Goal;

To understand general MOS I-V characteristics in the context of drift-diffusion equation

Ref : 1. J.R. Bruce, Solid State Electronics, pp. 345-355
2. YJ Park et al, VLSI Device Theory with NanoCAD, Chapter 7, Daeyoungsa, 2003,(Korean)

Consider a general bias condition and reference potential in Fig. 1 where source potential is set to 0V. In a circuit operation, source potential can be changed, however, the analysis adopted here can be applied with minor change.

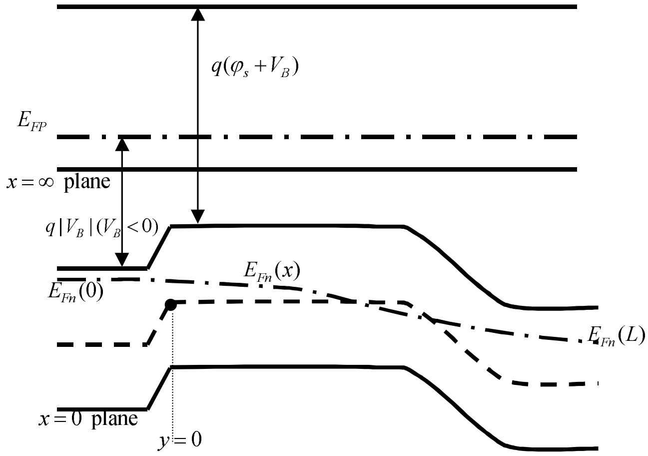

Fig. 1.

A. MOS Current Equation based on the Bruce's Charge sheet theory

This equation covers the current from the subthreshold region to above threshold region. Not only the drift, but also the 'diffusion' component of the current is considered.

However, students are not encouraged to read the paper (Ref. 1) thoroughly since the paper is a little to much involved. The followings are the essence of the paper, and other portions of the paper may be referred when they are needed.

$$qN_s(y) = C_{ox}(V_{GS} - V_{FB} - \psi_s(y)) - \sqrt{2}\,qN_a L_d \sqrt{\frac{\psi_s}{V_t} - 1} \qquad (1)$$

$$\begin{aligned} N_s(y) &= \int_{x=0}^{x_{inv}} n_i\, e^{(E_{Fn} - E_i)/kT} \\ &= A e^{(\psi_s(y) - \phi_F(y))/V_t} \end{aligned} \qquad (2)$$

$$I = Wq\mu^* n_s(y)\left[\frac{d\psi_s(y)}{dy} - V_t \frac{d\ln n_s}{dy}\right] \qquad (3)$$

at $x = L$, with V_{DS}. $\psi_{sL} = ?$

$$\frac{\psi_{sL}}{V_t} = \frac{\psi_{so}}{V_t} + \frac{V_{DS}}{V_t} + \ln\frac{n_s(L)}{n_s(0)} \qquad (4)$$

$$qn_s(L) = C_{ox}(V_G - V_{FB} - \psi_{sL}) - qN_a L_B \sqrt{2}\sqrt{\frac{\psi_{sL}}{V_t} - 1} \qquad (5)$$

Then

$$\int_0^y \frac{Id_y}{Wq\mu^*} = \int_0^y [n_s(y)\, d\psi_s(y) - V_t n_s d\ \ln n_s$$

Substituting

$y = L$ and $\psi_{sL} = \psi_{so} + V_D$

$$
\begin{aligned}
I \;=\; & V_t \frac{W}{L} \mu^* \Big[C_{ox} V_t (\Psi_{sL} - \Psi_{so}) - \frac{1}{2} C_{ox} (\Psi_s L^2 - \Psi_{so}^2) \\
& - qN_{SUB} L_D \frac{2\sqrt{2}}{3} (\frac{\Psi_{sL}}{V_t} - 1)^{0.5} - \Big((\frac{\Psi_{so}}{V_t} - 1)^{1.5} \Big) \\
& + qN_{sUB} L_p \sqrt{2} \Big((\frac{\Psi_{sL}}{V_t} - 1)^{0.5} - (\frac{\Psi_{so}}{V_t} - 1)^{0.5} \Big) \Big]
\end{aligned}
$$

HW:

Read Brews & Verify that it can be written as a diffusion equation in the subthreshlod region as,

$$I_{DS} \simeq WD_n \frac{C_D V_t}{L} \; e^{V_{GS}/nV_t} (1 - e^{-V_D/V_t})$$

where $n = 1 + \dfrac{C_D + C_{ss}}{C_{ox}}$ eq. (25) in Brew's.

