

# 재료상변태

## Phase Transformation of Materials

2008.09.25.

박은수

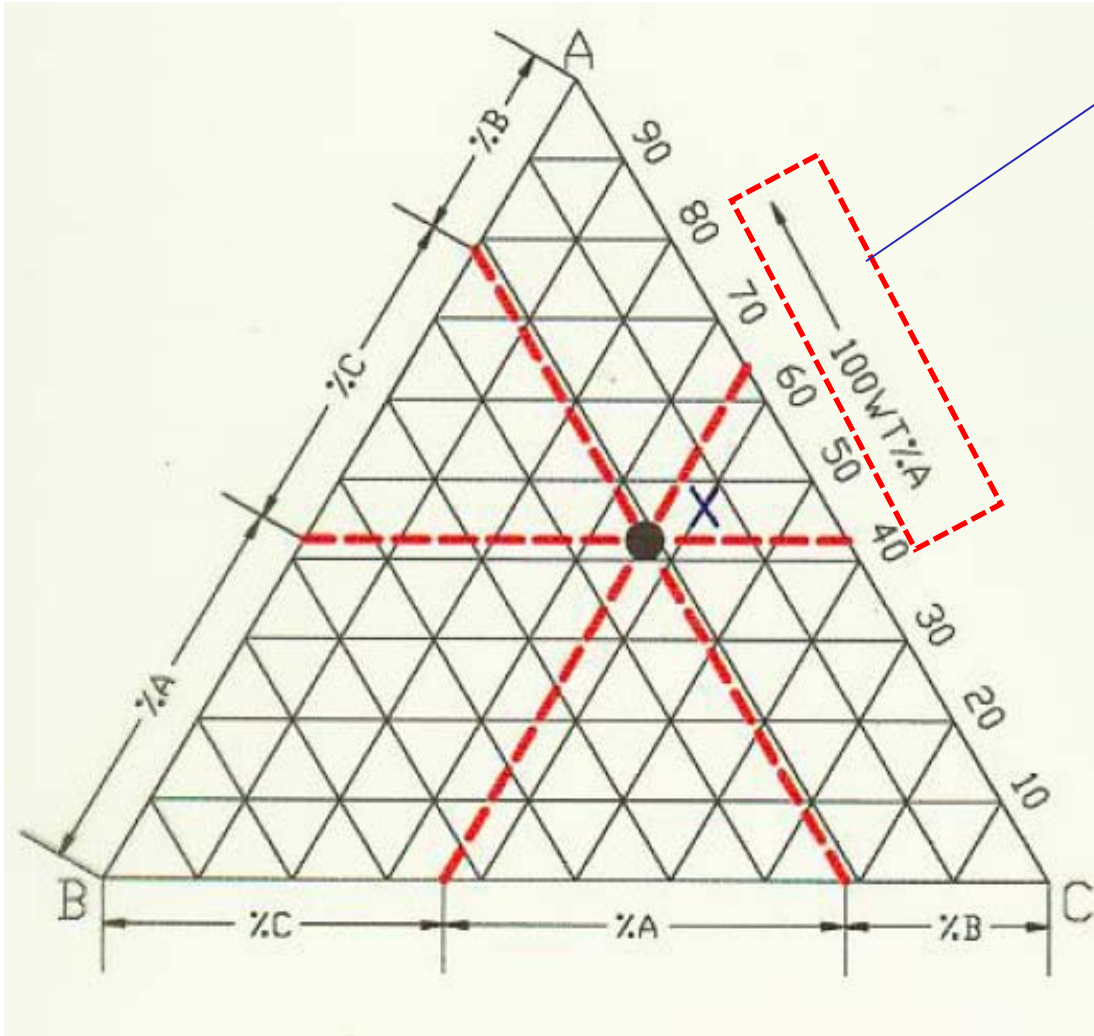
서울대학교 재료공학부

# Contents for previous class

- **Effect of Temperature on Solid Solubility**  $X_B^e = A \exp\left\{-\frac{Q}{RT}\right\}$
- **Equilibrium Vacancy Concentration**  $X_V^e = \exp\frac{-\Delta G_V}{RT}$
- **Influence of Interfaces on Equilibrium**  
 $\Delta G = \frac{2\gamma V_m}{r}$  Gibbs-Thomson effect
- **Gibbs-Duhem Equation:** Be able to calculate the change in chemical potential that result from a change in alloy composition.
- **Ternary Equilibrium: Ternary Phase Diagram**

# Gibbs Triangle

An Equilateral triangle on which the pure components are represented by each corner.

