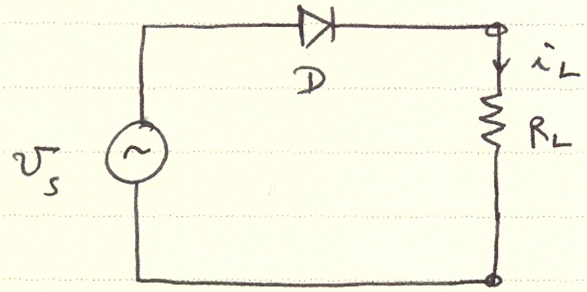
$$i_D = \frac{V_{SS}}{R} - \frac{v_D}{R}$$

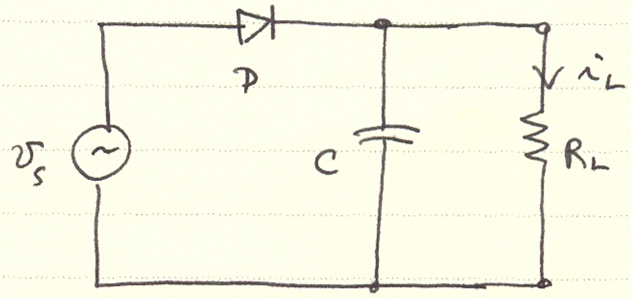
Nonlinear : (Diode) $i_D = I_S \cdot \left[\exp\left(\frac{v_D}{n V_T} \right) - 1 \right]$



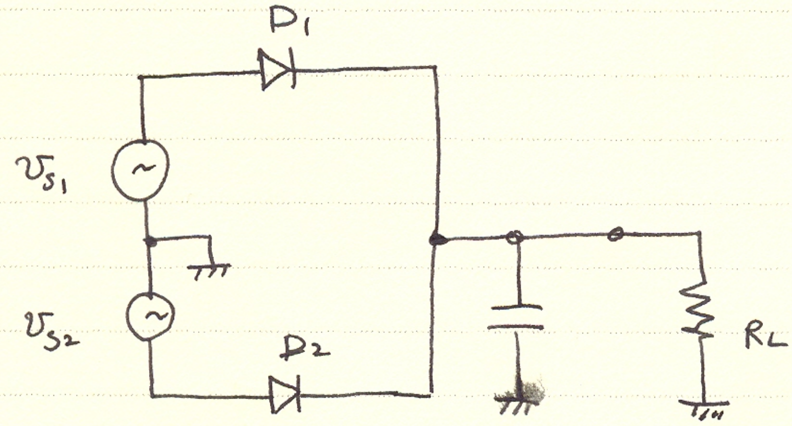
§10-6. Rectifier Circuits

∘ Half wave rectifier

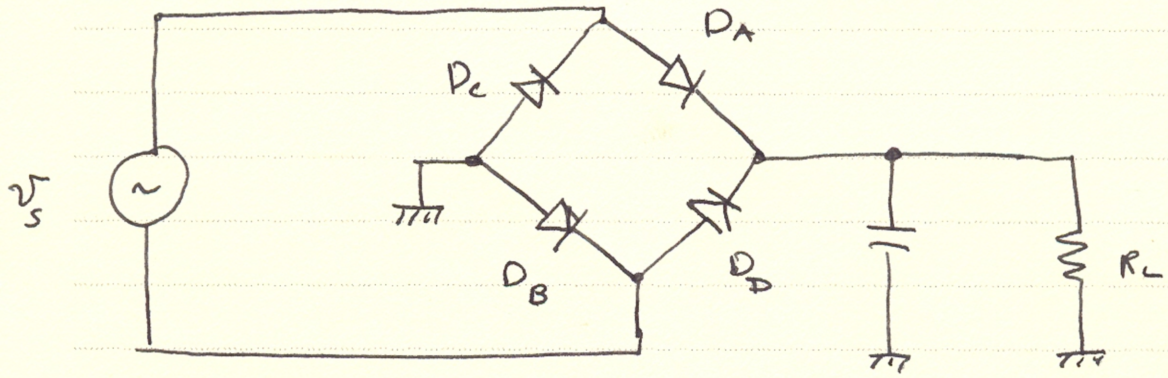




∘ Full wave rectifier

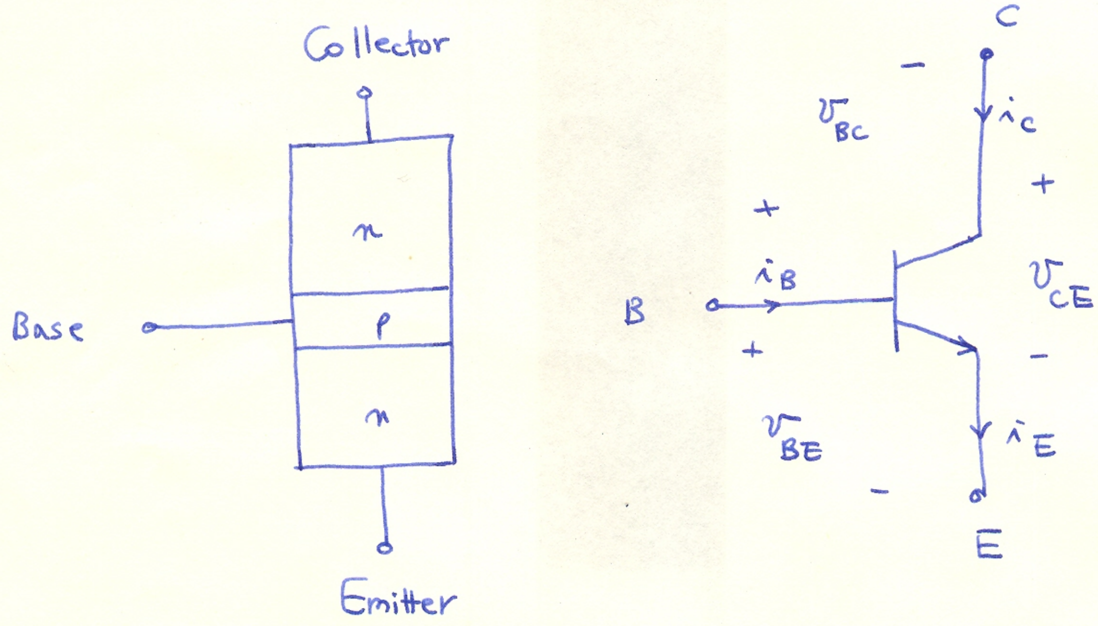


∘ Diode Bridge



§7-3. Bipolar Junction Transistor (BJT)

- Current and Voltage relationship.



- Bias $\begin{cases} \text{base} - \text{collector} : \text{reverse bias} \\ \text{base} - \text{emitter} : \text{forward bias} \end{cases}$

- Emitter current

$$i_E = I_{SE} \cdot \left[\exp\left(\frac{v_{BE}}{V_T} \right) - 1 \right] \quad \leftarrow \text{forward biased diode}$$

$$i_E = i_B + i_C$$

- Define α

$$\alpha = \frac{i_C}{i_E} \qquad (= 0.9 \sim 0.999)$$

- Collector current

$$i_C = \alpha \cdot i_E = \alpha \cdot I_{SE} \cdot \left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1 \right]$$

$$\approx I_S \cdot \exp\left(\frac{v_{BE}}{V_T}\right)$$

where $I_S = \alpha \cdot I_{SE}$

- Base current

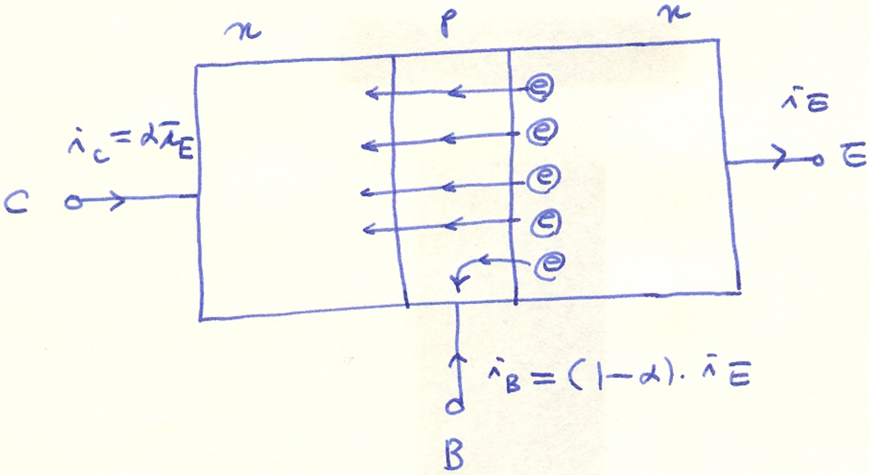
$$i_B = (1-\alpha) \cdot i_E = (1-\alpha) \cdot I_{SE} \cdot \left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1 \right]$$

