Chapter 3.3 Voltage Limitation

3-3. Voltage limitation
- Tunneling
- Avalanche Breakdown

Objective To understand the leakage current in the reverse bias PN junction other than the thermal generation studied in chap. 3.2. Most of them are due to tunneling and impact ionization mechanism.

1. Tunneling mechanism

Under the revere bias PN junction, especially with the highly doped regions, the valence electrons in the valence band has chances to tunnel to the Conduction band.
The reverse leakage current due to the tunneling mechanisms are
-the direct tunneling from VB to CB and (fig. 3-1 a)
-the indirect tunneling with the trap in the forbidden band as the tunneling step stone. (TAT: Trap Assisted Tunneling, fig. 3-2 b).

In the fig., notice that the valence electrons which can contribute to the tunneling are determined by the Eg, E(field) and Et.

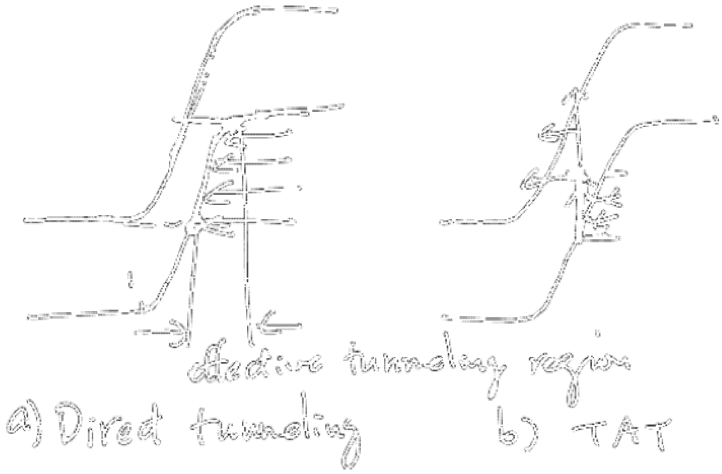

Fig. 3.1 a) Direct tunneling and b) TAT(Trap Assisted tunneling)

A. WKB theory from VB to CB

The most widely used model for the tunneling probability is based on WKB(Wentzel, Kramers and Brillouin).
Let A(x) be the wavefunction of an electron and consider the 1D potential energy given by U(x). Based on the effective mass approximation, the steady state Schrodinger equation can be written as,

$$[\ -\P^2/2m^*\ d^2/dx^2\ +\ U(x)]\ =\ E\ U(x) \qquad (1)$$

Assume the solution A(x) = exp(r(x)) and substitituing into (1), we obtain

$$-\P^2/2m^*\ [\ d^2r/dx^2 + (dr/dx)^2]\ =\ U(x)\ -E$$

In the WKB approximation,

$$d^2r/dx^2 << (dr/dx)^2$$

which is valid when r is slowly varying, (2) becomes

$$dr/dx\ C\ \{\ 2m^*\ /\P^2\ [U(x)\ -\ E]\}^{1/2}.$$

Integrating over the region where U(x)>E, the tunneling probability can be obtained by

$$P(E) = A(E)A(E^*) = \exp(+-2 \int_{x_r} \{ 2m^* / \P^2 [U(x) - E]\}^{1/2} dE.$$

B. TAT(Trap Assisted Tunneling)

ref.
1. G. Hurx et. al, "Anomalous behavior of surface leakage currents in heavily doped gated-diodes," TED, p.2273, 1993.
2. G. Hurx et.al, " A new recombination model for device simulation including tunneling," TED, p.331, Feb. 1992.
3. Hackbarth et. al., " Inherent and stress-induced leakage in heavily doped silicon junctions," TED, p.2108, Dec., 1988.

- Model

Net recombination rate is enhanced by the "tunneling enhanced recombination" and "tunneling enhanced emission of an electron."