B. Classical approximation above threshold

Above threshold, $Q_n(y)$ is approximated as,

$$
\begin{aligned}
Q_n(y) \;&=\; -C_{ox}(V_{GS} - V_{TH} - \Psi_s) - Q_d(\Psi_s) \\
&=\; -C_{ox}(V_{GS} - V_T - \Psi_s) + C_{ox}\sqrt{\Psi_s}
\end{aligned}
$$

After all the channel region is strongly inverted. Then

$$\Psi_s \simeq 2\Phi_{fp} + V$$

Here, $V(y)$ is the potential drop defined in Fig.1

$$J = -qn(x, y)\, v(x, y) = qn(x, y)\, \mu E_y(x, y) \tag{6}$$

$$\frac{I}{W} = \int J dx = \int qn(x,y)\,v(x,y)dx = -Q_n v_D^* = Q_n \mu_n E_y \qquad (7)$$

where

$$\begin{aligned} Q_n(y) &= -C_{ox}(V_{GS}-V_{FB}-\psi_s)-Q_d(y) \\ &= -C_{ox}(V_{GS}-V_{FB}-\psi_s)+C_{ox}\gamma\sqrt{\psi_s} \end{aligned} \qquad (8)$$

$$I = -WC_{ox}[(V_{GS}-V_{FB}-\psi_s)-\gamma\sqrt{\psi_s}]\mu_n(-\frac{\partial\psi_s}{\partial y}) \qquad (9)$$

$$\begin{aligned} I_{DS} = &\frac{W}{L}\mu_n C_{ox}\Big[\Big(V_{GS}-V_{FB}-2\phi_{fp}-\frac{V_{DS}}{2}\Big)V_{DS} \\ &-\frac{2}{3}\gamma[(V_{DS}+2\phi_{fp})^{3/2}-(2\phi_{fp})^{3/2}]\Big] \end{aligned} \qquad (10)$$

If V_{DS} is very small so that 2nd term of RHS in (10) is expanded in Taylor series expansion and the 1st term is retained.

$$\begin{aligned} I_{DS} &= \frac{W}{L}\mu_n C_{ox}\Big(V_{GS}-V_{FB}-2\phi_F-\gamma\sqrt{2\phi_F}-\frac{1}{2}V_{DS}\Big)V_{DS} \\ &= \frac{W}{L}\mu_n C_{ox}\Big(V_{GS}-V_T-\frac{1}{2}V_{DS}\Big)V_{DS} \end{aligned} \qquad (11)$$

Now consider the Q_n value at the drain side,

$$\begin{aligned} Q_n(L) &= -C_{ox}(V_{GS}-V_{FB}-2\phi_F-V_{DS})+\gamma C_{ox}\sqrt{V_{DS}+2\phi_F} \\ &= -C_{ox}(V_{GS}-V_T(L)) \end{aligned} \qquad (12)$$

The V_T at the drain side is larger than $V_T(0)$ and

$$V_T(L) = V_{FB}+V_{DS}+2\phi_F+\gamma\sqrt{V_{DS}+2\phi_F} \qquad (13)$$

$V_{DS,sat}$ is defined as the V_{DS} value to make $Q_n(L)=0$,

Or

$$V_{FB}+V_{DS,sat}+2\phi_{fp}+\gamma\sqrt{V_{DS,sat}+2\phi_{fp}} = V_{GS} \qquad (14)$$

C. Classical approximation below threshold (Subthreshold characteristics)

When $V_G < V_T$, current is assumed to be only due to diffusion. Then current is determined by the Q_n at the source region($x=0$) and at the drain region($x=L$).

From eq. (2) in §2.1, and for $V_t \ll \Psi_s \leq 2\Phi_{Fp}$

$$Q_n(0) = -\sqrt{2}\varepsilon_s \frac{V_t}{L_d}\sqrt{\frac{\Psi_s}{V_t} + e^{-2\Phi_{Fp}} e^{\Psi_s/V_t}} - Q_d(0) \quad (15)$$

$$\begin{aligned} Q_n(L) &= -\sqrt{2}\varepsilon_s \frac{V_t}{L_d}\sqrt{\frac{\Psi_s + V_D}{V_t} + e^{-2\Phi_{Fp}} e^{(\Psi_s - V_{DS})/V_t}} - Q_d(\Psi_s) \\ &= -\sqrt{2\varepsilon_s q N_A}\left[\sqrt{\Psi_s + V_{DS} + V_t\, e^{(\Psi_s - V_{DS} - 2\Phi_{Fp})/V_t}} - \sqrt{\Psi_s + V_{DS}}\right] \quad (16) \\ &= -C_d V_t\, e^{(\Psi_s - 2\Phi_{Fp} - V_{DS})/V_t} \end{aligned}$$