- Define β

$$\beta = \frac{i_C}{i_B} = \frac{\alpha}{1-\alpha} \qquad (= 10 \sim 1000)$$

$\rightarrow \quad i_C = \beta \cdot i_B$

- Principle of BJT



v_{BE} – forward bias

→ electrons are injected into base

→ base is thin

→ most of the electrons (α) are diffused into collector

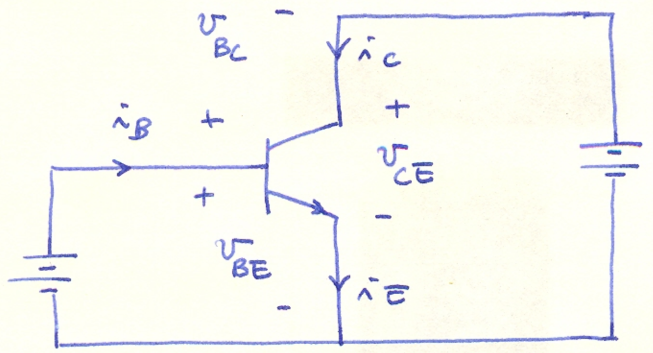
→ only $(1-\alpha)$ electrons reach base.

ratio α is constant.

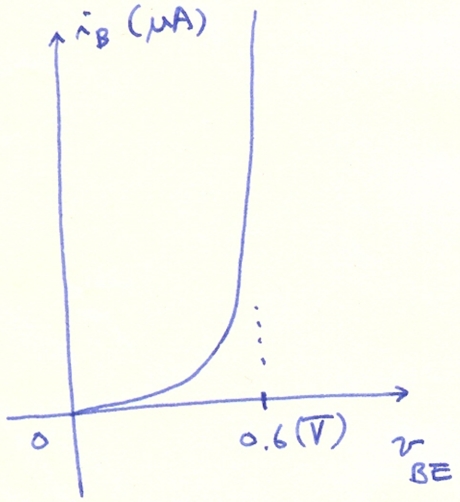
$$\rightarrow i_C = \frac{\alpha}{1-\alpha} \cdot i_B = \beta \cdot i_B$$

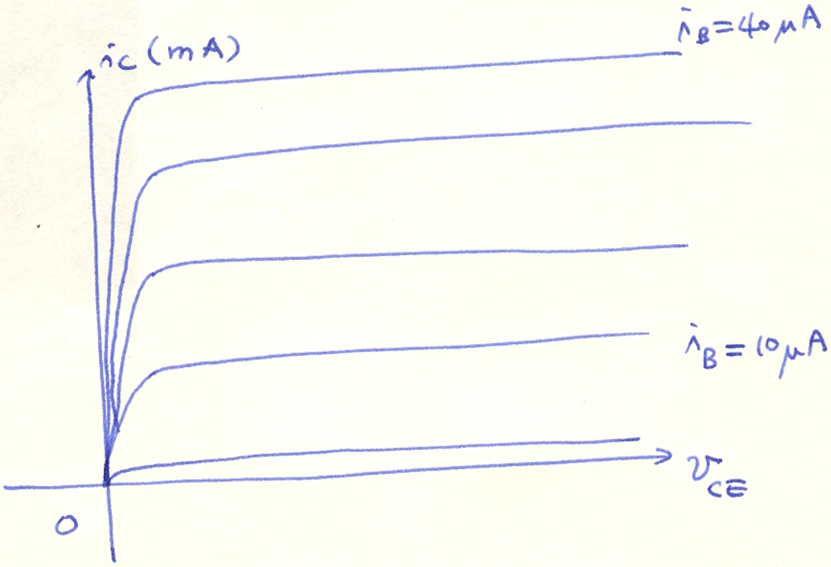
⇒ Collector current i_C can be controlled by small i_B.
Large

§ Common Emitter characteristics



• $v-i$ characteristics



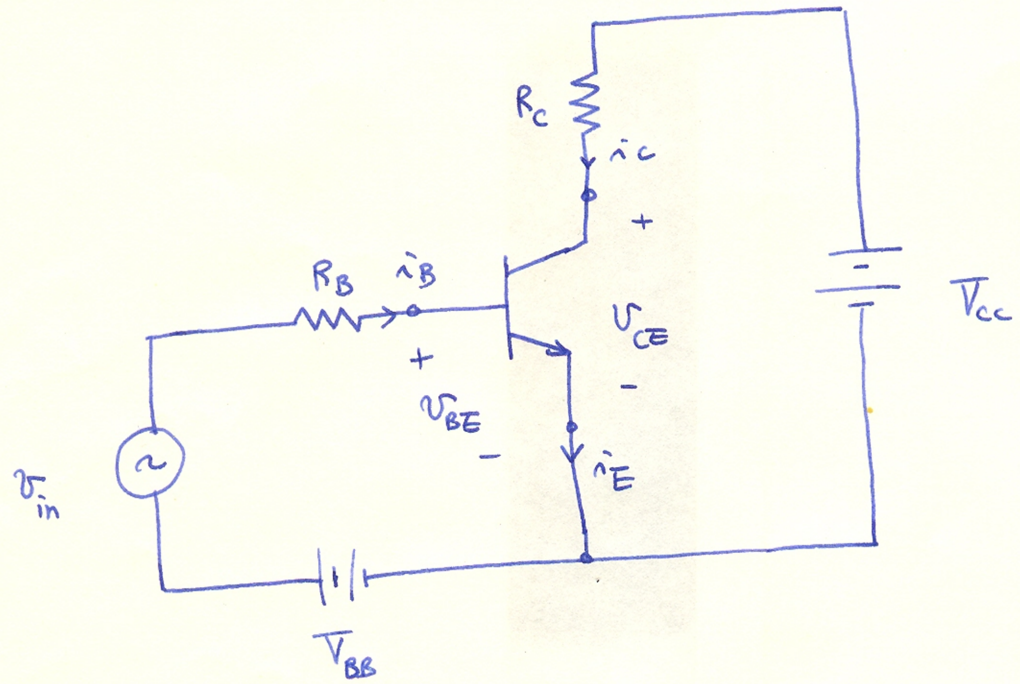


i_C is constant regardless v_{CE}.

$i_C = \beta \cdot i_B$

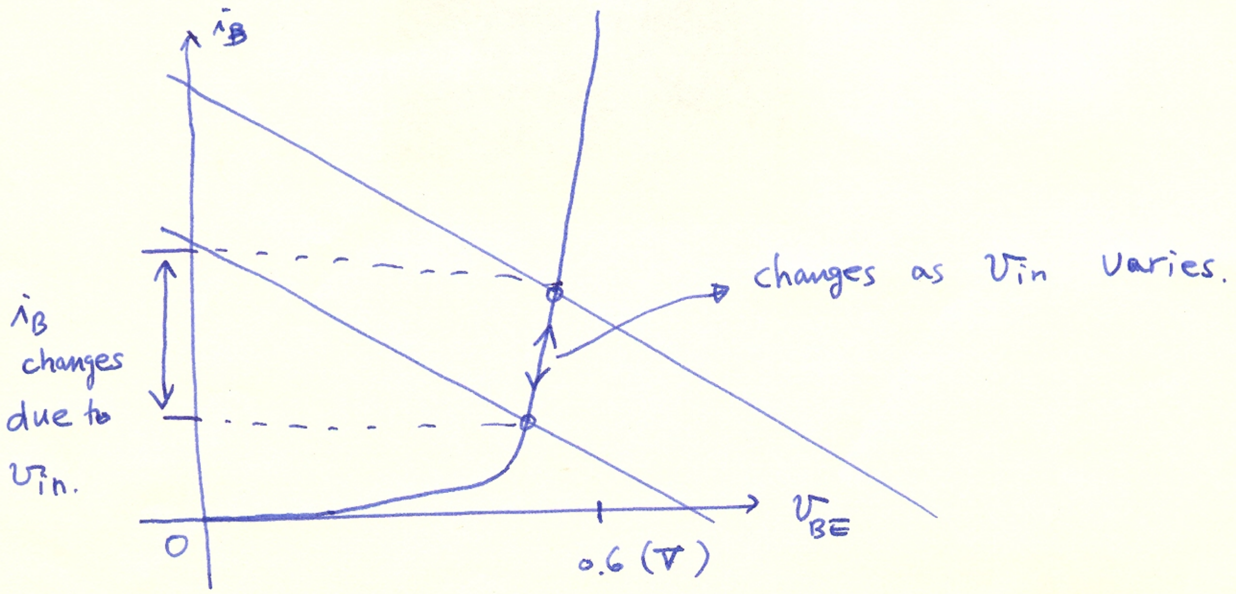
§ Load Line Analysis

. Common Emitter Amplifier.

