Chapter 5.

PN Junction Electrostatics

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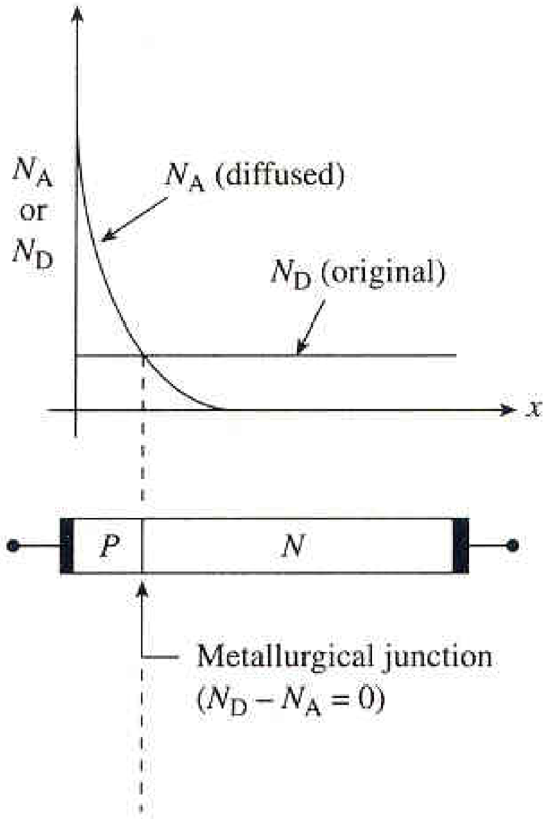
Subject

Analysis of Field and potential distribution in the junction.

- Quantitative Electrostatic Relationships

❑ Preliminaries

• Junction Terminology/Idealized Profiles



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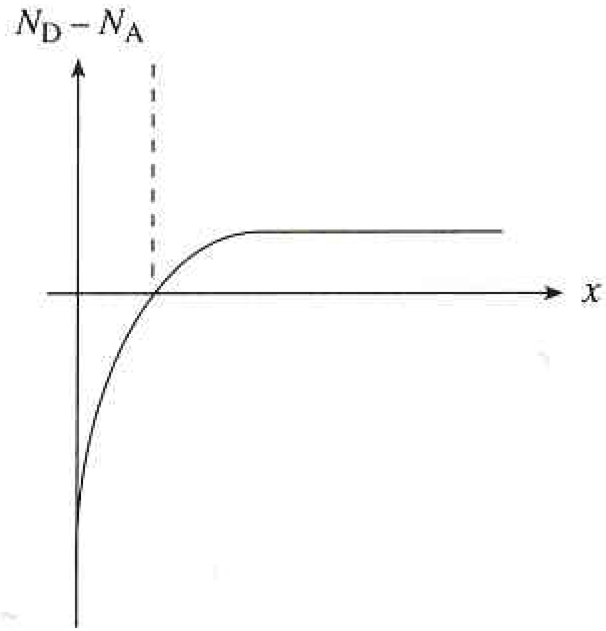
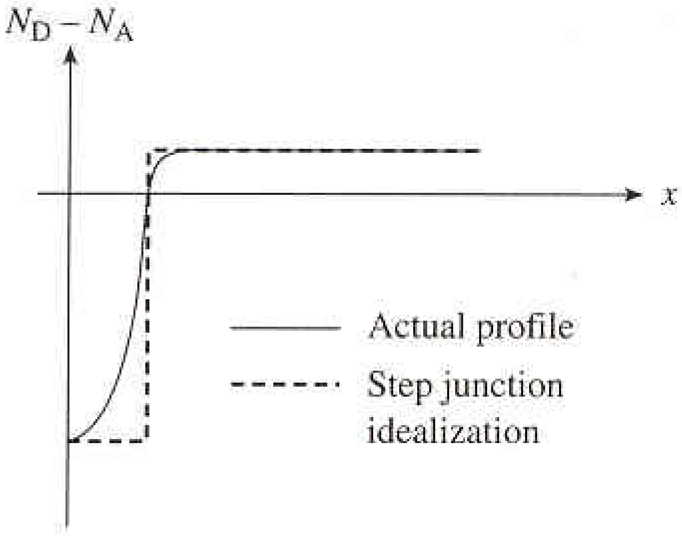


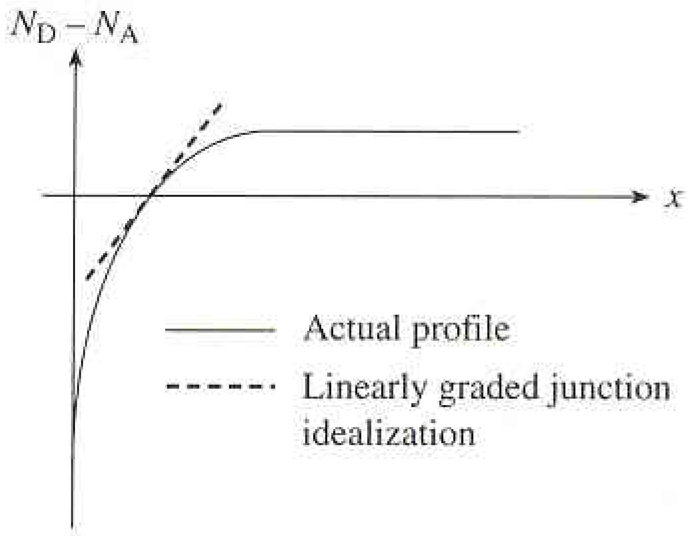
(b)

Net doping profile



(a)

Step (abrupt) junction



(b)

Linearly graded junction

✓ The step junction is an acceptable approximation to an ion-implantation or shallow diffusion into a lightly doped starting wafer

• Poisson's Equation

$$\nabla \cdot \mathscr{E} = \frac{\rho}{K_S \varepsilon_0} \xrightarrow{\text{1-Dimension}} \boxed{\frac{d\mathscr{E}}{dx} = \frac{\rho}{K_S \varepsilon_0}}$$

K_s is the semiconductor dielectric constant and ε_0 is the permittivity of free space. ρ is the charge density (charge/cm^3)

