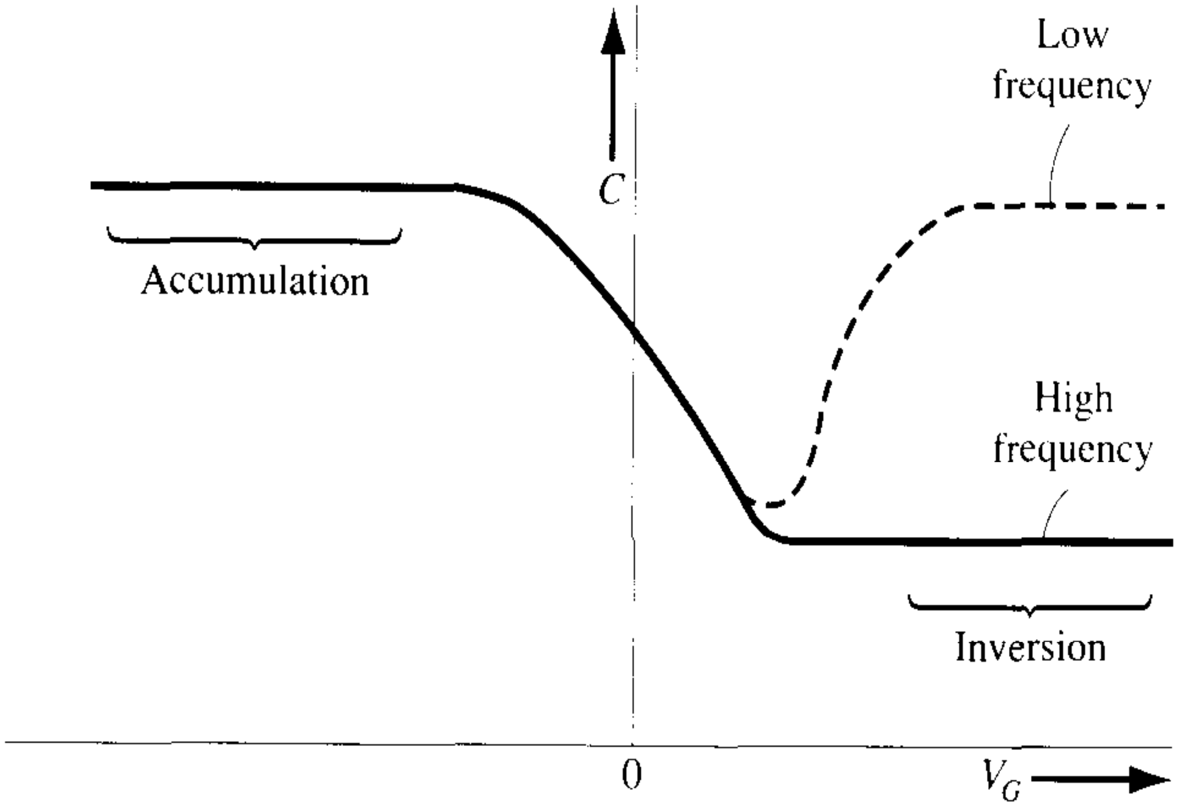
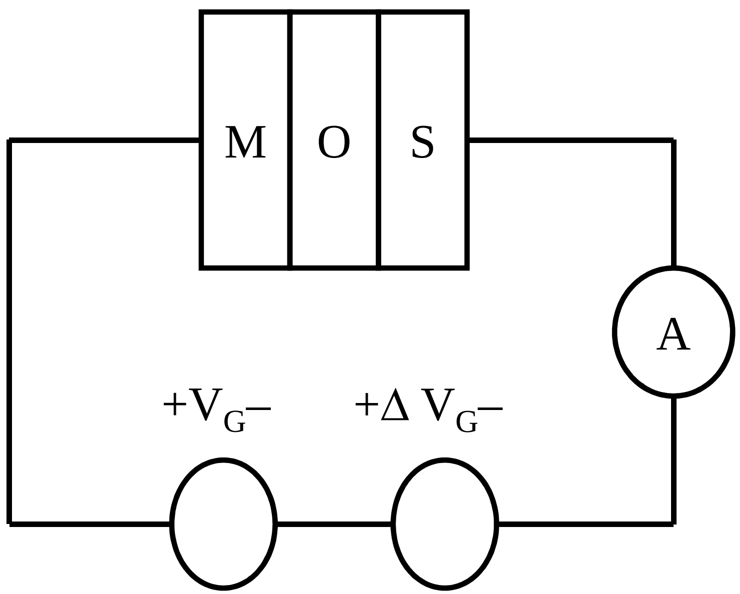
# Capacitance - Voltage Characteristics

## High- and low- frequency C-V characteristics.



- CV characteristic is of considerable practical importance.