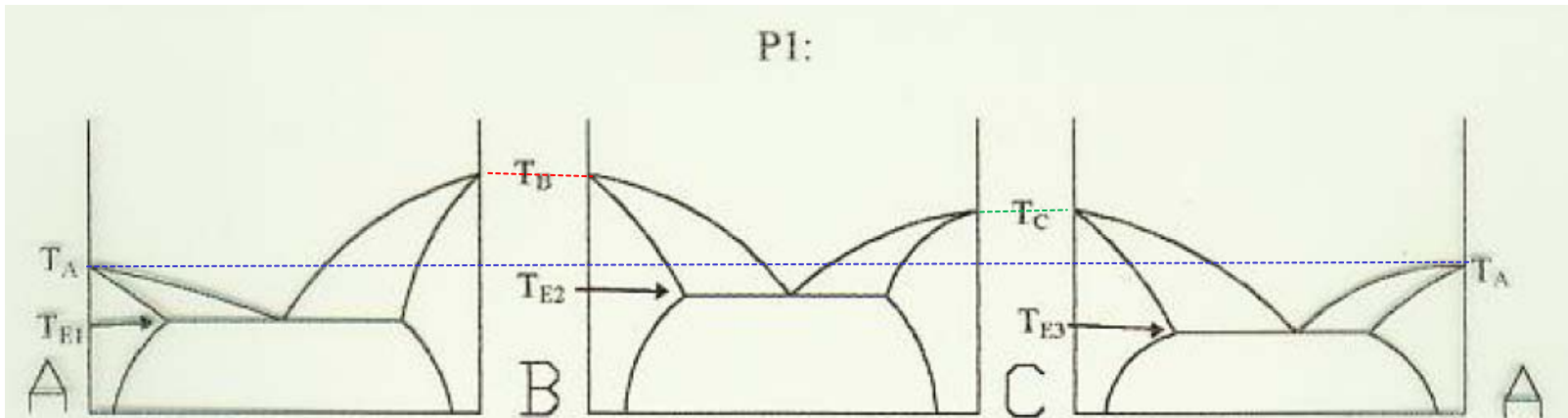


Concentration can be expressed as either "wt. %" or "at.% = molar %".