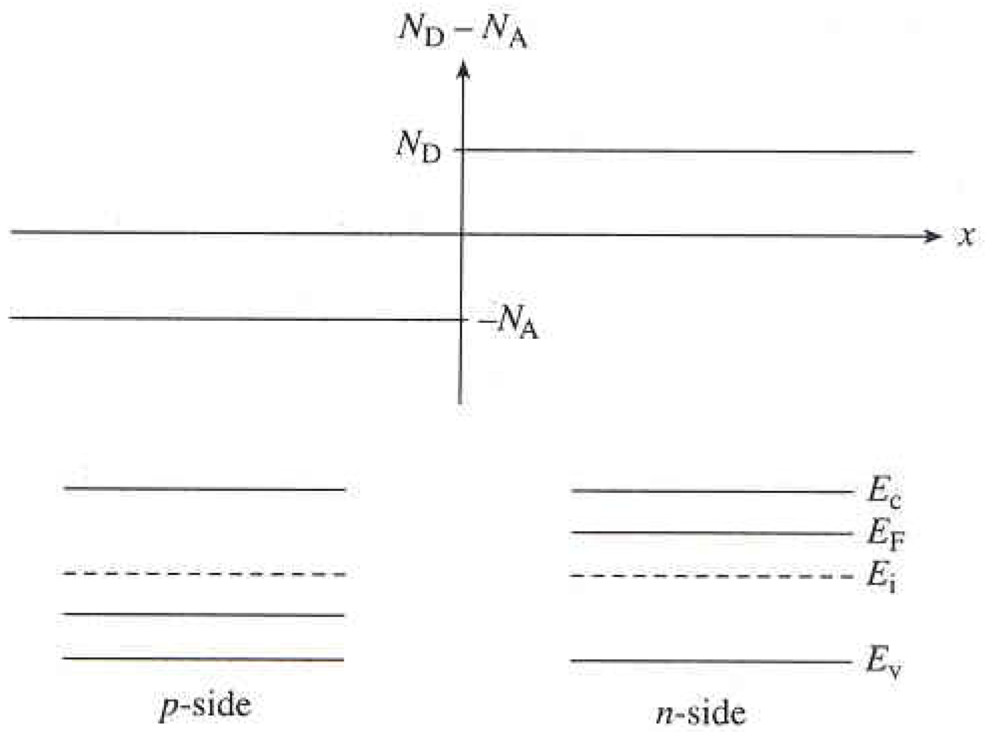
$$\boxed{\rho = q(p - n + N_D - N_A)}$$

$$\rho \;\; is \;\; proportional \;\; to \;\; \frac{dE}{dx}$$

• Qualitative Solution

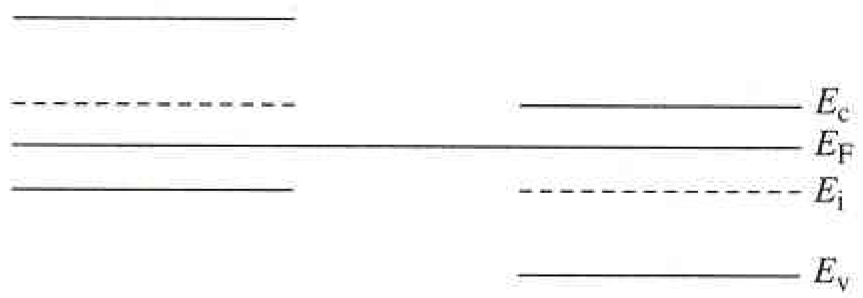
✓ Let us assume an equilibrium conditions

✓ It is reasonable to expect regions far removed from the metallurgical junction to be identical to an isolated semiconductor.



(a)

✓ Under equilibrium conditions, the Fermi level is a constant



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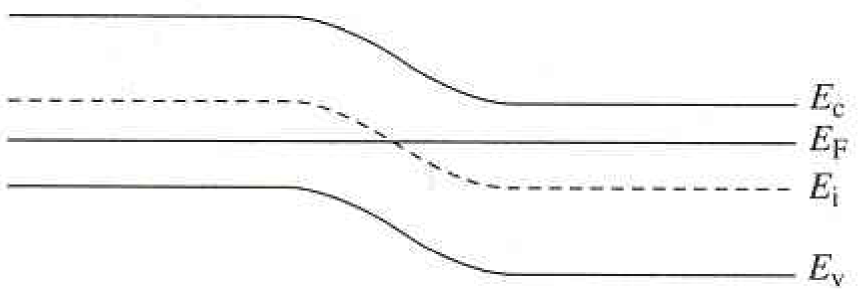
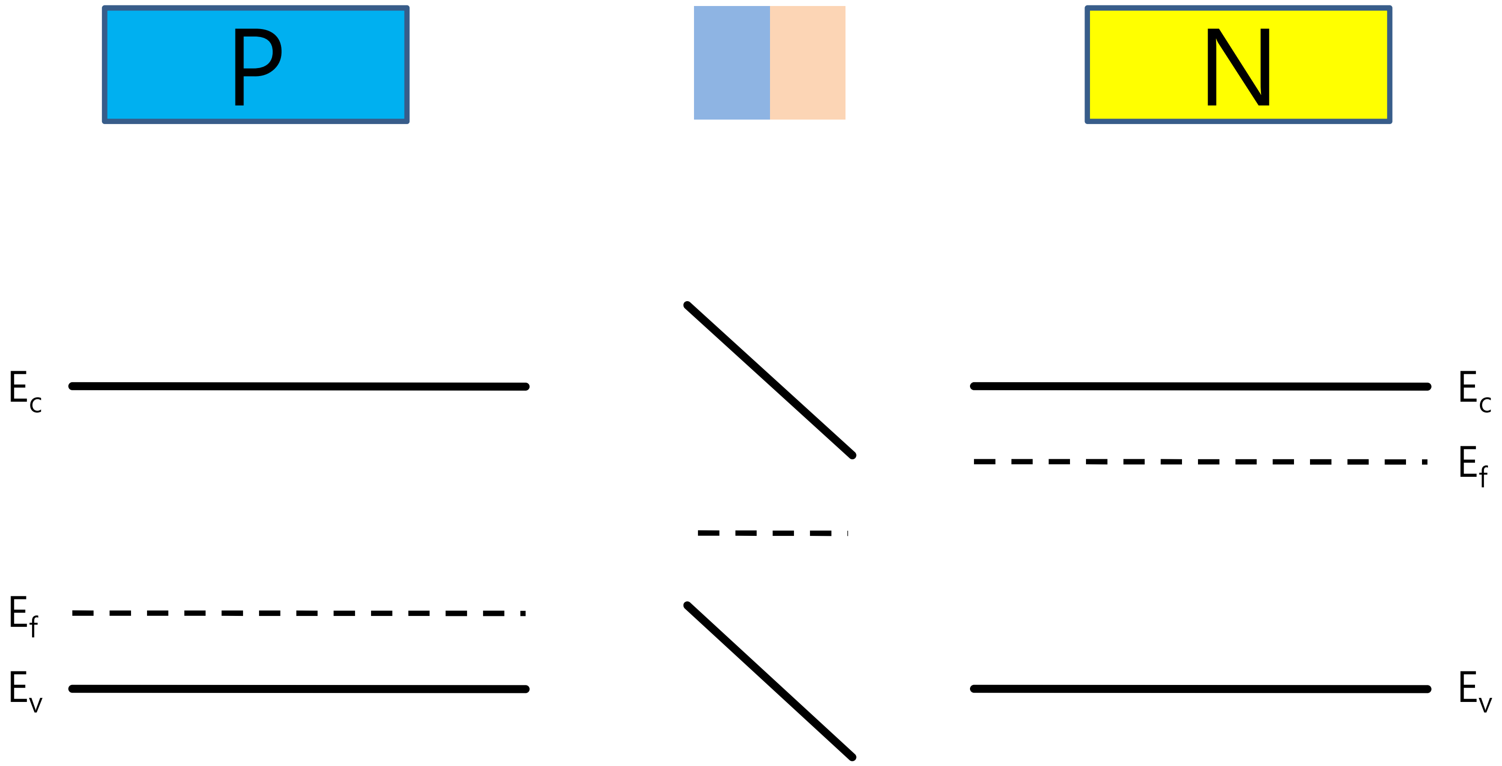
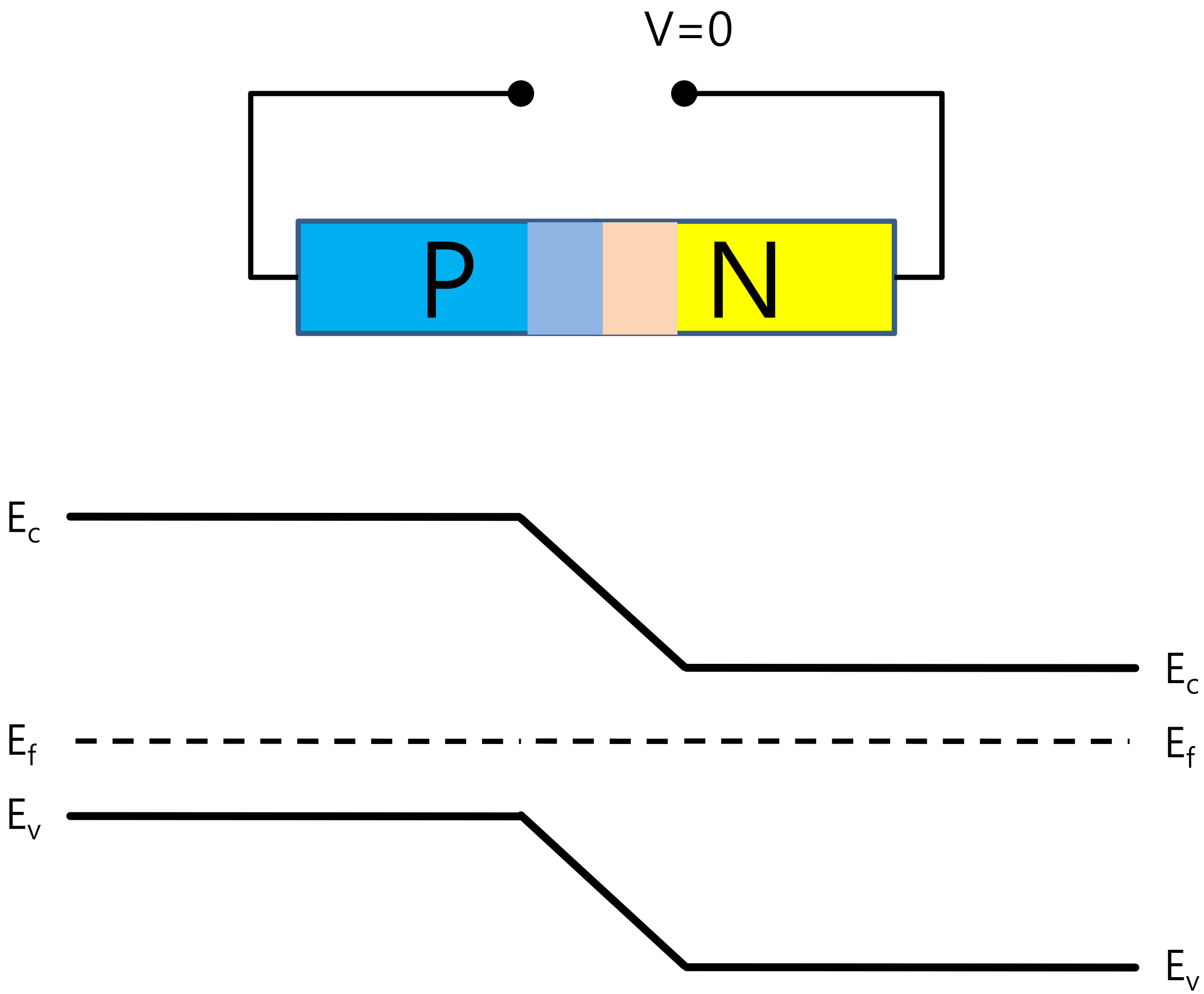