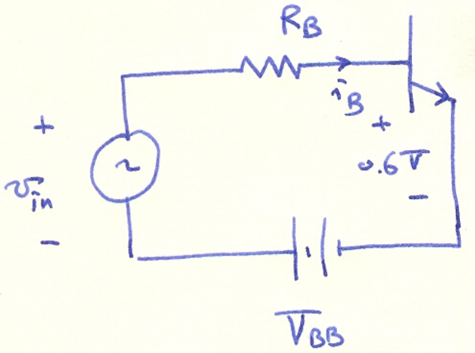


$V_{BB} + v_{in} = R_B \cdot i_B + v_{BE}$: input circuit

$V_{cc} = R_c \cdot i_c + v_{CE}$: output circuit

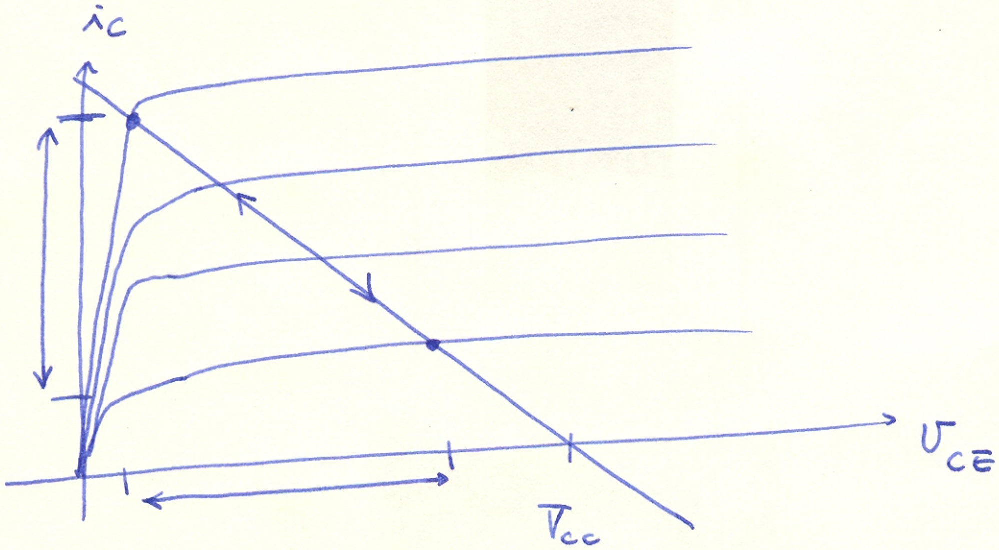


- Input circuit analysis : i_B vs. v_{BE}



$$i_B \cong \frac{v_{in} + V_{BB} - 0.6}{R_B}$$

- output circuit analysis : i_c vs. v_{CE}

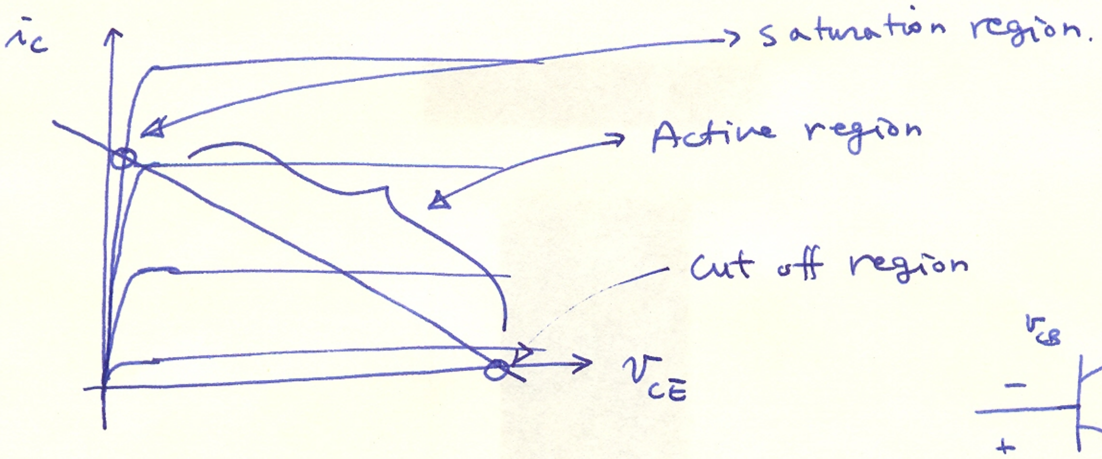


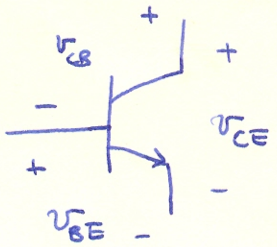
v_{in} changes i_B.

i_B changes i_c. and v_{CE}.

$v_{in} \longrightarrow v_{CE}$.

§ BJT Operating regions





- Active region.