$$X_A + X_B + X_C = 1$$

Used to determine  
the overall composition

# Ternary Eutectic System (with Solid Solubility)



$T_A$ : Melting Point Of Material A

$T_B$ : Melting Point Of Material B

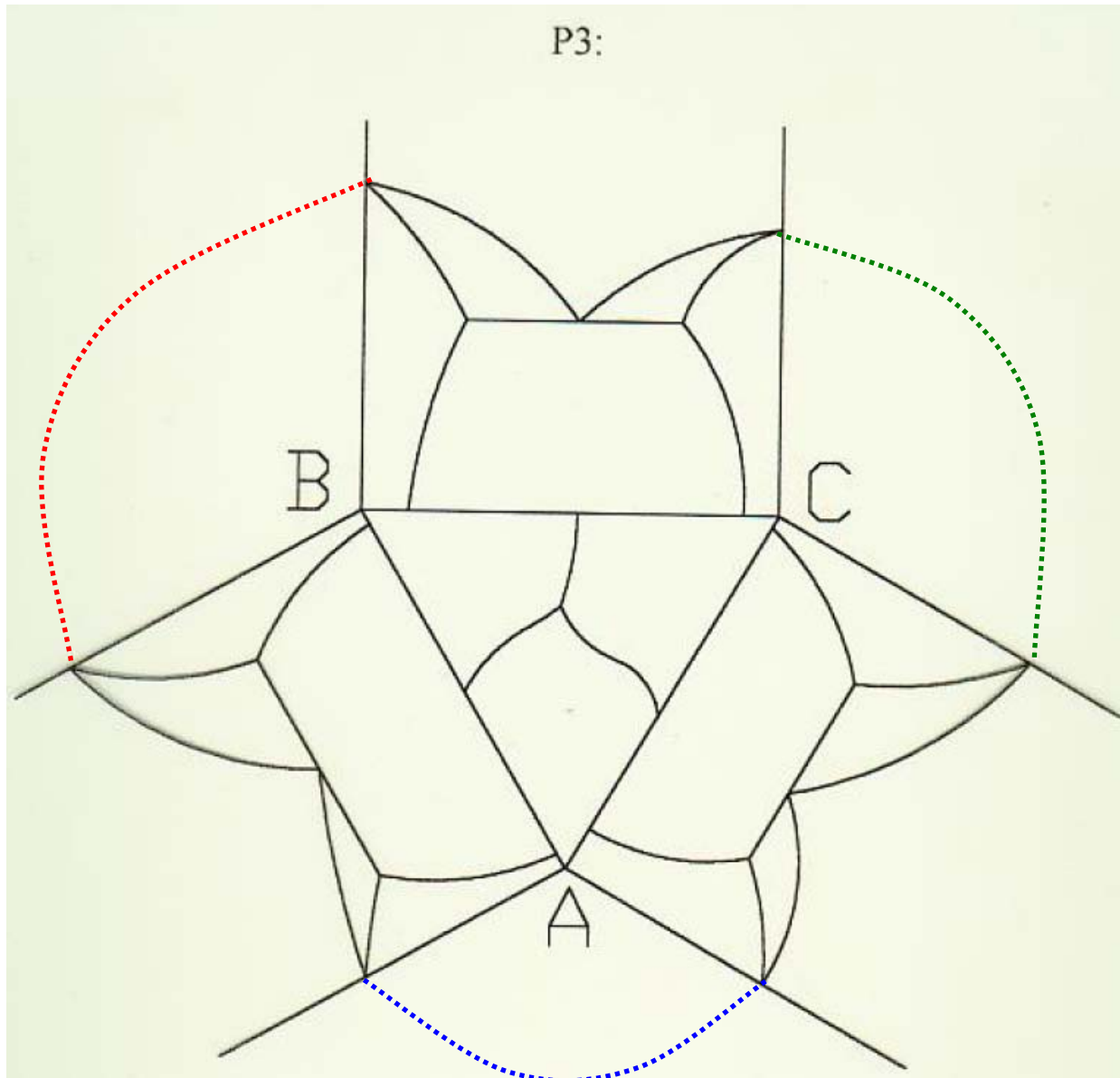
$T_C$ : Melting Point Of Material C

