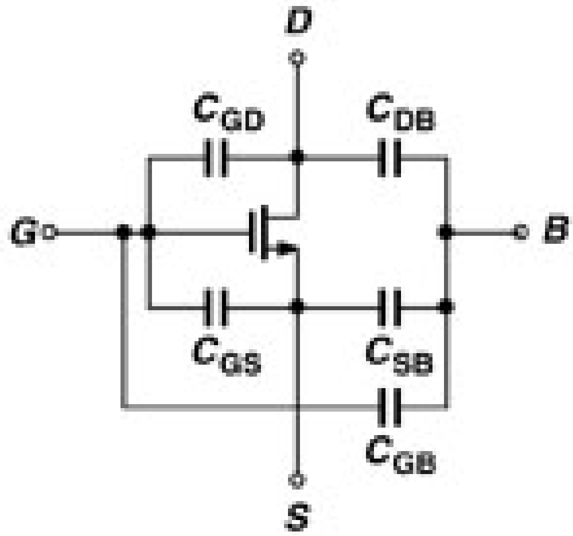
Chapter 6
Frequency Response of Amplifiers

MOSFET Capacitances-L22

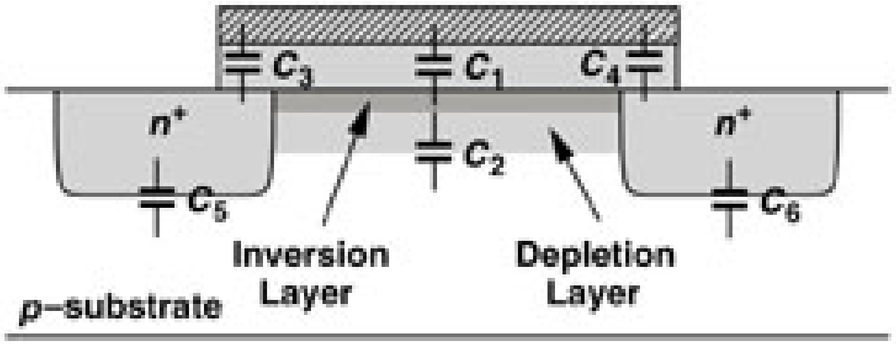
Device Capacitances

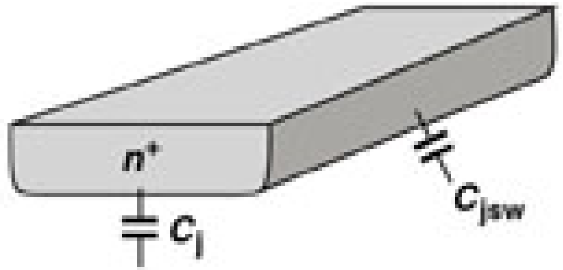


There is some capacitance between every pair of MOSFET terminals.

Only exception: We neglect the capacitance between Source and Drain.

Gate to Substrate capacitance





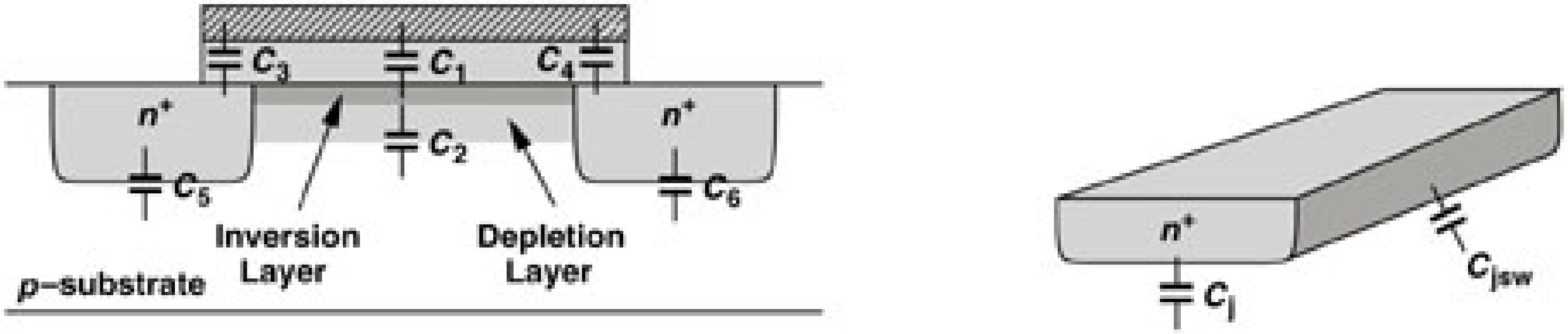
C_1 is the oxide capacitance (between Gate and channel)

$C_1 = WLC_{OX}$

C_2 is the depletion capacitance between channel and Substrate

$C_2 = WL(q\varepsilon_{si}N_{sub}/(4\Phi_F))^{1/2} = C_d$

Gate-Drain and Gate-Source Overlap Capacitance

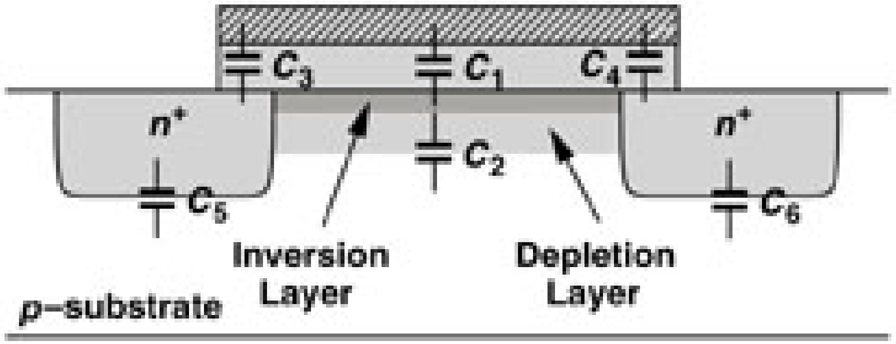


C_3 and C_4 are due to overlap between the gate poly-silicon and the Source and Drain regions.

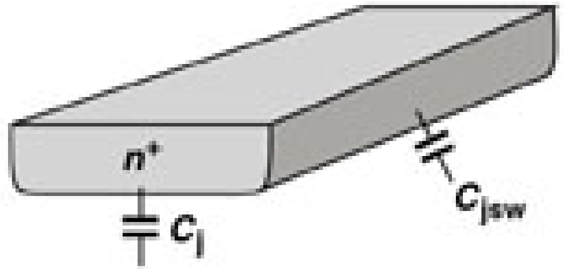
No simple formulas for C_3 and C_4: It is incorrect to write $C_3 = C_4 = WL_D C_{OX}$ because of fringing electric field lines.

We denote the overlap capacitance per unit width as C_{ov}

Source-Substrate and Drain-Substrate Junction Capacitances



(a)



(b)

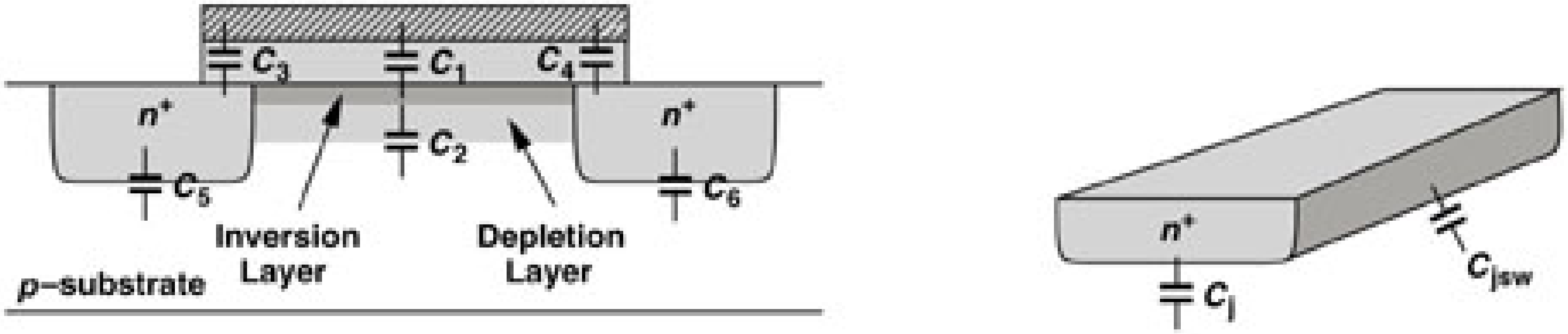
Again, no simple formulas for C_5 and C_6:

See right figure: Each capacitance should be decomposed into two components:

Bottom-plate capacitance, denoted as C_j, which depends on the junction area.

Side-wall capacitance C_{jsw} which depends on the perimeter of the junction.

Source-Substrate and Drain-Substrate Junction Capacitances

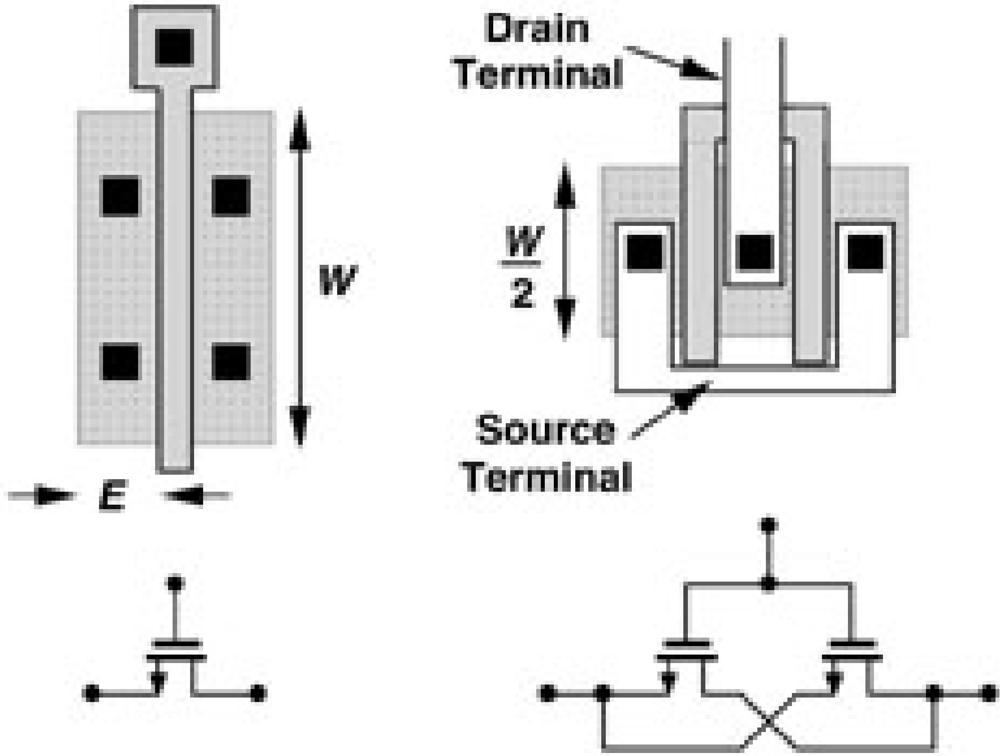


(a) (b)

In MOSFET models C_j and C_{jsw} are usually given as capacitance-per-unit-area, and capacitance-per-unit-length, respectively.

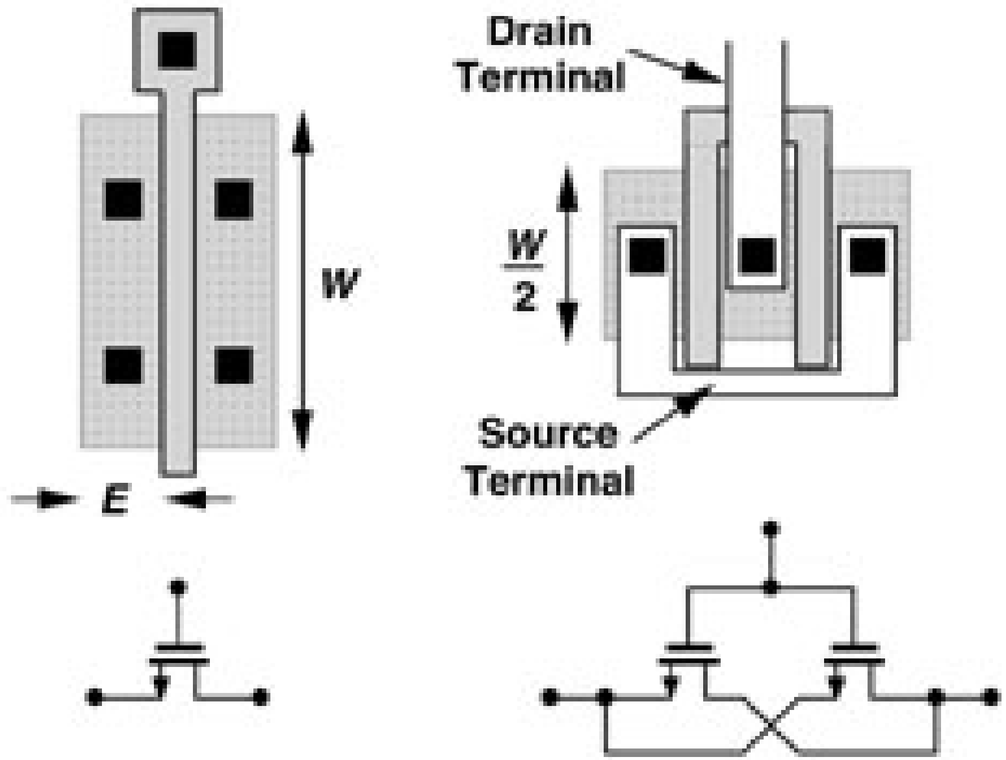
$C_j = C_{j0} / [1+V_R/\Phi_B]^m$ where V_R is the junction's reverse voltage, Φ_B is the junction built-in potential, and m typically ranges from 0.3 to 0.4

Examples (both transistors have the same C_j and C_{jsw} parameters)



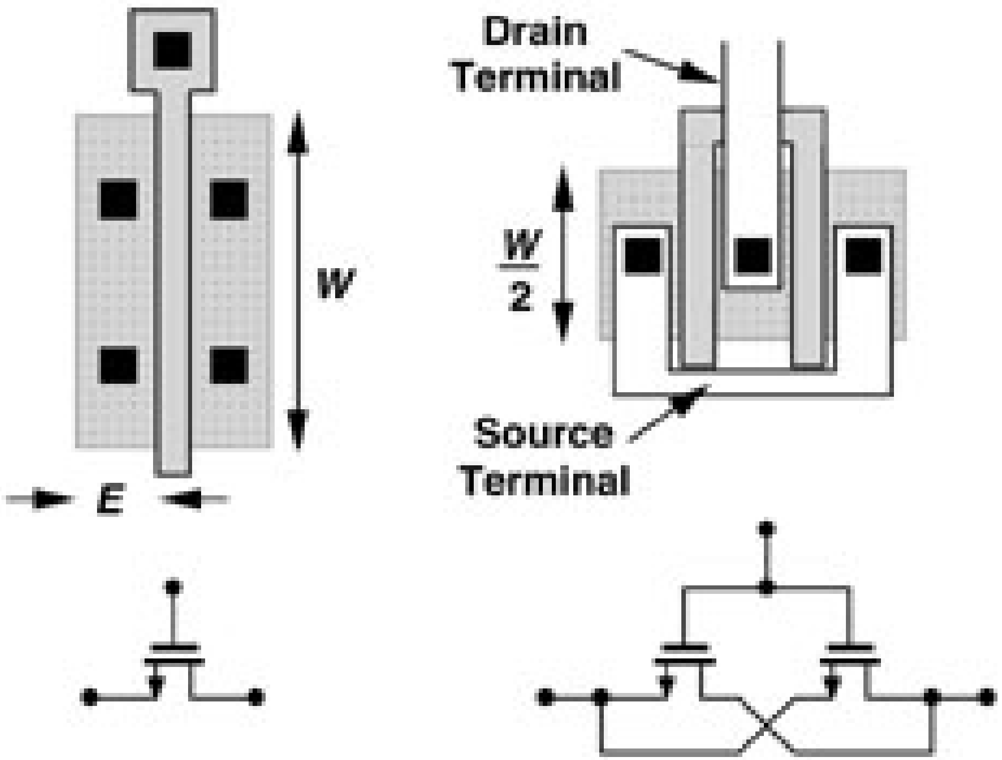
Calculate C_{DB} and C_{SB} for each structure

Left structure



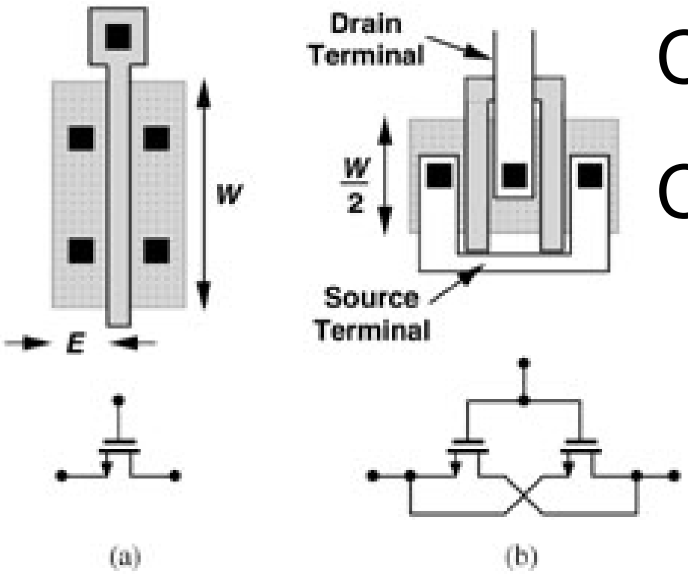
$C_{DB}=C_{SB} = WEC_j + 2(W+E)C_{jsw}$

Layout for Low Capacitance (see folded structure on the right)



Two parallel transistors with one common Drain

Layout for Low Capacitance (see folded structure on the right)



$C_{DB}=(W/2)EC_j+2((W/2)+E)C_{jsw}$

$C_{SB}=2((W/2)EC_j+2((W/2)+E)C_{jsw}=$

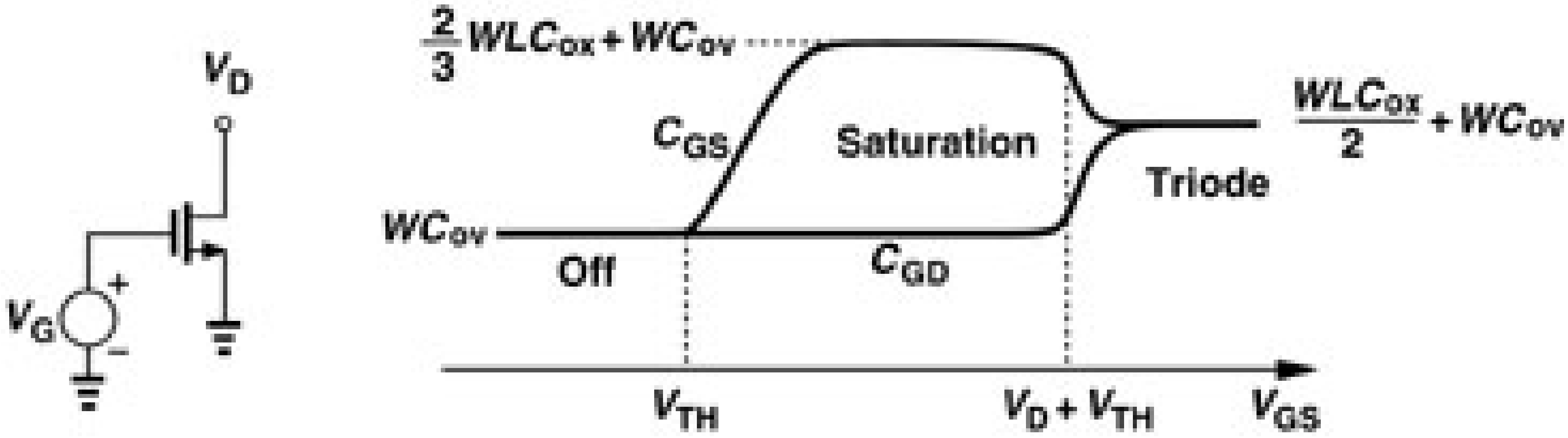
$= WEC_j +2(W+2E)C_{jsw}$

Compare to the left structure:

$C_{DB}=C_{SB} = WEC_j + 2(W+E)C_{jsw}$

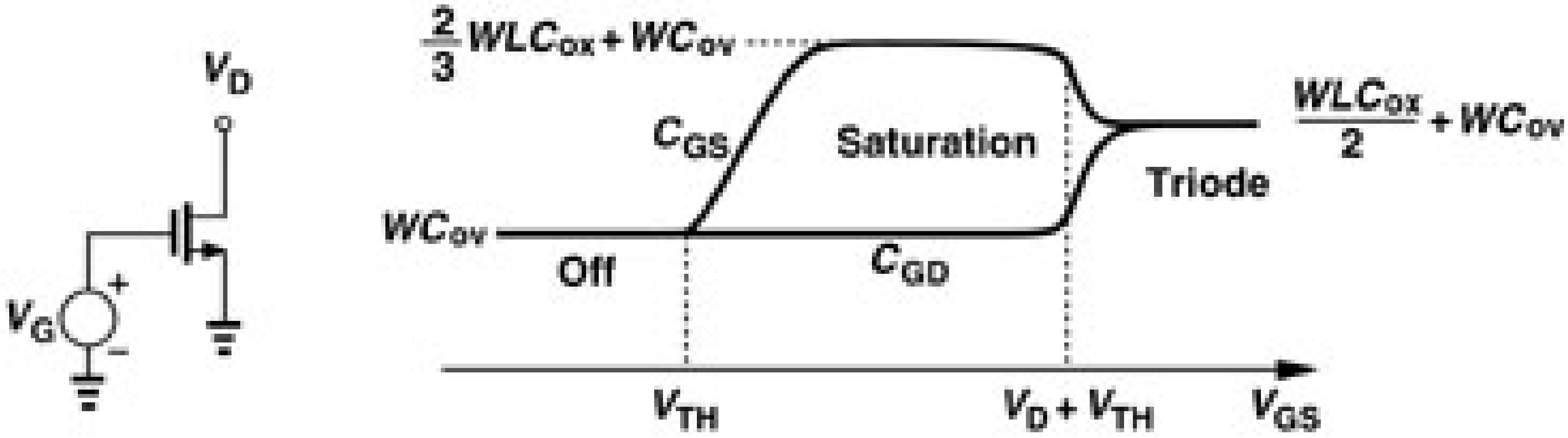
C_{SB} slightly larger, C_{DB} a lot smaller. Both transistors have same W/L

C_{GS},C_{GD} at Cutoff



C_{GD}=C_{GS}=C_{ov}W

C_{GB} at Cutoff

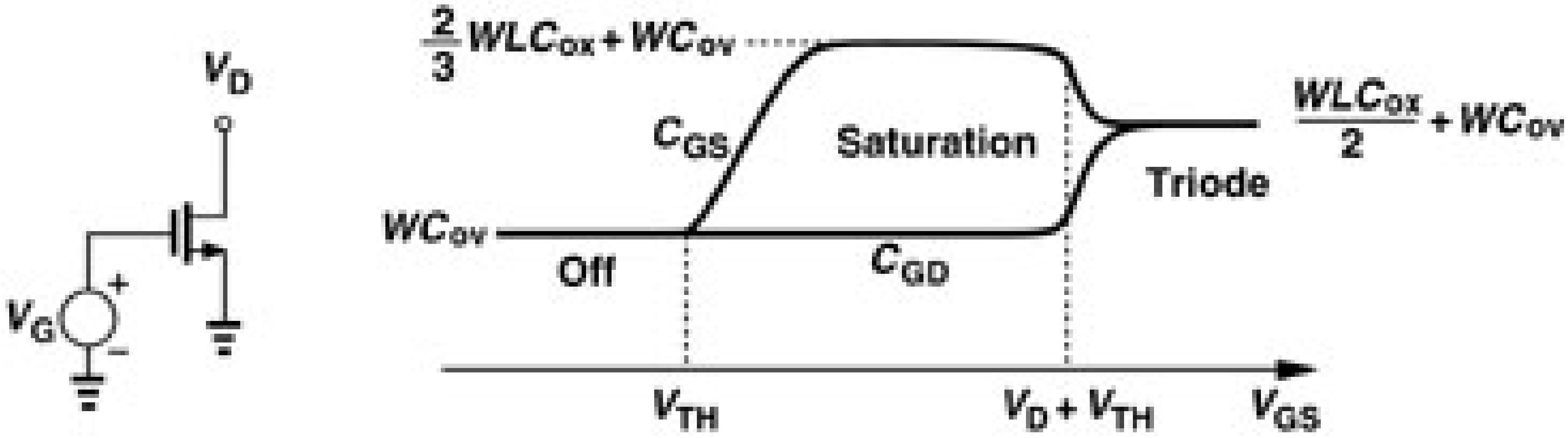


$C_1 = WLC_{OX}, C_2 = WL(q\varepsilon_{si}N_{sub}/(4\Phi_F))^{1/2} = C_d$

$C_{GB} = C_1C_2/(C_1+C_2) = WLC_{OX}C_d/(WLC_{OX}+C_d)$

L is the effective length of the channel.