$$i_C = \beta \cdot i_B$$

B−E : forward bias , $v_{BE} = 0.6\,V$
C−B : reverse bias

- Cut off region

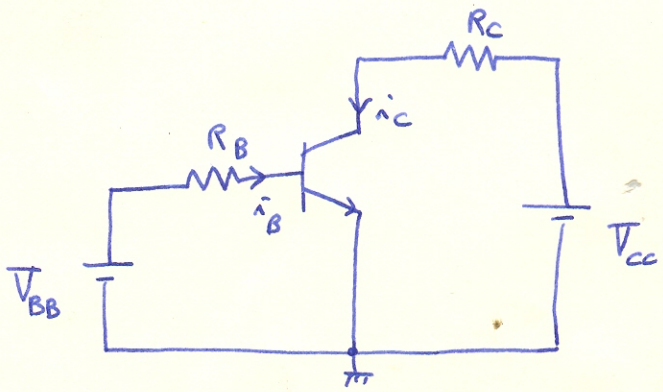
$$i_C = 0$$

B−E : reverse bias $\longrightarrow$ $i_B = 0$
C−B : reverse bias

- Saturated region

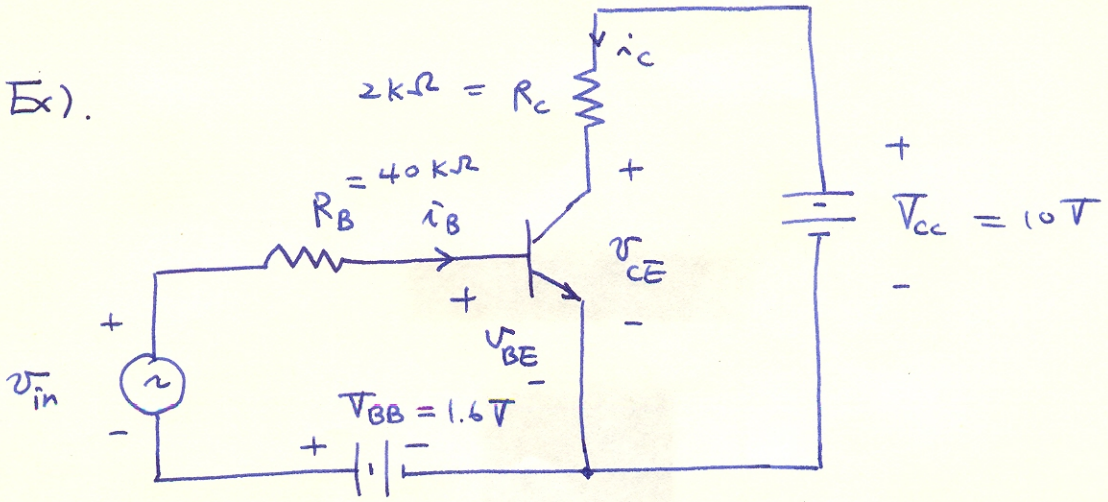
$$i_C < \beta \cdot i_B$$

B−E : forward bias
$v_{CE} \cong 0.2\,V$
C−B : forward bias

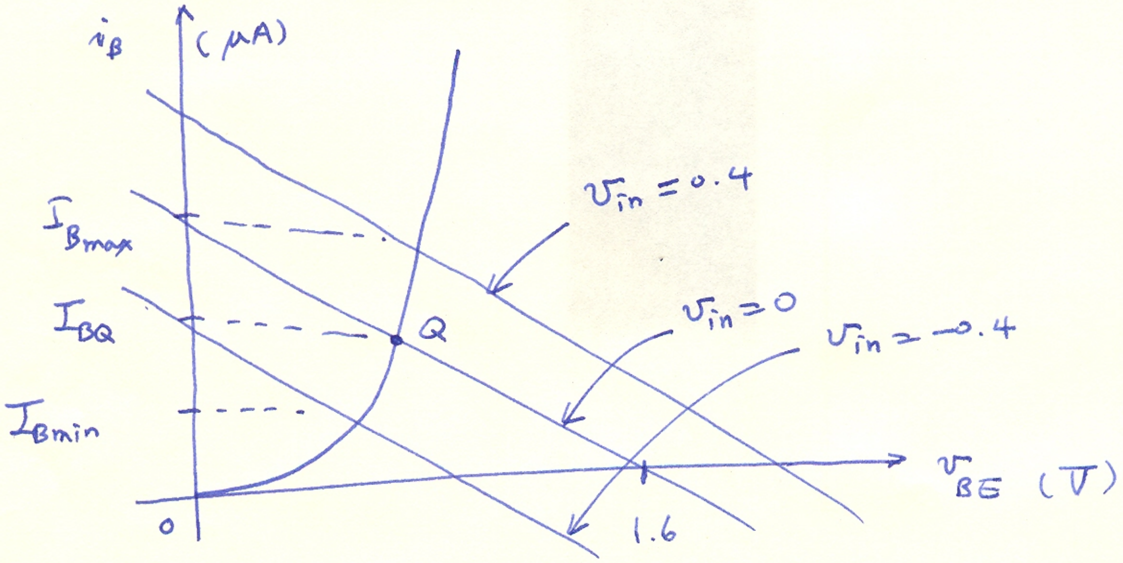


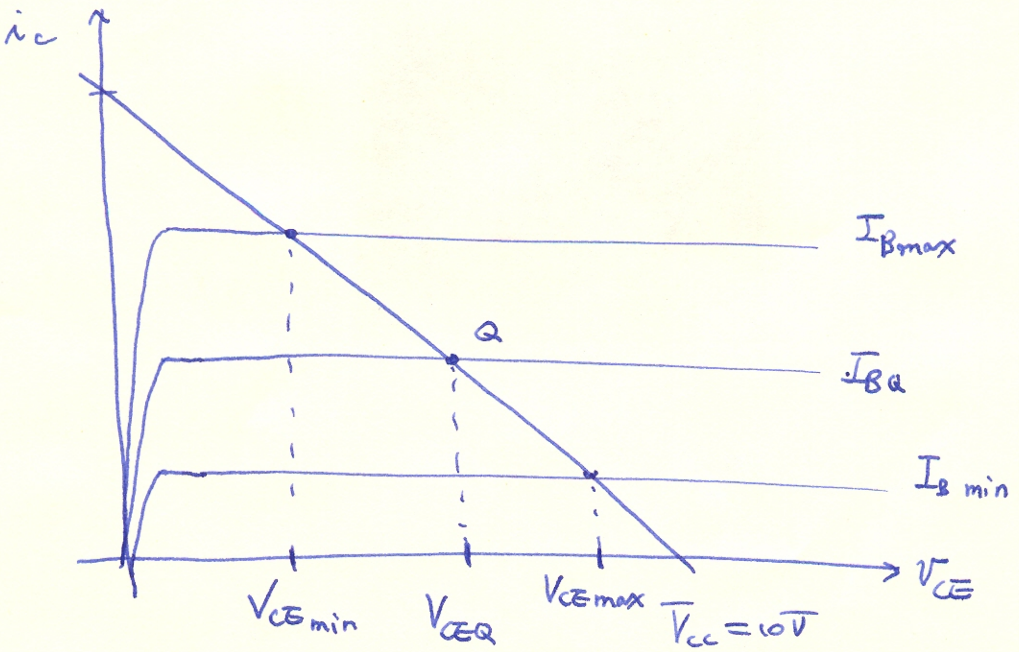
As i_B increases, $R_C \cdot i_C \to V_{CC}$.

Ex).



$$v_{in}(t) = 0.4 \cdot \sin(2000\pi t)$$

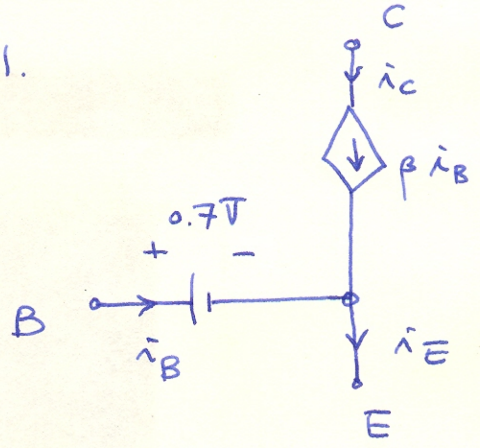




§ Large Signal DC model. (Bias Analysis)

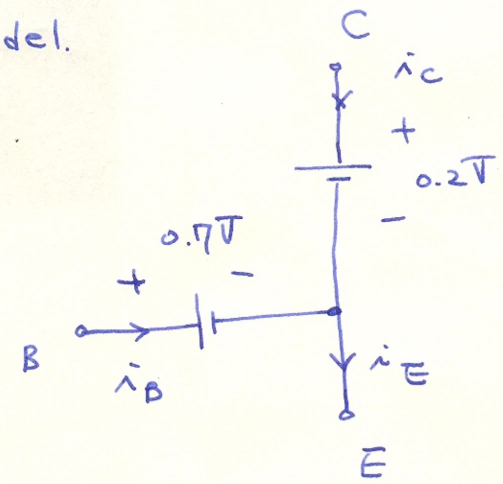
- Active region model.

$$\begin{cases} i_c = \beta \cdot i_B \\ v_{BE} = 0.7\ V \\ v_{CE} > 0.2\ V \end{cases}$$



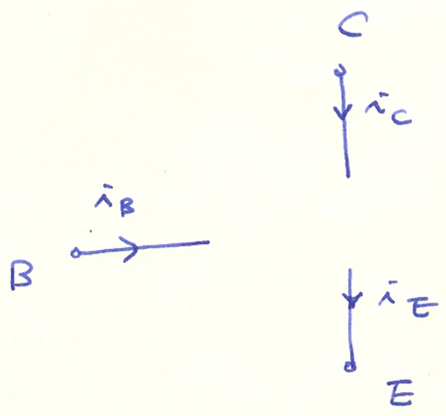
- Saturation region model.

$$\begin{cases} v_{BE} = 0.7\ V \\ v_{CE} \simeq 0.2\ V \\ 0 < i_c < \beta i_B \end{cases}$$



- Cutoff region model

$$\begin{cases} v_{BE} < 0.5\ V \\ v_{BC} < 0.5\ V \\ i_B \approx 0 \\ i_c \approx 0 \end{cases}$$



Ex.) $\beta = 100$

a) $I_B = 50\,\mu A$, $I_C = 3\,mA$

$i_C = 3\,mA < \beta \cdot i_B \longrightarrow$ saturation region.

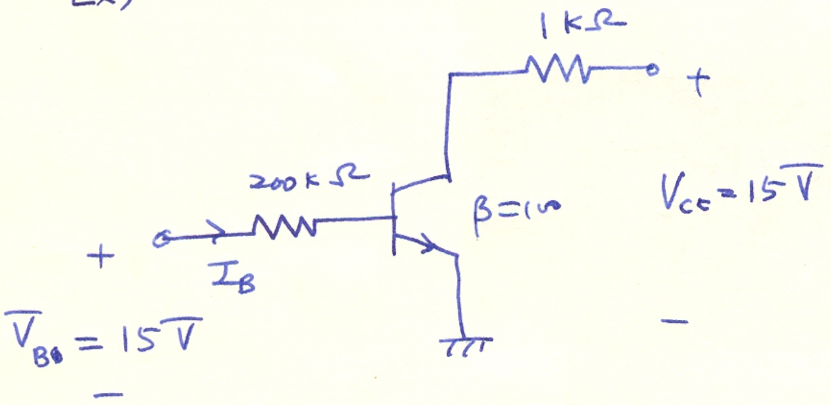
b) $i_B = 50\,\mu A$, $V_{CE} = 5\,V > 0.2\,V \longrightarrow$ active

c) $V_{BE} = -2\,V$, $V_{CE} = -1\,V \longrightarrow$ cut off.