(c)

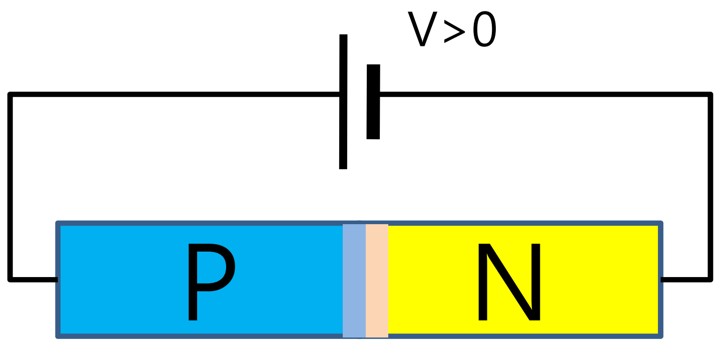
P-N Diode Junction Energy Band

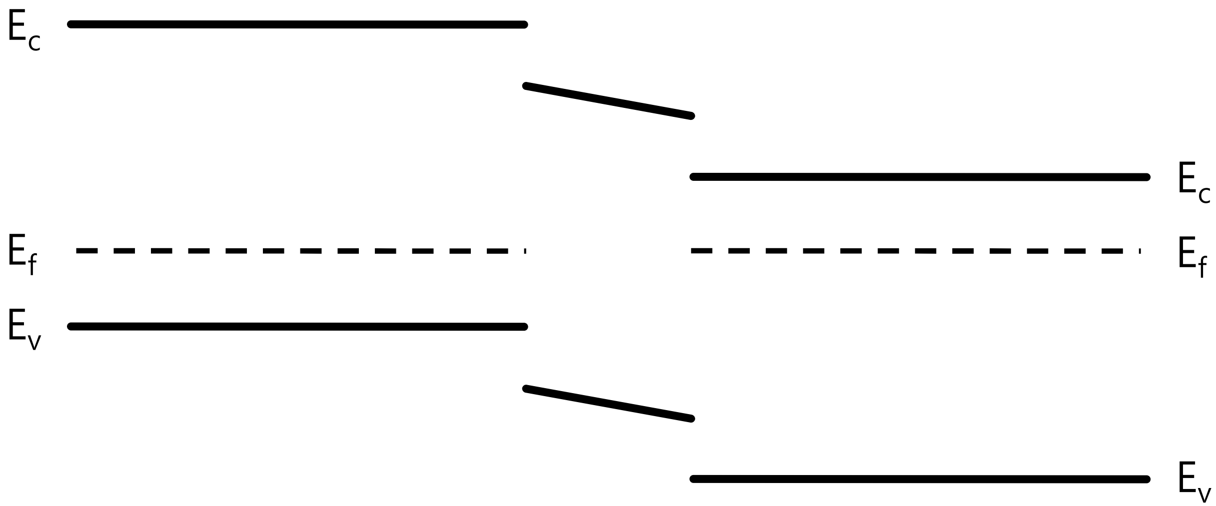


Equilibrium P-N Junction

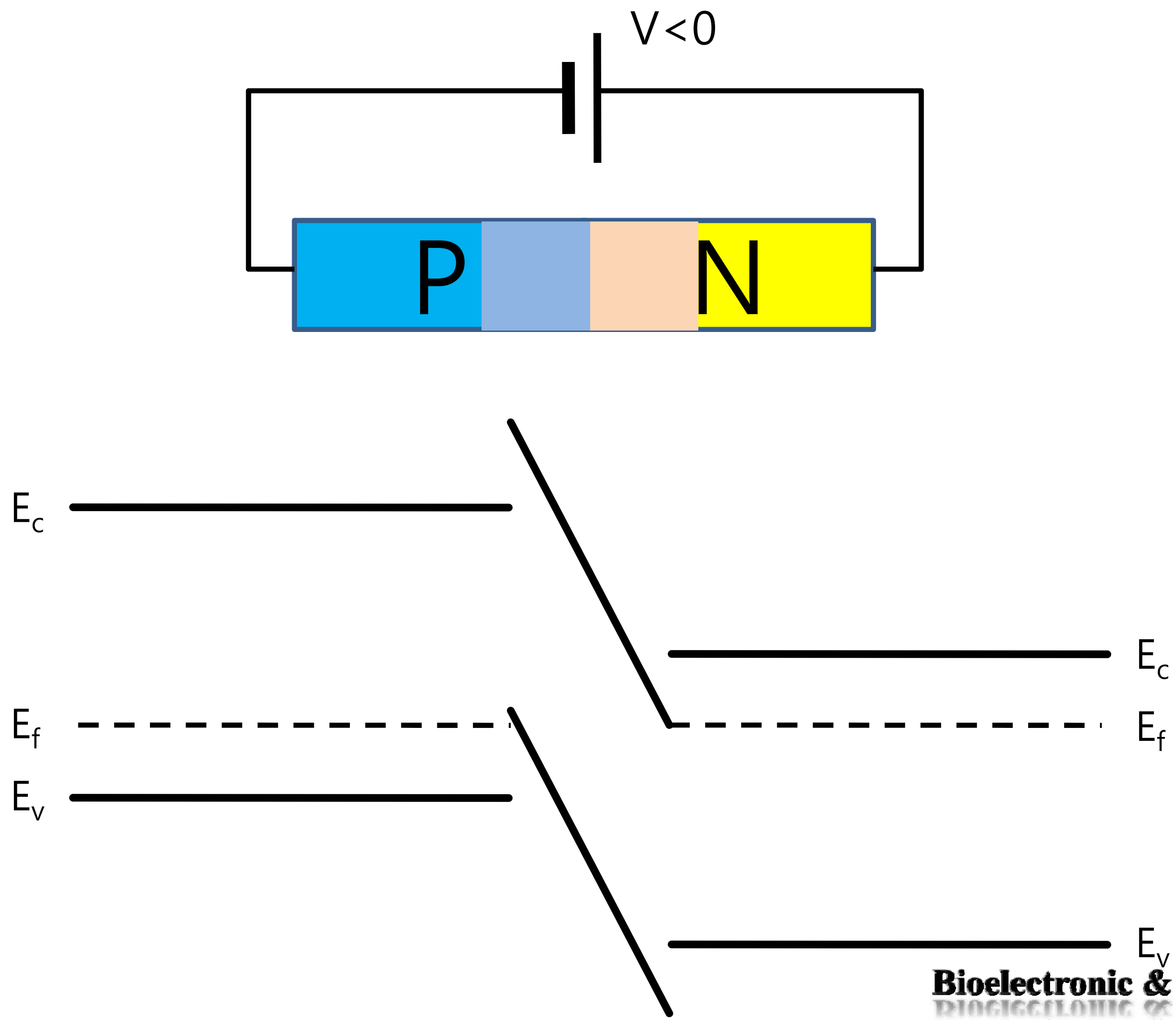


Forward Biased P-N Junction

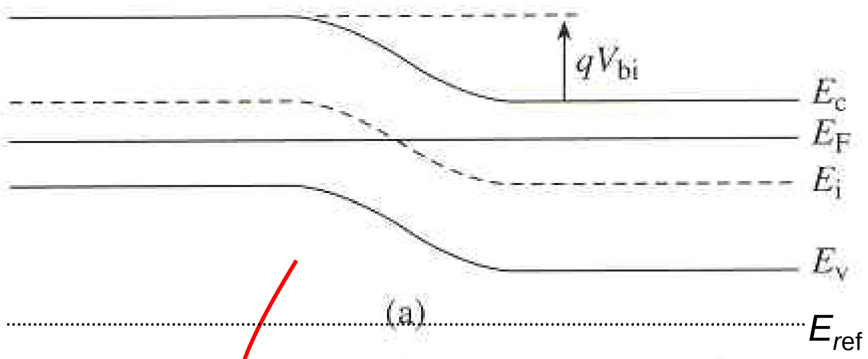




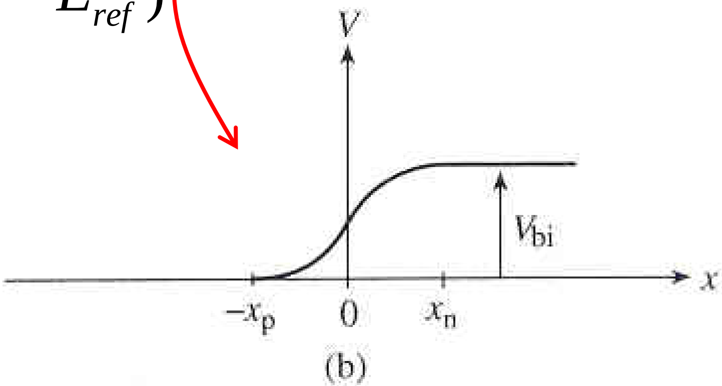
Reverse Biased P-N Junction



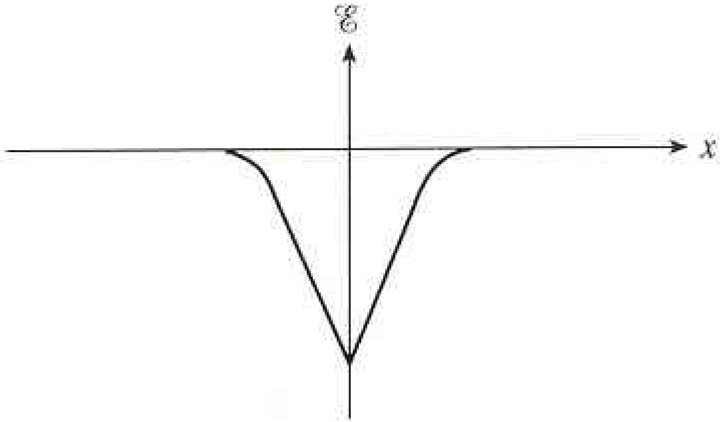
✓ V versus x relationship must have the same functional form as the " upside-down" of Ec