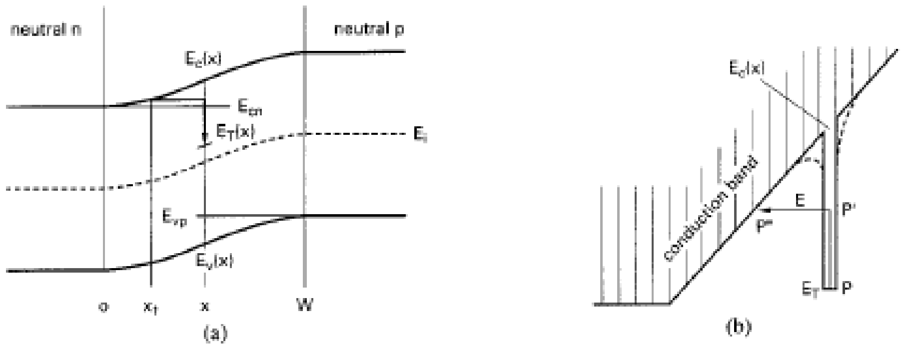

Fig. 2 from the ref. 2.

In the figure (a) above, the electron concentration at the position of x(the trap location) is effectively increased by the tunneling of the conduction electrons in the neighborhood. By assuming a constant field in the depletion region(ε_{eff}), analytic solution for the "effective electron concentration" at x (the location of the trap) can be obtained as

$$n_{eff}(x) = \int_{\delta}^{x} (-\frac{dn(x)}{dx})_{x=x_1} \frac{Ai^2[\gamma(x-x_1]}{Ai^2(0)} dx_1$$

(1)

where δ is the position of the nonzero tunneling probability begins in the depletion region, and $\gamma = (2q\varepsilon_{eff}m^* \hbar^{-2})^{1/3}$.

In the figure (b), the emission probability is also enhanced by the tunneling. The electrons in the trap may have to gain energy smaller than E_c-E_t by photon before tunneling so that the emission probability e_n is enhanced by,

$$e_{n_{eff}} = \frac{1}{k_B T} \int_0^{\Delta E_n} e^{-E/k_B T} \frac{Ai^2(2m^* r^{-2} \hbar^{-2} E)}{Ai^2(0)} dE$$

(2)

where ΔE_n is the energy range for possible process for phonon activation followed by tunneling (E_c-E_T in the case the fig. (b)).

Comparing $\frac{n_t}{n}$ and $\frac{e_{n_{eff}}}{e_{no}}$, we note that the factor is same as P where P can be written as after some approximation in the airy function as,

$$P = 2\sqrt{3\pi}\frac{\varepsilon}{\varepsilon_P} e^{(\varepsilon_{eff}/\varepsilon_P)^2}$$

where $\varepsilon_P = \frac{\sqrt{2qm^*(k_B T)^2}}{q\hbar}$.

- Model

Ref. 2 in the above propose a convenient model which is consistent with the SRH model, but captures most of the physics describing the enhancement of the generation and recombination rate due to tunneling. The model is

$$U = \frac{np - n_i^2}{C_p^{-1}\left(n + n_i e^{E_j/k_B T}\right) + C_n^{-1}\left(p + n_i e^{-E_j/k_B T}\right)}$$

The effects of the ehancement of the emission probability and capture cross section due to tunneling can be expressed by the Gamma function as,

$$C_\alpha = \sigma_\alpha v_{th}(1 + \Gamma_\alpha)$$

, where the Gamma function includes the tunneling from the valence band, generation via trap, and tunneling to the conduction band.

$$\Gamma_\alpha = \frac{\Delta E_\alpha}{k_B T}\int_0^1 \exp\left[\frac{\Delta E_\alpha}{k_B T}u - K_\alpha u^{3/2}\right] du$$

$$K_\alpha = \frac{4}{3}\frac{\sqrt{2m^*\Delta E_\alpha^3}}{q\hbar|E|}.$$

<reading material> Ref 2 and

Seonghoon Jin, *et.al,* ' Prediction of Data Retention Time Distribution of DRAM by Physics-Based Statistical Simulation', IEEE TED, VOL. 52, NO. 11, NOVEMBER 2005

2. Impact Ionization

As the electrons and holes are accelerated by high field in the PN junction, there are chances for carriers to generate electron hole pairs. The phenomenon is called the impact ionization and best described by 'three particle process' where,

e = e + e +h

. This process can be regarded as the 'reverse Auger process'.
In the process, the energy and momentum should be conserved.