§ Large Signal DC Analysis

- Bias Analysis (Q point
  operation mode

Ex)



$I_C = \beta I_B = 7.15\,mA$

$V_{CE} = 15 - 1000 \times 0.00715$

$= 7.85 > 0.2\,V$

$\rightarrow$ Active region.

Assume active region.

$$I_B = \frac{15 - 0.7}{200{,}000} = 71.5\,\mu A$$

Ex) $\beta = 300$

Assume active region.

$I_B = 71.5 \ \mu A$

$I_C = 21.45 \ mA$

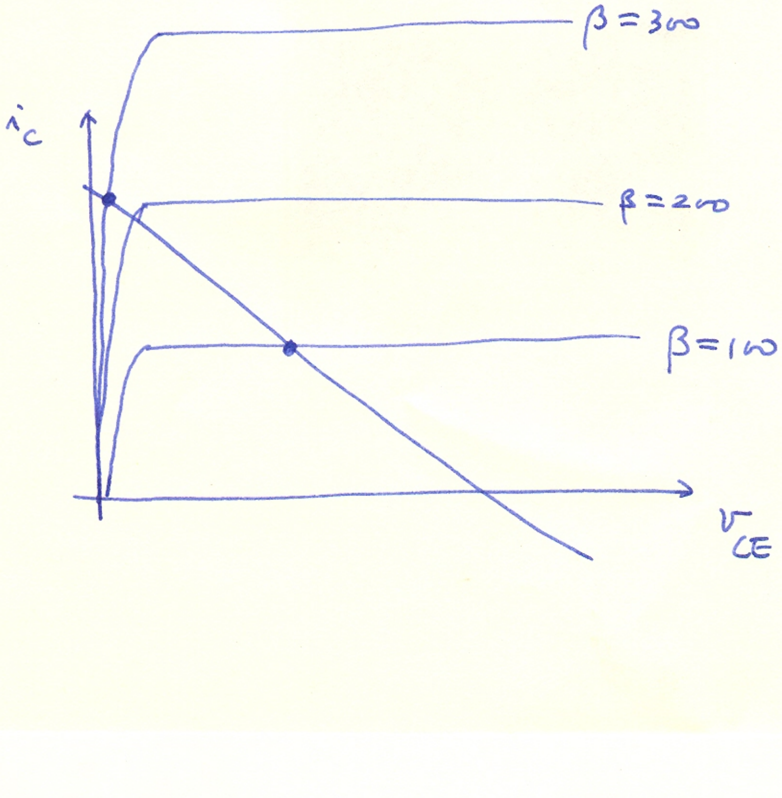
$V_{CE} = V_{CC} - R_C \cdot I_C = -6.45 < 0.2V \rightarrow$ saturation!

Assume saturation region.

$V_{CE} = 0.2 \ V$

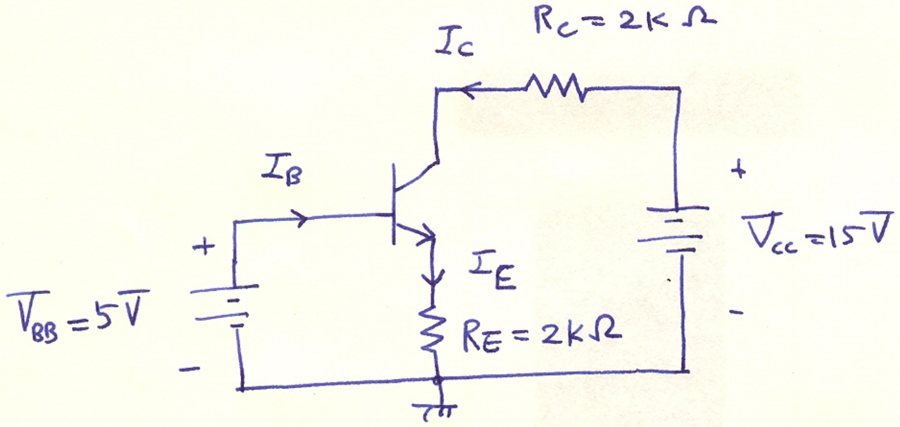
$$I_C = \frac{V_{CC} - 0.2}{R_C} = 14.8 \ mA < \beta I_B \rightarrow \text{saturation region.}$$

$\Rightarrow$ Operating point Q is sensitive to β.



for same i_B

Ex) $V_{cc} = 15V$, $V_{BB} = 5V$, $R_c = 2k\Omega$, $R_E = 2k\Omega$



a) $\beta = 100$:

Assume active, $I_E = \dfrac{V_{BB} - 0.7}{2000} = 2.15 \text{ mA}$

$I_E = (1+\beta) \cdot I_B \longrightarrow I_B = \dfrac{I_E}{(1+\beta)} = 21.3 \ \mu A$

$I_c = I_E - I_B = 2.13 \text{ mA}$

$V_{CE} = V_{cc} - R_c \cdot I_c - R_E \cdot I_E = 6.44V > 0.2V$

b) $\beta = 300$:

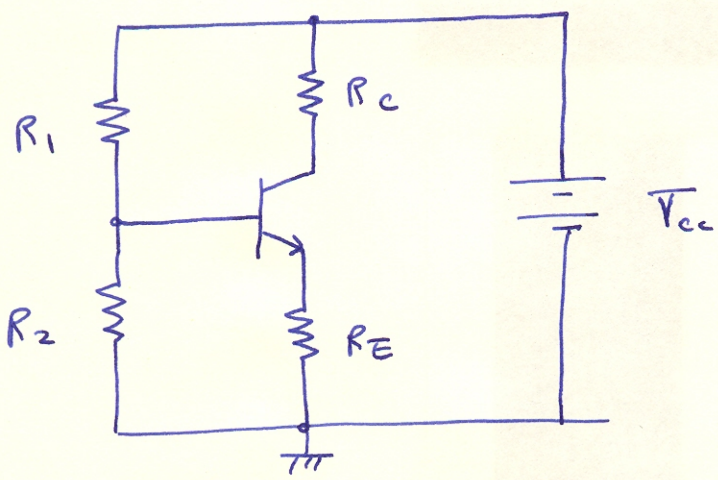
$I_B = 7.14 \ \mu A$

$I_c = 2.14 \text{ mA}$

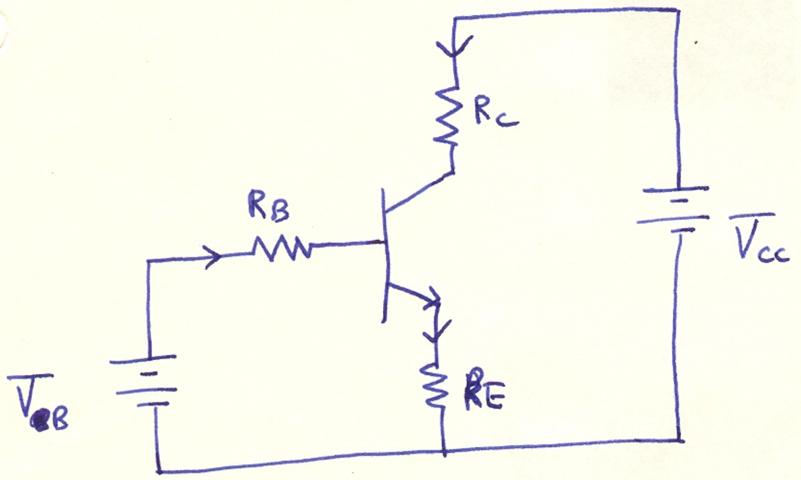
$V_{CE} = 6.42 V > 0.2$

$\longrightarrow$ Q is not sensitive to β.