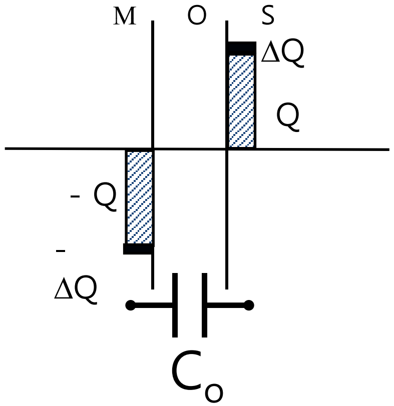



$V_G$ : Slowly changed

$\Delta V_G$ : AC signal ( high frequency/low frequency)

$$C \equiv \frac{\Delta Q}{\Delta V_G}$$

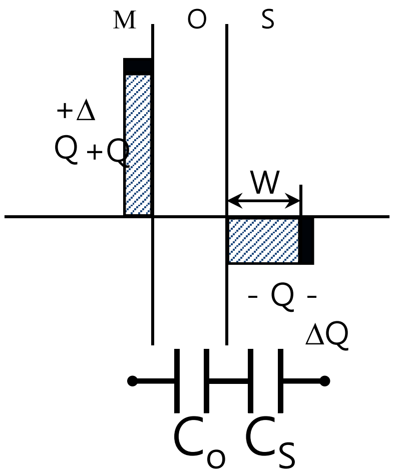
- Qualitative theory
  - Accumulation
    - The state of the system can be changed very rapidly. The majority carrier can equilibrate with a time constant on the order of $10^{-10}$ to $10^{-13}$ sec.
    - The small ac signal merely adds or subtracts a charge close to the edges of an insulator. parallel-plate capacitor, $C_O$.




$$C(acc) = C_o\text{(oxide capacitance)}$$

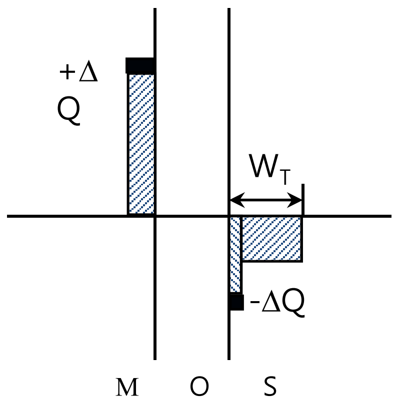
$$= \frac{K_o \varepsilon_0 A_G}{x_o}$$

– Depletion
- Withdrawal of majority carriers
- The charge state can be changed very rapidly.
- The depletion width quasi-statically fluctuates about its dc value two parallel plate capacitors ($C_o$ and $C_s$; oxide and semiconductor capacitance) in series.
- DC bias ↑ W ↑ C(depl)↓




$$C(depl) = \frac{C_O C_S}{C_O + C_S} = \frac{C_O}{1 + \dfrac{K_O W}{K_S\, x_O}}$$

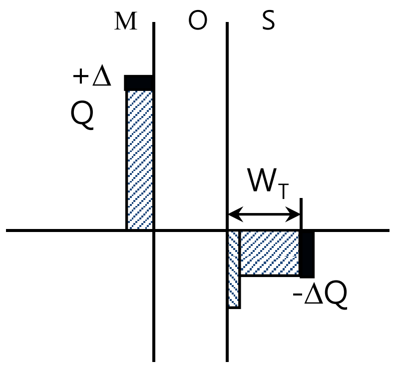
- Inversion
  - $W = W_T$
  - The charge fluctuation depends on the frequency of the AC signal
- Low frequency :
  - Minority carriers can be generated or annihilated in response to the ac signal. Just as in accumulation, charge is added or subtracted close to the edges of insulator.