$$V = -\frac{1}{q}(E_c - E_{ref})$$

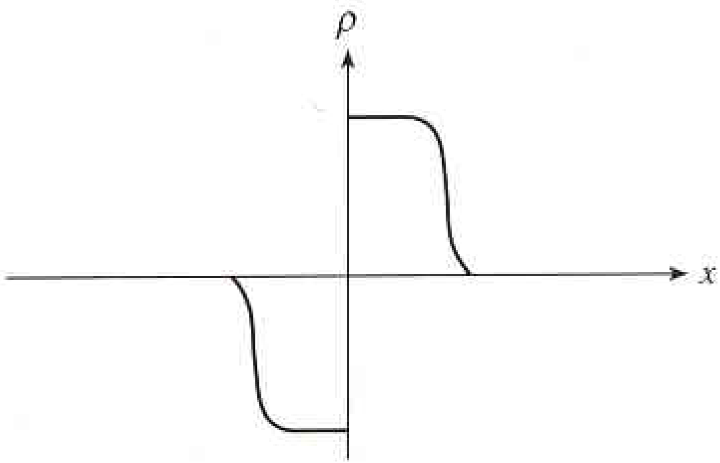


$$\mathscr{E} = -\frac{dV}{dx}$$



(c)

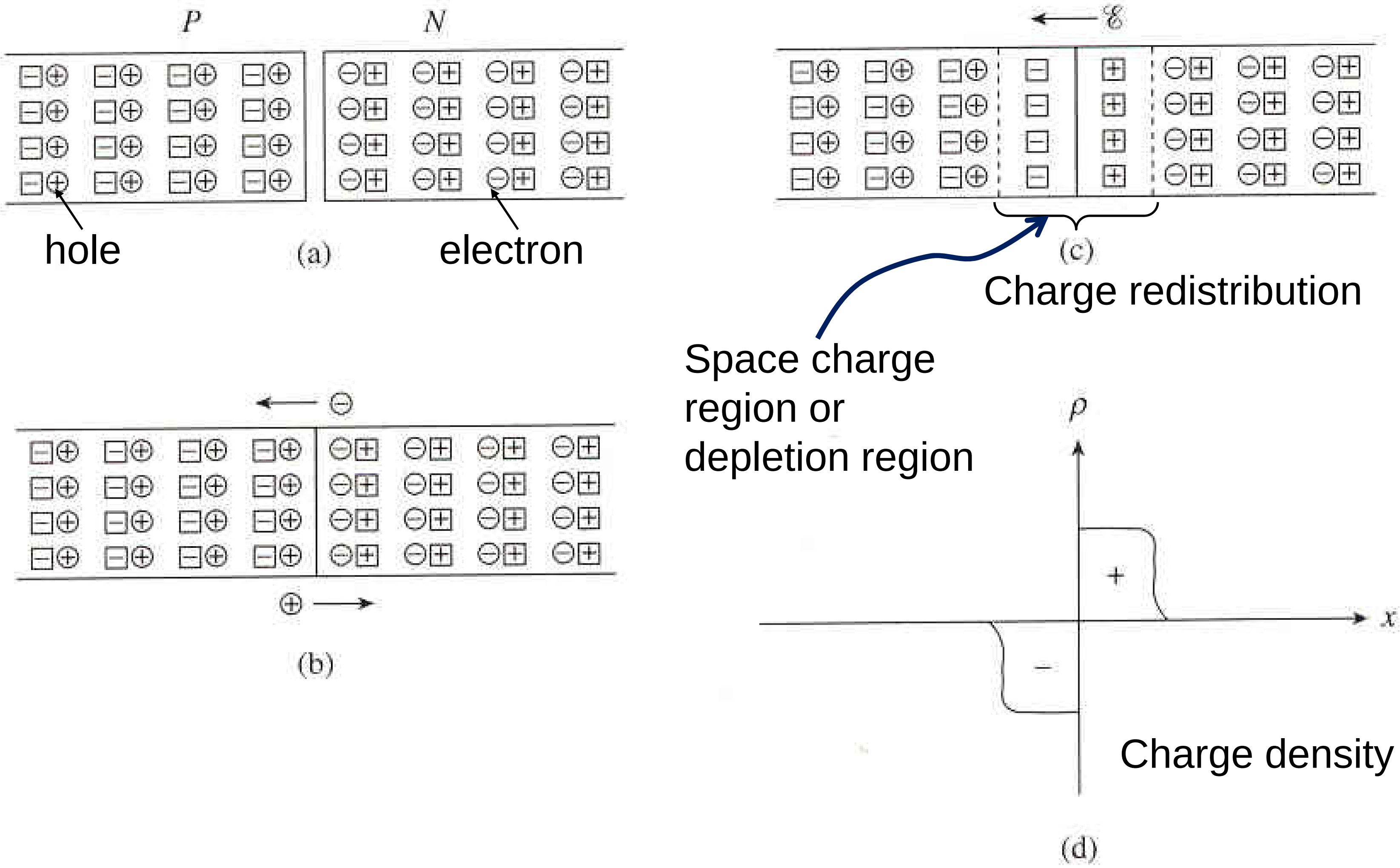
$$\frac{d\mathscr{E}}{dx} = \frac{\rho}{K_S \varepsilon_0}$$



(d)

- ✓ The voltage drop across the junction under equilibrium conditions and the appearance of charge near the metallurgical boundary
- ✓ Where does this charge come from?