§ Four Resistor Bias Circuit



Note) R_1 and R_2 are voltage divider



$$V_B = \frac{R_2}{R_1 + R_2} \cdot V_{cc}$$

$$R_B = R_1 \| R_2$$

$$V_B = R_B \cdot I_B + V_{BE} + R_E \cdot I_E$$

$$I_E = (1+\beta) \cdot I_B$$

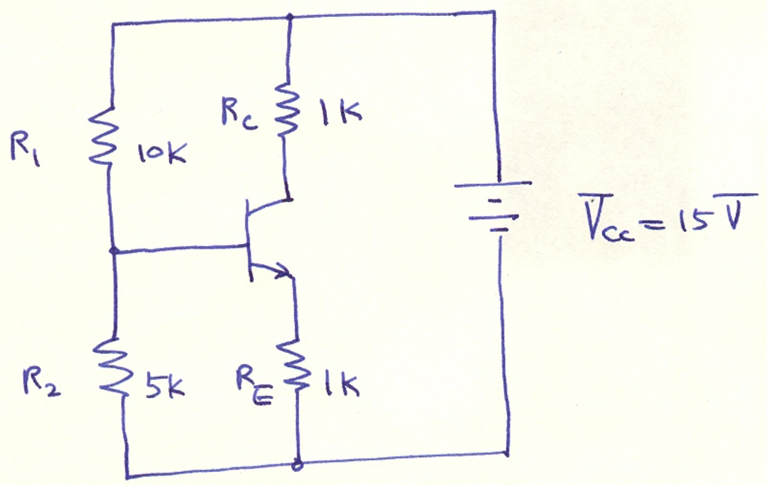
$$\rightarrow \quad I_B = \frac{V_B - V_{BE}}{R_B + (1+\beta) \cdot R_E}$$

$$V_{cc} = R_C \cdot I_C + V_{CE} + R_E \cdot I_E$$

$$I_C = \beta I_B = \frac{\beta (V_B - V_{BE})}{R_B + (\beta+1) \cdot R_E} \approx \frac{(V_B - V_{BE})}{R_E}$$

$\rightarrow$ Q point is not sensitive to β.

Ex)



a) $\beta = 100$

$$R_B = 10k \| 10k = 3.33\ k\Omega$$

$$V_B = \frac{5}{10+5} \times 15 = 5\ V$$

$$I_B = \frac{V_B - V_{BE}}{R_B + (1+\beta) \cdot R_E} = 41.2\ \mu A$$

$$I_C = \beta \cdot I_B = 4.12\ mA$$

$$V_{CE} = 6.72\ V$$

b) $\beta = 300$

$$I_B = 14.1\ \mu A$$

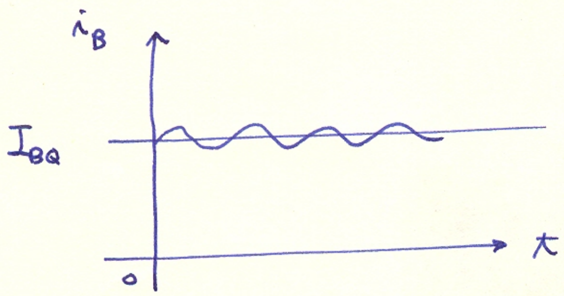
$$I_c = \beta \cdot I_B = 4.24\ mA$$

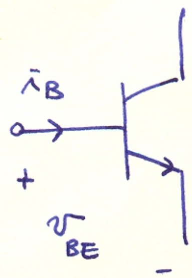
$$V_{CE} = 6.51\ V$$

§ Small Signal Equivalent Circuit

Consider DC biased signal i_B current

$$\underset{\substack{\uparrow \\ \text{DC bias}}}{i_B = I_{BQ}} + \underset{\substack{\uparrow \\ \text{AC signal}}}{i_b}$$





$$i_B = (1-\alpha) \cdot I_{ES} \cdot \left[\exp\left(\frac{v_{BE}}{V_T} \right) - 1 \right]$$

$$I_{BQ} + i_b = (1-\alpha) \cdot I_{ES} \cdot \exp\left(\frac{V_{BEQ} + v_{be}}{V_T} \right)$$

$$= I_{BQ} \cdot \exp\left(\frac{v_{be}}{V_T} \right) \simeq I_{BQ} \cdot \left(1 + \frac{v_{be}}{V_T} \right)$$

$$\rightarrow \quad i_b = \left(\frac{I_{BQ}}{V_T}\right) \cdot v_{be} = \frac{v_{be}}{r_\pi}$$

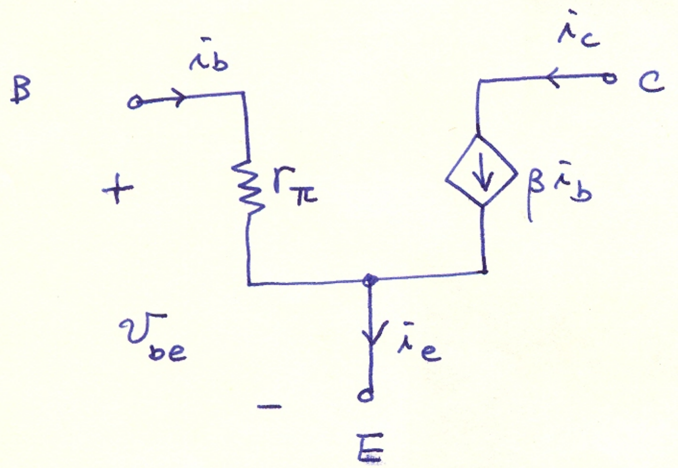
$$\boxed{r_\pi = \frac{V_T}{I_{BQ}} = \frac{\beta V_T}{I_{CQ}}} \quad \text{where } V_T \approx 0.026\, V$$

$$i_C = \beta i_B$$

$$I_{CQ} + i_c = \beta I_{BQ} + \beta i_b$$

$$\rightarrow \quad i_c = \beta i_b$$

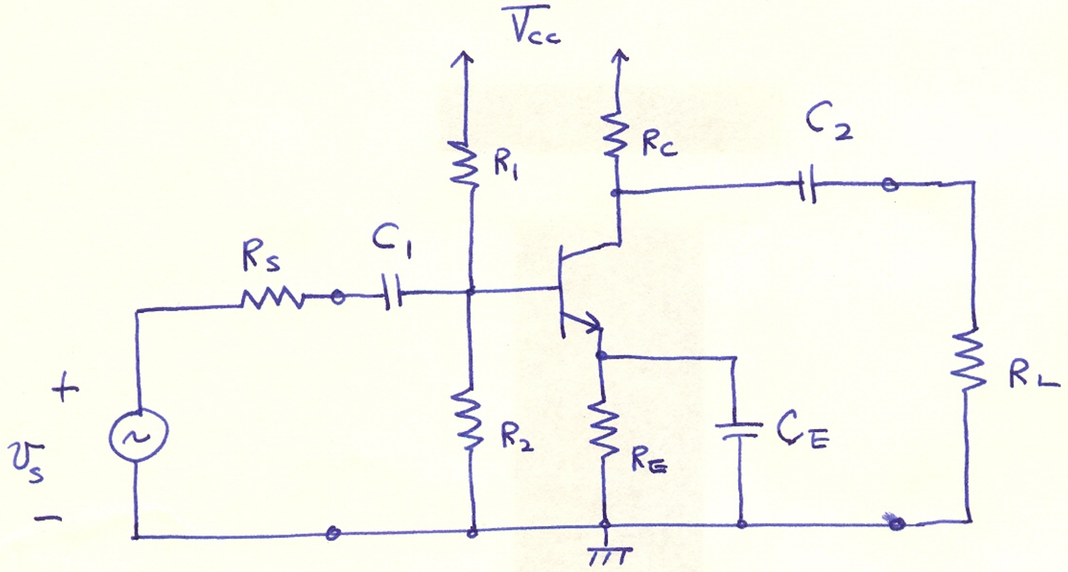
- Small Signal Equivalent Circuit.



Note) Two stage Analysis for BJT

1. DC bias analysis $\rightarrow$ Q point
2. Small signal analysis $\rightarrow$ Ac signal analysis

§ Common Emitter Amplifier.