$T_{E1}$ : Eutectic Temperature Of A-B

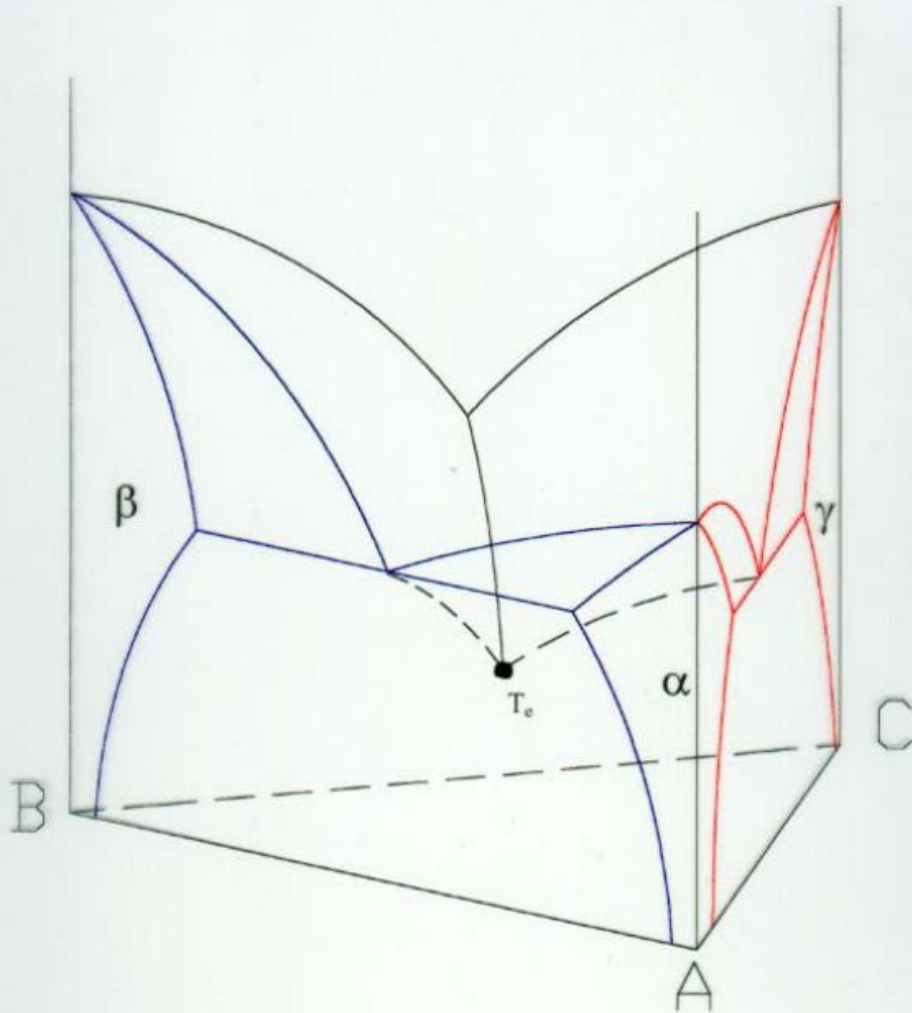
$T_{E2}$ : Eutectic Temperature Of B-C

$T_{E3}$ : Eutectic Temperature Of C-A

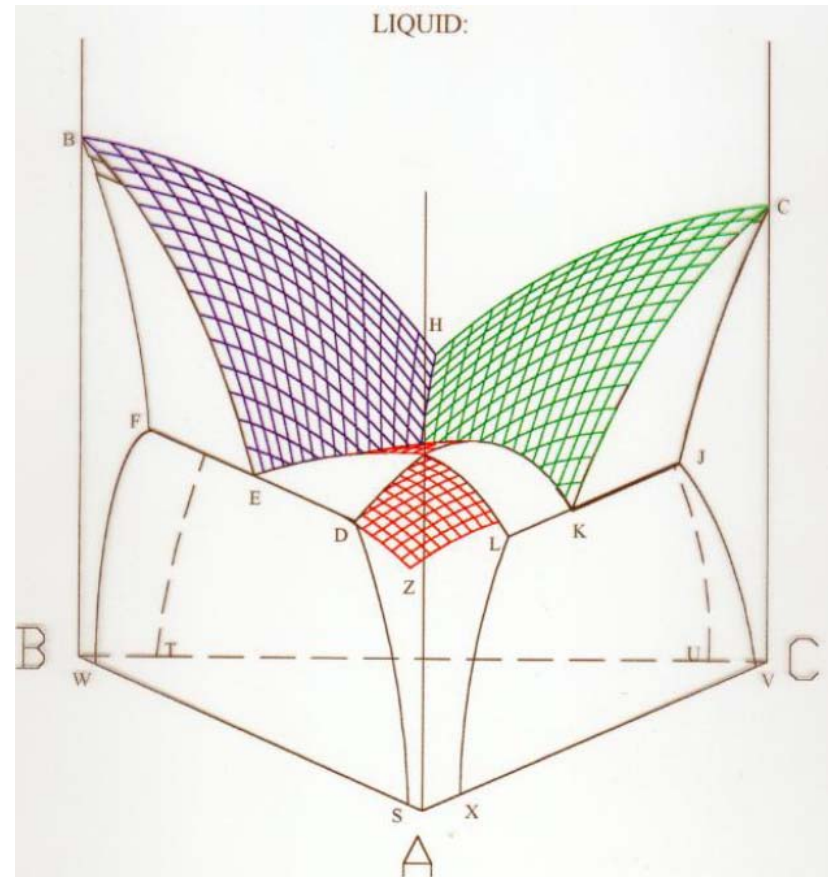
# Ternary Eutectic System (with Solid Solubility)