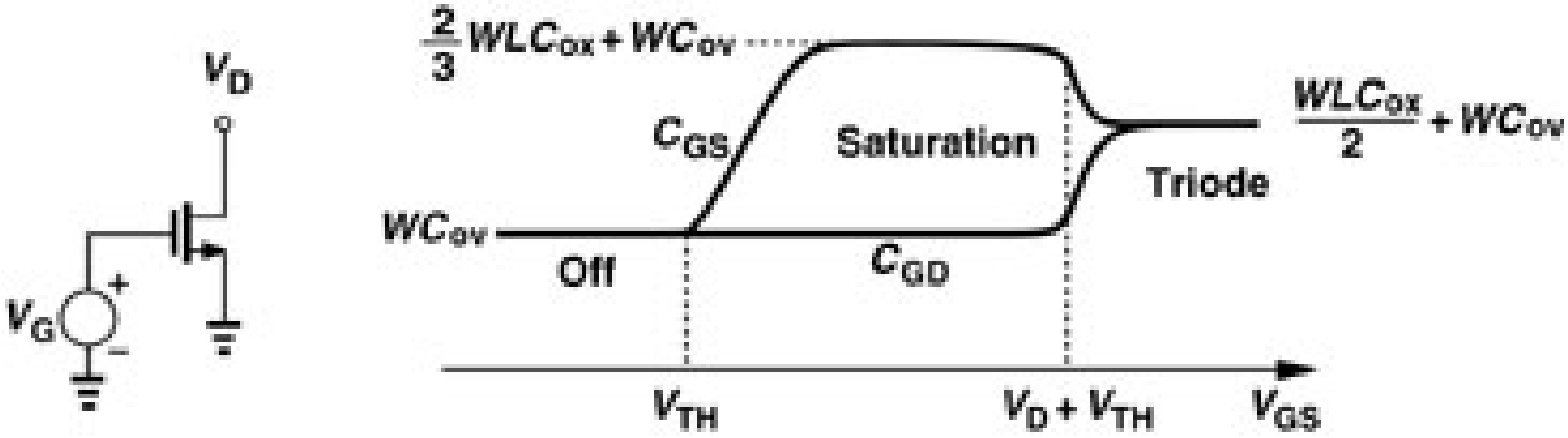
C_{DB} and C_{SB} at all modes



Both depend on the reverse voltages Drain to Substrate and Source to Substrate

Recall $C_i = C_{i0} / [1+V_R/\Phi_B]^m$

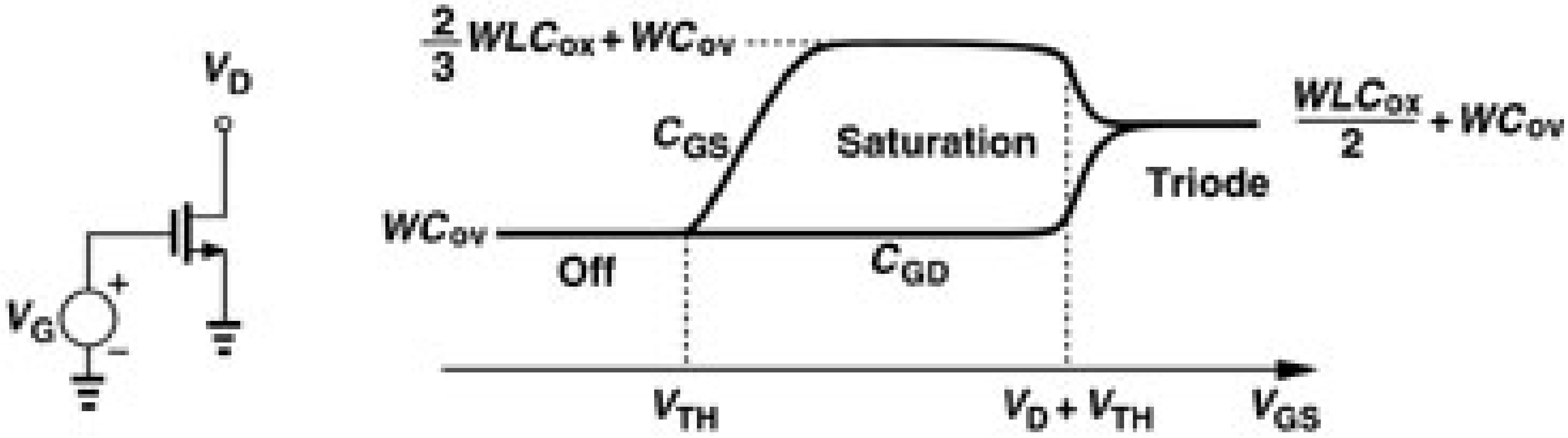
C_{GS},C_{GD} at Triode Mode



If V_{DS} is small enough, we can assume V_{GS}≈V_{GD} → Channel uniform → Gate-Channel capacitance WLC_{OX} is distributed uniformly

$$C_{GD}=C_{GS}\approx (WLC_{OX})/2+C_{ov}W$$

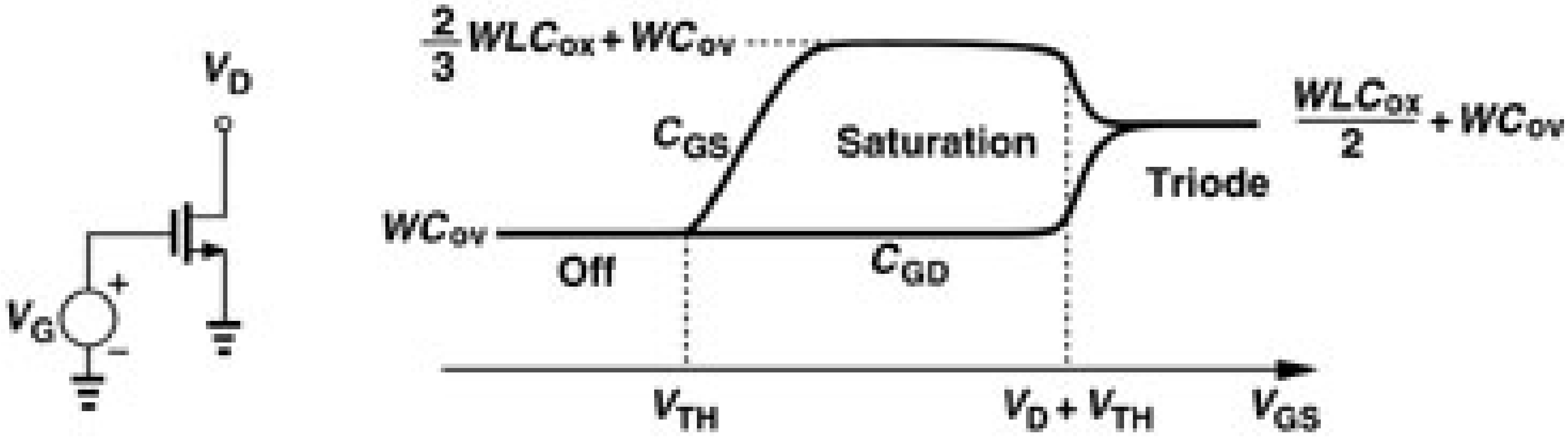
C_{GS},C_{GD} at Saturation Mode



$C_{GD} \approx C_{ov}W$ because there isn't much of a channel near Drain.

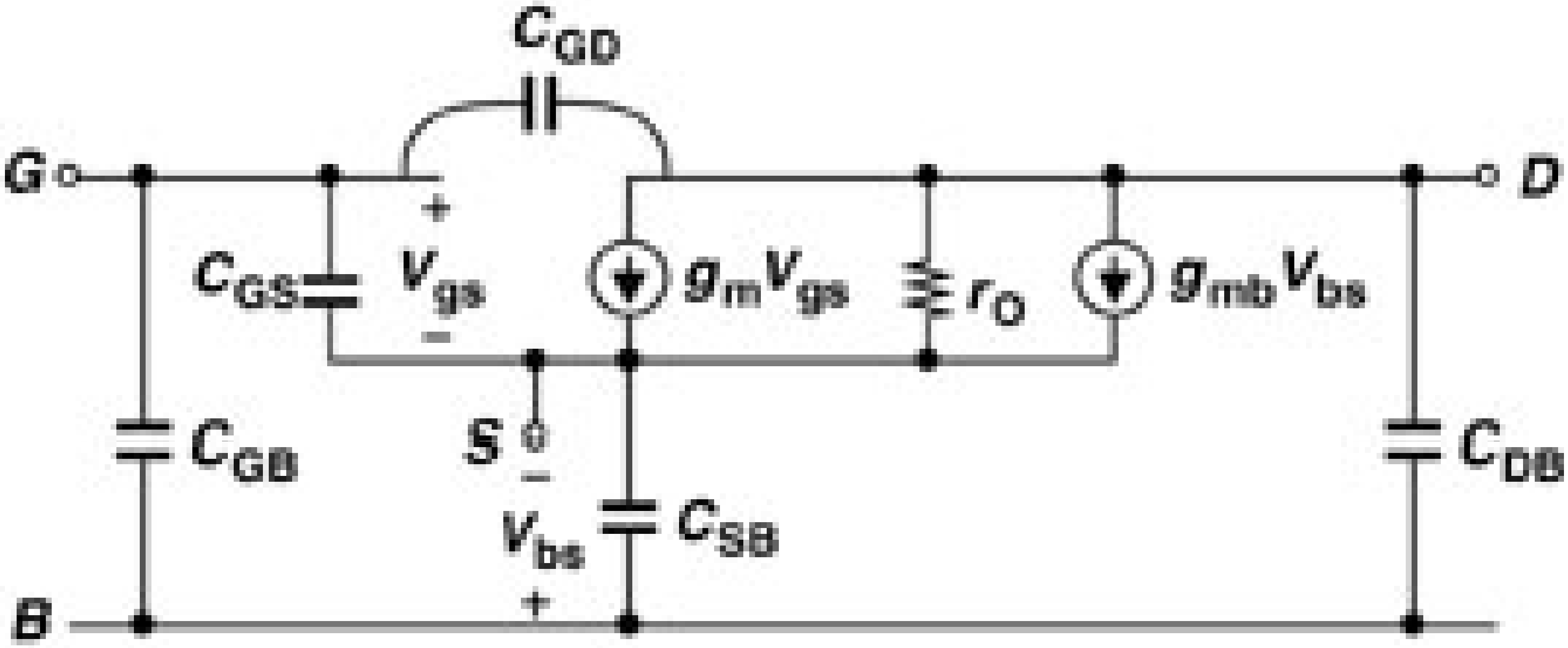
(It can be proved):
$C_{GS}=2WL_{eff}C_{OX}/3+WC_{ov}$

C_{GB} at Triode and Saturation Modes

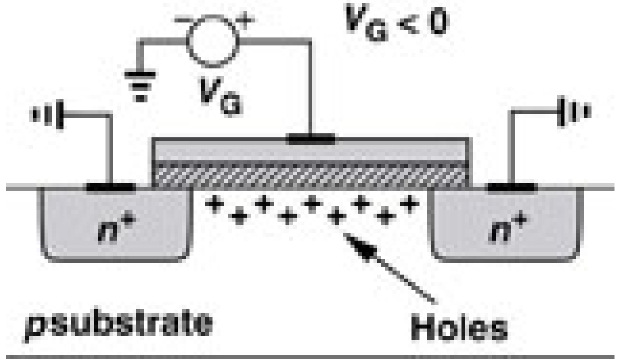


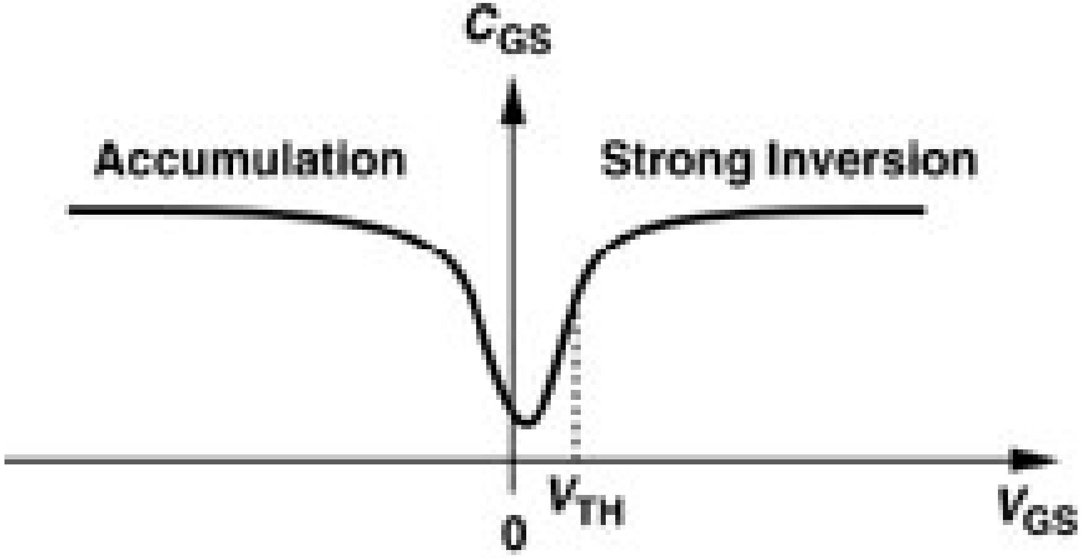
C_{GB} is negligible, because channel acts as a "shield" between Gate and Substrate. That is, if V_G varies, change of charge comes from Source and Drain, and not from the Substrate!

MOS Small Signal Model with Capacitance



C-V of NMOS



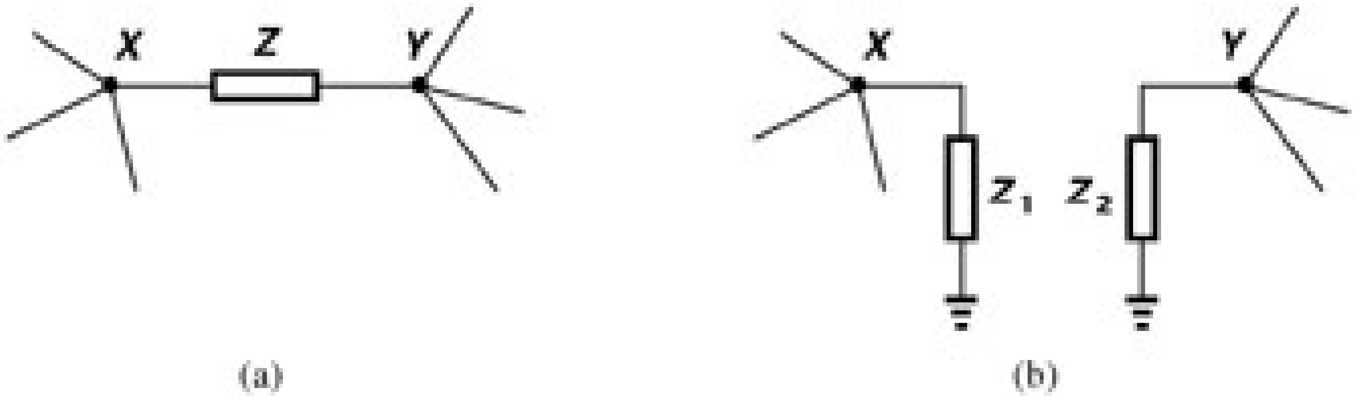


In "Weak Inversion" C_{GS} is a series combination of C_{OX}-capacitance and C_{dep}-capacitance → Low capacitance peak near $V_{GS} \approx V_{TH}$

High-Frequency Response of Amplifiers-L23

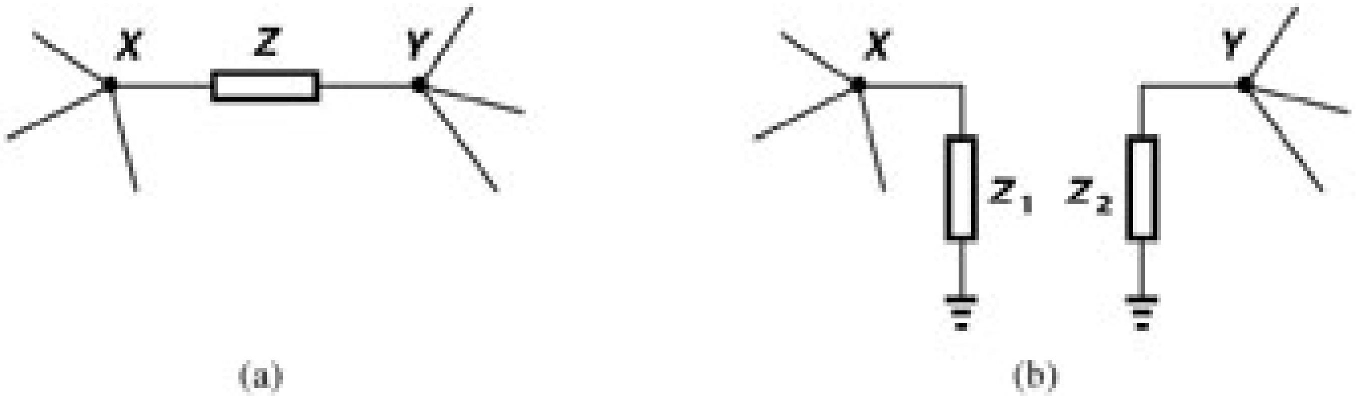
Basic Concepts

Miller's Theorem



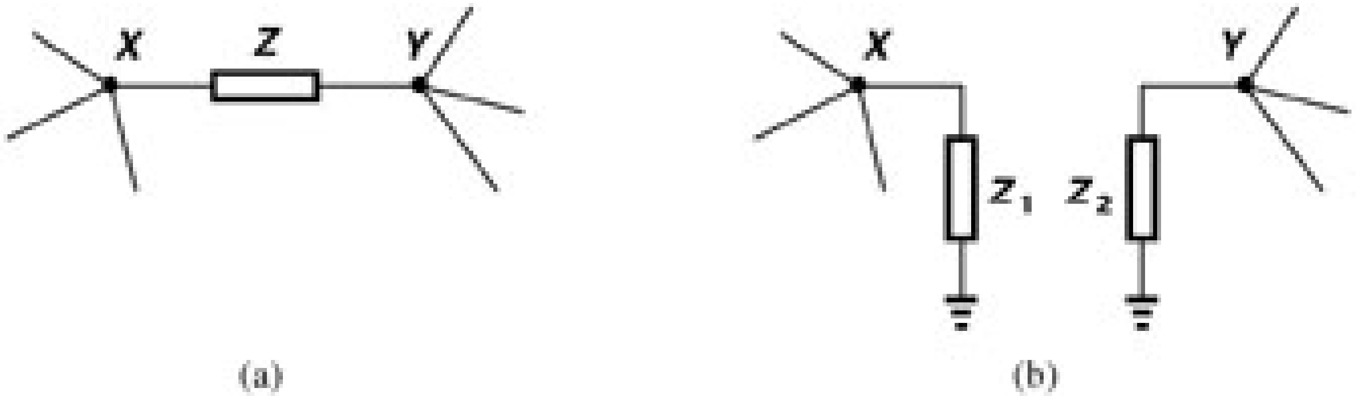
Replacement of a bridging component (typically a capacitor) with equivalent input and output impedances, for the sake of facilitating high-frequency response analysis.

Miller's Theorem Statement



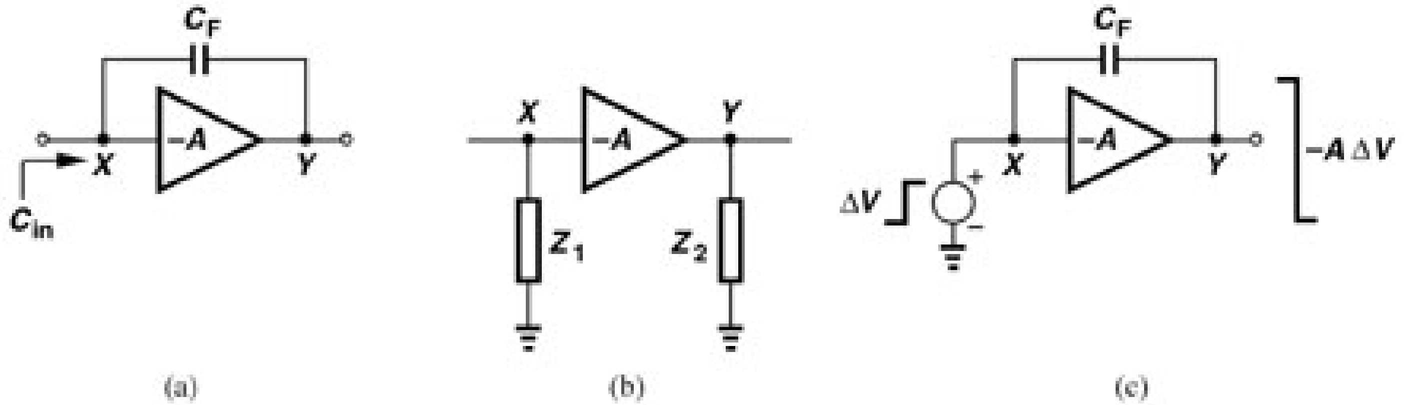
- Let $A_V = V_Y/V_X$ be known.
- Then (a) and (b) above are equivalent if:
- $Z_1=Z/(1-A_V)$ and $Z_2=Z/(1-A_V^{-1})$
- Equivalence in V_X,V_Y and currents to Z's.

Proof of Miller's Theorem



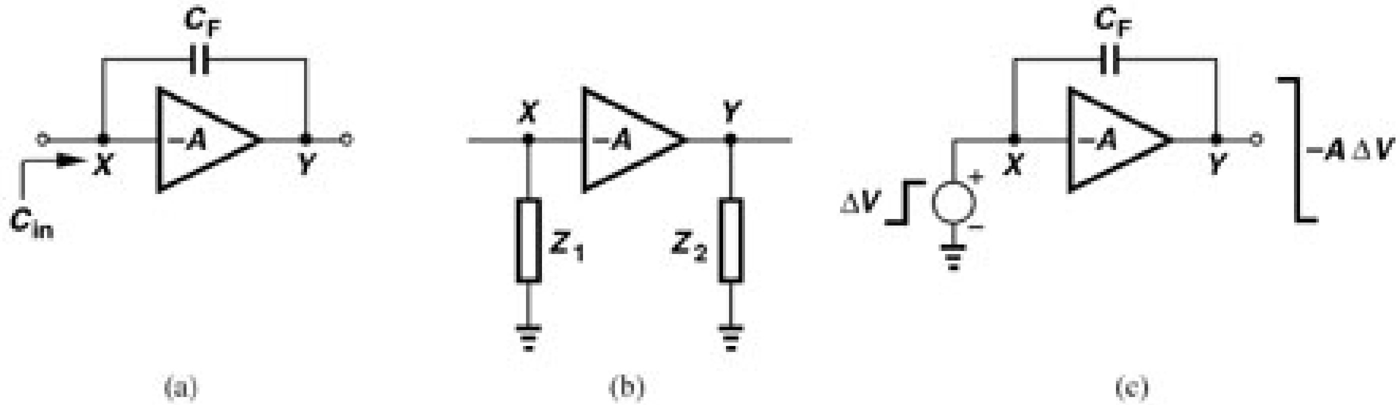
- Current equivalence: $(V_X - V_Y)/Z = V_X/Z_1$
- That is: $Z_1 = Z/(1 - V_Y/V_X)$ etc.
- Similarly: $(V_Y - V_X)/Z = V_Y/Z_2 \rightarrow Z_2 = Z/(1 - V_X/V_Y)$
- A simple-looking result, but what does it mean?

The most common application



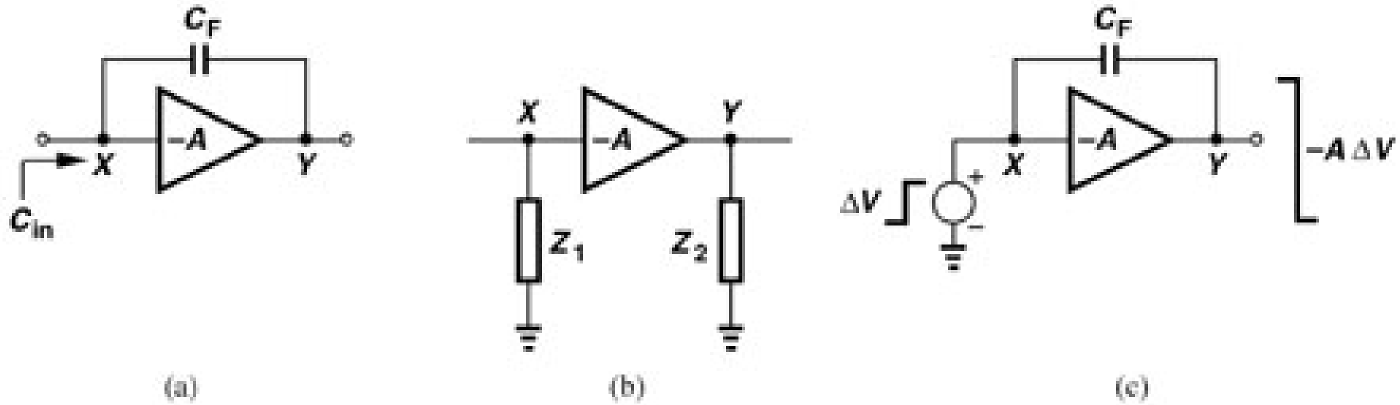
- Mid-band gain of amplifier is known to be –A.
- We want to know (at least approximately) how does the bridging capacitor C_F influence the amplifier's bandwidth.

Exact vs. Approximate Analysis



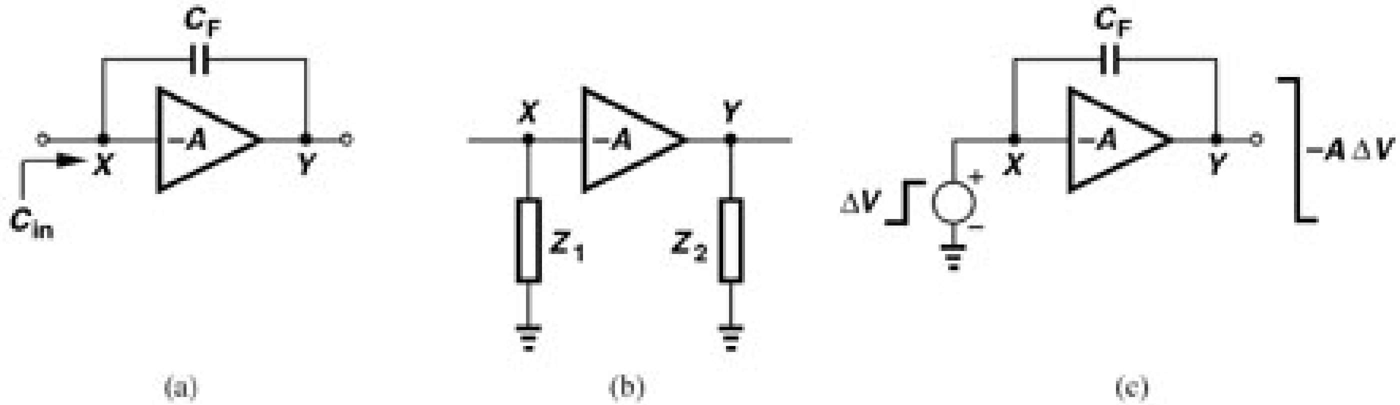
- For exact analysis, replace C_F with its impedance $Z_C=1/sC_F$ and find transfer function $V_Y(s)/V_X(s)$ of circuit.
- Approximate analysis: Use Miller's theorem to break capacitor into two capacitances, and then study the input circuit's time constant and the output circuit's time constant.