$$C_{LF}(inv) = C_O$$

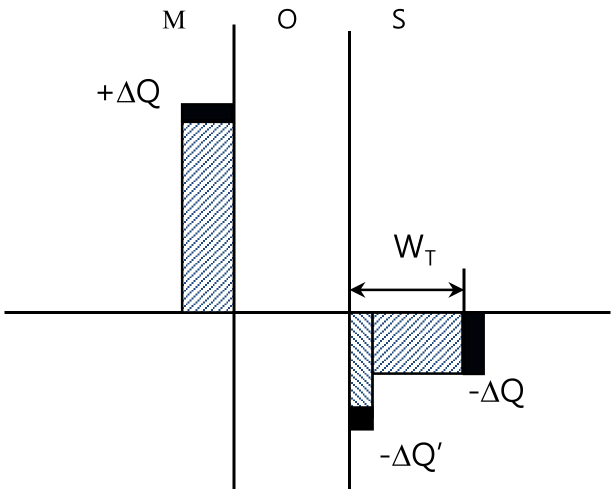
– High frequency :

- The relatively sluggish generation-recombination process can't supply or eliminate minority carriers in response to the ac signal. The number of minority carriers in the inversion layer remains fixed and the depletion width fluctuates about the $W_T$ dc value. Two parallel-plate capacitors in series.
- Since $W_T$ *=constant*, $C_{HF}(inv) = C(depl)_{minimum} = constant$




$$C_{HF}(inv) = \frac{C_o C_s}{C_o + C_s} = \frac{C_o}{1 + \dfrac{K_o W_T}{K_s x_o}}$$

– For medium frequency
  - A portion of the inversion layer can be created/annihilated.
  - $C_{HF}(inv) < C_{MF}(inv) < C_{LF}(inv)$



- Delta-Depletion Analysis (skip)
  - Depletion bias

$$W = \frac{K_s}{K_o} x_o \left[ \sqrt{1 + \frac{V_G}{V_\delta}} - 1 \right]$$

$$\text{where} V_\delta \equiv \frac{q}{2} \frac{K_s x_o}{K_o{}^2 \varepsilon_o} N_A$$

$$C = \frac{C_o}{\sqrt{1 + \frac{V_G}{V_\delta}}}$$

$$W = \left[ \frac{2K_S \varepsilon_0}{qN_A} \phi_S \right]^{1/2} \longrightarrow \phi_S = \frac{qN_A}{2K_S \varepsilon_0} W^2$$

$$V_G = \phi_S + \frac{K_s}{K_o} x_o \sqrt{\frac{2qN_A}{K_s \varepsilon_0} \phi_S}$$

