1.2. Semiconductor statistics based on general band diagram

Goal ;

Express n&p in the following form

$$n = n_i(x)\, e^{q(E_{Fn} - E_i(x))/k_B T}$$

$$p = n_i(x)\, e^{q(E_i(x) - E_{Fp})/k_B T}$$

A. Statistics for n, p, and Nd, Na, Nt

In equilibrium, $E_{Fn}=E_{Fp}=E_F$ so that $np = n_i^2(x)$, and n_i^2 is determined by the material property. The reason why $n_i(x)$ is not constant is that the device may have different

- impurity concentration (heavy doping effect)
- lattice composition(such as different mole fraction in SiGe or GaAs, as examples)

— $n(x) = \sum f(k) = \int_{E_C} DOS\ f(E)dE$

$$= \int DOS \frac{1}{1+e^{(E-E_{Fn})/k_B T}} dE$$

$$= N_C\ e^{(E_{Fn} - E_{c,eff})/k_B T}$$

comment) Notice that Nc is the effective density of states

— In the above equation, N_c, $E_{c,eff}$ include all the corrections to be made in using the simple "Maxwell-Boltzmann" type form.

B. n, p statistics expressed in the form of Maxwell Boltzmann statistics

If the band is spherical band,

$$n = \frac{2}{\sqrt{\pi}} N_C F_{1/2}(\eta_n), \qquad p = \frac{2}{\sqrt{\pi}} N_V F_{1/2}(\eta_p)$$

$$\text{where} \quad \eta_n \equiv \frac{E_F - E_C}{k_B T}, \quad \eta_p \equiv \frac{E_V - E_F}{k_B T}$$

— See YJP p.58 for the correction to be made in using MB statistics instead of FD statistics.

$$\begin{aligned} n &= \frac{2}{\sqrt{\pi}} N_C F_{1/2}(\eta_n) \\ &= C N_C e^{(E_{Fn} - E_C)/k_B T} \\ &= N_C e^{(E_{Fn} - E_C + \Delta E_g)/k_B T} \end{aligned}$$

$$\text{where} \quad C \equiv \frac{\frac{2}{\sqrt{\pi}} N_C F_{1/2}(\eta_n)}{N_C e^{(E_F - E_C)/k_B T}} = e^{-E_g/k_B T}$$

Figure 1-18
in ; Semiconductor Device with NANOCAD

— As doping increases, the electrical bandgap decreases.
Rigid bandgap narrowing theory assumes that the band shape (spherical energy band, for example) does not change as doping concentration increases. In this context, n(x) can be written as,

$$n(x) = N_C \, e^{(E_{Fn} - E_{c,eff})/k_B T}$$

where $E_{c,eff} \equiv E_C - \Delta E_g|_1 - \Delta E_g|_2$,

$\Delta E_g|_1$: bandgap correction to include the FD statistics.

$\Delta E_g|_2$: bandgap narrowing due to the heavy doping effect.

Notice in the above equation that the form for n does not change, which means that the "Rigid band theory" is assumed.

C. Statistics expressed with n_{ieff}

D. Statistics for impurity concentration

- The statistics fo N_d^{+} out of given N_d can be obtained from the method leading to n in the conduction band, which is the Fermi Dirac statistics.

Only the difference is that

- The impurity state is 'localized state' so that the state cannot occupied by two electrons due to Coulomb repulsion.
- There are more than 2 degeneracy for silicon , for example, because there are 6 equivalent valleys in silicon.

Considering these, we N_d^0/N_d and Na-/Na as,

$$N_d^0/N_d = 1/\left(1+(1/g_d)\, e^{(E_d - E_f)}\right)$$

and $$Na^-/Na = 1/\left(1+(g_a)\, e^{(E_d - E_f)}\right).$$

For Silicon, gd =12 and ga = 4.

<reading material #3, Wolfe, Physical Properties of Semiconductors, pp 112-118>

E. Net charge in the semiconductor devices.

There are several types of charge we have to consider.

In the bulk, n, p, N_d^{+} and N_a^{+}. If there exists the trap states(Nt), which can trap electrons(and holes), then their contribution to net charge can be written as,

Donor like states : + if electron is empty
neutral otherwise.

Acceptor like states: : - if electron is occupied.

neutral otherwise.

Then net charge(C/cm^3) '-' is
- = p-n+Nd+ - Na- + Nt+-Nt-. ------------(E-1)

- n, p equations for charge neutral region.

If the semiconductor is in the thermal equilibrium, and
- =0 (charge neutrallity) is valid, we can obtain n, p from

- = p-n+Nd+ - Na- + Nt+-Nt-.(equation for 1 unknown).

- Condition for full ionization.
Nd+= Nd and Na=Na-, meaning that all the impurities are fully ionized. The condition is satisfied for most of the semiconductor devices operating near the room temperature.
However, for the cases of high doping, low temperature operation the condition is not met. In this case , you have to use computer to solve the equation in eq. E-1.

<comment: Study the what is the carrier freeze out effect>

Homework>
The semiconductor bar with the length of 1um is doped with
$N_{D}(x) = N_{D0}$ exp(-x/ xd) where xd=0.1um. Here ND0=1E20/cm**3.

1) Draw the band diagram of the bar from x=0 to 1um.
   Assume full ionization of the carrier. Use the band gap narrowing parameter found in YJP p.78(fig. 1-22) or ref. .
2) Obtain the hole concentration p(x)
   a) considering the band gap narrowing

b) neglecting the band gap narrowing.