Miller Effect – input capacitance



- Input circuit's added capacitance due to C_F: Since Z_1 is smaller than Z by $(1-A_V)$, and since C_F appears in denominator of Z, it means that input capacitance $C_1=C_F(1-A_V)$. If $|A_V|$ is large, C_1 may become pretty large.

Miller Effect – output capacitance



- Output circuit's added capacitance due to C_F: Since Z_2 is smaller than Z by $(1-A_V^{-1})$, and since C_F appears in denominator of Z, it means that output capacitance $C_2=C_F(1-A_V^{-1})$. If $|A_V|$ is large, C_2 will be almost equal to C_F .

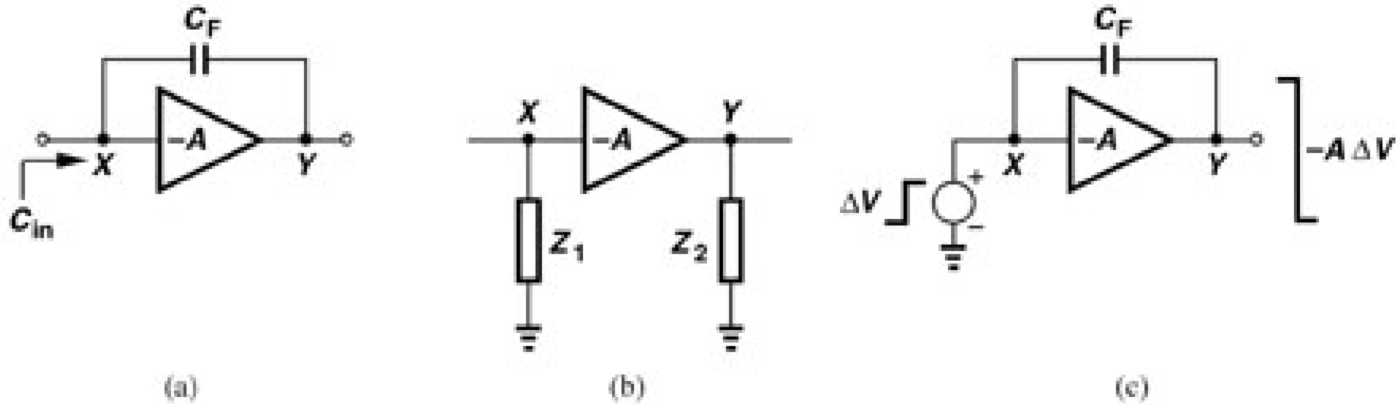
Bridging component restriction

- Both sides of the bridging component must be fully floating, for Miller's theorem to be valid.
- If voltage, on either side, if fixed then there is no Miller effect.
- Later we'll use Miller's theorem to deal with C_{GD}. Therefore in CG and Source Follower amplifiers, in which one side of C_{GD} is ground, there will be no Miller effect.
- Main impact of Miller is on CS amplifiers.

Meaning of “approximated analysis”

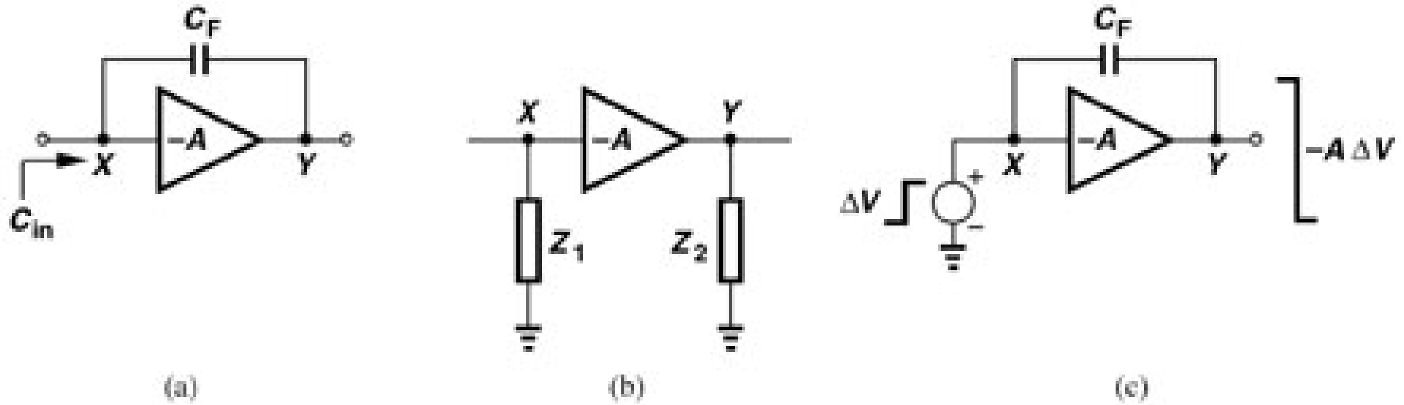
- There is little use for the complete high-frequency transfer function (all its poles and zeros).
- We are interested in the amplifier only at mid-band frequencies (at which all MOSFET capacitances are considered open circuit), and may be a little bit beyond, near high-frequency cutoff (-3db) frequency.

Approximate Analysis Meaning



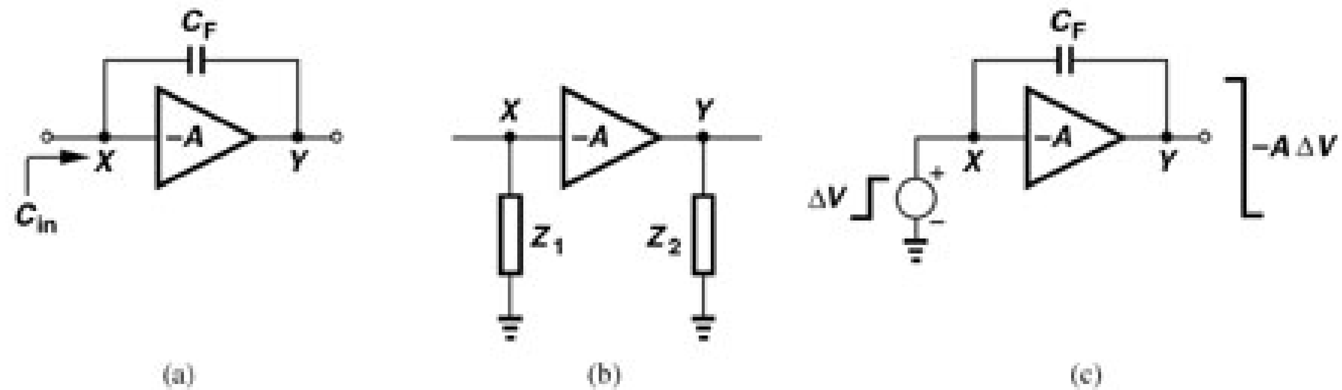
- We calculate input circuit’s and output circuit’s time constant for the sake of determining the amplifier’s bandwidth.
- We use the known mid-band voltage gain in Miller’s theorem.

Validity of Miller's analysis at HF



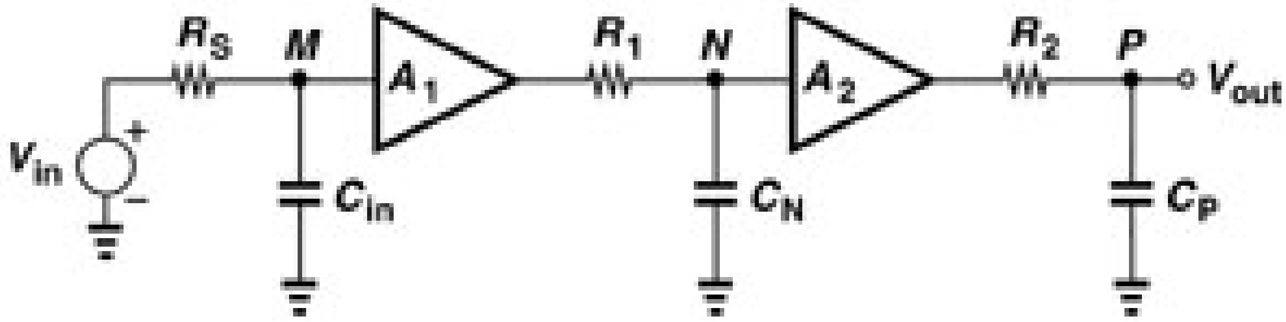
- We use the known mid-band voltage gain in Miller's theorem.
- At high frequencies, much higher than the -3db frequency, gain is much lower → Miller's theorem, using mid-band gain, is not valid!

Bandwidth Calculation



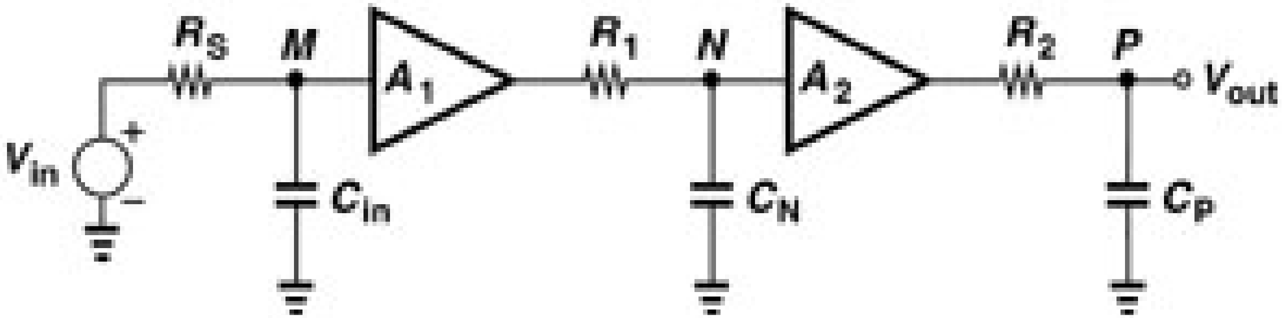
- Typically, the two time constants are rarely of the same order of magnitude.
- The larger time constant determines the amplifier's -3db frequency.
- The smaller time constant (if much smaller than other) should be discarded as invalid.

Poles and Zeros



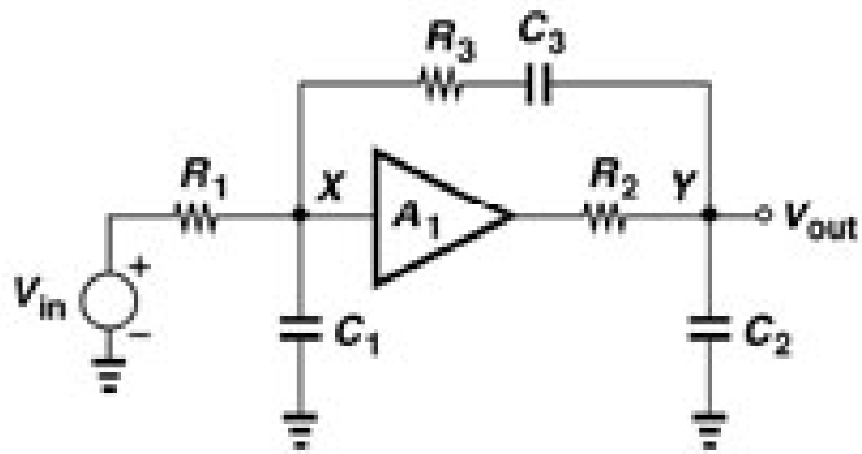
- A time constant $\tau = 1/RC$ means that the transfer function has a pole at s=-1/τ.
- Typically, every capacitor contributes one pole to the transfer function.

Example below: All poles are real; No zeros



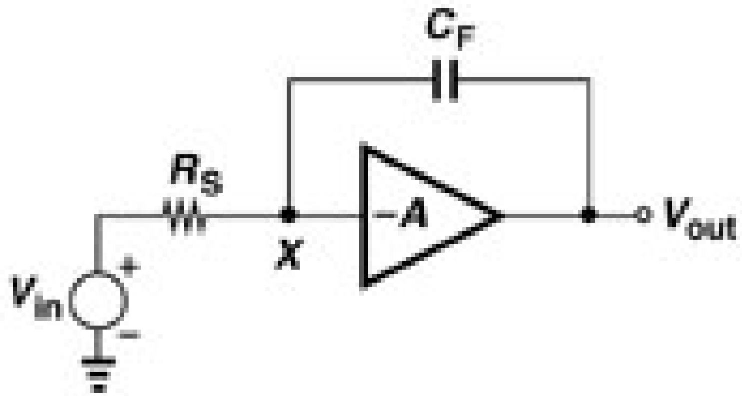
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{A_1 A_2}{(1+sR_S C_{in})(1+sR_1 C_N)(1+sR_2 C_P)}$$

Complicated example



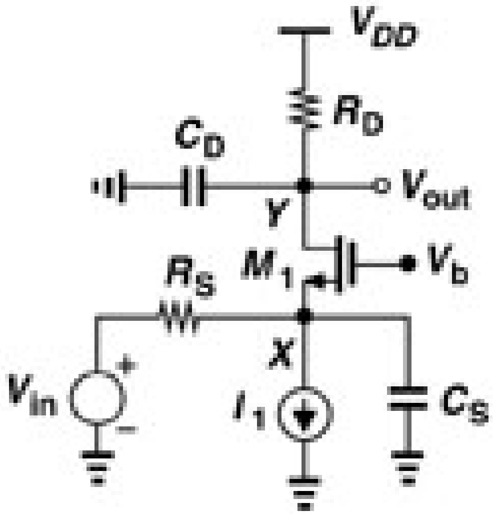
- There are three poles due to the three capacitors. It’s hard to determine poles location by inspection (some poles may be complex)
- There may be some zeros.
- Difficult due to V_{in},V_{out} interaction (via R_3,C_3)

Example: Feedback capacitor over finite gain



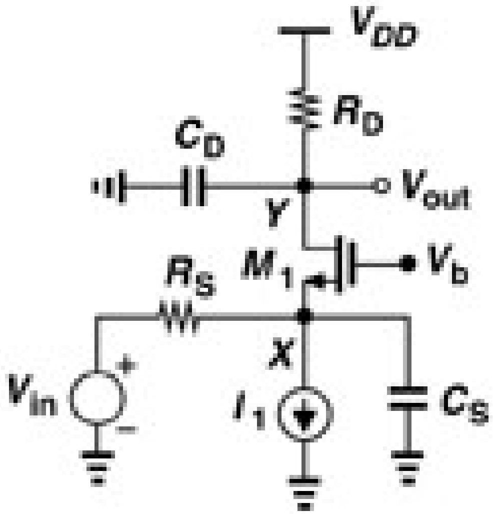
- $V_{out}(s)/V_X(s) = -A$
- $(V_{in}(s)-V_X(s))/R_S = sC_F(V_X(s)-V_{out}(s))$
- Exact Result: $V_{out}(s)/V_{in}(s) = -A/[1+sR_SC_F(1+A)]$

Example: CG Amplifier



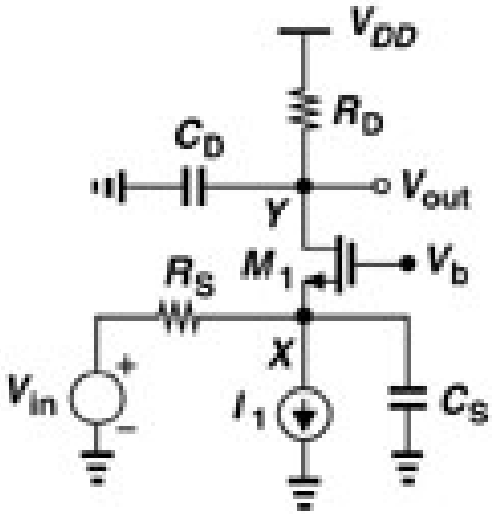
- $C_S = C_{GS1} + C_{SB1}$ Capacitance “seen” from Source to ground.
- $C_D = C_{DG} + C_{DB}$ Capacitance “seen” from Drain to ground.

Example: CG Amplifier



- Need to determine (by inspection) the two poles contributed, one by C_S and another by C_D.
- This is easy to do only if we neglect r_{01}.
- Strategy: Find equivalent resistance "seen" by each capacitor, when sources are nulled.

Example: CG Amplifier

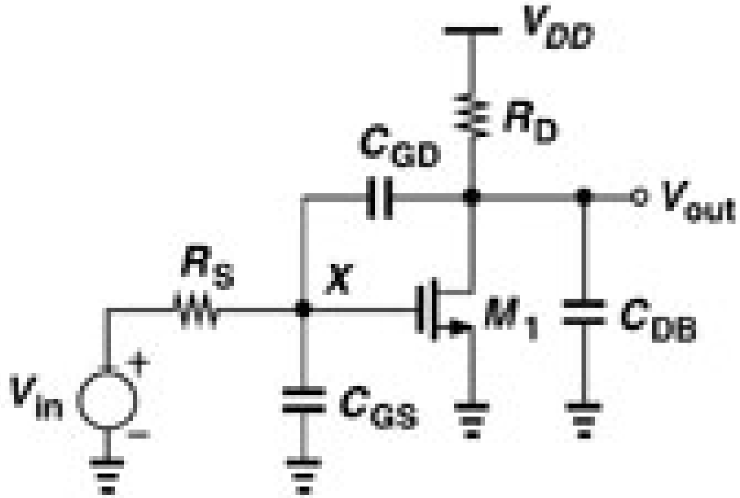


- $\tau_S = C_S R_{S,eq} = C_S\{R_S || [1/(g_{m1}+g_{mb1})]$
- $\tau_D = C_D R_{D,eq} = C_D R_D$
- $A = (g_{m1}+g_{mb1}) R_D / (1+(g_{m1}+g_{mb1}) R_S)$
- $V_{out}(s)/V_{in}(s) = A/[(1+s\tau_S)(1+s\tau_D)]$

High-Frequency Response-L24

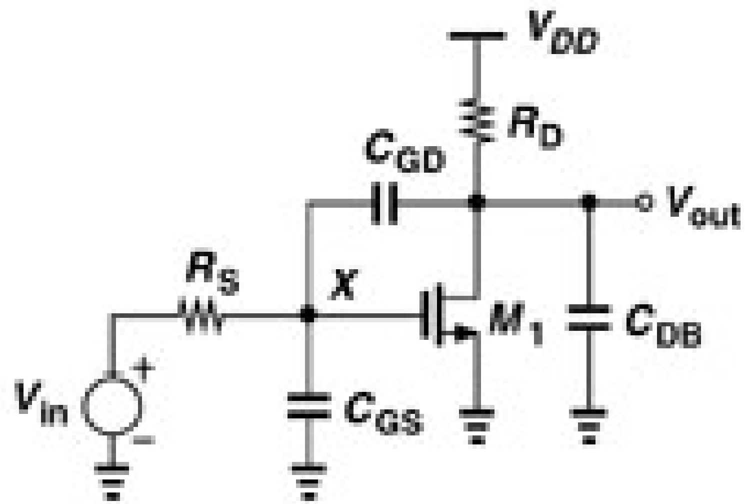
Common-Source Amplifier

High Frequency Model of CS Amplifier



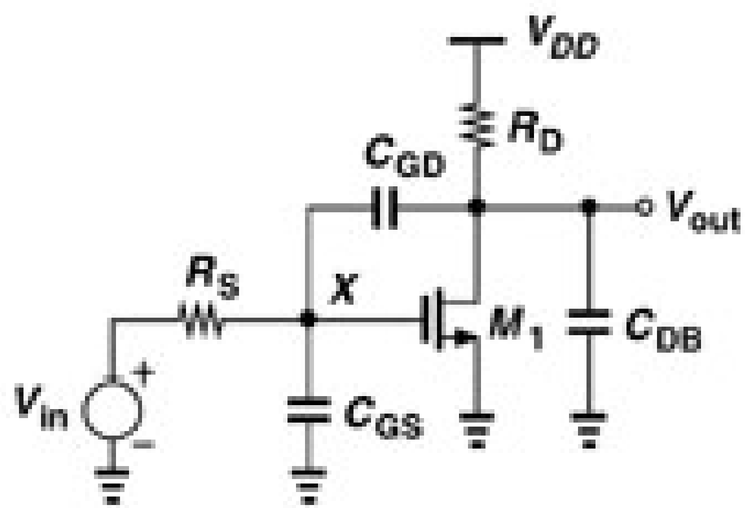
- C_{GD} creates a Miller Effect.
- It is essential to assume a nonzero signal source resistance R_S, otherwise signal voltage changes appear directly across C_{GS}

C_{GD} Miller Effect on input



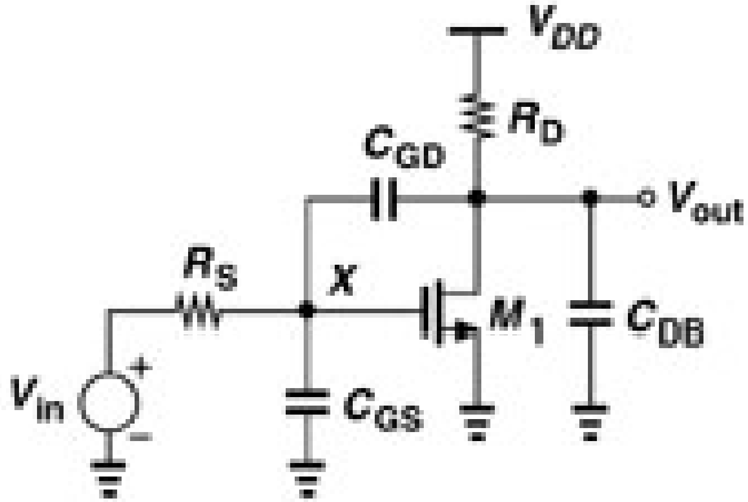
- Let A_V be the low frequencies voltage gain of the CS amplifier, such as $A_V \approx -g_m R_D$.
- Capacitance added to C_{GS} is $C_{GD}(1-A_V)$.
- $\tau_{in}=R_S[C_{GS}+C_{GD}(1-A_V)]$

C_{GD} Miller Effect on output



- Capacitance added to C_{DB} is $C_{GD}(1-A_V^{-1}) \approx C_{GD}$.
- $\tau_{out} = R_D(C_{GD}+C_{DB})$
- If τ_{in} and τ_{out} are much different, then the largest one is valid and the smaller one is invalid.