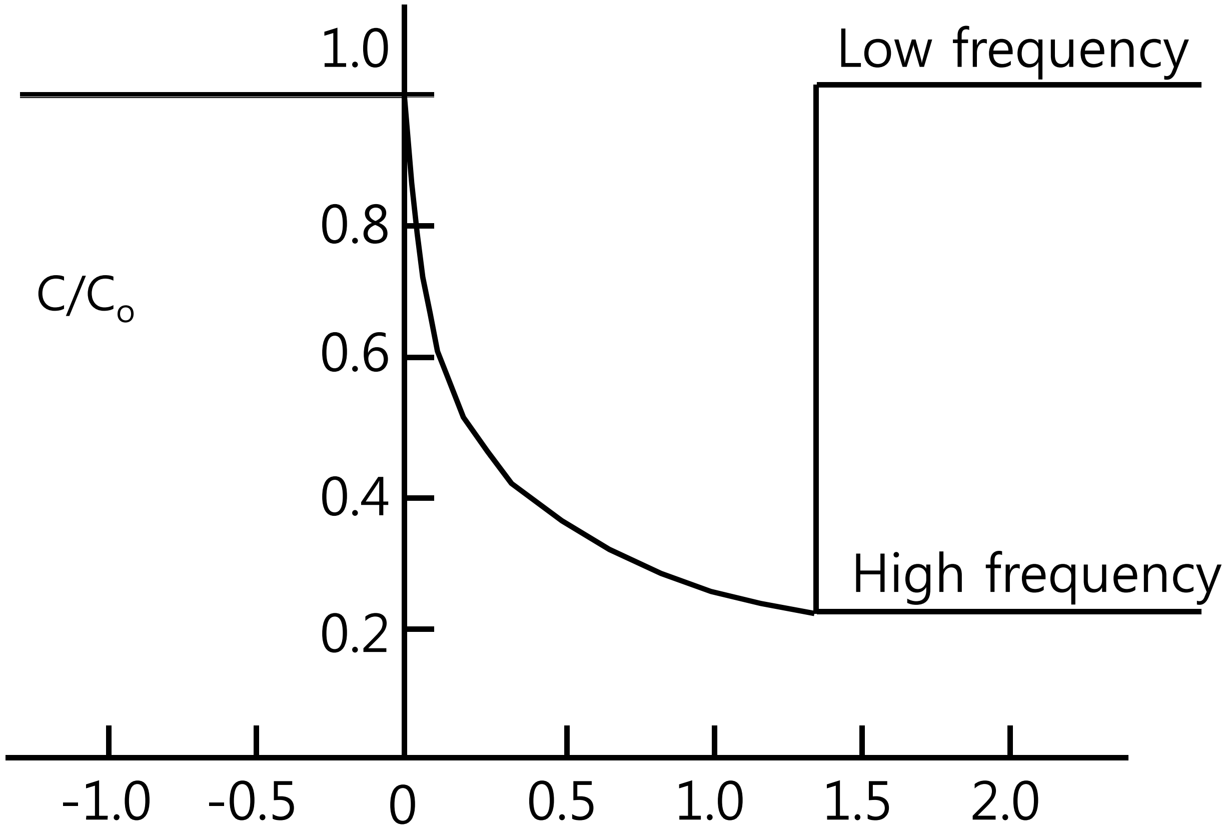
$$\frac{qN_A}{2K_s \varepsilon_0} W^2 + \frac{K_s}{K_o} x_o \frac{qN_A}{K_s \varepsilon_0} W - V_G = 0$$

$$W = -\frac{K_s}{K_o} x_o \pm \sqrt{\left( \frac{K_s}{K_o} x_o \right)^2 + \frac{2K_s \varepsilon_0}{qN_A} V_G}$$

$$W = \frac{K_s}{K_o} x_o \left[ + \sqrt{1 + \frac{2K_s \varepsilon_0 K_o}{qN_A K_s x_o} V_G} - 1 \right]$$