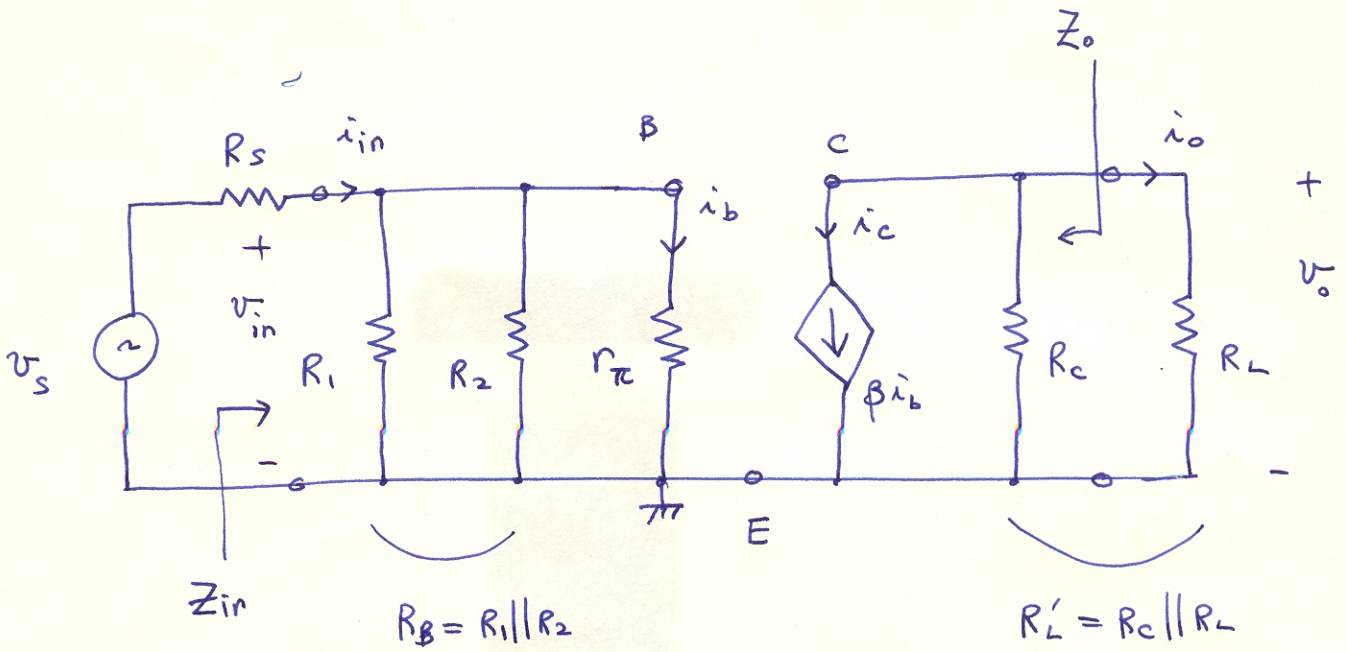


- DC Analysis :

  (C_1, C_2, C_E are open for DC.
  Find I_{BQ} and I_{CQ} $\rightarrow$ r_π.

- AC Analysis :

  (C_1, C_2, C_E are short for AC.
  $V_{CC} \longrightarrow 0\,V$ for AC.
  small signal BJT Equivalent circuit.



- Voltage gain.

$$A_v = \frac{v_o}{v_{in}}$$

$$v_{in} = v_{be} = r_\pi \cdot i_b$$

$$v_o = -i_c \cdot R_L' = -\beta \cdot R_L' \cdot i_b$$

$$A_v = \frac{-R_L' \cdot \beta}{r_\pi}$$

- Open circuit voltage gain: $R_L \to \infty$, $R_L' \to R_c$

$$A_{vo} = \left.\frac{v_o}{v_{in}}\right|_{R_L \to \infty} = -\frac{R_c \cdot \beta}{r_\pi}$$

- Input impedance

$$Z_{in} = \frac{v_{in}}{i_{in}} = R_B || r_\pi$$

- Current gain.

$$A_i = \frac{i_o}{i_{in}} = \left(\frac{v_o}{R_L}\right) / \left(v_{in}/z_{in}\right) = A_v \cdot \frac{Z_{in}}{R_L}$$

$$Z_{in} = R_1 \| R_2 \| r_\pi = R_B \| r_\pi$$

- Power gain

$$G = A_v \cdot A_i$$

- Output impedance

$$Z_o = \frac{v_o}{-i_o}\bigg|_{v_{in}=0} = R_c$$

Ex) $v_s(t) = 0.001 \cdot \sin(\omega t)$

$R_s = 520\,\Omega$, $R_1 = 10\,k\Omega$, $R_2 = 5\,k\Omega$,

$R_C = 1\,k\Omega$, $R_E = 1\,k\Omega$, $R_L = 2\,k\Omega$

$V_{BE} = 0.7\ V$, $\beta = 100$

By DC Analysis,

$I_{CQ} = 4.12\ mA$, $V_{CE} = 6.72\,V$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = 631\ \Omega$$

For AC Analysis,

$$R_B = R_1 \| R_2 = 3.33\ k\Omega$$

$$R_L' = R_L \| R_C = 667\ \Omega$$

$$A_v = \frac{v_o}{v_{in}} = -\frac{R_L'\beta}{r_\pi} = -106$$

$$A_{v_o} = \frac{v_o}{v_{in}} = -\frac{R_C\beta}{r_\pi} = -158$$

$$Z_{in} = \frac{v_{in}}{i_{in}} = R_B \| r_\pi = 531\ \Omega$$

$$A_i = \frac{i_o}{i_{in}} = A_v \cdot \frac{Z_{in}}{R_L} = -28.1$$

$$G = A_v \cdot A_i = 2980$$

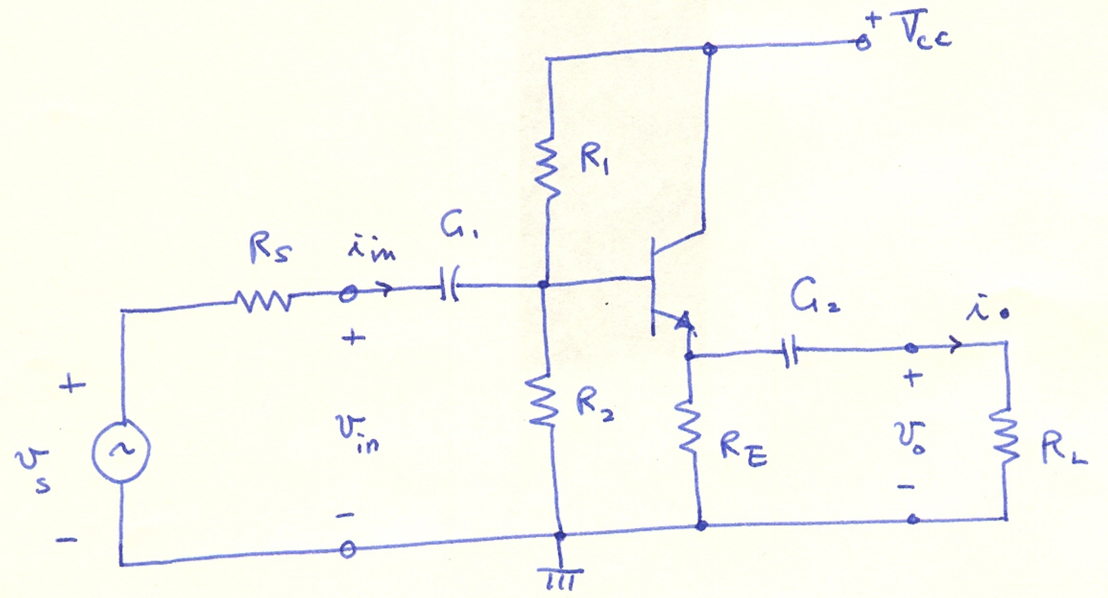
$$Z_o = R_c = 1\,k\Omega$$

$$v_{in} = v_s \cdot \frac{Z_{in}}{Z_{in} + R_s} = 0.515 \cdot v_s$$

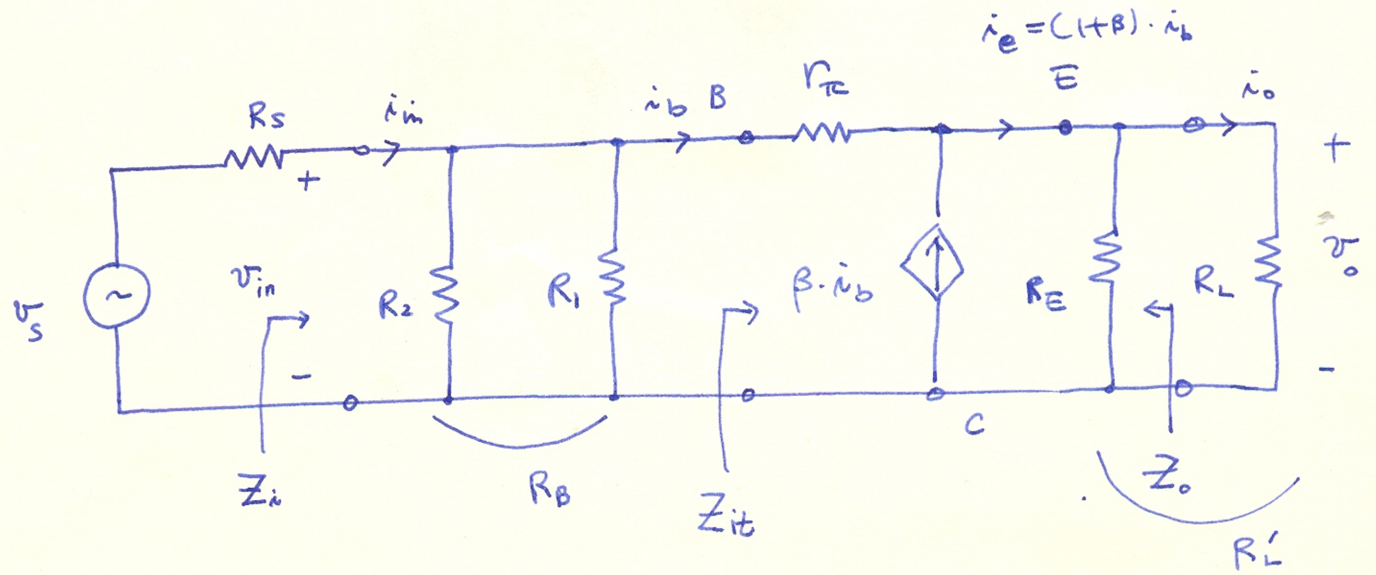
$$v_o = A_v \cdot v_{in} = -106 \times 0.515 \cdot v_s$$