Common Source (must be in Saturation Mode)

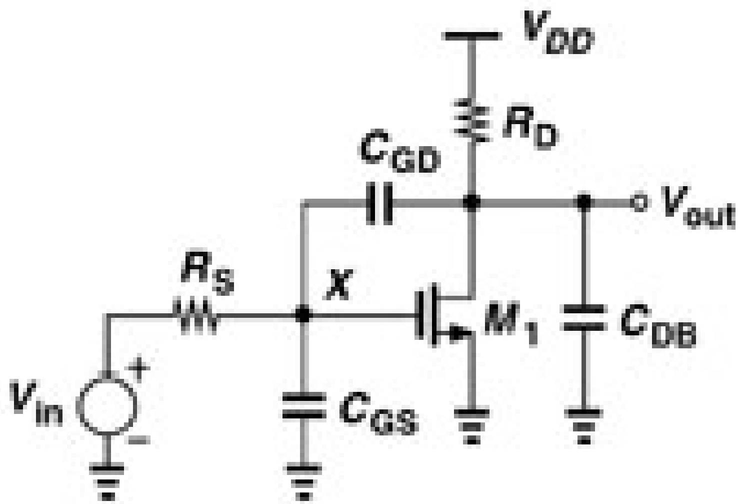


Neglecting input/output interaction,

$$f_{p,in} = \frac{1}{2\pi R_S\left[C_{GS} + (1 + g_m R_D)C_{GD}\right]}$$

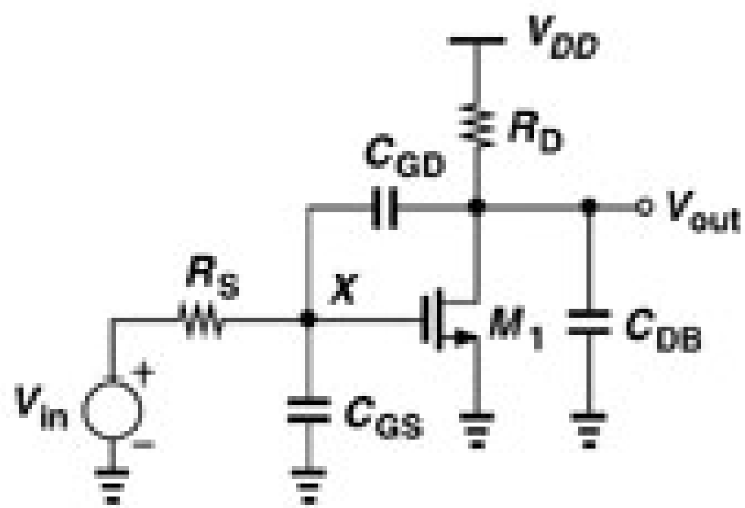
$$f_{p,out} = \frac{1}{2\pi\left[(C_{GD} + C_{DB})R_D\right]}$$

CS Amplifier's Bandwidth if R_S large



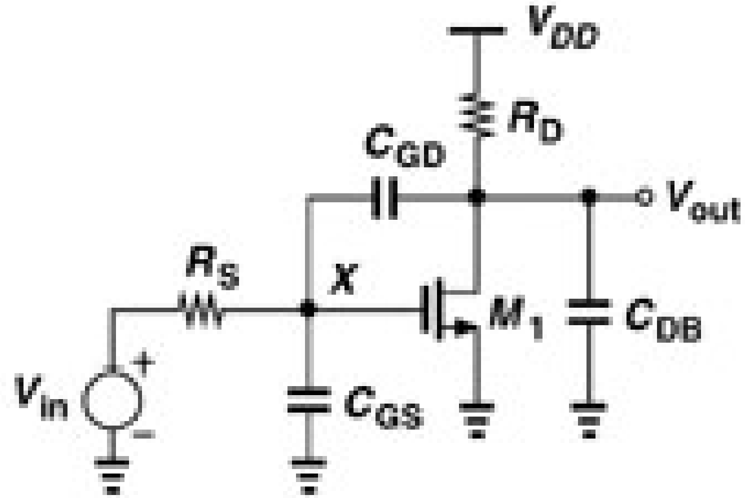
- $\tau_{out} = R_D(C_{GD}+C_{DB})$; $\tau_{in}=R_S[C_{GS}+C_{GD}(1-A_V)]$; $A_V=-g_mR_D$
- If all transistor capacitances are of the same order of magnitude, and if R_S is of the same order of magnitude of R_D (or larger) then $A_V(s)\approx -g_mR_D/(1+s\tau_{in})$: Input capacitance dominates. $BW=f_h=1/2\pi\tau_{in}$

CS Amplifier's Bandwidth if V_{in} is almost a perfect voltage source



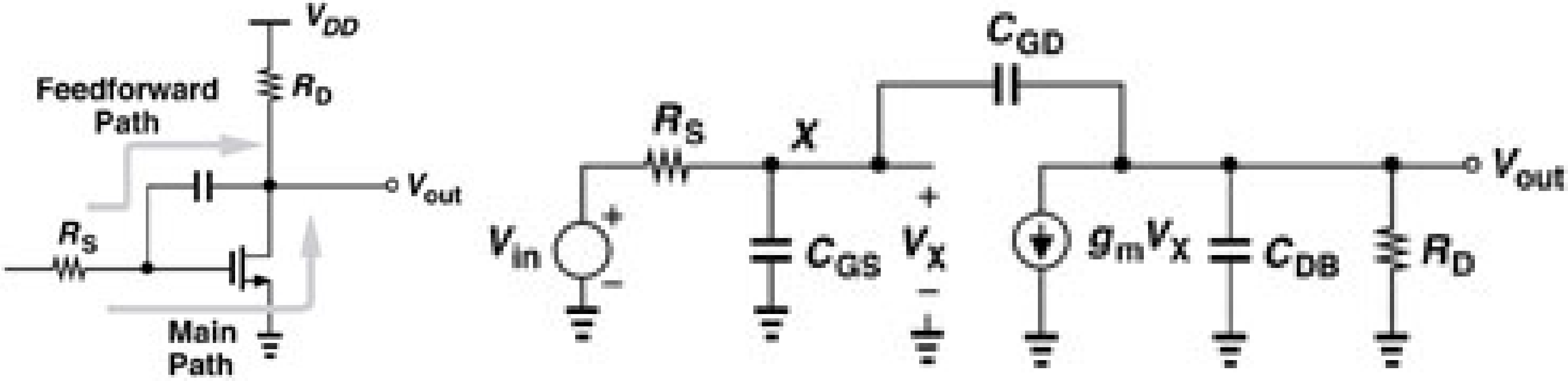
- $\tau_{out} = R_D(C_{GD}+C_{DB})$; $\tau_{in}=R_S[C_{GS}+C_{GD}(1-A_V)]$; $A_V=-g_mR_D$
- If all transistor capacitances are of the same order of magnitude, and if R_S is very small : $A_V(s)\approx -g_mR_D/(1+s\tau_{out})$: Output capacitance dominates. $BW=f_h=1/2\pi\tau_{out}$

High Frequency Response of CS Amplifier – "Exact" Analysis



- Use small-signal diagram, and replace every capacitor by its impedance 1/sC ; Neglect r_o
- Result: Only two poles, and one zero in $V_{out}(s)/V_{in}(s)$!

CS HF Response: Exact Result



$$\frac{V_{out}(s)}{V_{in}(s)}=\frac{(sC_{GD}-g_m)R_D}{s^2R_SR_D(C_{GS}C_{GD}+C_{GS}C_{SB}+C_{GD}C_{DB})+s\left[R_S(1+g_mR_D)C_{GD}+R_SC_{GS}+R_D(C_{GD}+C_{DB})\right]+1}$$

$$Assume\ \ D=\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)=\frac{s^2}{\omega_{p1}\omega_{p2}}+\frac{s}{\omega_{p1}}+1\ ,\quad \omega_{p2}>>\omega_{p1}$$

$$f_{p,in}=\frac{1}{2\pi\left(R_S\left[C_{GS}+(1+g_mR_D)C_{GD}\right]+R_D(C_{GD}+C_{DB})\right)}$$

CS Exact Analysis (cont.)

$$f_{p,out} = \frac{R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD}+C_{DB})}{2\pi R_S R_D(C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB})}$$

$$f_{p,out} \approx \frac{1}{2\pi R_D(C_{GD}+C_{DB})}\ , \quad \text{for large } C_{GS}$$

$$f_{p,out} \approx \frac{g_m R_S R_D C_{GD}}{2\pi R_S R_D(C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB})}$$

$$\approx \frac{g_m}{2\pi(C_{GS}+C_{DB})},\ \text{for large } C_{GD}$$

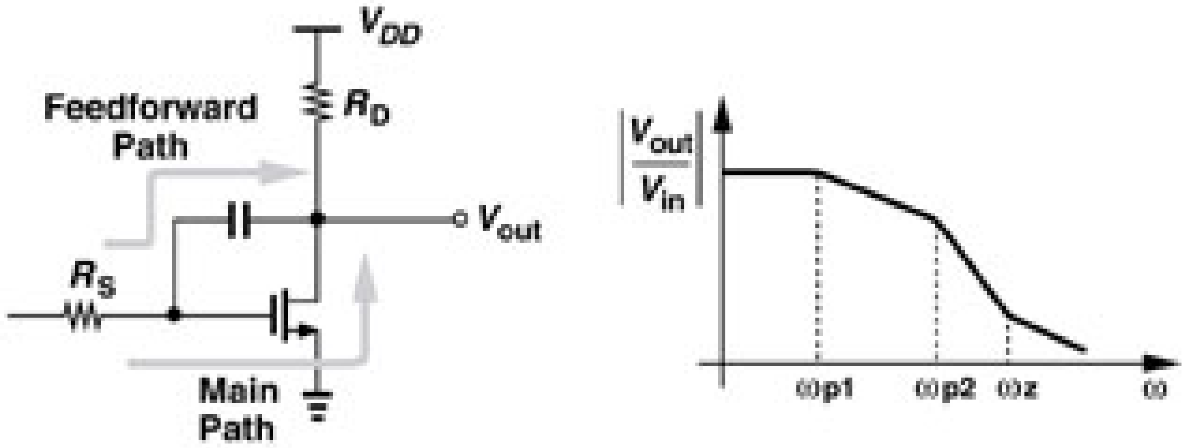
CS Amplifier RHP Zero

Right half plane zero, from the numerator of $V_{out}(s)/V_{in}(s)$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{(sC_{GD} - g_m)R_D}{s^2 R_S R_D (C_{GS}C_{GD} + C_{GS}C_{SB} + C_{GD}C_{DB}) + s\left[R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB)}\right] + 1}$$

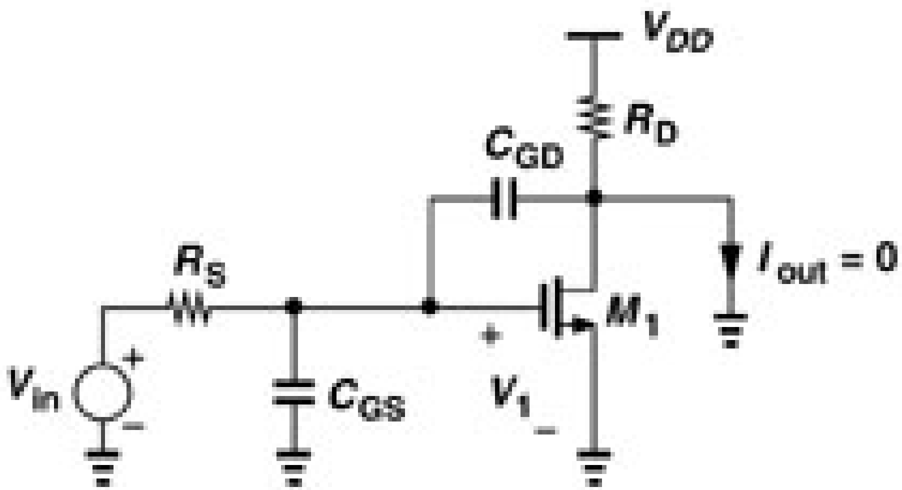
$$\frac{sC_{GD} - g_m}{\cdots} \quad \rightarrow \quad f_z = \frac{+g_m}{2\pi C_{GD}}$$

Zero Creation



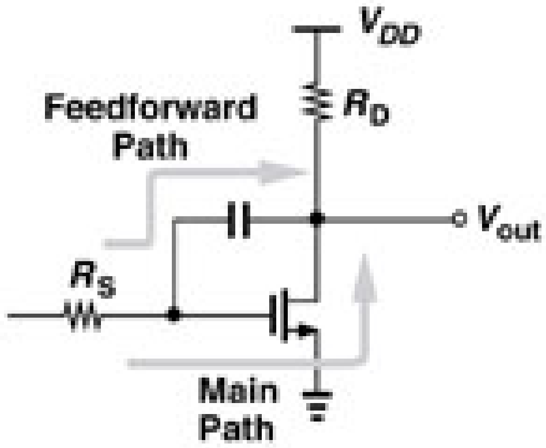
- A Zero indicates a frequency at which output becomes zero.
- The two paths from input to output may create signals that perfectly cancel one another at one specific frequency.

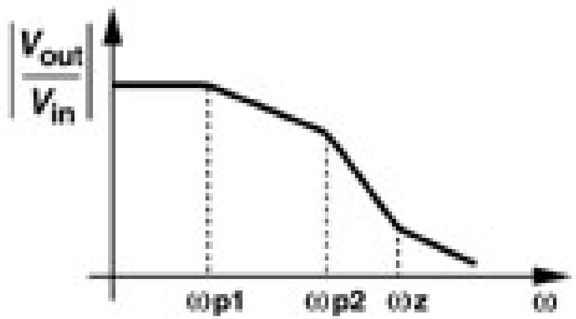
Zero Calculation



- At frequency at which output is zero ($s=s_z$) , the currents through C_{GD} and M_1 are equal and opposite. Can assume that at this frequency (only) output node is shorted to ground.
- $V_1/(1/s_zC_{GD})=g_mV_1$ → Zero expression results.

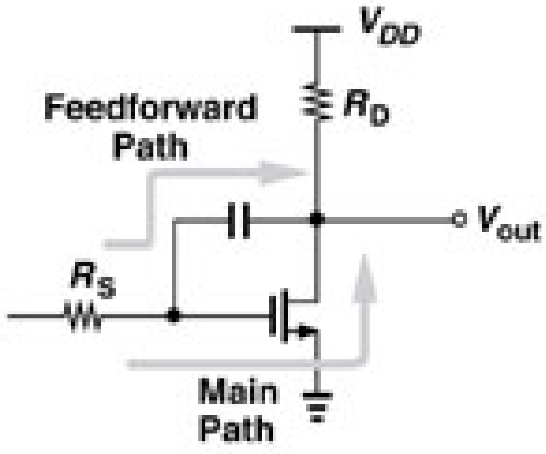
RHP Zero Effect on the Frequency Response

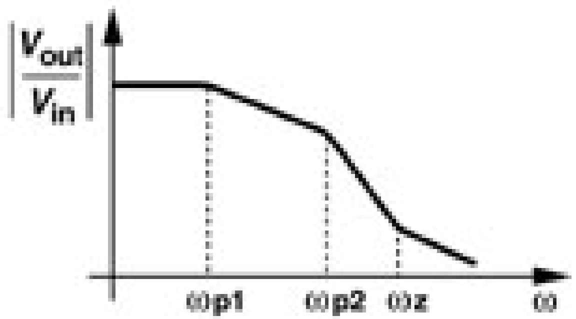




- RHP Zero (like LHP zero) contributes positively to the slope of magnitude frequency response curve → reduces the roll-off slope.
- RHP Zero (contrary to LHP zero) adds NEGATIVE phase shift.

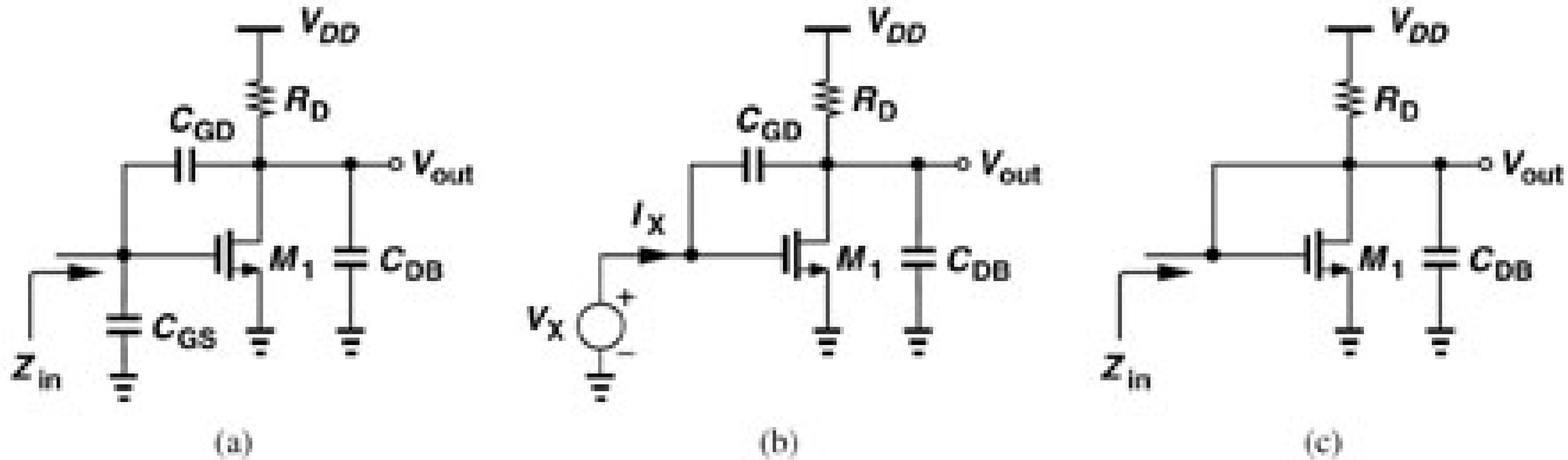
RHP Zero Impact





- It is often negligible, as it happens in very high frequencies.
- It becomes important if CS amplifier is part of multi-stage amplifiers creating a very large open-loop gain. It may affect stabilization compensation.

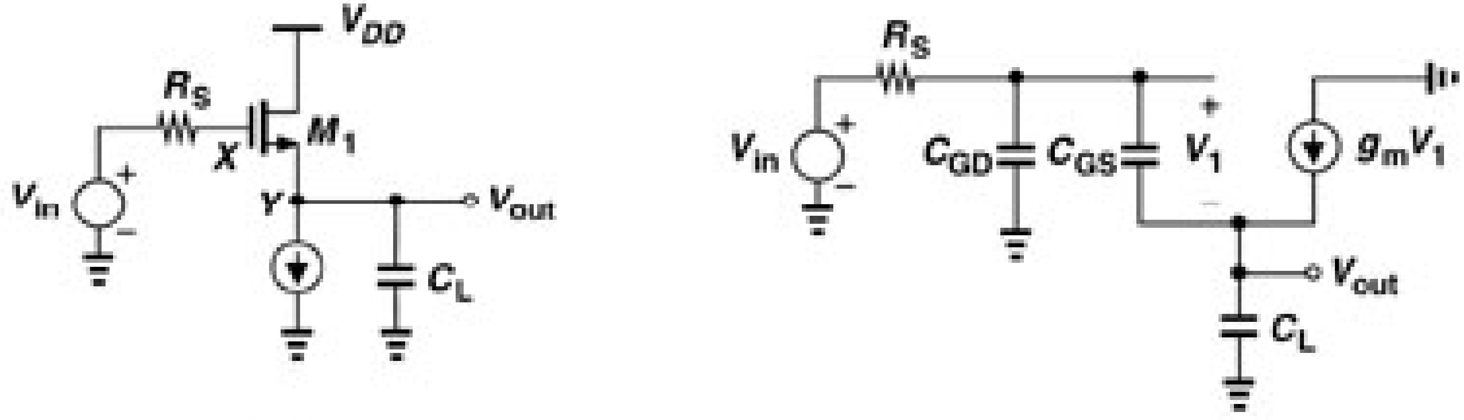
Input Impedance of CS stage



High-Frequency Response-L25

Source Follower

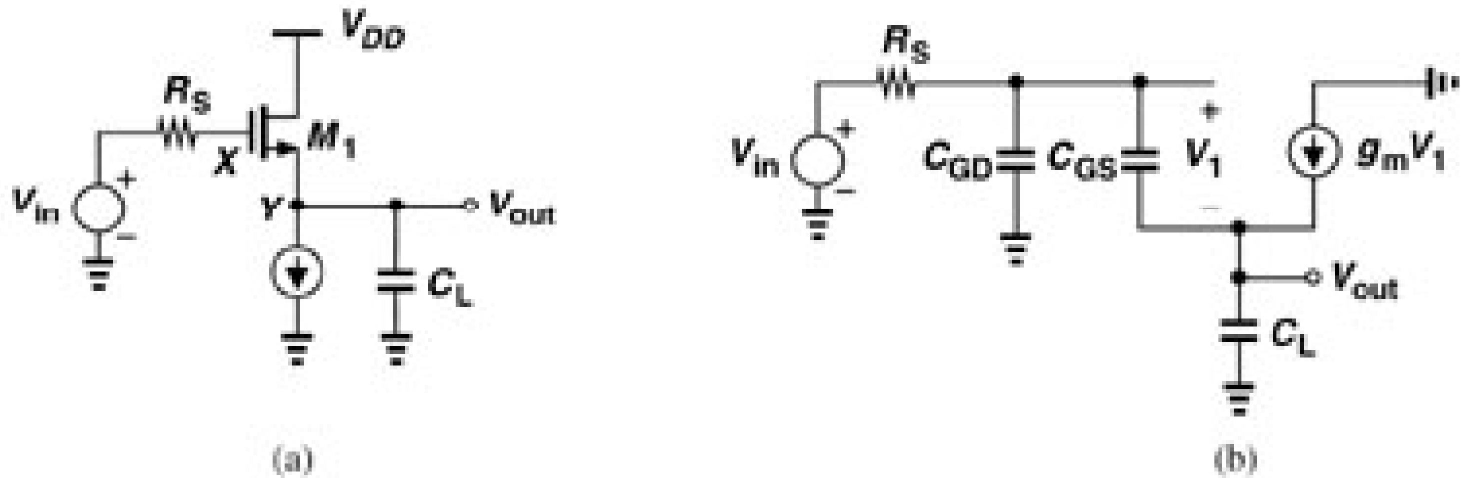
High-Frequency Response of Source Follower – No Miller



(a) (b)

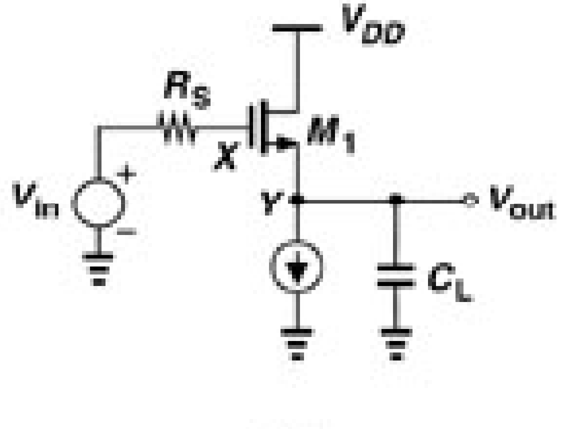
- No Miller Effect – C_{GD} is not a bridge capacitor (as Drain side is grounded).
- C_L is a combination of several capacitances: C_{SB1}, $C_{DB,SS}$, $C_{GD,SS}$ and C_{in} of next stage.

High-Frequency Response of Source Follower – No solution "by inspection"

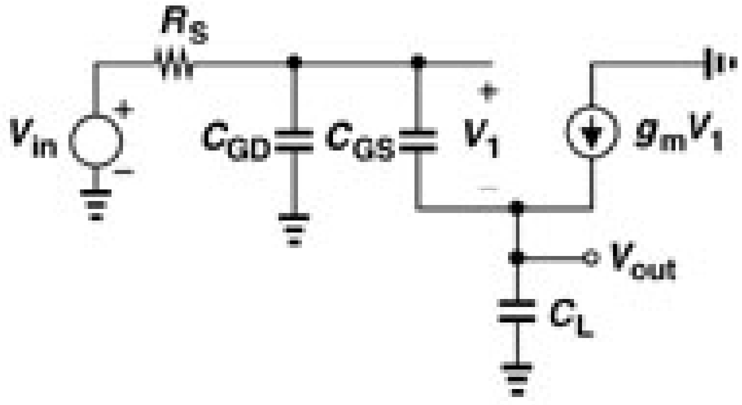


- Because of C_{GS} that ties input to output, it is impossible to find various time constants, one capacitor at a time – combined effects of C_{GS} and $C_{L.}$

Source Follower HF Response Derivation Assumptions



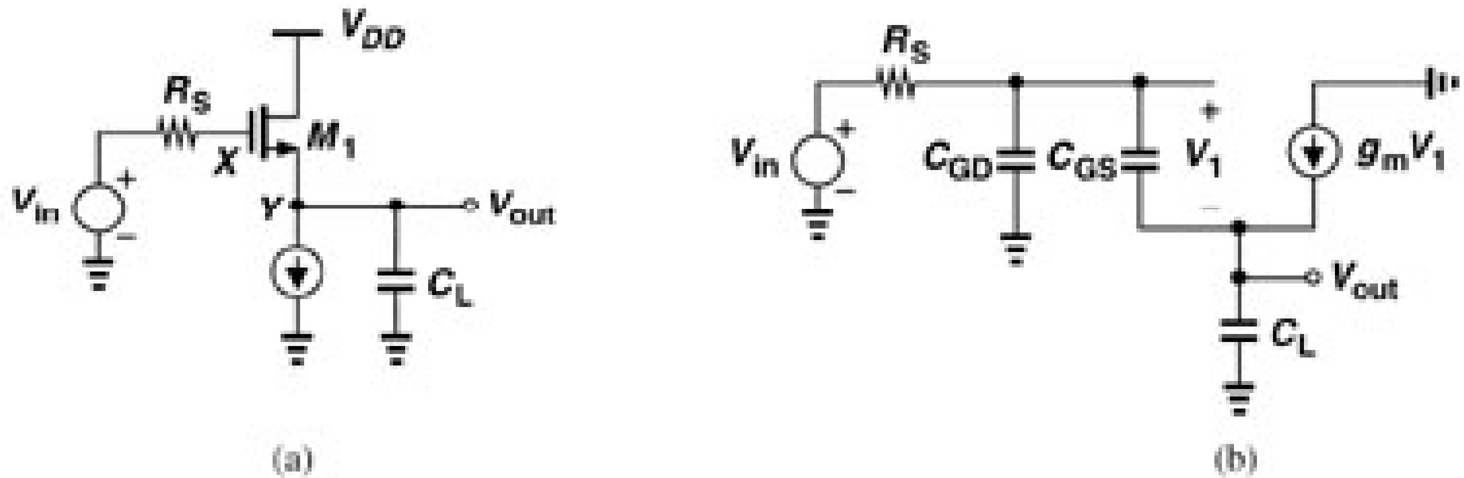
(a)



(b)

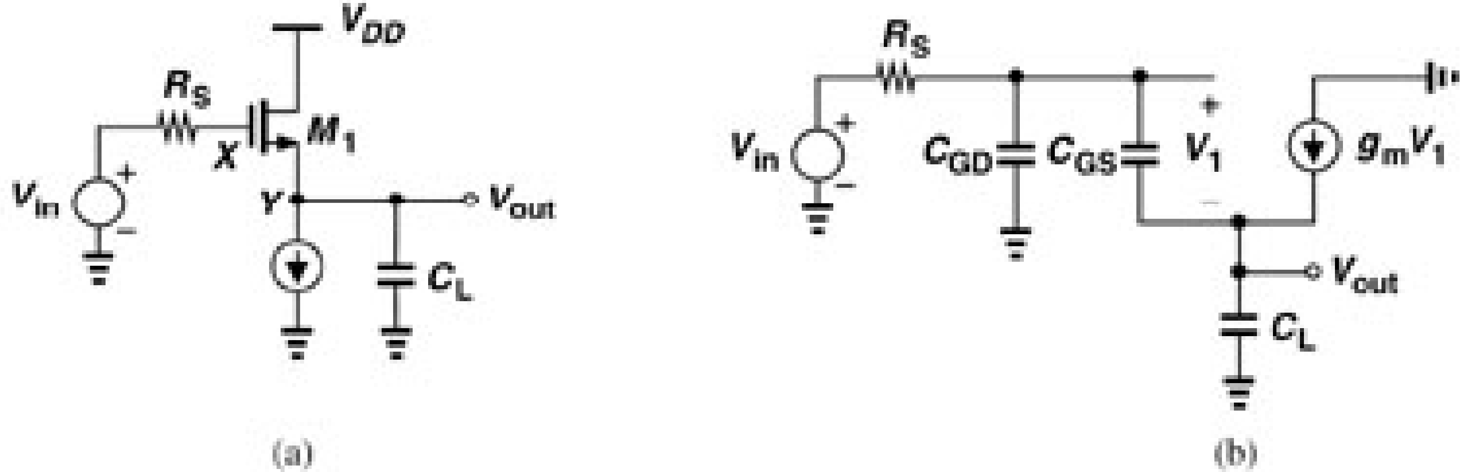
- Use small-signal diagram and 1/sC terms.
- Simplifying assumptions: We neglect effects of r_o and γ (body effect).