✓ Charge neutrality is assumed to prevail in the isolated, uniformly doped semiconductors



✓ The build-up of charge and the associated electric field continues until the diffusion is precisely balanced by the carrier drift

✓ The individual carrier diffusion and drift components must of course cancel to make J_N and J_P separately zero

- The Built-in Potential (V_{bi})
  - ✓Consider a nondegenerately-doped junction

$$\mathscr{E} = -\frac{dV}{dx}$$

✓ Integrating

$$-\int_{-x_\mathrm{p}}^{x_\mathrm{n}} \mathscr{E}\, dx = \int_{V(-x_\mathrm{p})}^{V(x_\mathrm{n})} dV = V(x_\mathrm{n}) - V(-x_\mathrm{p}) = V_\mathrm{bi}$$

$$J_\mathrm{N} = q\mu_\mathrm{n} n\mathscr{E} + qD_\mathrm{N}\frac{dn}{dx} = 0$$

✓ Solving for $\mathscr{E}$ and making use of the Einstein relationship, we obtain

$$\mathscr{E} = -\frac{D_\mathrm{N}}{\mu_\mathrm{n}}\frac{dn/dx}{n} = -\frac{kT}{q}\frac{dn/dx}{n}$$

$$V_{\mathrm{bi}} = -\int_{-x_{\mathrm{p}}}^{x_{\mathrm{n}}} \mathscr{E}\, dx = \frac{kT}{q}\int_{n-(x_{\mathrm{p}})}^{n(x_{\mathrm{n}})} \frac{dn}{n} = \frac{kT}{q}\ln\left[\frac{n(x_{\mathrm{n}})}{n(-x_{\mathrm{p}})}\right]$$

$$n(x_{\mathrm{n}}) = N_{\mathrm{D}}, \quad n(-x_{\mathrm{p}}) = \frac{n_{\mathrm{i}}^2}{N_{\mathrm{A}}}$$

$$V_{\mathrm{bi}} = \frac{kT}{q}\ln\left(\frac{N_{\mathrm{A}} N_{\mathrm{D}}}{n_{\mathrm{i}}^2}\right)$$

- The Depletion Approximation
  - ✓ It is very hard to solve

$$\frac{d\mathscr{E}}{dx} = \frac{\rho}{K_s\varepsilon_0} = \frac{q}{K_s\varepsilon_0}(p - n + N_D - N_A)$$

(1) The carrier concentrations are negligible in $-x_p \le x \le x_n$

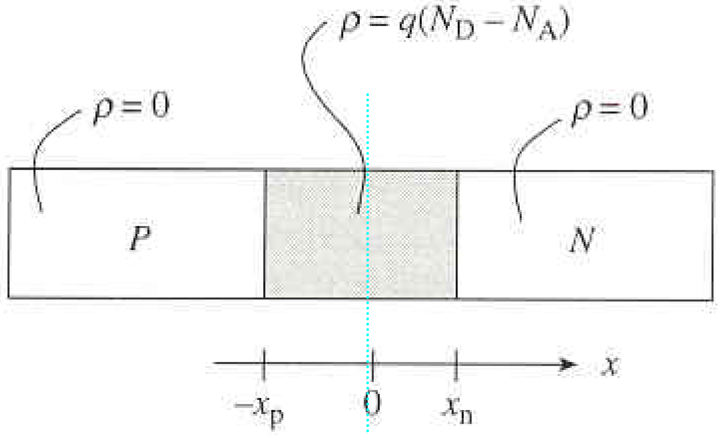
(2) The charge density outside the depletion region=0

✓ Exact

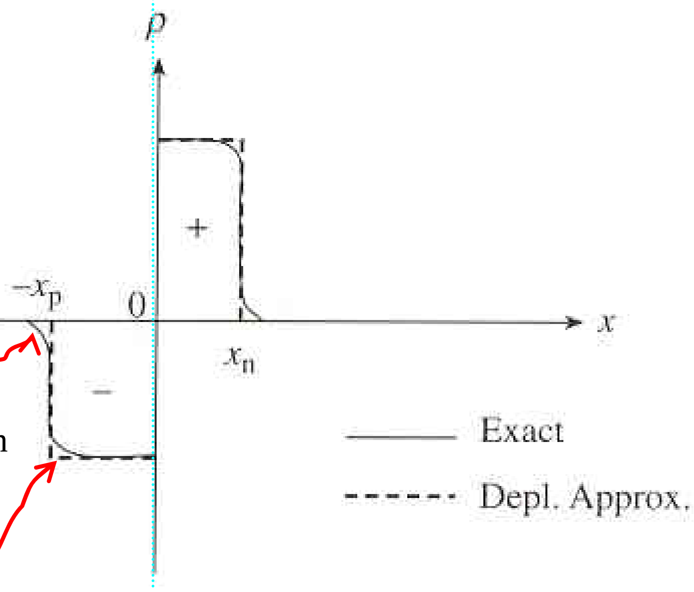
$$\frac{d\mathscr{E}}{dx} = \frac{\rho}{K_S \varepsilon_0} = \frac{q}{K_S \varepsilon_0}(p - n + N_D - N_A)$$

✓ Depletion Approximation

$$\frac{d\mathscr{E}}{dx} \cong \begin{cases} \dfrac{q}{K_S \varepsilon_0}(N_D - N_A) & \ldots -x_p \le x \le x_n \\ 0 & \ldots\ x \le -x_p \text{ and } x \ge x_n \end{cases}$$



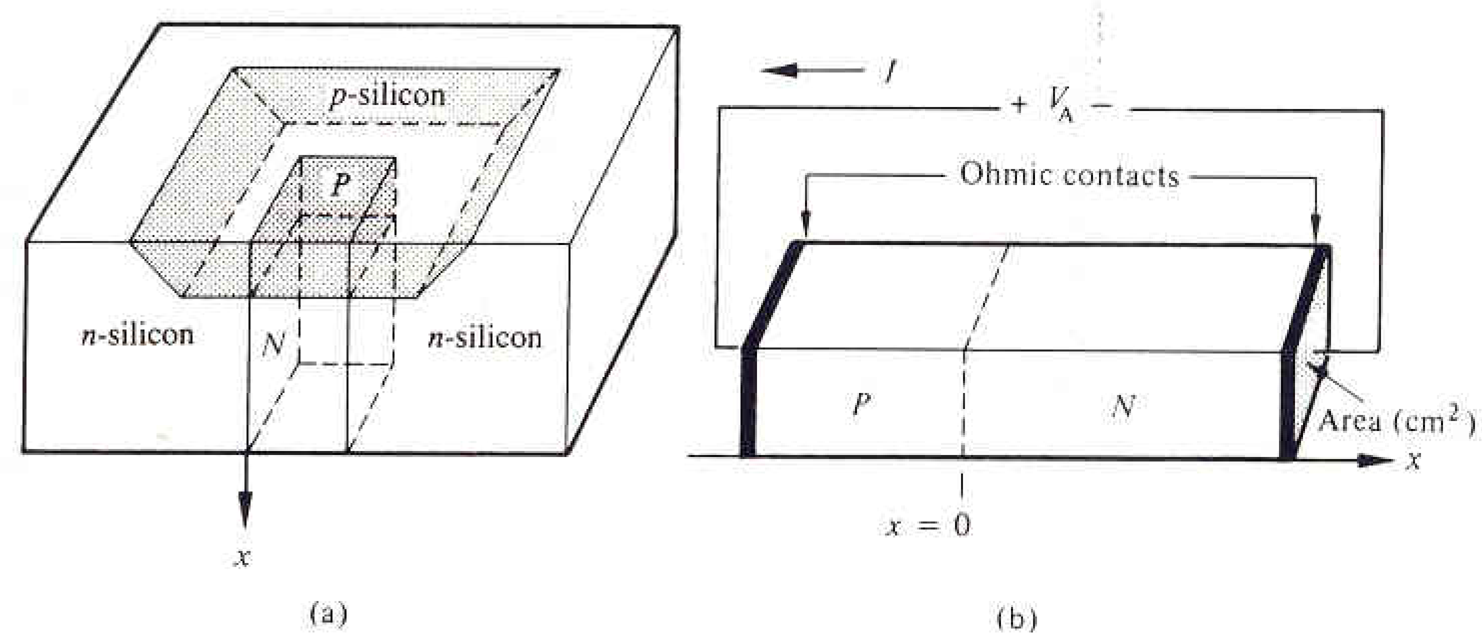
(a)