Γ_n and Γ_p are defined as the impact ionization rates for electrons and holes. It is defined as the number of e-p generation even when an electron(or an hole) travels in the high electric field region per 1cm.

Then Gii (generation rate due to the impact ionization event) be written as,

A. local and nonlocal model for Γ

ref. > M. Valdinoci, et.al., Impact-ionization in silicon at large operating temperature, SISPAD, 1998 and references therein.

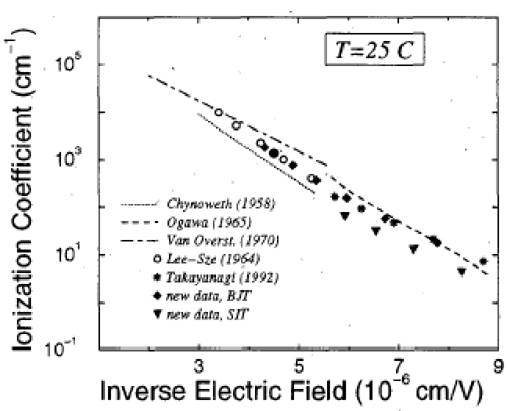

Fig. 1: Electron ionization coefficient vs. inverse electric field at different temperatures: comparison with literature data.

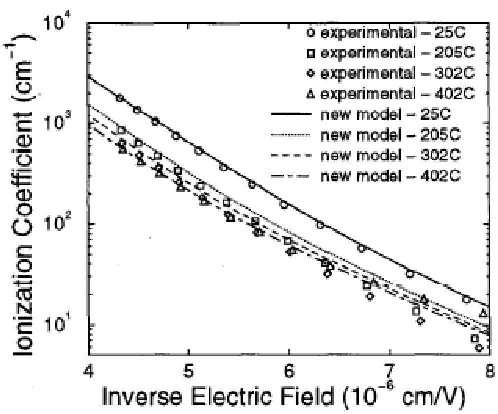

Fig. 2: Electron ionization coefficient: experimental data (symbols) compared with the new impact-ionization model (continuous lines).

As the electric field increases with the increase of reverse bias condition, electrons(holes) gain energy and the energy can be losed by way of impact ionization. The rate is written as,

$$Gii = \alpha_n |F_n| * \alpha_p |F_p| \quad [/cm**3\text{-}sec]$$

where α_n and α_p are the impact ionization constants and usually determined by experiments as the function of local field. However, in the highly nonuniform field where electrical field changes abruptly in space(as in the drain section of MOSFET), electron energy does not respond to the field change. In this case, the impact ionization should be modeled by solving the hydro dynamic equations.

3. Reverse breakdown

Due to either the tunneling or the impact ionization, the reverse leakage current increases abruptly when the reverse bias voltage reaches 'some critical ' value.
The value is called the reverse breakdown. Once the breakdown occurs, the junction may or may not recover according to the damage created by the energy dissipation taking place in the PN junction.

A. Multiplication avalache breakdown

Once Gii is known, the multiplication factor M and the breakdown voltage may be obtained rather easily. See, for example, YJP's book where $_n$ is same as $_p$.
The breakdown mechanism due to the impact ionization is the forward feedback mechanism where one event of the impaction ionization causes another event of the impact ionization by the generated by the original impact ionization event. The phenomenon is called the 'avalanche breakdown' .

B. Zener Breakdown

The breakdown mechanism caused by the tunneling mechanism is called the Zener breakdown. The tunneling current can be found by integrating hitting event of the valence electrons per unit time multiplied by the tunneling probability along the effective area for tunneling shown in fig. 1(a).