Source Follower HF Response Derivation



- Sum currents at output node (all functions of s):
- $V_1C_{GS}s+g_mV_1=V_{out}C_Ls$, yielding:
- $V_1=[C_Ls/(g_m+C_{GS}s)]V_{out}$
- KVL: $V_{in}=R_S[V_1C_{GS}s+(V_1+V_{out})C_{GD}s]+V_1+V_{out}$

Source Follower Transfer Function



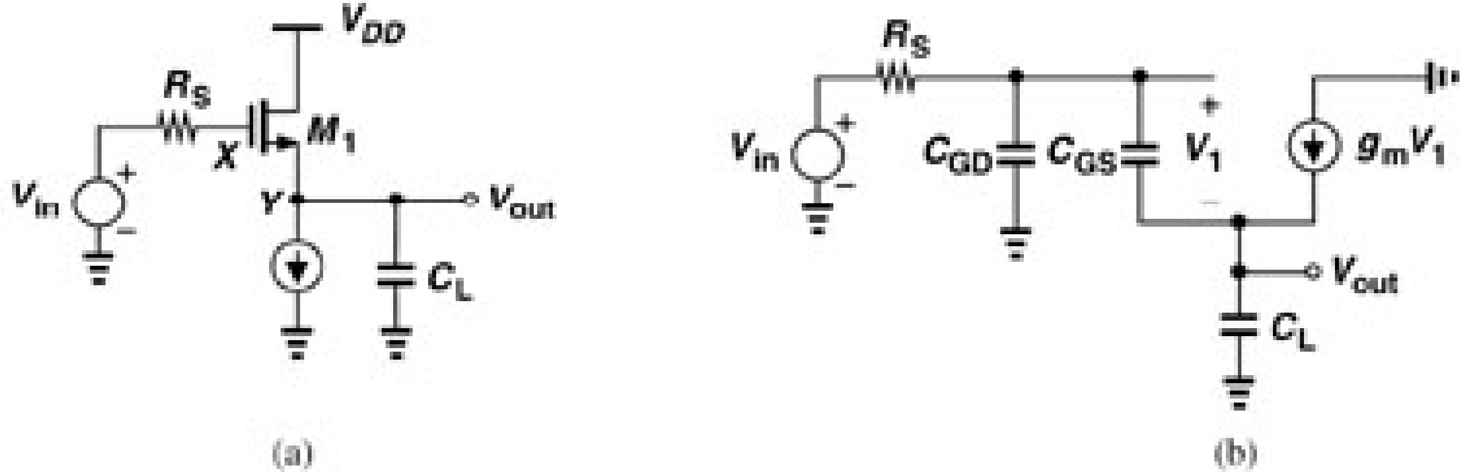
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{g_m + sC_{GS}}{s^2 R_S (C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L) + s\left(g_m R_S C_{GD} + C_{GD} + C_{GS}\right) + g_m}$$

Idea for detecting "dominant pole":

$$\frac{1}{(1+s\tau_1)(1+s\tau_2)} = \frac{1}{\tau_1\tau_2 s^2 + (\tau_1+\tau_2)s + 1} \approx \frac{1}{\tau_1\tau_2 s^2 + \tau_1 s + 1}$$

If $\tau_1 >> \tau_2$ → Can find τ_1 by inspection of the s coefficient

Source Follower Dominant Pole (if it exists…)

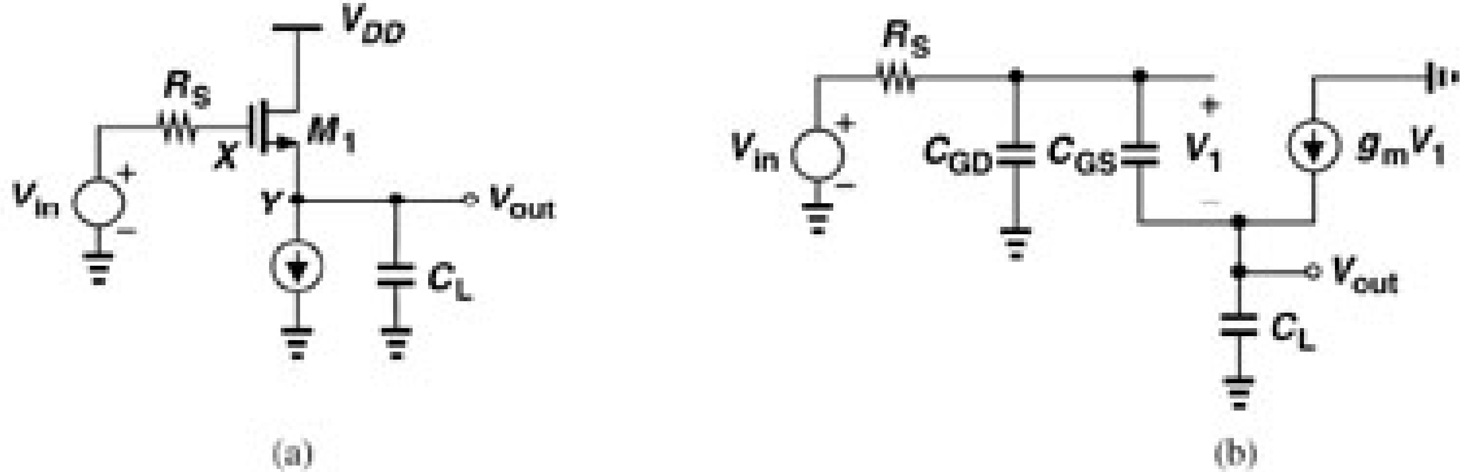


$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{g_m + sC_{GS}}{s^2 R_S (C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L) + s\left(g_m R_S C_{GD} + C_{GD} + C_{GS}\right) + g_m}$$

$$f_{p1} \approx \frac{g_m}{2\pi\left(g_m R_S C_{GD} + C_L + C_{GS}\right)} \text{ , assuming } f_{p2} >> f_{p1}$$

$$= \frac{1}{2\pi\left(R_S C_{GD} + \frac{C_L + C_{GS}}{g_m}\right)}$$

Source Follower Dominant Pole (if R_S is very small)



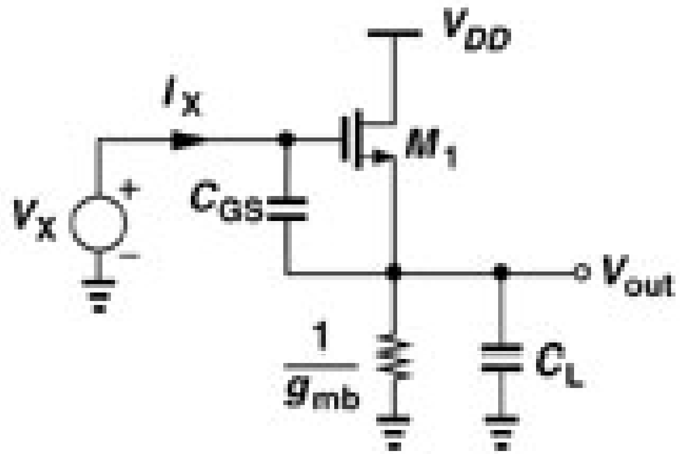
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{g_m + sC_{GS}}{s^2 R_S (C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L) + s(g_m R_S C_{GD} + C_{GD} + C_{GS}) + g_m}$$

$$f_{p1} \approx \frac{g_m}{2\pi(C_L + C_{GS})} = \frac{1}{2\pi\left(\dfrac{C_L + C_{GS}}{g_m}\right)}$$

Source Follower Input Impedance

- At low frequencies we take R_{in} of CS, Source Follower, Cascode and Differential amplifiers to be practically infinite.
- At higher frequencies, we need to study the Input Impedance of the various amplifiers.
- Specifically for a Source Follower: Is Z_{in} purely capacitive?

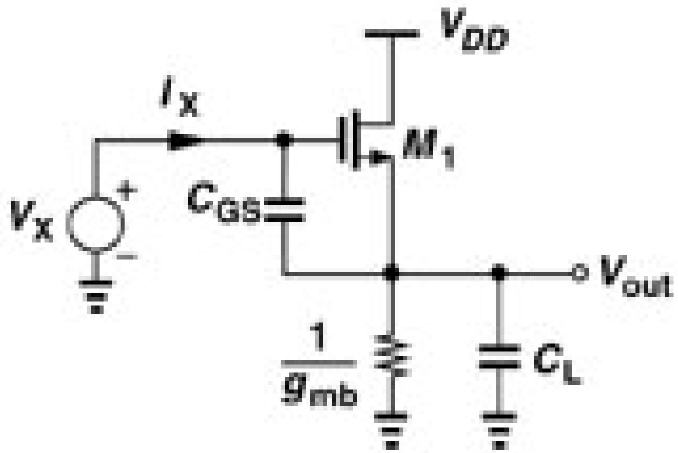
Source Follower Input Impedance Derivation



Laying C_{GD} aside, as it appears in parallel to the remaining part of Z_{in}:

$$V_X \approx \frac{I_X}{sC_{GS}} + \left(I_X + \frac{g_m I_X}{sC_{GS}} \right) \left(\frac{1}{g_{mb}} \parallel \frac{1}{C_L s} \right)$$

Source Follower Input Impedance



Neglecting C_{GD} ,

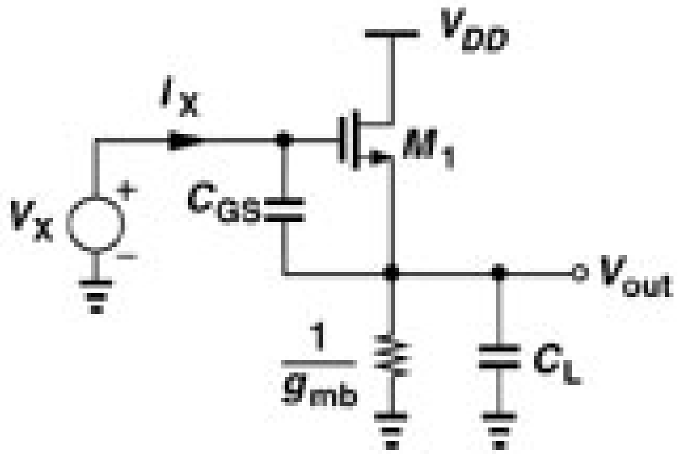
$$Z_{in} \approx \frac{1}{sC_{GS}} + \left(1 + \frac{g_m}{sC_{GS}}\right)\frac{1}{g_{mb} + sC_L}$$

At low frequencies, $g_{mb} >> | sC_L |$

$$Z_{in} \approx \frac{1}{sC_{GS}}\left(1 + (g_m / g_{mb})\right) + 1 / g_{mb}$$

$$\therefore C_{in} = C_{GS} g_{mb} / (g_m + g_{mb}) + C_{GD} \ \ (same\ as\ Miller)$$

"Miller Effect" of C_{GS}



Low frequency gain is $g_m/(g_m+g_{mb})$

$C_{G,Miller}=C_{GS}(1-A_V)$

At low frequencies, $g_{mb} >> | sC_L |$

$$Z_{in} \approx \frac{1}{sC_{GS}}\left(1+(g_m / g_{mb})\right)+1/ g_{mb}$$

$$\therefore C_{in} = C_{GS} g_{mb} /(g_m + g_{mb}) + C_{GD} \ \ (same\ as\ Miller)$$

Effect not important: Only a fraction of C_{GS} is added to C_{in}

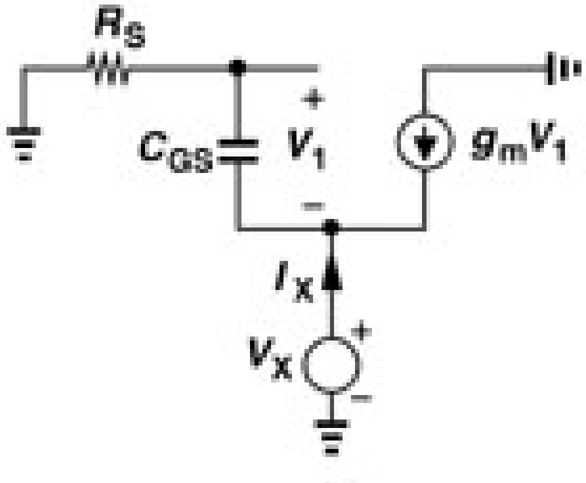
Source Follower HF Z_{in}

At high frequencies, $g_{mb} << \ |sC_L|$

$$Z_{in} \approx \frac{1}{sC_{GS}} + \frac{1}{sC_L} + \frac{g_m}{s^2 C_{GS} \mathrm{C_L}}$$

At a particular frequency, input impedance includes C_{GD} in parallel with series combination of C_{GS} and C_L and a *negative* resistance equal to
$-g_m/(C_{GS}C_L\omega^2)$.

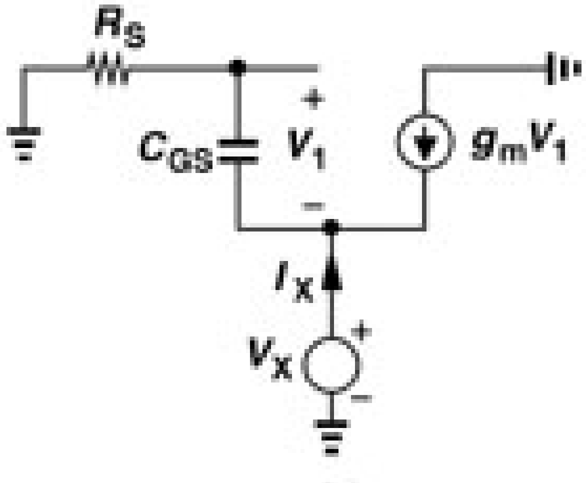
Source Follower Output Impedance Derivation Assumptions



Body effect and C_{SB} contribute an impedance which is a parallel portion of Z_{out} – we'll keep it in mind. We also neglect C_{GD}.

$$Z_{OUT} = V_X / I_X = ?$$

Source Follower Output Impedance Derivation



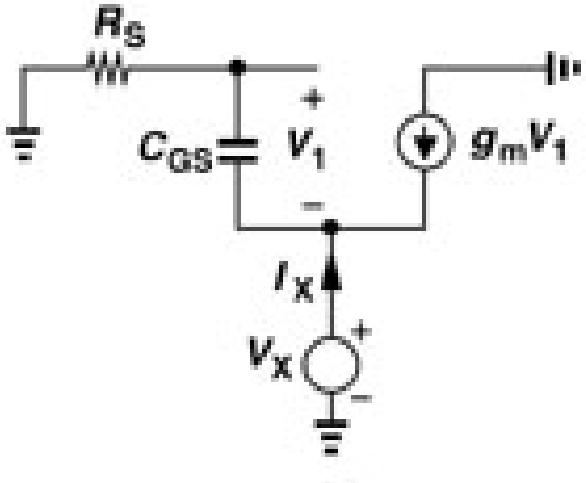
$$V_1 C_{GS} s + g_m V_1 = -I_X$$

$$V_1 C_{GS} s R_S + V_1 = -V_X$$

$$Z_{OUT} = V_X / I_X$$

$$= \frac{s R_S C_{GS} + 1}{g_m + s C_{GS}}$$

Source Follower Output Impedance - Discussion



$$Z_{OUT} = V_X / I_X$$

$$= \frac{sR_SC_{GS} + 1}{g_m + sC_{GS}}$$

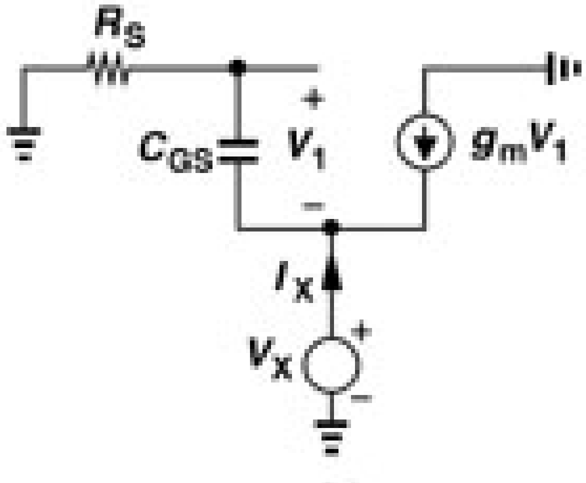
$$\approx 1/g_m \text{, at low frequencies}$$

$$\approx R_S \text{, at high frequencies}$$

We already know that at low frequencies $Z_{out} \approx 1/g_m$

At high frequencies, C_{GS} short-circuits the Gate-Source, and that's why Z_{out} depends on the Source Follower's driving signal source.

Source Follower Output Impedance - Discussion



$$Z_{OUT} = V_X / I_X$$

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$$\approx 1/g_m \text{, at low frequencies}$$

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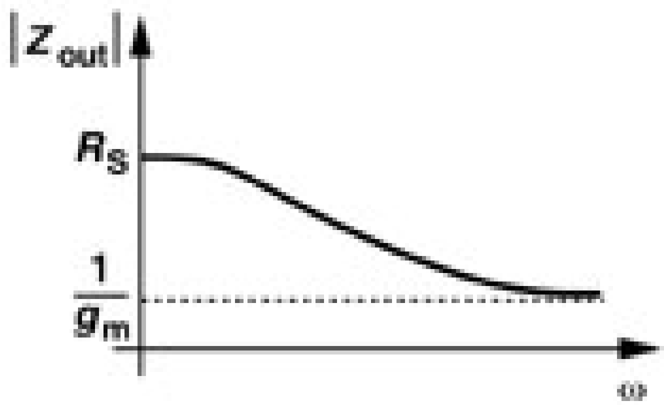
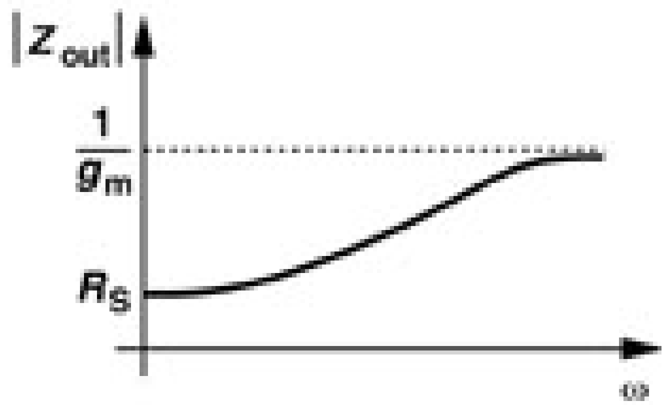
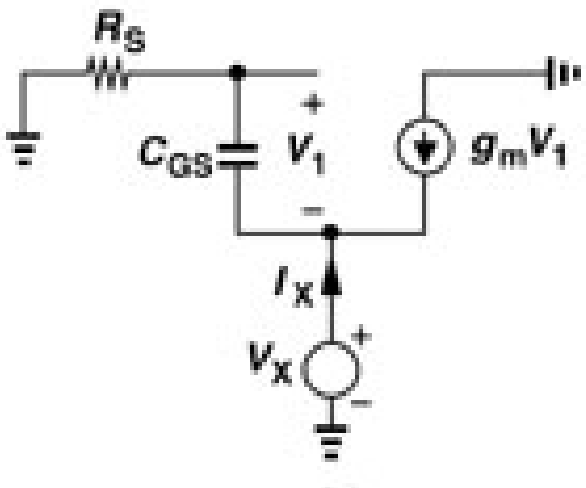


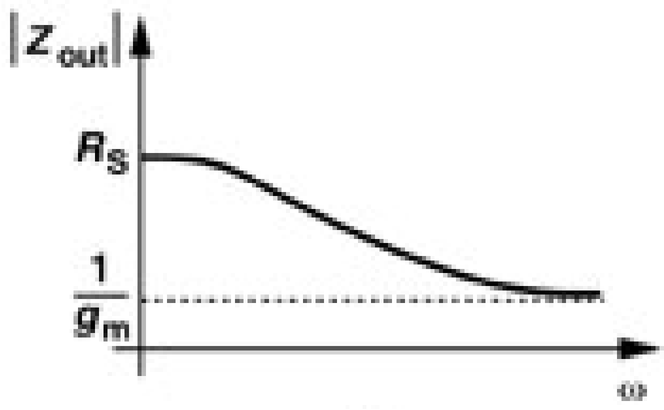


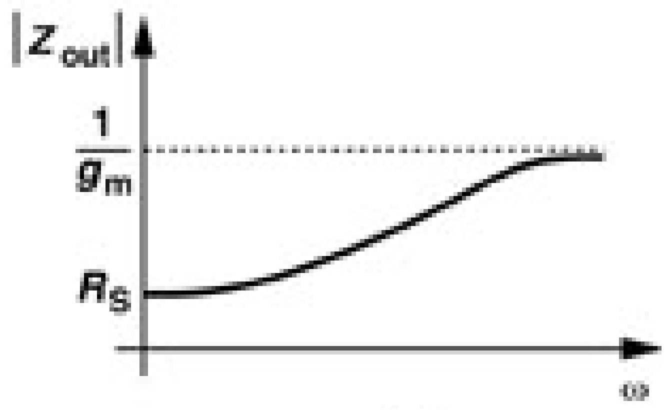
Source Follower Output Impedance - Discussion



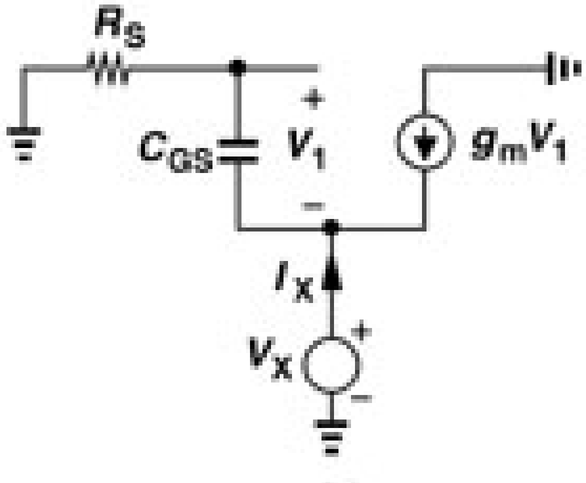
Typically $R_S > 1/g_m$

Therefore $|Z_{out}(\omega)|$ is increasing with ω.

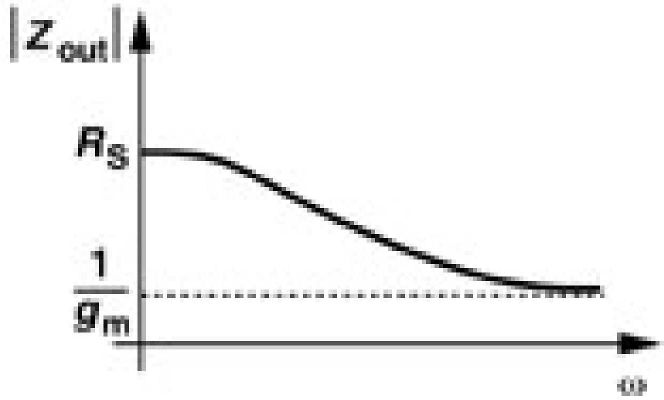


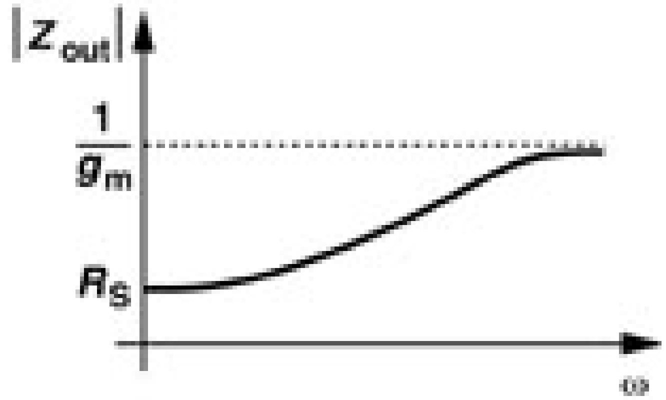


Source Follower Output Impedance - Discussion

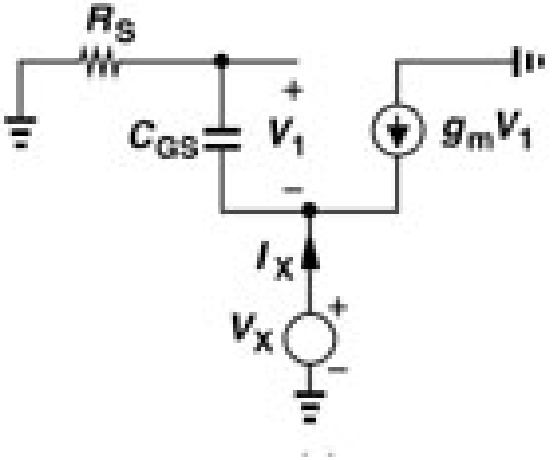


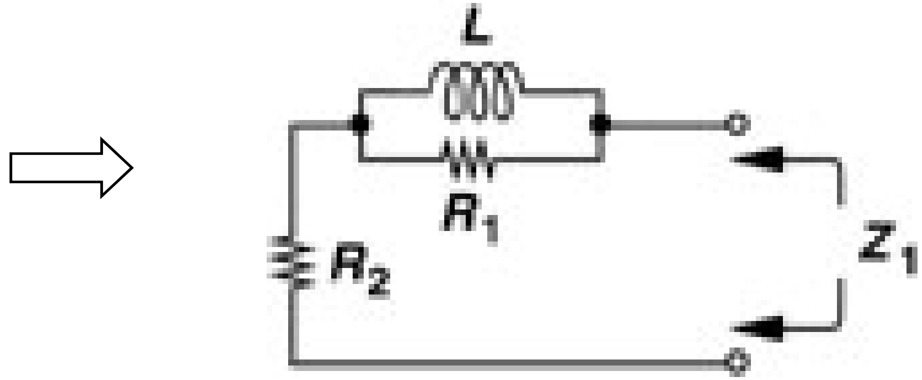
If $|Z_{out}(\omega)|$ is increasing with ω, then it must have an INDUCTIVE component!