(b)

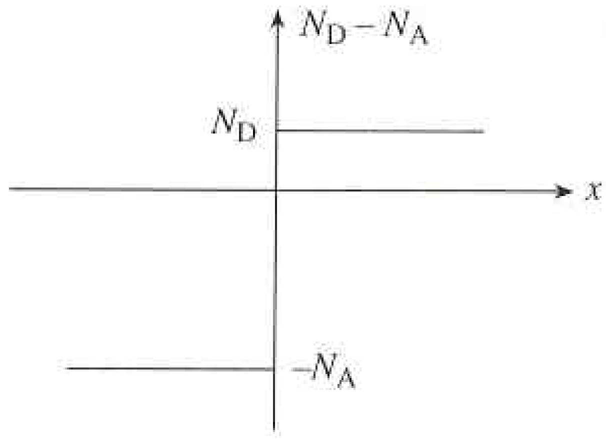
❑ Quantitative Electrostatic Relationships

• Assumptions/definitions



- Step Junction with V_A=0
  - ✓ Solution for ρ

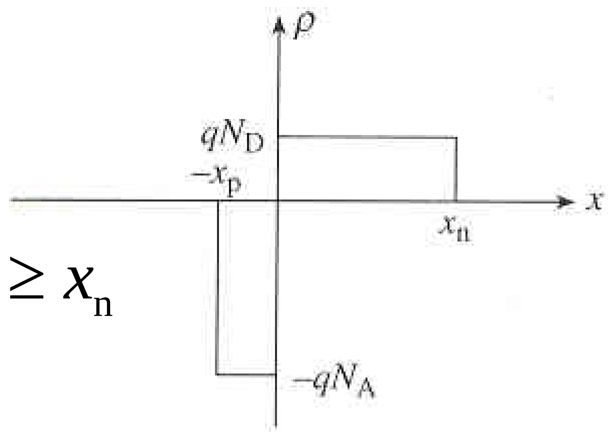
$$\rho \begin{cases} -qN_A & \ldots -x_p \le x \le 0 \\ qN_D & \ldots 0 \le x \le x_n \\ 0 & \ldots x \le -x_p \text{ and } x \ge x_n \end{cases}$$



(a)

  - ✓ Solution for $\mathscr{E}$

$$\frac{d\mathscr{E}}{dx} \begin{cases} -qN_A / K_S\varepsilon_0 & \ldots -x_p \le x \le 0 \\ qN_D / K_S\varepsilon_0 & \ldots 0 \le x \le x_n \\ 0 & \ldots x \le -x_p \text{ and } x \ge x_n \end{cases}$$



(b)

✓ For the *p*-side of the depletion region

$$\int_0^{\mathscr{E}(x)} d\mathscr{E}' = -\int_{-x_p}^{x} \frac{qN_A}{K_S\varepsilon_0} dx'$$

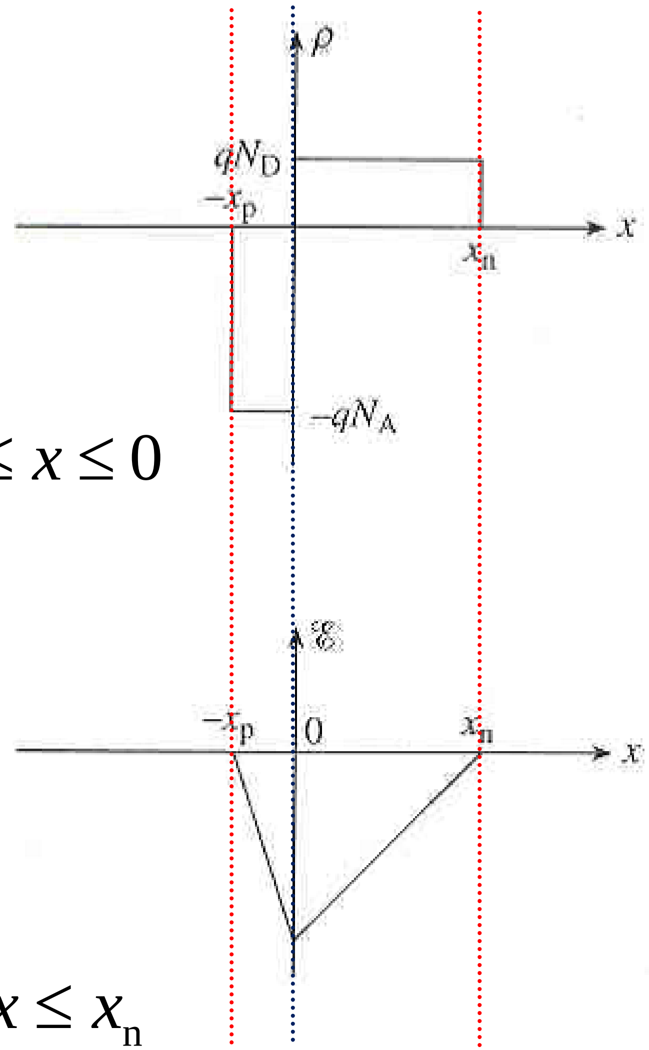
$$\mathscr{E}(x) = -\frac{qN_A}{K_S\varepsilon_0}(x_p + x) \quad \ldots -x_p \le x \le 0$$

✓ Similarly on the n-side

$$\int_{\mathscr{E}(x)}^{0} d\mathscr{E}' = -\int_{x}^{x_n} \frac{qN_D}{K_S\varepsilon_0} dx'$$

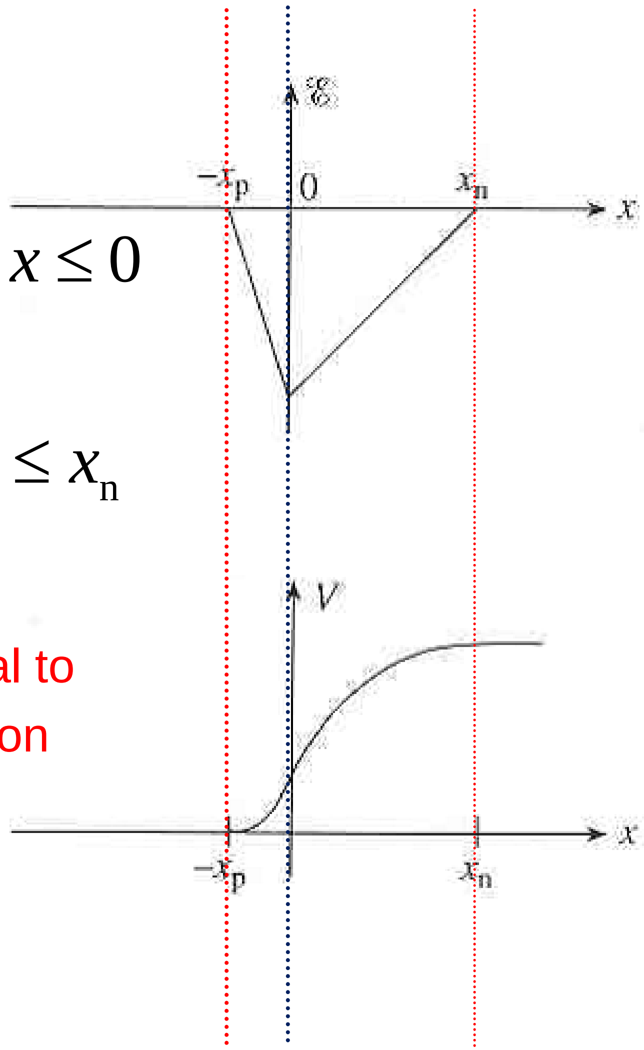
$$\mathscr{E}(x) = -\frac{qN_D}{K_S\varepsilon_0}(x_n - x) \quad \ldots\ 0 \le x \le x_n$$