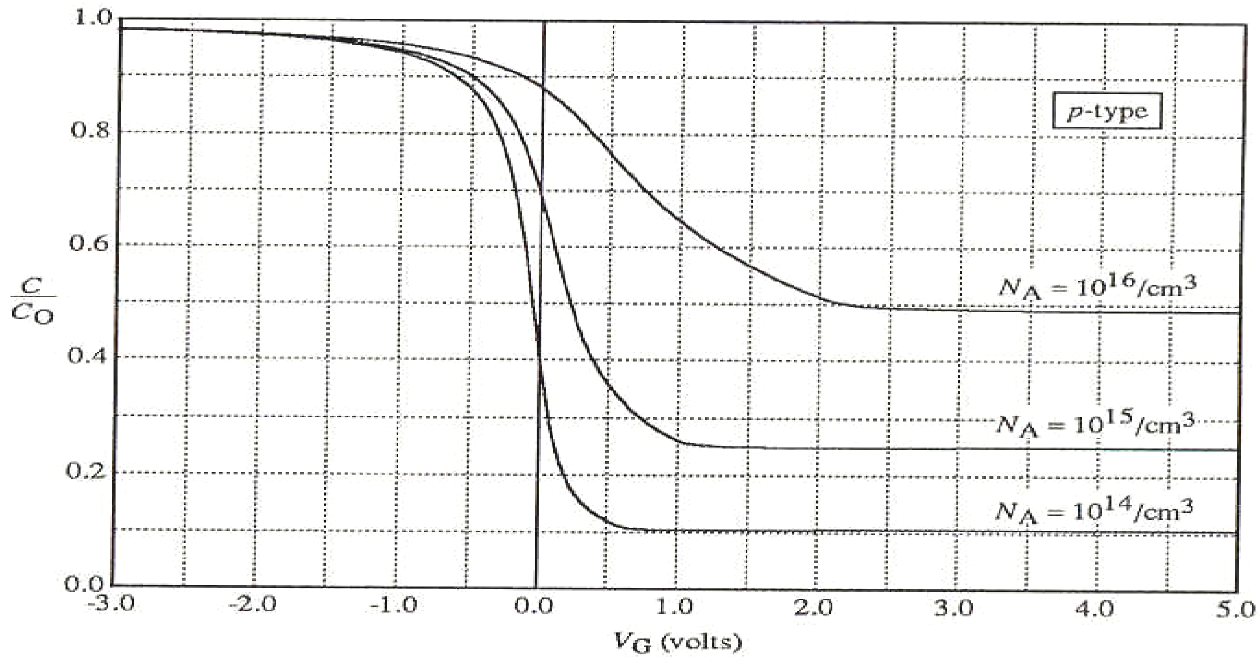
# the delta-depletion theory

## ($x_o$=0.01μm, $N_A$=$10^{17}$/cm$^3$,T=300K )



- Exact Calculation
  - Doping dependence

With increased doping, the high-frequency inversion capacitance increases significantly and the depletion bias region widens substantially.



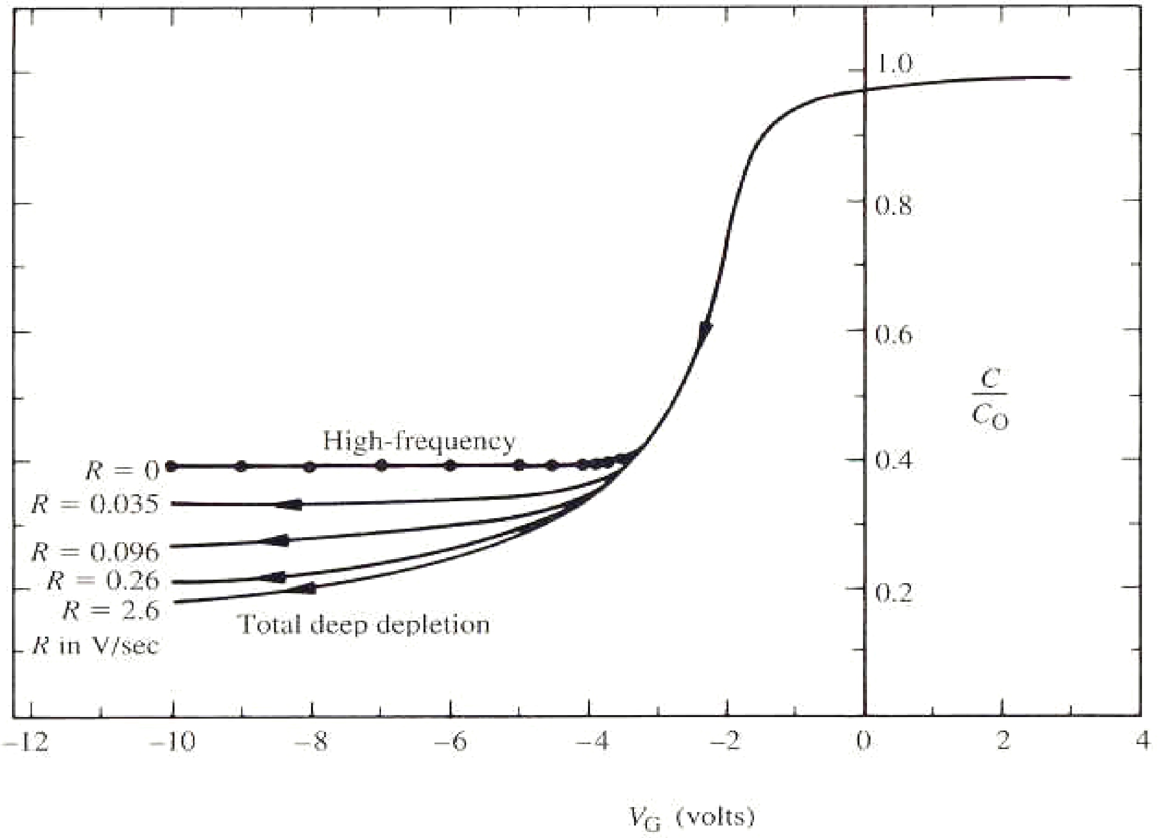
- Low frequency characteristics
  - Given modern-day MOS-Cs with long carrier lifetimes and low carrier generation rates, even freq. as low as several Hz will yield high-freq. C-V.
  - If a low-freq. characteristic is required, the quasi-static technique must be employed.
  - The quasi-static displacement current flowing through the device is proportional to the low-freq. capacitance.