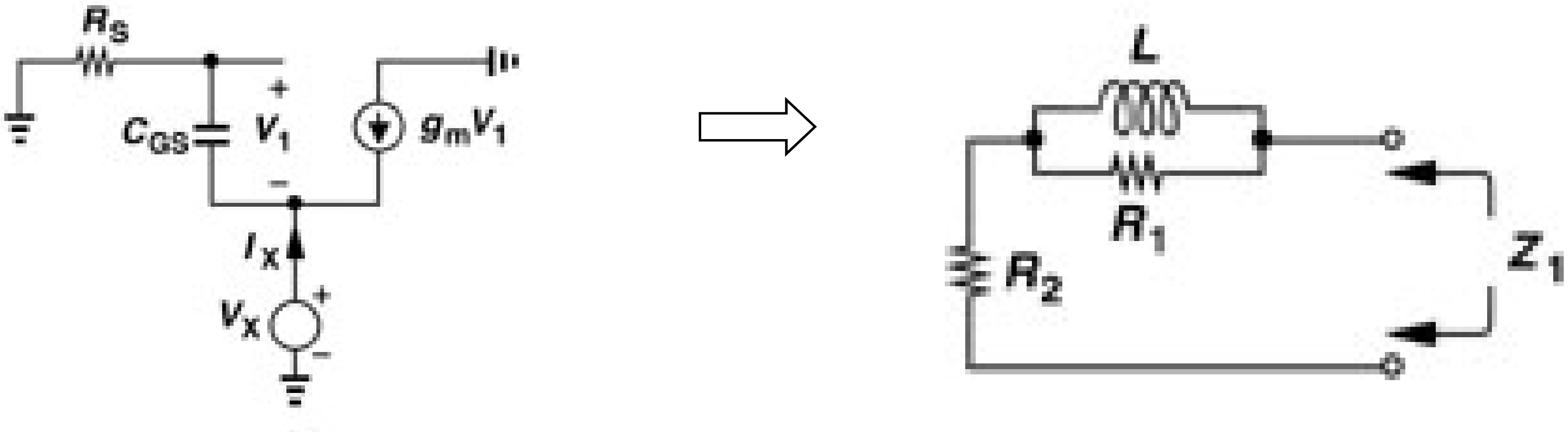
Source Follower Output Impedance – Passive Network Modeling?





Can we find values for R_1, R_2 and L such that Z_1=Z_{out} of the Source Follower?

Source Follower Output Impedance – Passive Network Modeling Construction



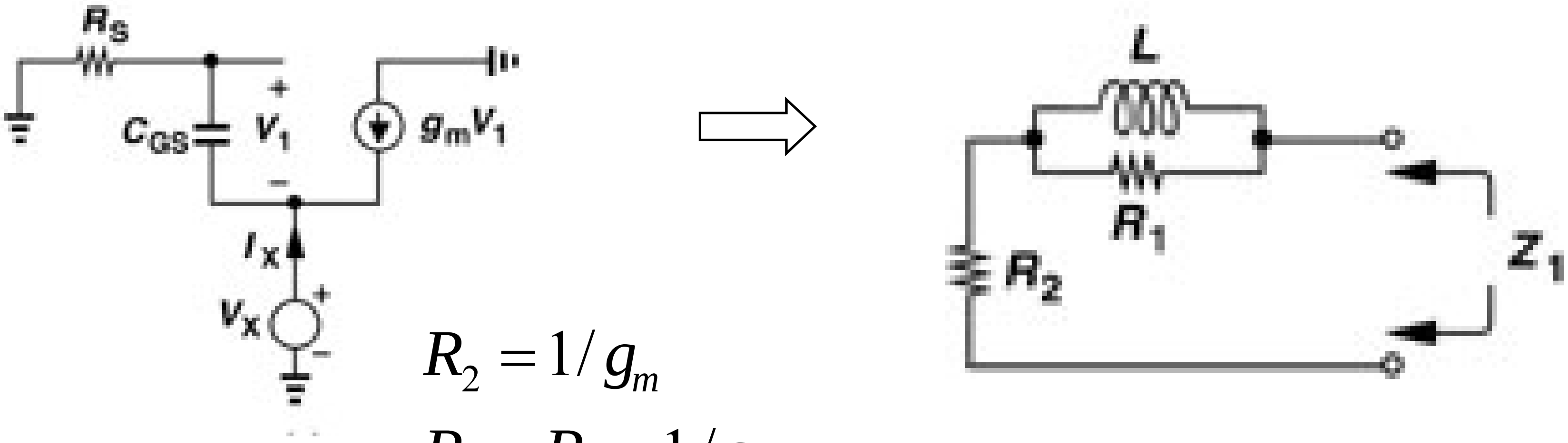
Clues:

At $\omega=0$ $Z_{out}=1/g_m$; At $\omega=\infty$ $Z_{out}=R_S$

At $\omega=0$ $Z_1=R_2$; At $\omega=\infty$ $Z_1=R_1+R_2$

Let $R_2=1/g_m$ and $R_1=R_S-1/g_m$ (here is where we assume $R_S>1/g_m$!)

Source Follower Output Impedance – Passive Network Modeling Final Result



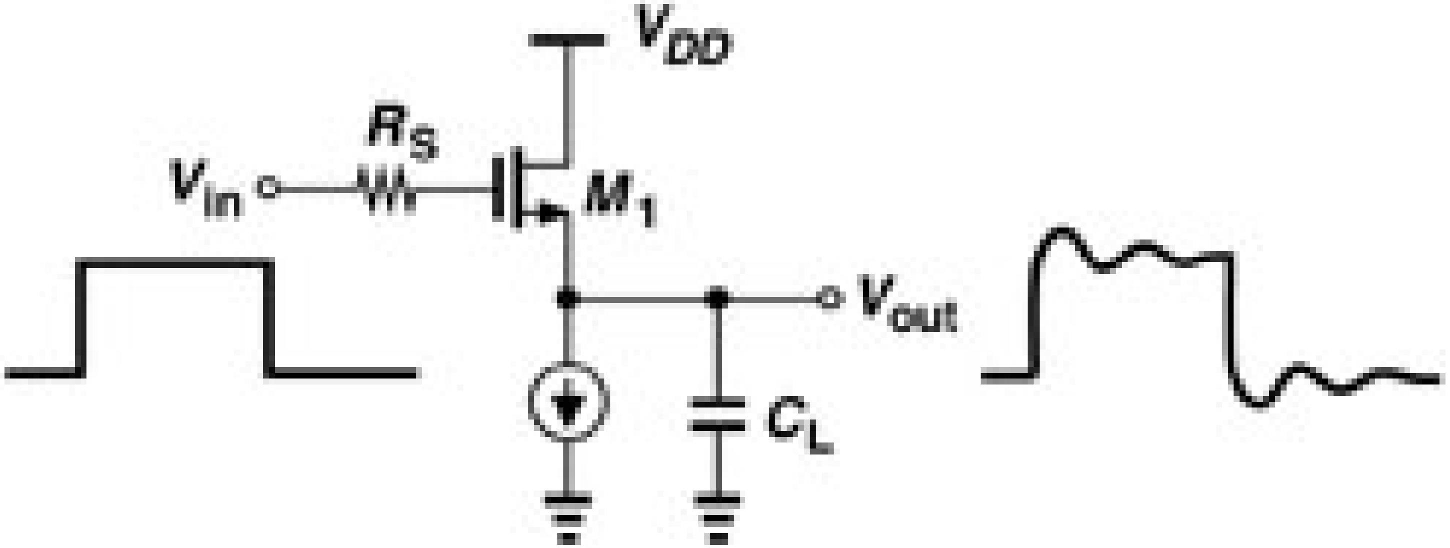
$$R_2 = 1/g_m$$

$$R_1 = R_S - 1/g_m$$

$$L = \frac{C_{GS}}{g_m}\left(R_S - 1/g_m\right)$$

Output impedance inductance dependent on source impedance, R_S (if R_S large) !

Source Follower Ringing

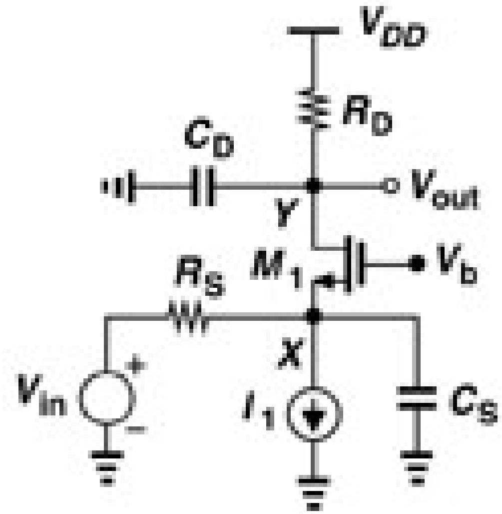


Output ringing due to tuned circuit formed with C_L and inductive component of output impedance. It happens especially if load capacitance is large.

High Frequency Response- L26

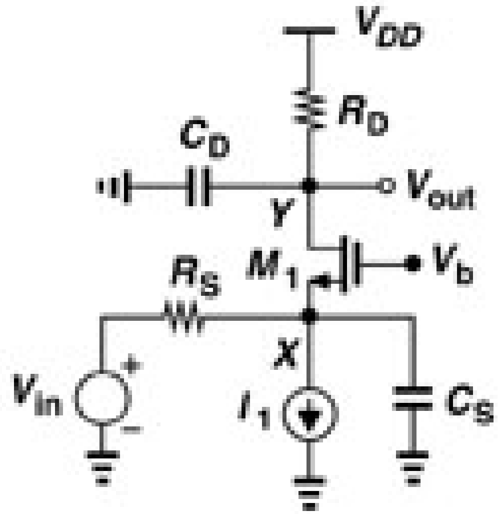
Common Gate Amplifier

CG Amplifier neglecting r_o



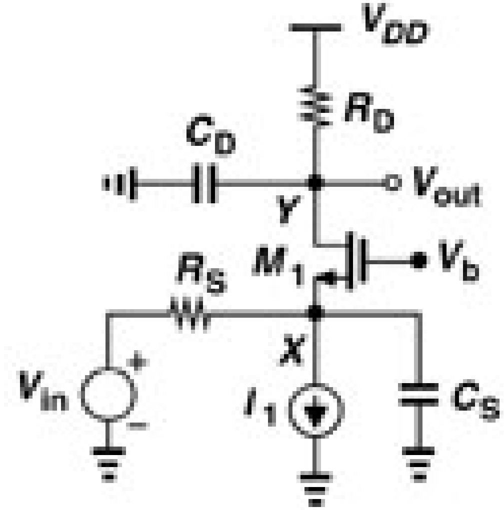
- $C_S = C_{GS1} + C_{SB1}$ Capacitance "seen" from Source to ground.
- $C_D = C_{DG} + C_{DB}$ Capacitance "seen" from Drain to ground.

CG Amplifier if r_o negligible



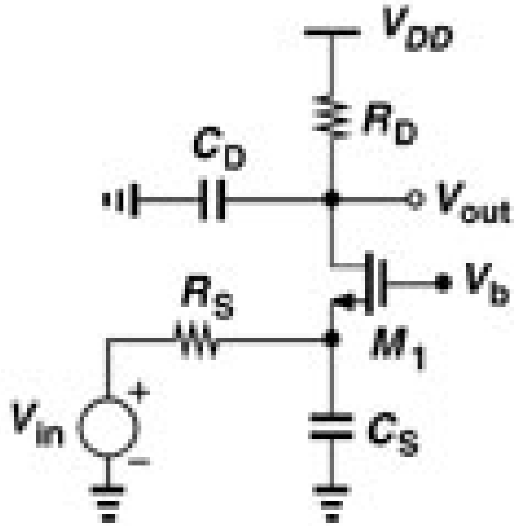
- Can determine (by inspection) the two poles contributed, one by C_S and another by C_D.
- This is easy to do only if we neglect r_{01}.
- Strategy: Find equivalent resistance “seen” by each capacitor, when sources are nulled.

CG Amplifier Transfer Function if r_o is negligible



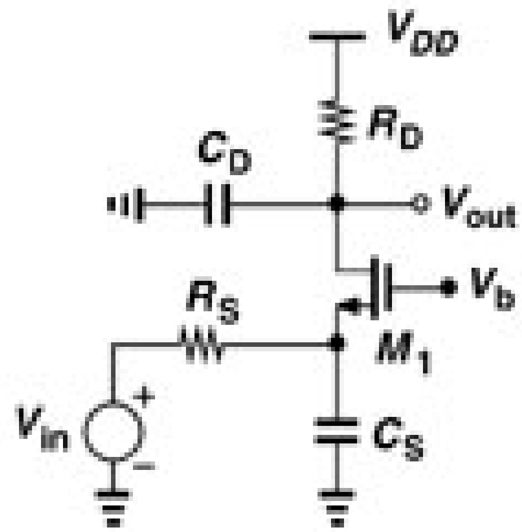
- $\tau_S = C_S R_{S,eq} = C_S \{R_S || [1/(g_{m1}+g_{mb1})]$
- $\tau_D = C_D R_{D,eq} = C_D R_D$
- $A = (g_{m1}+g_{mb1}) R_D / (1+(g_{m1}+g_{mb1}) R_S)$
- $V_{out}(s)/V_{in}(s) = A/[(1+s\tau_S)(1+s\tau_D)]$

CG Bandwidth taking r_o into account



Can we use Miller's theorem for the bridging r_o resistor?

CG Bandwidth taking r_o into account

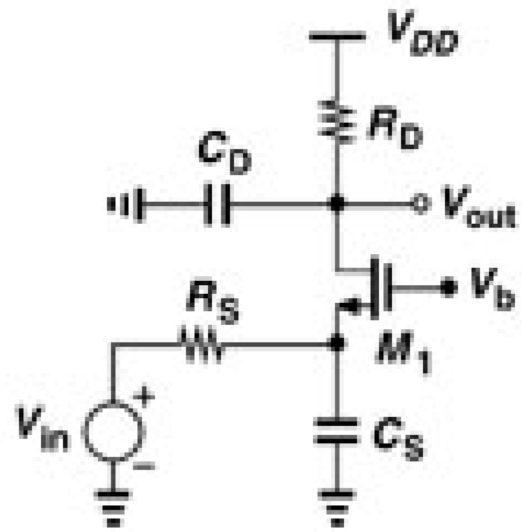


Can we use Miller's theorem for the bridging r_o resistor? Well, no.

Effect of r_o at input: Parallel resistance $r_o/(1-A_V)$.

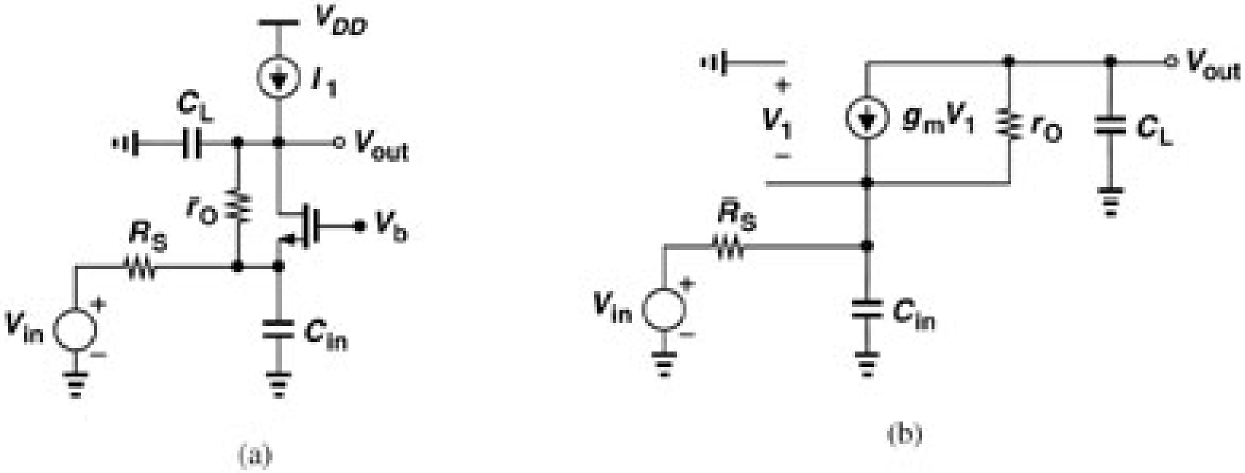
Recall: A_V is large and positive → Negative resistance! → How to compute time constants?

CG Bandwidth taking r_o into account



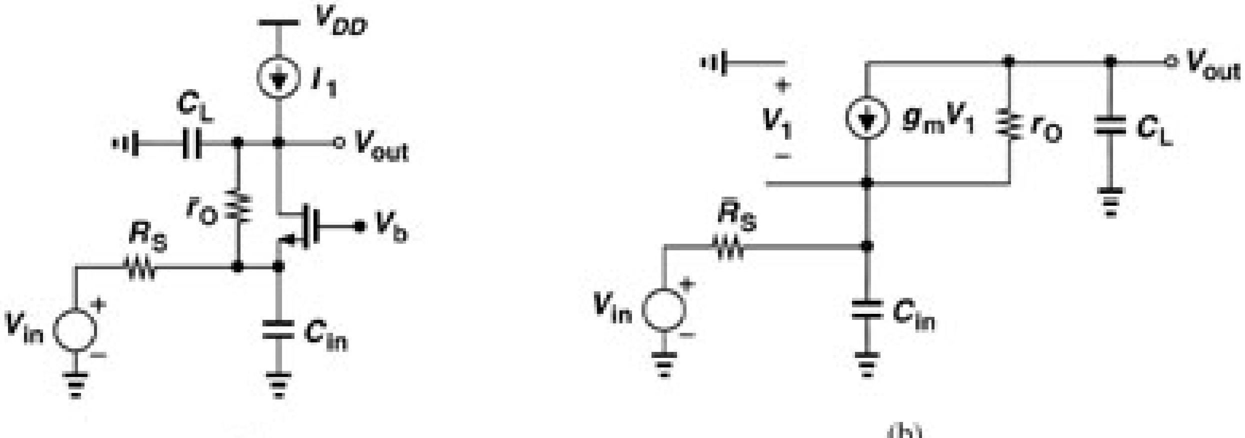
Need to solve exactly, using impedances 1/sC, and Kirchhoff’s laws.

Example: r_o effect if R_D is replaced with a current source load



- Current from V_{in} through R_S is: $V_{out}C_Ls+(-V_1)C_{in}s$
- Adding voltages: $(-V_{out}C_Ls+V_1C_{in}s)R_S+V_{in}=-V_1$
- $V_1= -[-V_{out}C_LsR_S+V_{in}]/(1+C_{in}R_Ss)$
- Also: $r_o(-V_{out}C_Ls-g_mV_1)-V_1=V_{out}$

Example: r_o effect if R_D is replaced with a current source load – final result



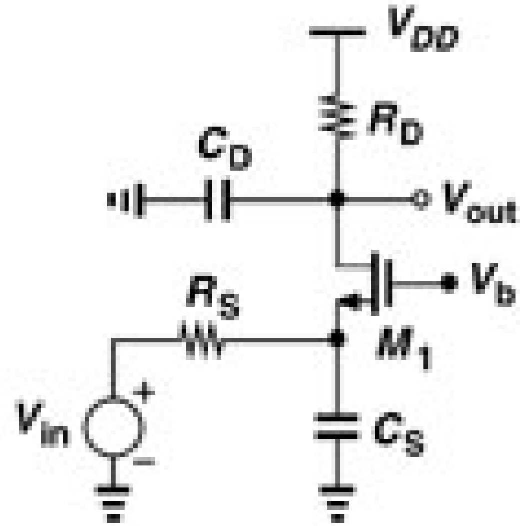
(a) (b)

$V_{out}(s)/V_{in}(s)=(1+g_mr_o)/D(s)$, where:

$D(s)=r_oC_LC_{in}R_Ss^2+[r_oC_L+C_{in}R_S+(1+g_mr_o)C_LR_S]s+1$

The effect of g_{mb} can now be added simply by replacing g_m by g_m+g_{mb}.

CG with R_D load Input Impedance

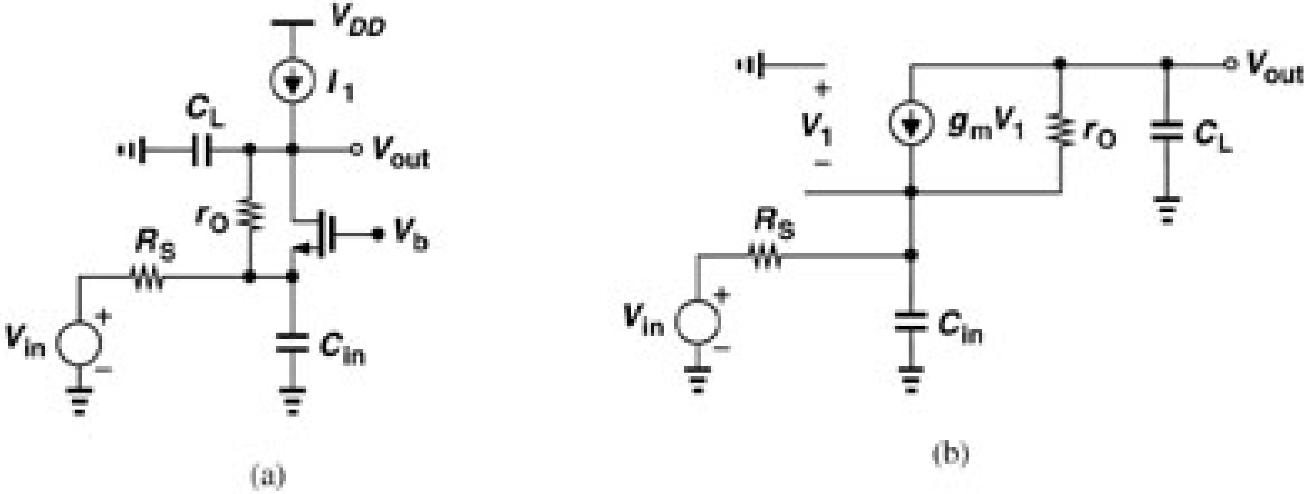


Recall the exact R_{in} formula at low frequencies:

$R_{in}=[R_D/((g_m+g_{mb})r_o)]+[1/(g_m+g_{mb})]$.

Simply replace R_{in} by Z_{in} and R_D with $Z_L=R_D||(1/sC_D)$

CG with current source load Input Impedance

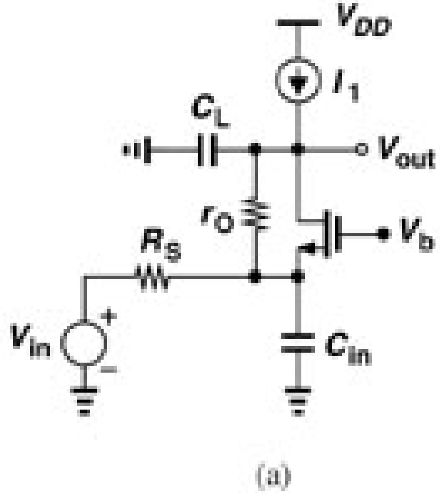


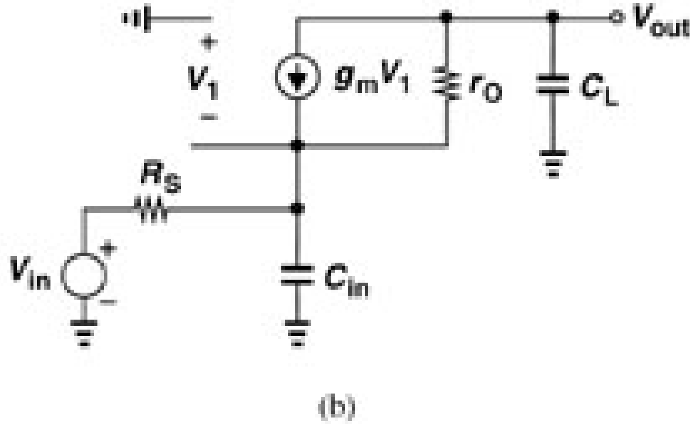
Recall the exact R_{in} formula at low frequencies:

$R_{in}=[R_D/((g_m+g_{mb})r_o)]+[1/(g_m+g_{mb})]$.

Here replace R_{in} by Z_{in} and R_D with $Z_L=1/sC_L$

CG with current source load Input Impedance - Discussion





Z_{in}=[1/sC_L/((g_m+g_{mb})r_o)]+[1/(g_m+g_{mb})].

At high frequencies, or if C_L is large, the effect of C_L at input becomes negligible compared to the effect of C_{in} (as $Z_{in}\rightarrow$ 1/(g_m+g_{mb}))

CG and CS Bandwidth Comparison

- If R_S is large enough, bandwidth is determined by the input time constant
- CG: $\tau_{in}=(C_{GS}+C_{SB})[R_S||(1/(g_m+g_{mb}))]$
- CS: $\tau_{in}=[C_{GS}+(1+g_mR_D)C_{GD}]R_S$
- Typically -3db frequency of CG amplifier is by an order of magnitude larger than that of a CS amplifier.
- If R_S is small, τ_{out} dominates. It is the same in both amplifiers.