# Ternary Eutectic System (with Solid Solubility)

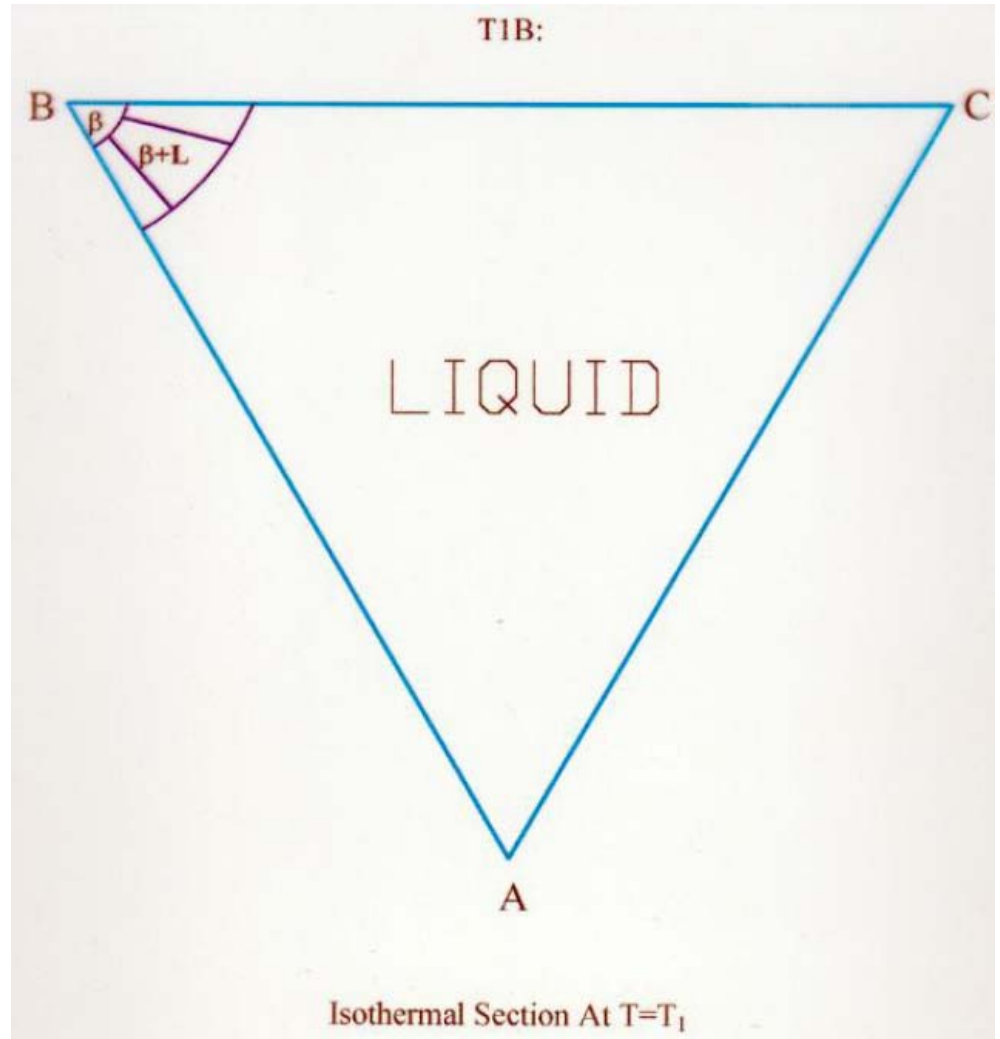
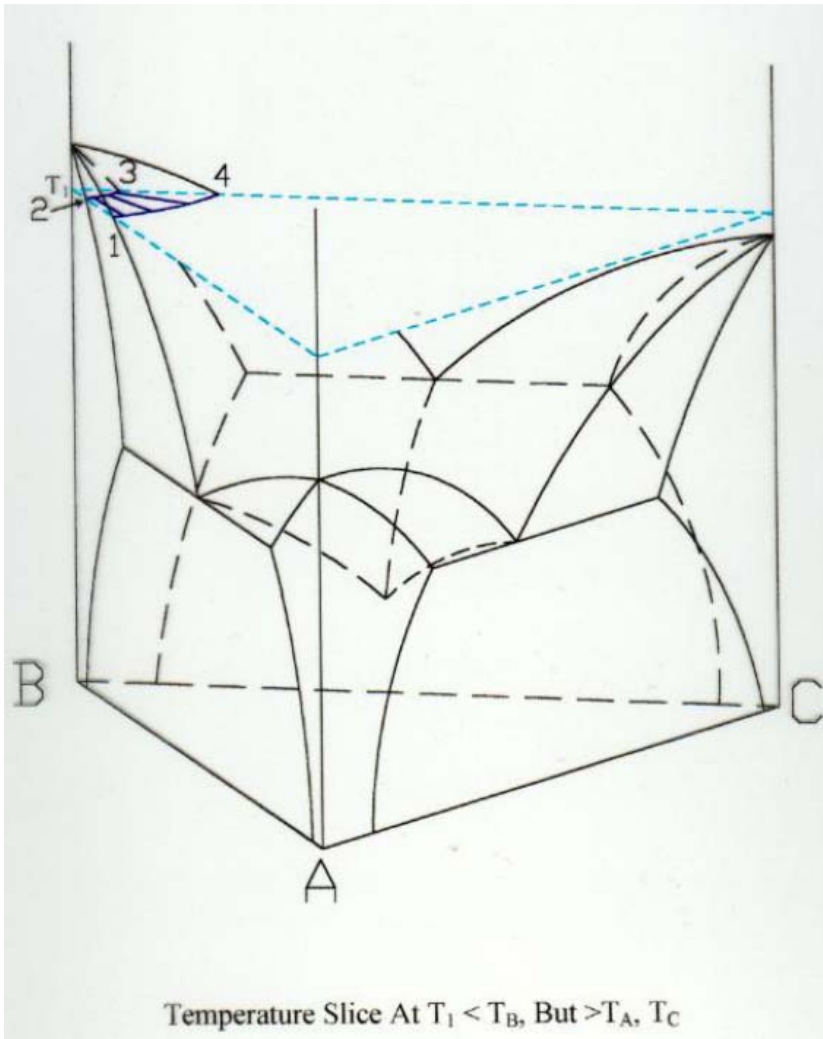