$$C = \frac{\Delta Q}{\Delta V} = \frac{I \Delta t}{\Delta V} = \frac{I}{\Delta V / \Delta t} = \frac{I}{R}$$

(I : displacement current, R : voltage ramp rate)

- High frequency characteristics
  - Normal measurement frequency ~ 1MHz.
  - In the ramped-measurement, a deep-depletion phenomenon may appear.

- Deep depletion




Note that at even the slowest ramp rates one does not properly plot out the inversion portion of the high-freq. characteristic.

- In accumulation or depletion, only majority carriers are involved Charge configuration rapidly reacts to the changing gate bias.
- In inversion region, minority carriers should be generated
- The generation process is sluggish and has difficulty supplying the minority carriers needed for the structure to equilibrate.

– Deep depletion : the nonequilibrium condition where there is a deficit of minority carriers and a depletion width in excess of $W_T$ $C < C_{HF}(inv)$

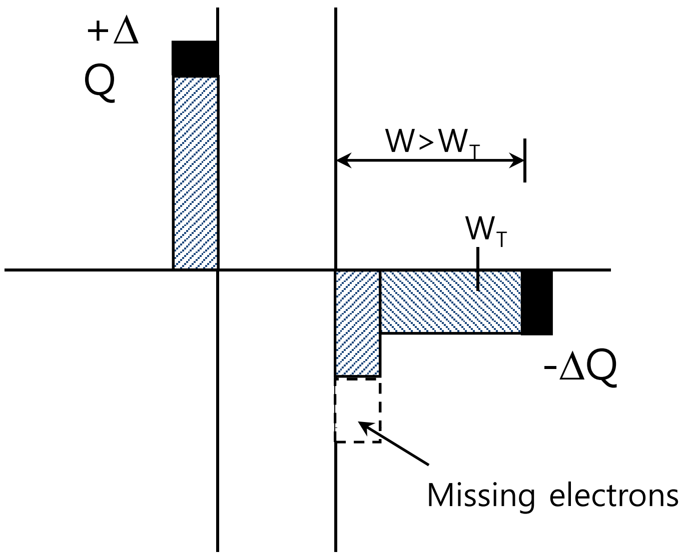
– Ramp rate ↑ C ↓

– The limiting case occurs when the semiconductor is totally devoid of minority carriers - totally deep depleted.

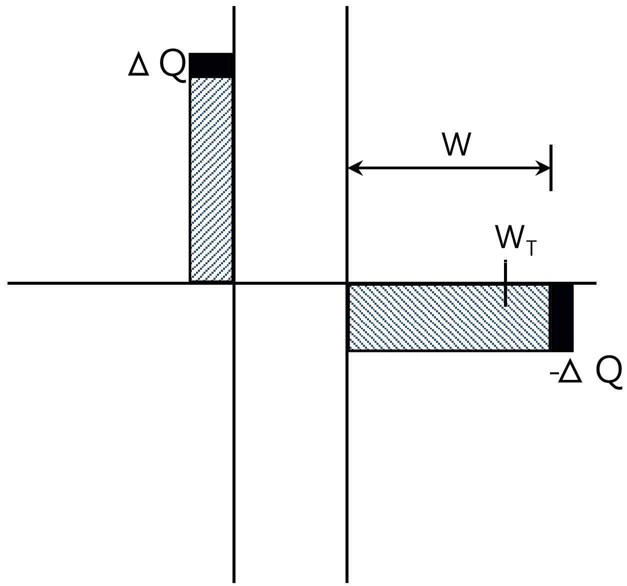
$$C = \frac{C_0}{\sqrt{1 + \frac{V_G}{V_\delta}}}$$

Same form as simple depletion case

# Non-equilibrium charge configuration inside an p-type MOS-C




(a) under deep depletion




(b) under total deep depletion