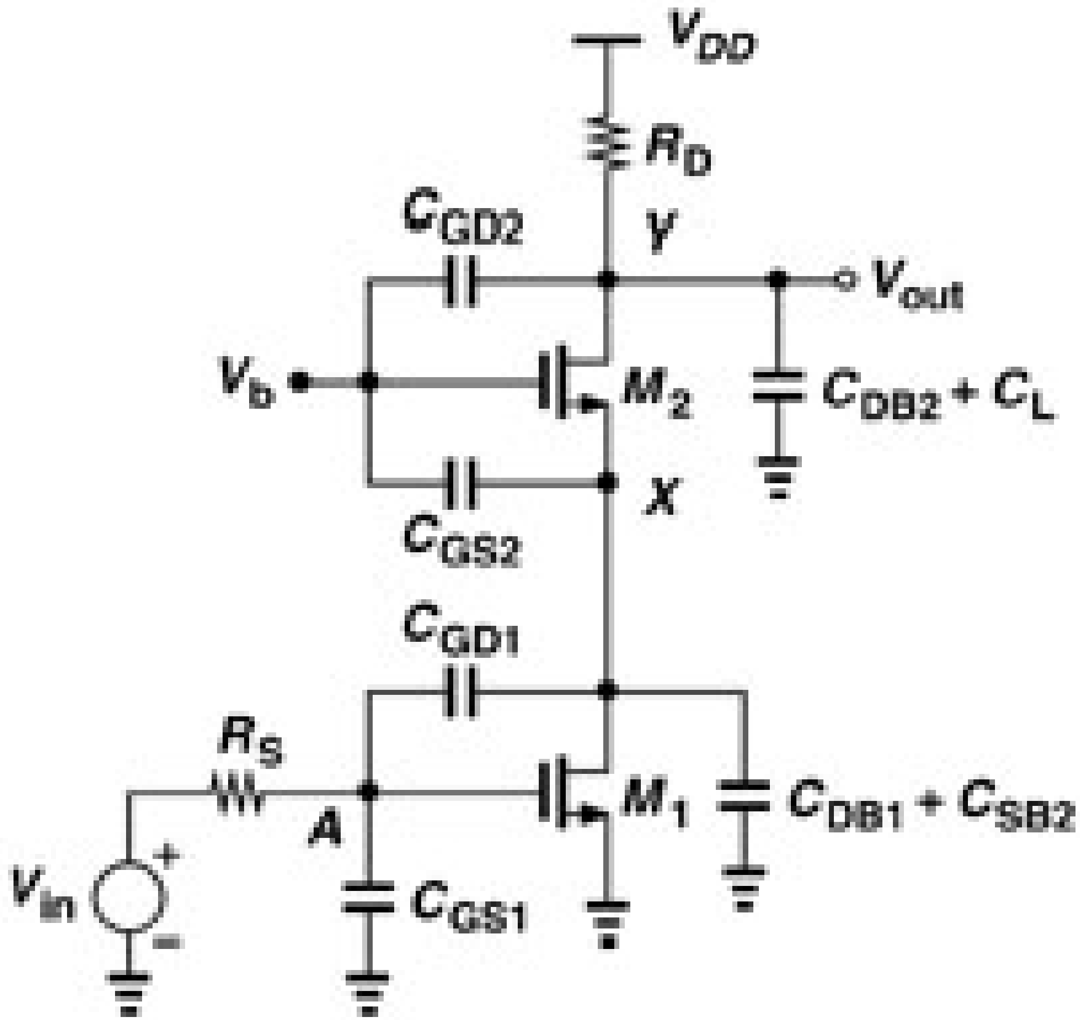
High Frequency Response-L27

Cascode Amplifier

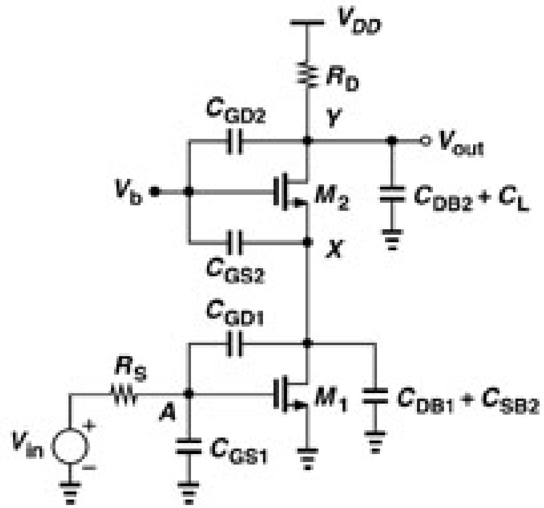
Key Ideas:

- Cascode amplifier has the same input resistance and voltage gain as those of a CS amplifier.
- Cascode amplifier has a much larger bandwidth than CS amplifier.
- Cascode = CS→CG. CS has a much reduced Miller Effect because CS gain is low (near -1).

Cascode Amplifier Capacitances

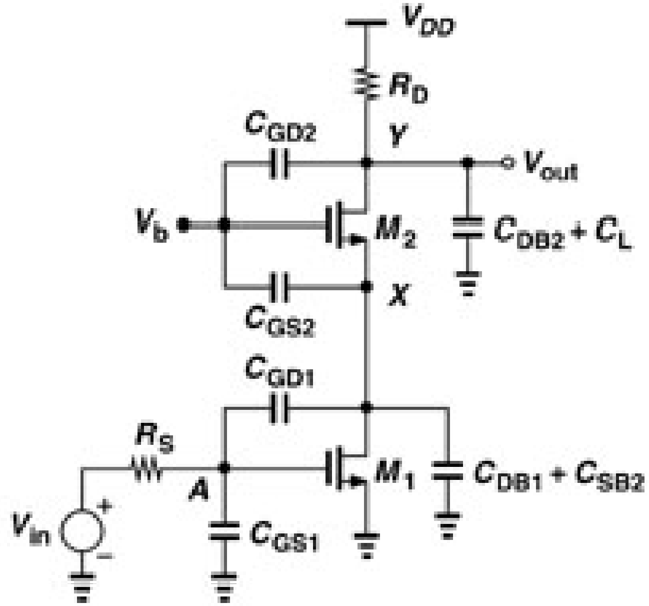


C_{GD1} Insignificant Miller Effect



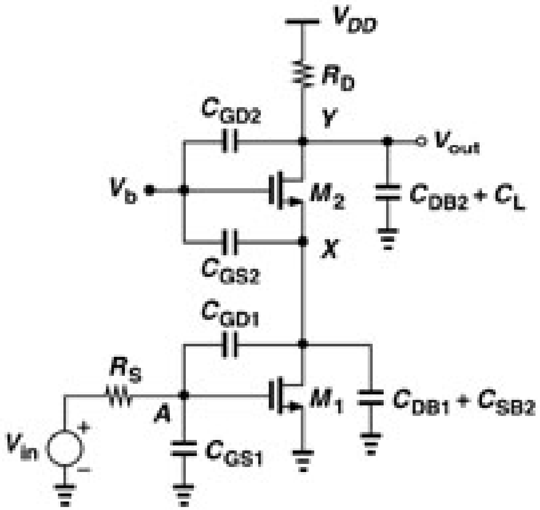
- M_1 is CS with a load of $1/(g_{m2}+g_{mb2})$ (the input resistance of the CG stage).
- $A_V= -g_{m1} /(g_{m2}+g_{mb2})$ which is a fraction.
- Effect of C_{GD1} at input is at most $2C_{GD1}$

Cascode Input Pole (if dominant)



$$f_{p,A} = \frac{1}{2\pi R_S [C_{GS1} + (1 + \frac{g_{m1}}{g_{m2} + g_{mb2}})C_{GD1}]}$$

Cascode Mid-section Pole (if dominant)

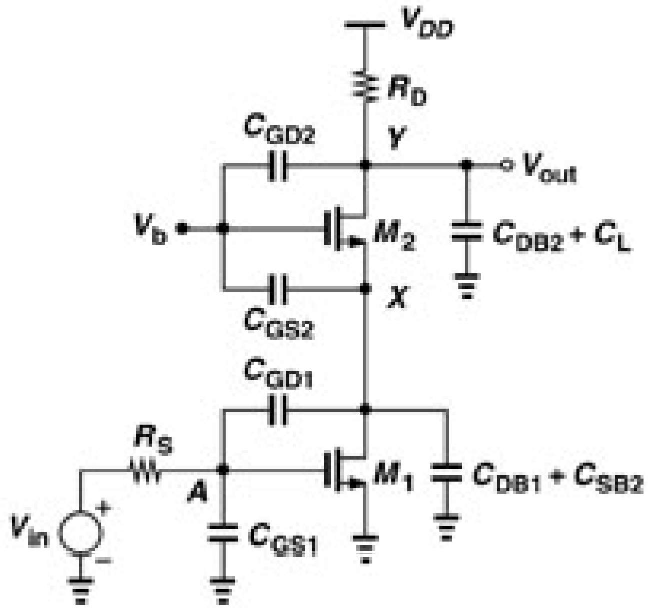


Total resistance seen at Node X is M_2 input resistance.

By Miller: C_{GD1} contribution is at most C_{GD1}

$$f_{p,X} \approx \frac{g_{m2} + g_{mb2}}{2\pi\left(C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}\right)}$$

Cascode Output Pole (if dominant)



$$f_{p,Y} = \frac{1}{2\pi R_D\left(C_{DB2} + C_L + C_{GD2}\right)}$$

Which of the Cascode's three poles is dominant?

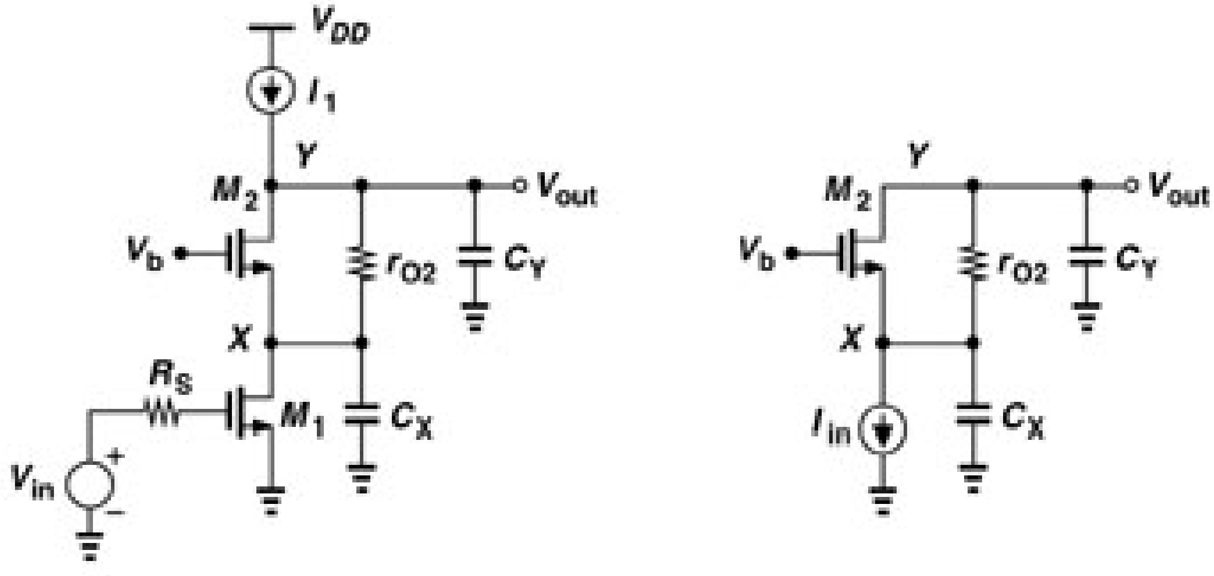
$$f_{p,A} = \frac{1}{2\pi R_S\left[C_{GS1} + C_{GD1}\left(1 + \frac{g_{m1}}{g_{m2} + g_{mb2}}\right)\right]}$$

$$f_{p,X} = \frac{g_{m2} + g_{mb2}}{2\pi\left(C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}\right)}$$

$$f_{p,Y} = \frac{1}{2\pi R_D\left(C_{DB2} + C_L + C_{GD2}\right)}$$

Either $f_{p,A}$ or $f_{p,Y}$ dominates. $f_{p,X}$ is typically a higher frequency.

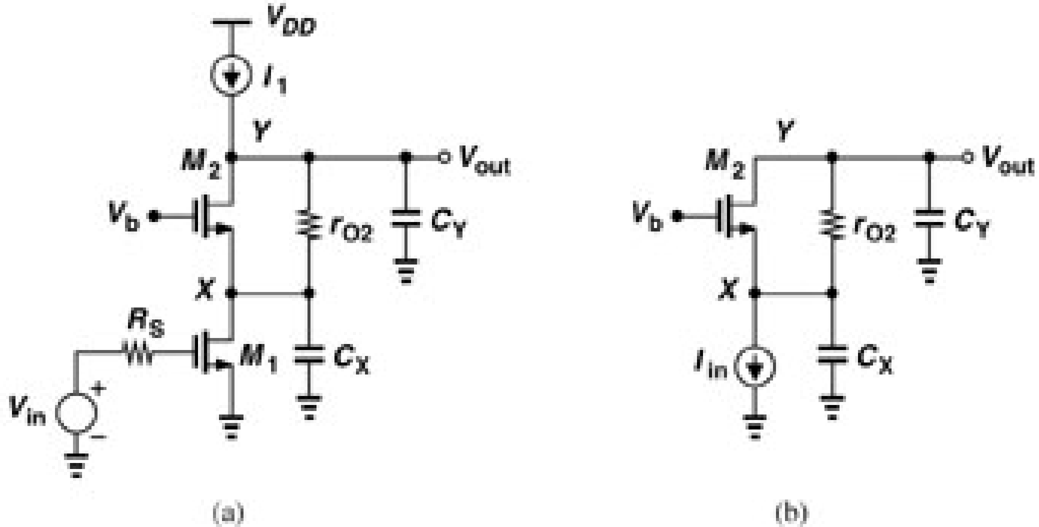
Cascode with current source load



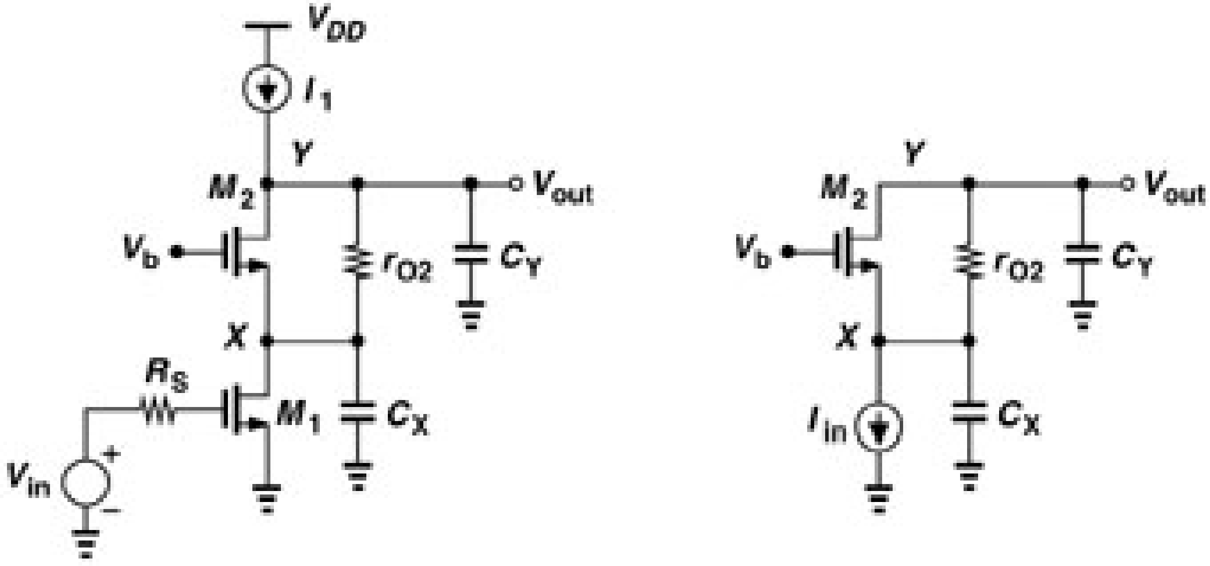
(a) (b)

- This is done whenever we want larger voltage gains.
- However we pay by having a lower bandwidth:

Lower Bandwidth of Cascode with current source load



- Key problem: If a CG amplifier feeds a current source load, its R_{in} may be quite large.
- It causes an aggravation of Miller effect at input.
- Also: The impact of C_X (mid-section capacitance) may no longer be negligible.

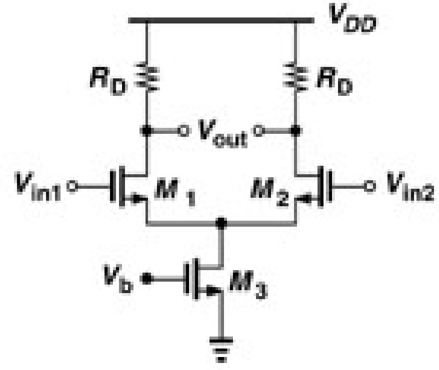


(a) (b)

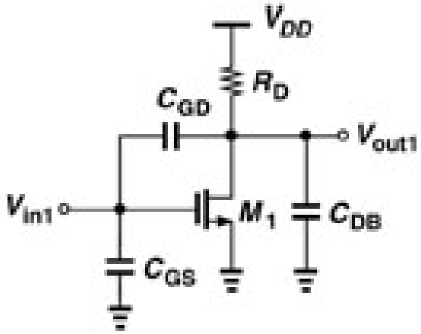
High Frequency Response-L28

Differential Amplifiers

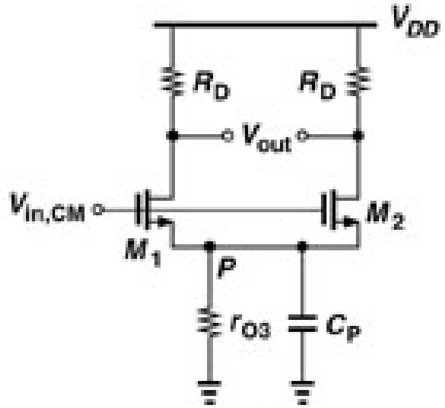
Analysis Strategy



(a)



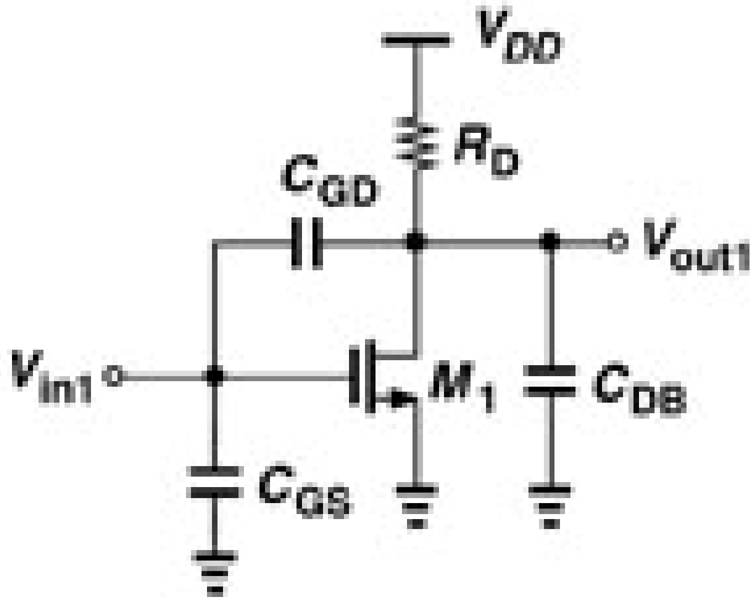
(b)



(c)

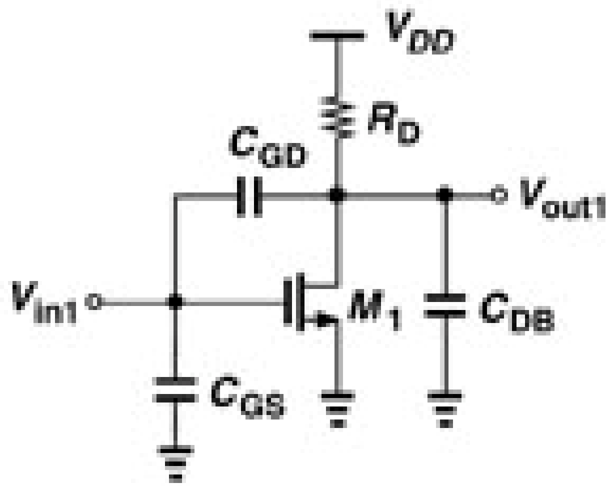
Do separate Differential-Mode and Common-Mode high-frequency response analysis

Differential-Mode – Half-Circuit Analysis



Differential-Mode HF response identical to that of a CS amplifier.

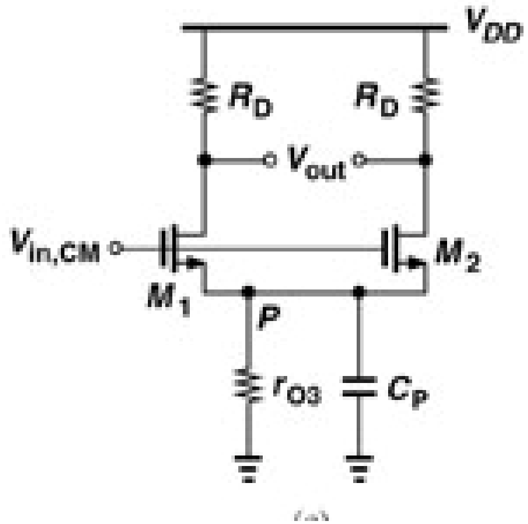
Differential-Mode HF response Analysis



Relatively low A_{DM} bandwidth due to C_{GD} Miller effect.

Remedy: Cascode Differential Amplifier.

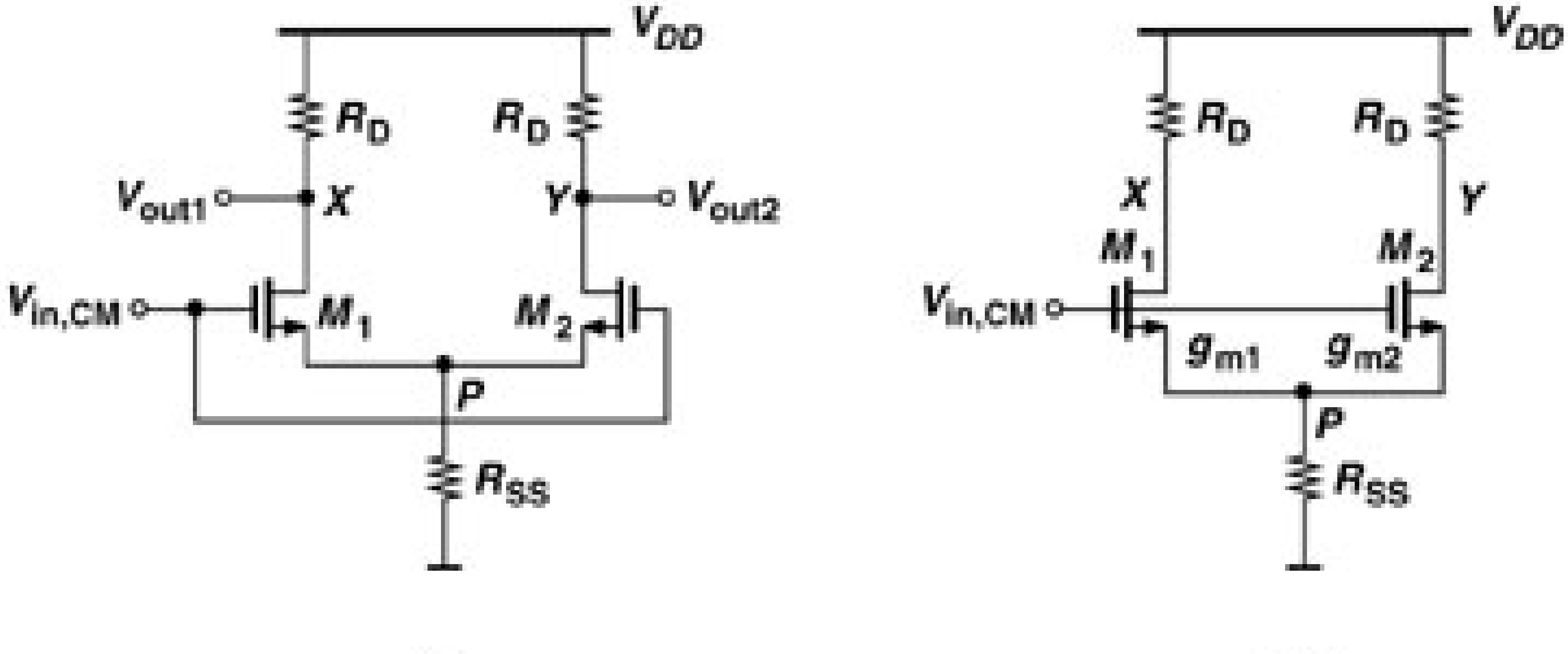
Common-Mode HF Response Analysis



C_P depends on C_{GD3}, C_{DB3}, C_{SB1} and C_{SB2}

How large is it? Can be substantial if (W/L)'s large

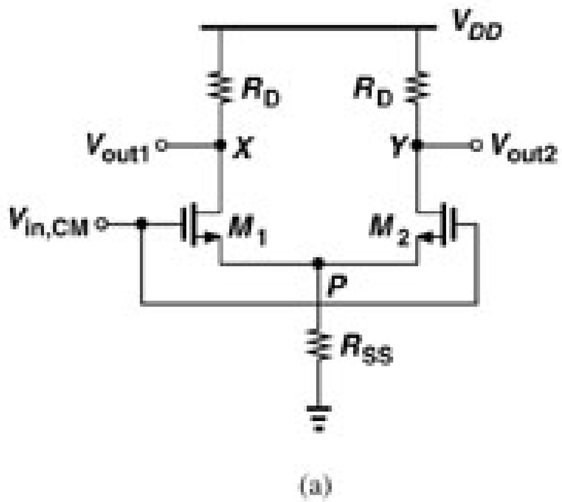
Recall: Mismatches in W/L, V_{TH} and other transistor parameters all translate to mismatches in g_m



(a) (b)

$$A_{V,CM-DM} = \frac{V_X - V_Y}{V_{in,CM}} = -\frac{g_{m1} - g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D$$

Common-Mode HF response if M_1 and M_2 mismatch (solution approach)



$$A_{V,CM-DM} = \frac{V_X - V_Y}{V_{in,CM}} = -\frac{g_{m1} - g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D$$

Replace: R_{SS} with $r_{o3}||(1/C_P s)$, R_D with $R_D||(1/C_L\ s)$

Common-Mode HF response if M_1 and M_2 mismatch - Result

$$A_{V,CM-DM}(s) = \frac{V_X - V_Y}{V_{in,CM}} = -\frac{(g_{m1} - g_{m2})[R_D \parallel \frac{1}{C_L s}]}{(g_{m1} + g_{m2})[r_{o3} \parallel \frac{1}{C_P s}] + 1} =$$

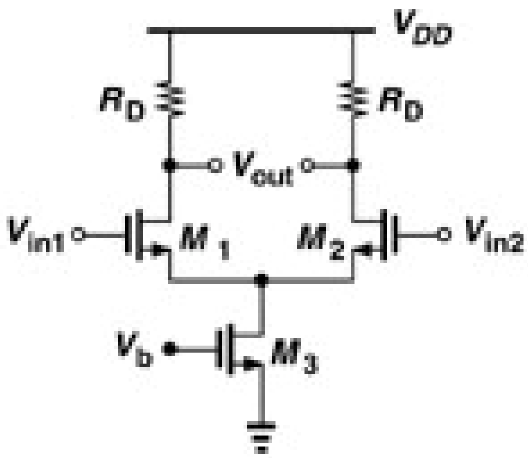
$$= -\frac{(g_{m1} - g_{m2})R_D}{(g_{m1} + g_{m2})r_{o3} + 1} \cdot \frac{(1 + s r_{o3} C_P)}{(1 + sR_D C_L)(1 + s\frac{r_{o3} C_P}{(g_{m1} + g_{m2})r_{o3} + 1})}$$

Zero dominates! $A_{V,CM-DM}$ begins to rise at $f_z = 1/(2\pi r_{o3} C_P)$

Overall Differential Amplifier's Bandwidth

- At a certain high frequency $f=f_{P,DM}$ differential gain A_{DM} begins to fall.
- At another high-frequency $f=f_{Z,CM}$ common-mode gain A_{CM} begins to rise.
- Whichever of the above 3db frequencies is lower determines the amplifier's bandwidth – we are really interested in the frequency at which the amplifier's CMRR begins to fall.