$$= -54.6 \cdot \sin(\omega t) \; (mV)$$

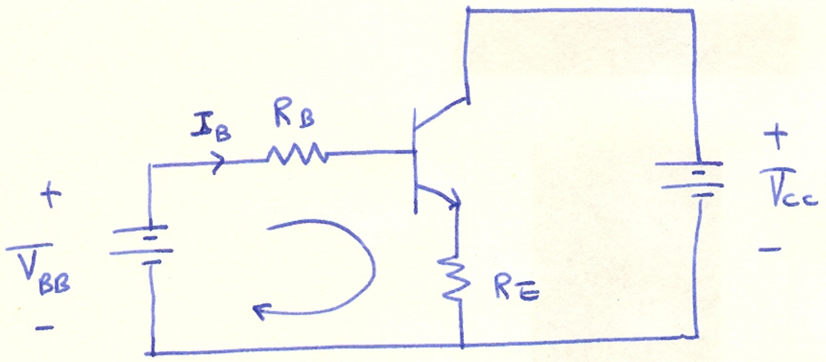
§ Emitter Follower (Common Collector Amplifier)



. AC Small Signal Equivalent Circuit



· DC Bias Analysis



$$V_{BB} = R_B \cdot I_B + v_{BE} + R_E (1+\beta) \cdot I_B$$

∘ AC Analysis

$$R_B = R_1 \| R_2$$

$$R_L' = R_E \| R_L$$

$$v_o = R_L' \cdot (1+\beta) \cdot i_b$$

$$v_{in} = r_\pi \cdot i_b + v_o = \left[r_\pi + (1+\beta) \cdot R_L' \right] \cdot i_b$$

$$A_v = \frac{(1+\beta) \cdot R_L'}{r_\pi + (1+\beta) \cdot R_L'} \simeq 1$$

$\longrightarrow$ $v_o \simeq v_{in}$: Buffer

Input Impedance

$$Z_i = R_B \parallel Z_{it}$$

$$Z_{it} = \frac{v_{in}}{i_b} = r_\pi + (1+\beta)\cdot R_L' \longrightarrow \text{High}$$

Current Gain

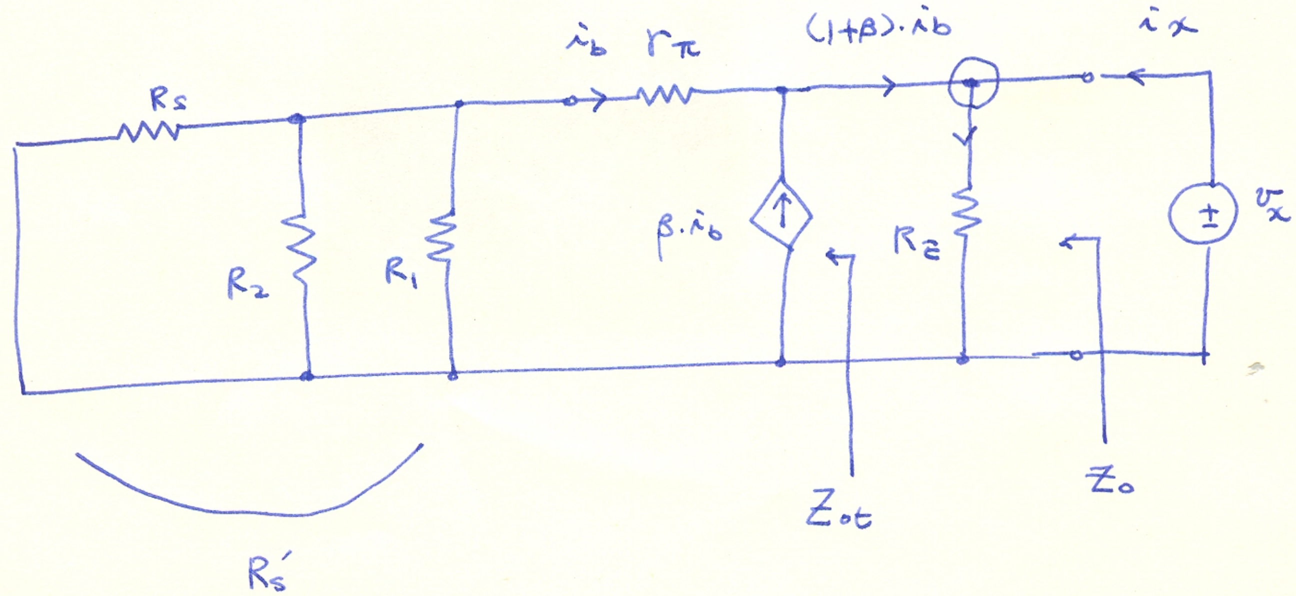
$$A_i = \frac{i_o}{i_{in}} = \frac{v_o / R_L}{v_{in} / Z_{in}} = \left(\frac{v_o}{v_{in}}\right)\cdot\left(\frac{Z_{in}}{R_L}\right) = A_v\left(\frac{Z_{in}}{R_L}\right)$$

Power Gain

$$G = A_v \cdot A_i$$

Output Impedance

$$Z_o = \frac{v_o}{-i_o}\Bigg|_{v_s = 0}$$



$$Z_o = \frac{v_x}{i_x}$$

$$\begin{cases} (1+\beta)\cdot i_b + i_x = \dfrac{v_x}{R_E} \\ R_s'\cdot i_b + r_\pi \cdot i_b + v_x = 0 \end{cases}$$

$$Z_o = \frac{v_x}{i_x} = \frac{1}{\left[\dfrac{(1+\beta)}{(R_s' + r_\pi)}\right] + \left[\dfrac{1}{R_E}\right]} = Z_{ot} \| R_E \rightarrow \text{small.}$$

Ex) $R_s = 10\,K\Omega$, $R_1 = 100\,K\Omega$, $R_2 = 100\,K\Omega$

$R_E = 2K\Omega$. $R_L = 1K\Omega$. $\beta = 200$, $V_{BEQ} = 0.7V$

- DC Bias Analysis

$$V_B = R_B . I_{BQ} + V_{BEQ} + R_E \cdot (1+\beta)\cdot I_{BQ}$$

$I_{BQ} = 20.6\ \mu A$, $I_{CQ} = \beta\, I_{BQ} = 4.12\,mA$,

$V_{CEQ} = V_{cc} - I_{EQ}\cdot R_E = 11.7\ V$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = 1260\ \Omega$$

- AC Small Signal Analysis

$R_B = R_1 || R_2 = 50\ K\Omega$

$R_L' = R_L || R_E = 667\ \Omega$

$$A_v = \frac{(1+\beta)\cdot R_L'}{r_\pi + (1+\beta) R_L'} = 0.991 \longrightarrow \text{near } 1$$

$Z_{it} = r_\pi + (1+\beta)\cdot R_L' = 135 K\Omega$

$$Z_i = \frac{1}{\frac{1}{R_B} + \frac{1}{Z_{it}}} = R_B || Z_{it} = 36.5\ K\Omega$$

$R_s' = R_s || R_1 || R_2 = 8.33\ K\Omega$

$$Z_o = \frac{1}{(1+\beta)/(R_s' + r_\pi) + 1/R_E} = 46.6\ \Omega$$

$$A_i = A_v \cdot \frac{Z_s}{R_L} = 36.2$$

$G = A_v \cdot A_i = 35.8$