$$N_A x_p = N_D x_n$$



✓ Solution for V ($\mathscr{E} = -dV/dx$)

$$\frac{dV}{dx} = \begin{cases} \frac{qN_A}{K_S\varepsilon_0}(x_p + x) & \ldots -x_p \le x \le 0 \\ \frac{qN_D}{K_S\varepsilon_0}(x_n - x) & \ldots\ 0 \le x \le x_n \end{cases}$$



✓ With the arbitrary reference potential set equal to zero at x=-x_p and V_{bi} across the depletion region equilibrium conditions

$$V = 0 \quad \text{at} \quad x = -x_p$$

$$V = V_{bi} \quad \text{at} \quad x = x_n$$

✓ For the *p*-side of the depletion region

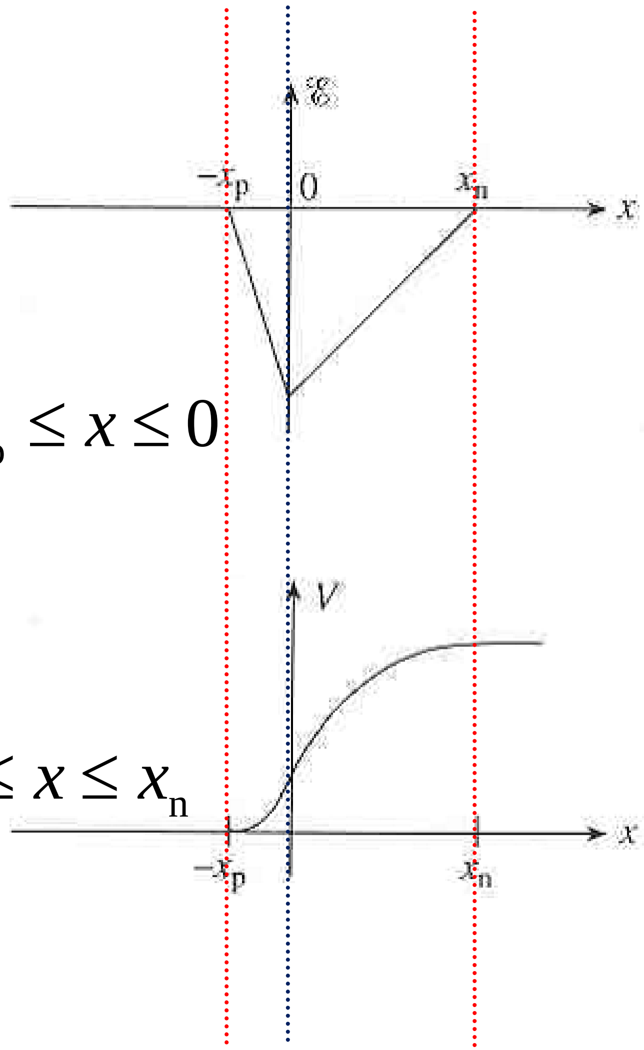
$$\int_0^{V(x)} dV' = \int_{-x_p}^{x} \frac{qN_A}{K_S\varepsilon_0}(x_p + x')dx'$$

$$V(x) = \frac{qN_A}{2K_S\varepsilon_0}(x_p + x)^2 \quad \ldots -x_p \le x \le 0$$

✓ Similarly on the n-side of the junction

$$V(x) = V_{bi} - \frac{qN_D}{2K_S\varepsilon_0}(x_n - x)^2 \quad \ldots 0 \le x \le x_n$$

$$\frac{qN_A}{2K_S\varepsilon_0}x_p^2 = V_{bi} - \frac{qN_D}{2K_S\varepsilon_0}x_n^2 \qquad \text{@ x=0}$$



✓ Solution for x_n and x_p

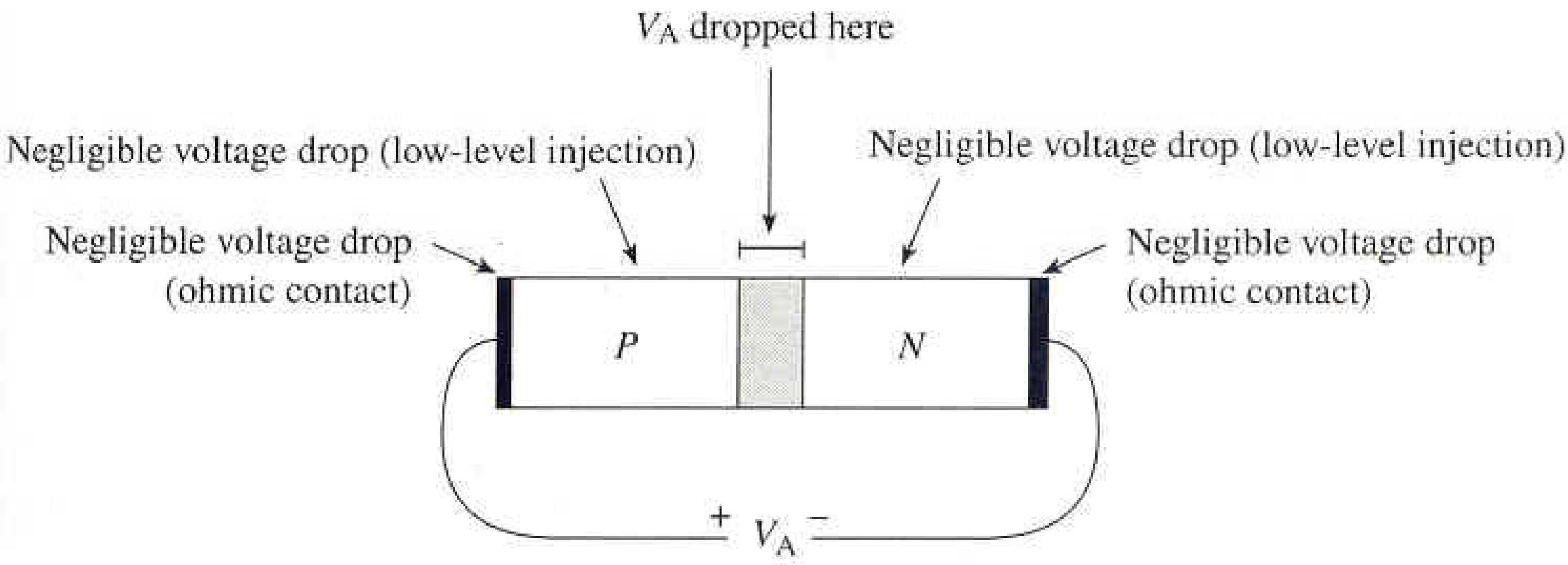
$$\because N_A x_p = N_D x_n$$

$$x_n = \left[\frac{2K_S\varepsilon_0}{q} \frac{N_A}{N_D(N_A + N_D)} V_{bi} \right]^{1/2}$$

$$x_p = \frac{N_D x_n}{N_A} \left[\frac{2K_S\varepsilon_0}{q} \frac{N_D}{N_A(N_A + N_D)} V_{bi} \right]^{1/2}$$

$$W \equiv x_n + x_p = \left[\frac{2K_S\varepsilon_0}{q} \left(\frac{N_A + N_D}{N_A N_D} \right) V_{bi} \right]^{1/2}$$ Depletion width