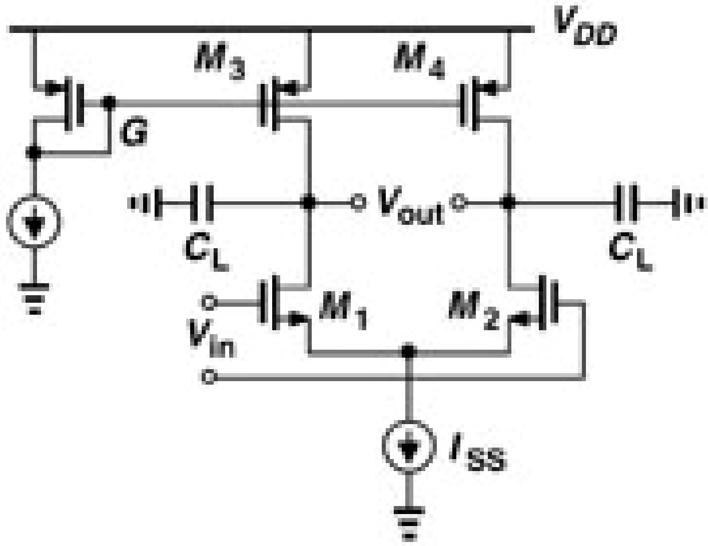
CMRR vs Bandwidth Tradeoff



- In order to reduce the voltage drop $I_D R_D$ and still obtain a large enough A_{DM} we typically want gate widths to be relatively large (for W/L to be large enough)
- The larger W's the smaller is bandwidth.
- Issue is more severe the smaller V_{DD} gets.

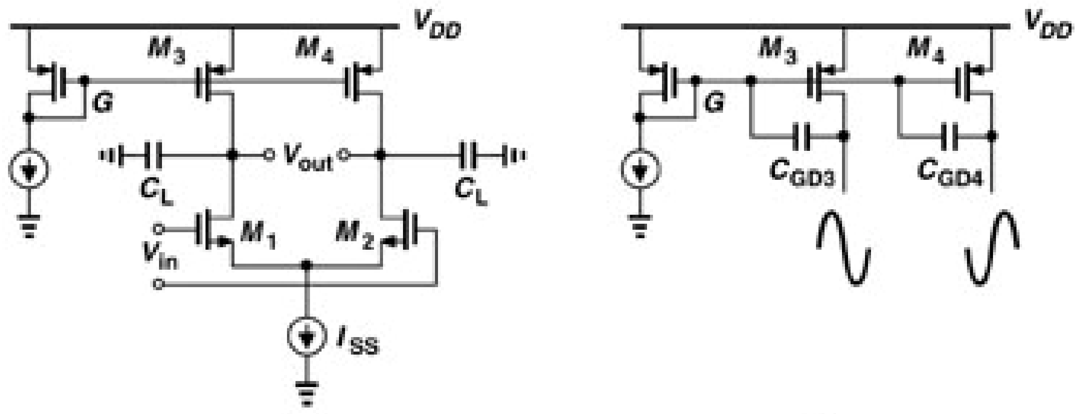
Frequency Response of Differential Pairs With High-Impedance Load

Differential output



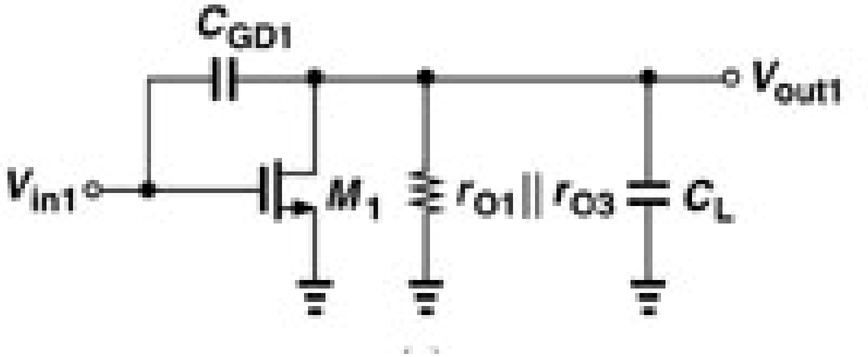
Here C_L include C_{GD} and C_{DB} of PMOS loads

Differential Mode Analysis: G Node



- For differential outputs, C_{GD3} and C_{GD4} conduct equal and opposite currents to node G.
- Therefore node G is ground for small-signal analysis.
- In practice: We hook up by-pass capacitor between G and ground.

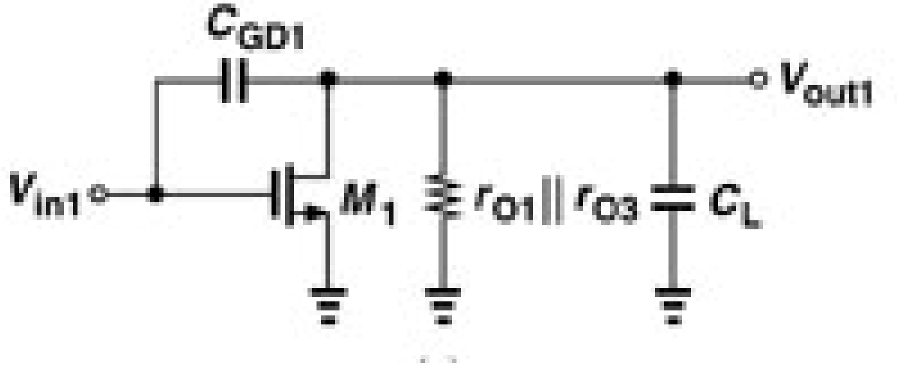
Differential Mode Half Circuit Analysis



$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{(sC_{GD} - g_m)R_D}{s^2 R_S R_D (C_{GS}C_{GD} + C_{GS}C_{SB} + C_{GD}C_{DB}) + s\left[R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})\right] + 1}$$

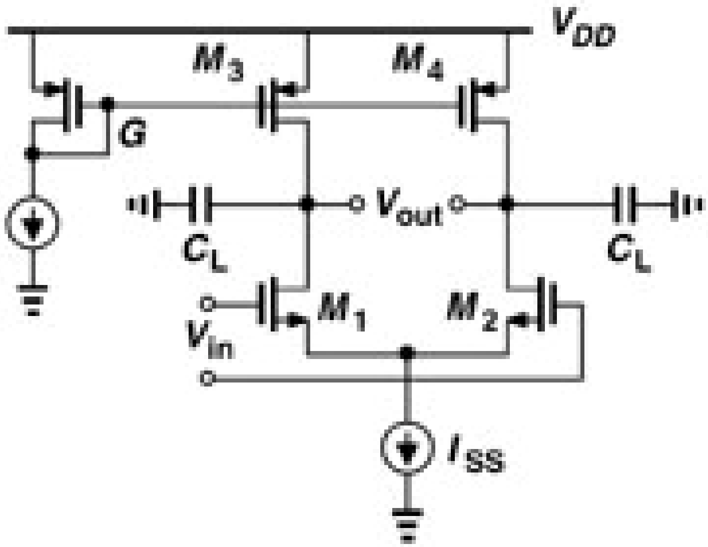
Above (obtained earlier) "exact" transfer function applies, if we replace R_D by $r_{o1}||r_{o3}$

Differential Mode Half Circuit Analysis – final result



- Here, because C_L is quiet large and because the output time constant involves $r_{o1}||r_{o3}$ which is large, it is the output time constant that dominates.
- $f_h \approx 1/2\pi C_L(r_{o1}||r_{o3})$

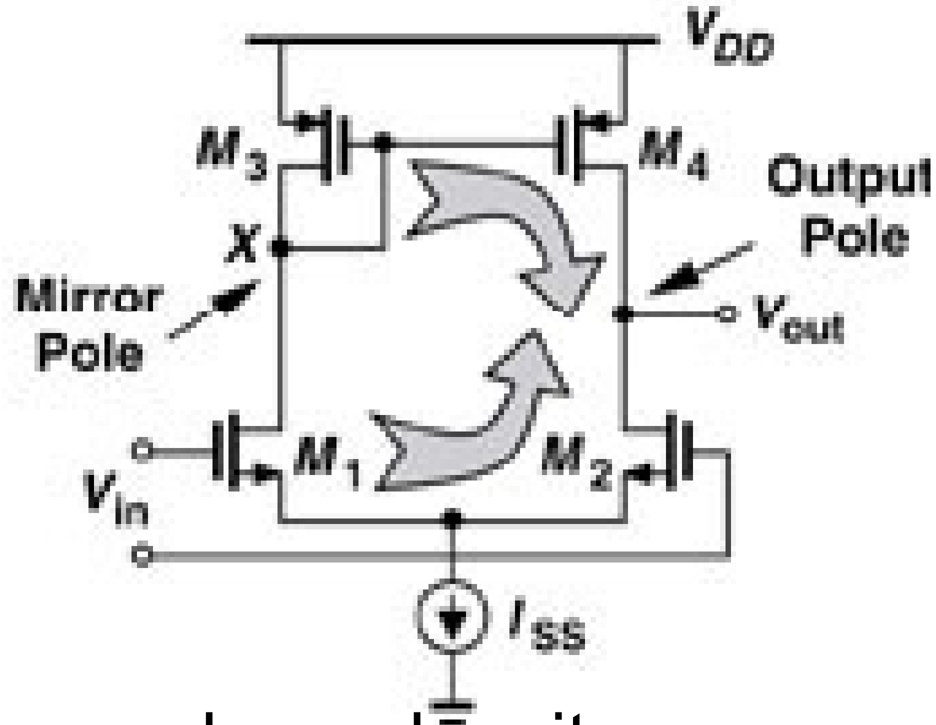
Common-Mode HF Analysis



Result identical to that of a differential amplifier with R_D load.

Recall dominant zero due to C_P (source)

Differential Pair with Active Current Mirror Load

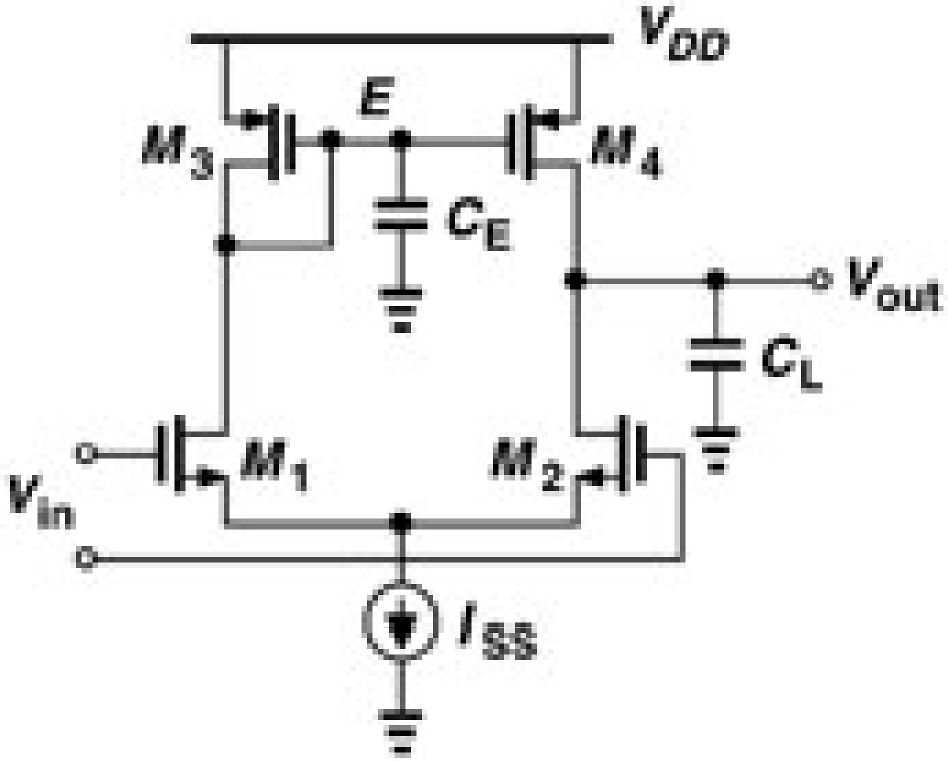


Single-ended output

There are two signal paths, each one has its own transfer function.

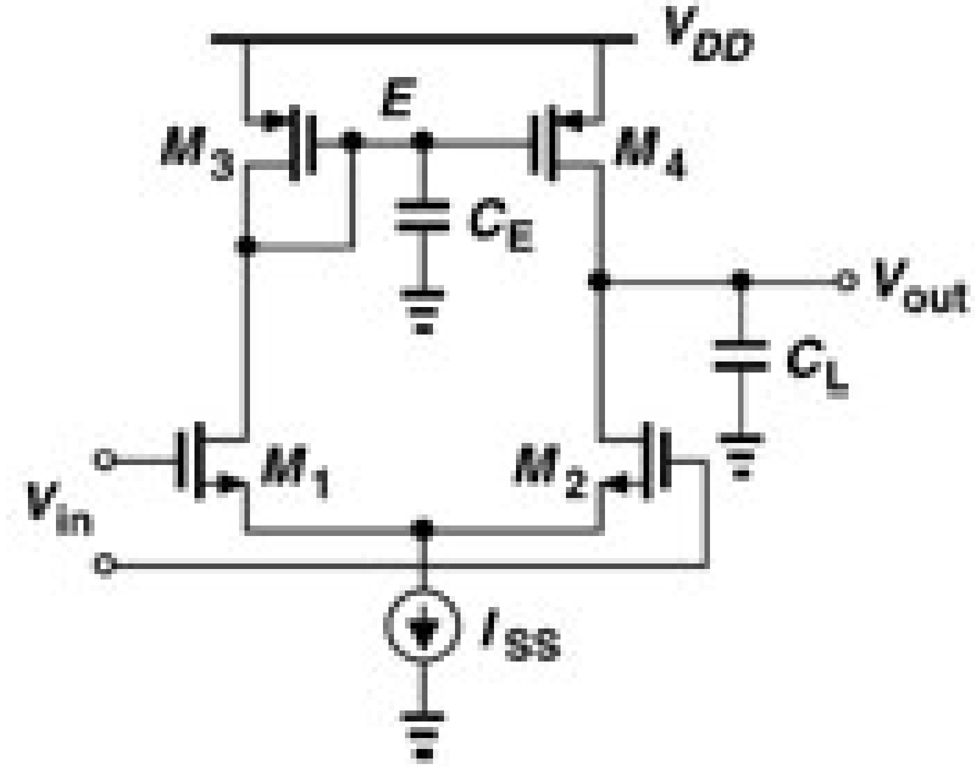
Overall transfer function is the sum of the two paths' transfer functions

The Mirror Pole



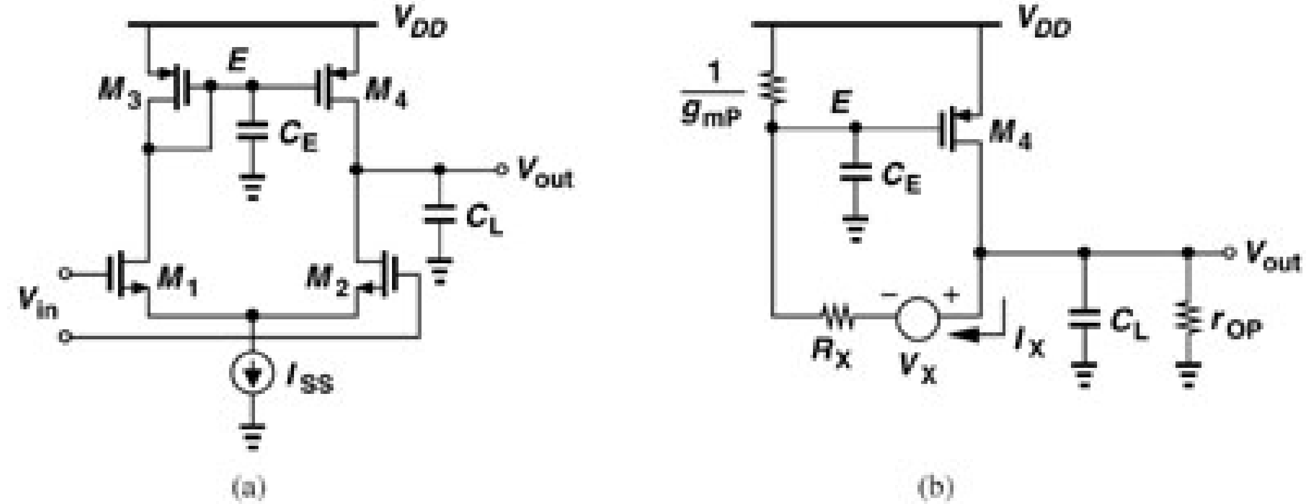
C_E arises from C_{GS3},C_{GS4},C_{DB3},C_{DB1} and Miller effect of C_{GD1} and C_{GD4}. Pole is at $s=-g_{m3}/C_E$

Simplified diagram showing only the largest capacitances:



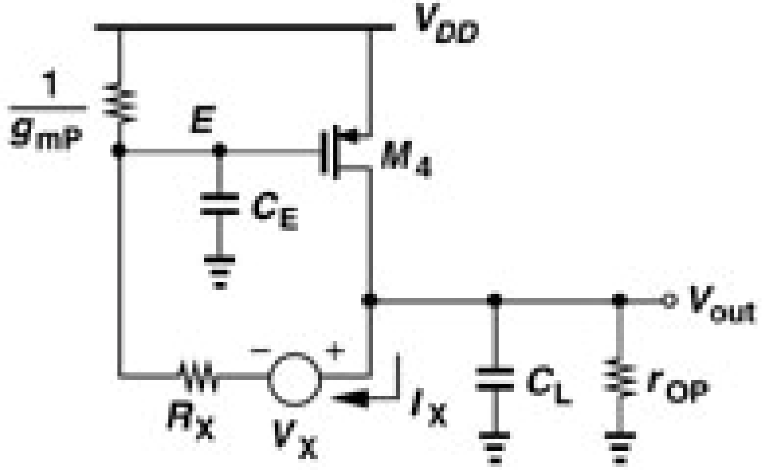
Solution approach: Replace V_{in}, M_1 and M_2 by a Thevenin equivalent

Thevenin equivalent of bottom circuit (for diff. mode analysis)



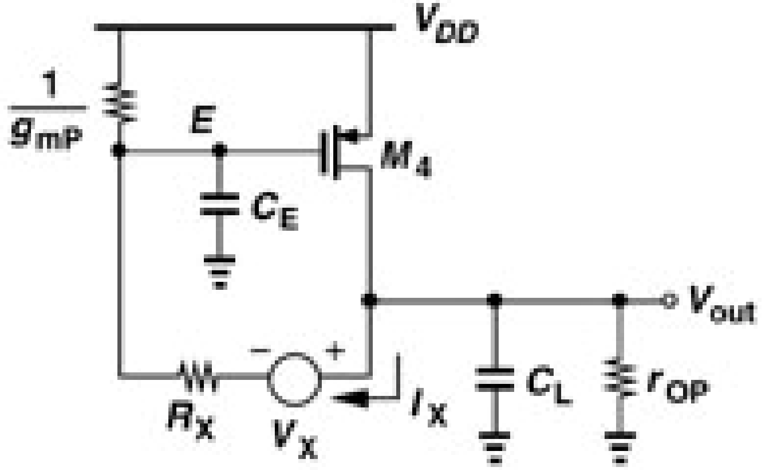
- We can show that:
- $V_X = g_{m1} r_{o1} V_{in} = g_{mN} r_{oN} V_{in}$
- $R_X = 2r_{o1} = 2r_{oN}$

Differential Mode analysis



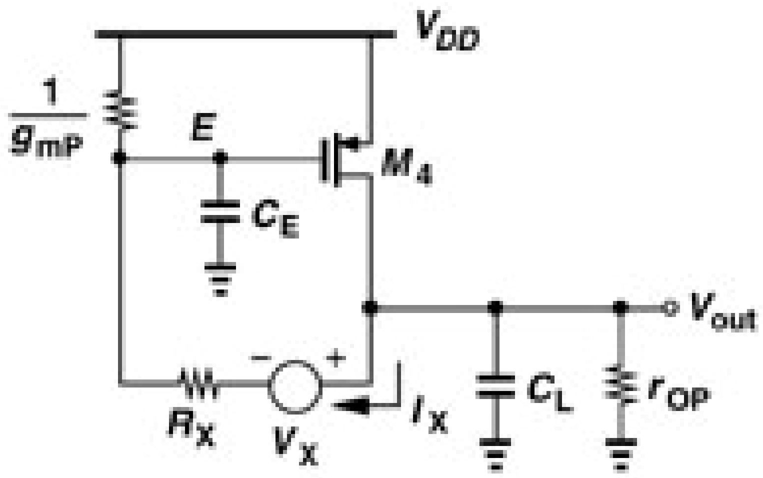
- We assume that $1/g_{mP} << r_{oP}$
- $V_E=(V_{out}-V_X)[(1/(C_E s+g_{mP})] / [R_X+(1/(C_E s+g_{mP})]$ voltage division

Differential Mode analysis (cont'd)



- Current of M_4 is $g_{m4}V_E$
- $-\ g_{m4}V_E - I_X = V_{out}(C_L s + r_{oP}^{-1})$

Differential Mode Analysis - Result



$$\frac{V_{out}}{V_{in}} = \frac{g_{mN} r_{oN} (2 g_{mP} + C_E s)}{2 r_{oP} r_{oN} C_E C_L s^2 + a s + 2 g_{mP} (r_{oN} + r_{oP})}$$

$$a = (2 r_{oN} + r_{oP}) C_E + r_{oP} (1 + 2 g_{mP} r_{oN}) C_L$$

Differential Mode Analysis – Result Interpretation

$$\frac{V_{out}}{V_{in}} = \frac{g_{mN} r_{oN} (2g_{mP} + C_E s)}{2 r_{oP} r_{oN} C_E C_L s^2 + as + 2 g_{mP} (r_{oN} + r_{oP})}$$

$$a = (2 r_{oN} + r_{oP}) C_E + r_{oP} (1 + 2 g_{mP} r_{oN}) C_L$$

This type of circuits typically produce a "dominant pole", as the output pole often dominates the mirror pole.

In such a case we can use "a" (above) to estimate the dominant pole location.

Differential Mode Analysis – Dominant and Secondary Poles

$$f_h \approx \frac{2g_{mP}(r_{oN}+r_{oP})}{(2r_{oN}+r_{oP})C_E + r_{oP}(1+2g_{mP}r_{oN})C_L} \approx$$

$$\approx \frac{2g_{mP}(r_{oN}+r_{oP})}{r_{oP}(1+2g_{mP}r_{oN})C_L} \approx \frac{2g_{mP}(r_{oN}+r_{oP})}{2r_{oP}g_{mP}r_{oN}C_L} =$$

$$= \frac{1}{(r_{oP} \| r_{oN})C_L}$$

By substituting f_h into the exact transfer function we can find the higher frequency pole: $f_{p2}=g_{mP}/C_E$

Single-ended differential amplifier summary of results

$$f_{p1} \approx \frac{1}{2\pi(r_{oN} \parallel r_{oP})C_L}$$

$$f_{p2} = \frac{g_{mP}}{2\pi C_E}$$

$$f_Z = 2 f_{p2} = \frac{2 g_{mP}}{2\pi C_E}$$

In summary

Differential amplifiers with current mirror load and differential output have a superior high frequency response over differential amplifiers with current mirror load and single-ended output.