Then

$$\begin{aligned} I_D &= WD_n \frac{Q_n(0) - Q_n(L)}{L} \\ &= WD_n \frac{1}{L} C_d V_t\, e^{(\Psi_s - 2\Phi_{Fp})/V_t}(1 - e^{-V_{DS}/V_t}) \end{aligned} \quad (17)$$

– $Q_n - V_{GS}$ relationship.

For $V_t \ll \Psi_s \ll 2\Phi_{fp}$,

Q_n can be written, from eq. (2) of §2.1, as,

$$\begin{aligned} Q_s &= -\varepsilon_s \frac{2V_t}{L_d}\sqrt{\frac{\Psi_s}{V_t} + e^{-2\Phi_{fp}/V_t} e^{\Psi_s/V_t}} \\ &= -\sqrt{2\varepsilon_s q N_a}\sqrt{\Psi_s + V_t\, e^{\Psi_s - 2\Phi_{fp}}} \end{aligned} \quad (18)$$

$$Q_n = Q_s - Q_d = -\sqrt{2\varepsilon_s q N_A}\ \left\{\sqrt{\Psi_s + V_t e^{\Psi_s - 2\Phi_{fp}}} - \sqrt{\Psi_s}\right\} \quad (19)$$

$$Q_n \simeq -\sqrt{2\varepsilon_s qN_A}\sqrt{\Psi_s}\left(1+\frac{V_t}{2\Psi_s}e^{\Psi_s-2\Phi_{fp}}-1\right) \tag{20}$$
$$= -C_d V_t e^{(\Psi_s-2\Phi_{fp})/V_t}$$

Now let us express Ψ_s in terms of V_{GS} (applied voltage). From MOS equation at $x=0$ including the surface states capacitance,

$$V_{GS}-V_{FB} = -\frac{Q_s(\Psi_s)+Q_{ss}(\Psi_s)}{C_{ox}+\Psi_s} \tag{21}$$

Expanding RHS arounds

$$\Psi_s = 2\Phi_{fp},$$
$$Q_s(\Psi_s) = Q_s(2\Phi_{fp})+\left.\frac{\partial Q_s}{\partial\Psi_s}\right|_{\Psi_s=2\Phi_{fp}}(\Psi_s-2\Phi_{fp})$$
$$Q_{ss}(\Psi_s)=Q_{ss}(2\Phi_{fp})+\left.\frac{\partial Q_{ss}}{\partial\Psi_s}\right|_{\Psi_s=2\Phi_{fp}}(\Psi_s-2\Phi_{fp}) \tag{22}$$

So that

$$V_{GS}-V_{FB} = -\frac{Q_s(2\Phi_{fp})+Q_{ss}(2\Phi_{fp})}{C_{ox}} + \left.\left(\frac{\partial Q_s}{\partial\Psi_s}+\frac{\partial Q_{ss}}{\partial\Psi_s}+1\right)\right|_{\Psi_s=2\Phi_{fp}}(\Psi_s-2\Phi_{fp})+2\Phi_{fp} \tag{23}$$

So, $V_{GS}=V_T+n(\Psi_s-2\Phi_{fp})$

$$n = \left.\left[1-\frac{\partial(Q_s+Q_{ss})}{C_{ox}\partial\Psi_s}\right]\right|_{\Psi_s=2\Phi_F}=1+\frac{C_s+C_{ss}}{C_{ox}} \tag{24}$$
$$\simeq 1+\frac{C_d+C_{ss}}{C_{ox}}\simeq 1+\frac{C_d}{C_{ox}}+\frac{qN_{ss}(2\Phi_F)}{C_{ox}}.$$

Now Q_n can be written as,

$$Q_n(V_G)=-C_d V_t e^{\frac{V_{GS}-V_t}{nV_t}}. \tag{25}$$

Substituting the above eq. to eq. (17),

$$I_{DS} = \frac{W}{L} D_n C_d (2\Phi_F) V_t e^{\frac{V_{GS} - V_t}{nV_t}}. \quad (26)$$

It is interesting to notice that $Q_n(0)$ can be approximated as,

$$Q_n = C_d V_t e^{\Psi_s / V_t}. \quad (27)$$