- Step Junction with $V_A \neq 0$
  - ✓ When $V_A > 0$, the externally imposed voltage drop lowers the potential on the n-side relative to the p-side



✓ The voltage drop across the depletion region, and hence the boundary condition at $x=x_n$, becomes $V_{bi}-V_A$

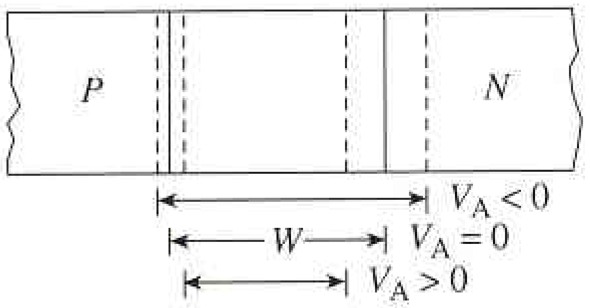
$$x_p = \left[\frac{2K_S\varepsilon_0}{q}\frac{N_D}{N_A(N_A+N_D)}(V_{bi}-V_A)\right]^{1/2}$$

$$x_n = \left[\frac{2K_S\varepsilon_0}{q}\frac{N_A}{N_D(N_A+N_D)}(V_{bi}-V_A)\right]^{1/2}$$

$$W = \left[\frac{2K_S\varepsilon_0}{q}\left(\frac{N_A+N_D}{N_AN_D}\right)(V_{bi}-V_A)\right]^{1/2}$$

- Examination/Extrapolation of Results
  - ✓ Depletion widths decrease under forward biasing and increase under reverse biasing
  - ✓A decreased depletion width when $V_A > 0$ means less charge around the junction and a correspondingly smaller $\mathcal{E}$-field. Similarly, the potential decreases at all points when $V_A > 0$
  - ✓ The Fermi level is omitted from the depletion region

$$E_{Fp} - E_{Fn} = -qV_A$$



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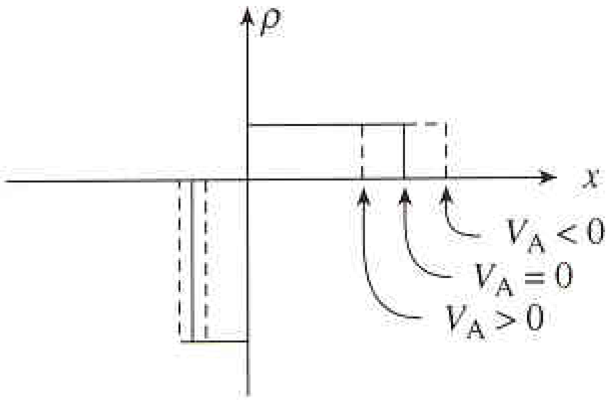

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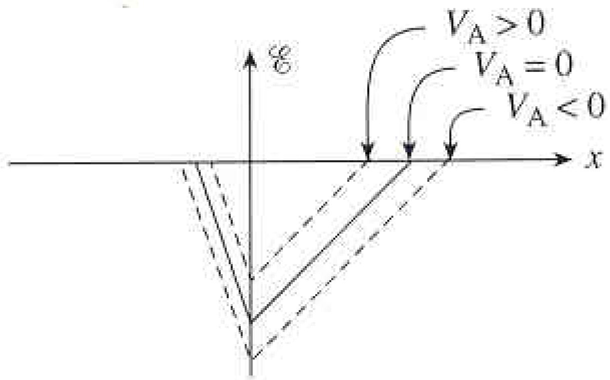

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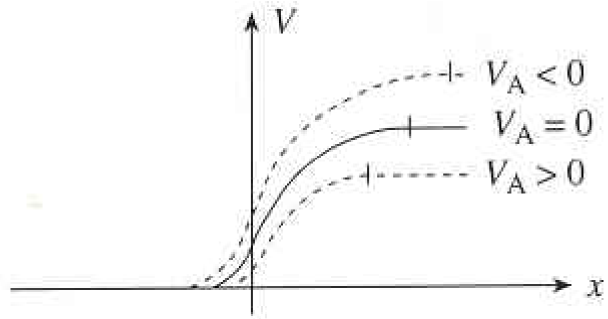

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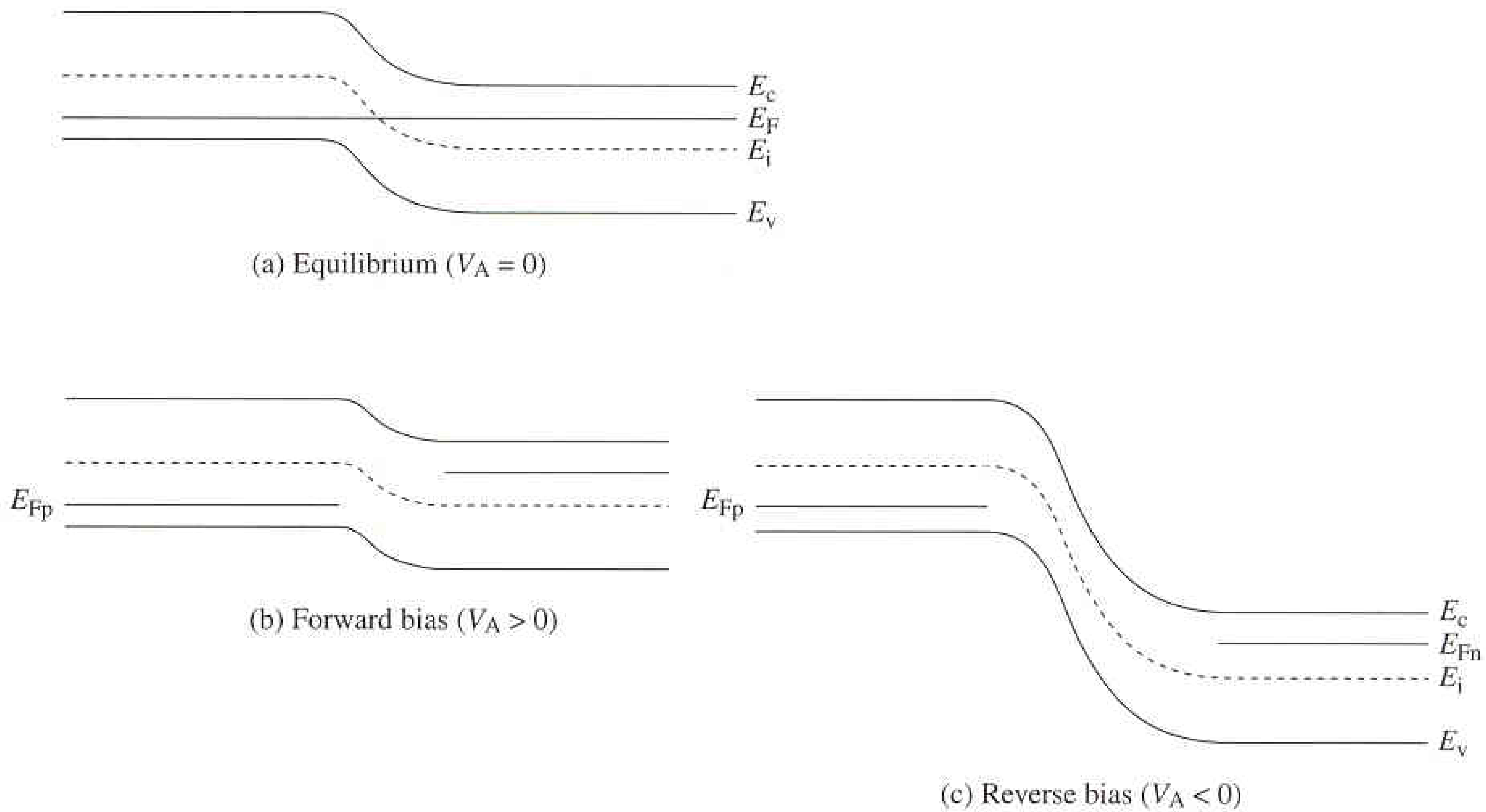

pn junction energy band diagrams.