Main outline of Ternary Phase Diagram with Ternary Eutectic ( $T_e$ ) and Solid Single Phase Regions Shown



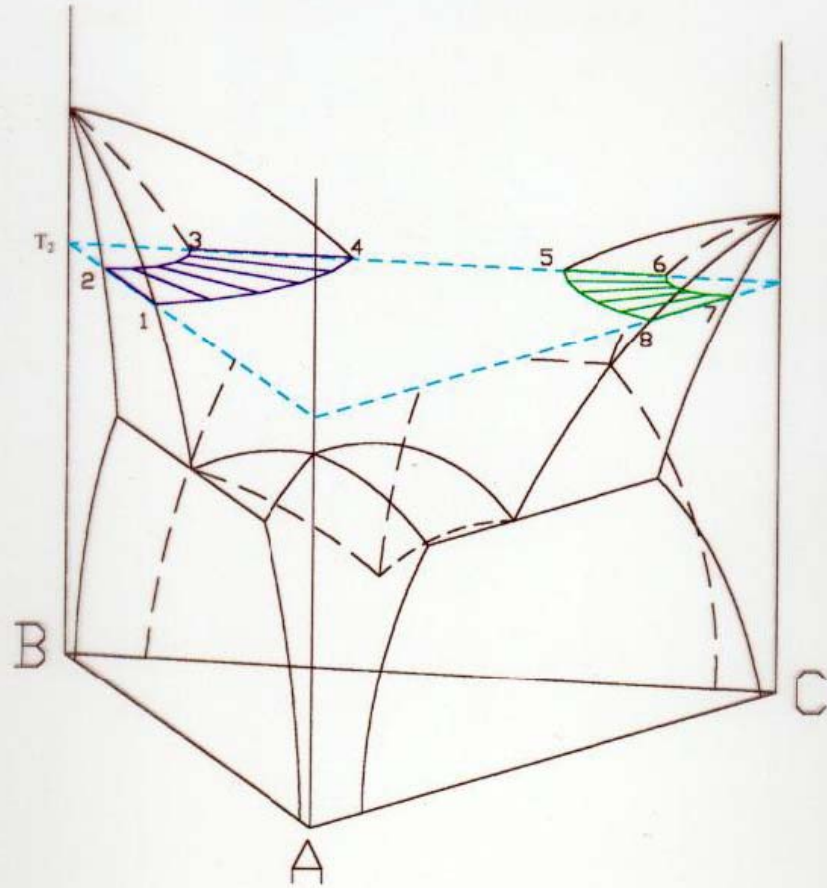
All Liquidus surfaces ( $\alpha+L$ -Red,  $\beta+L$ -Purple,  $\gamma+L$ -Green)

# Ternary Eutectic System (with Solid Solubility)



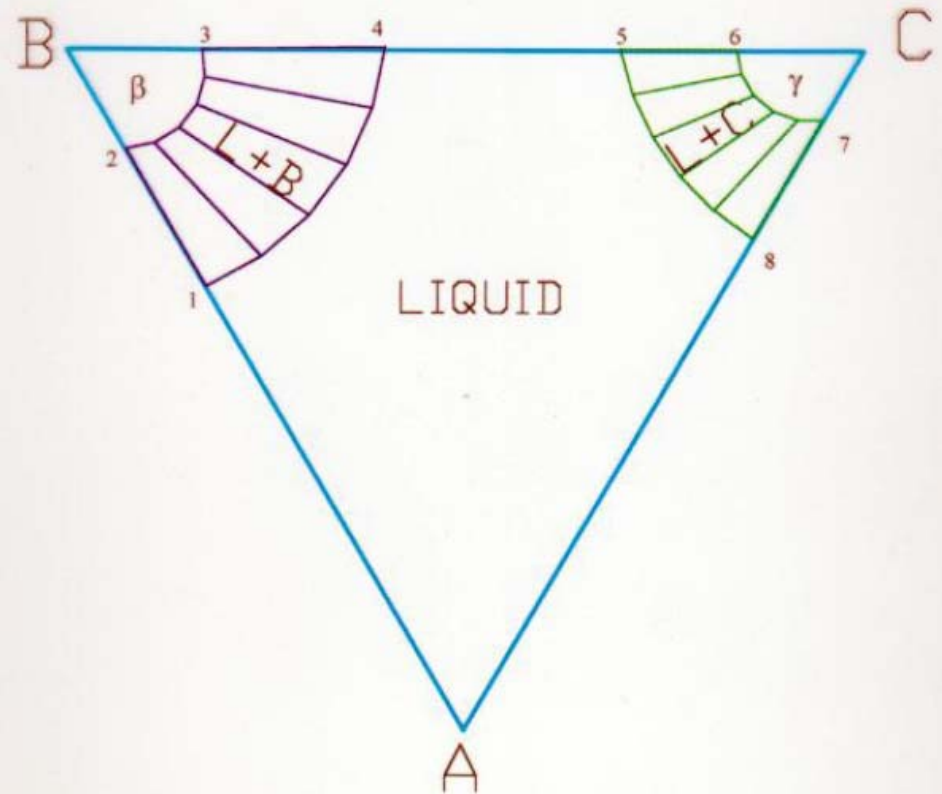
# Ternary Eutectic System (with Solid Solubility)

T2A



Temperature Slice At  $T_2 > T_A$  But,  $T_2 < T_B, T_C$

T2B

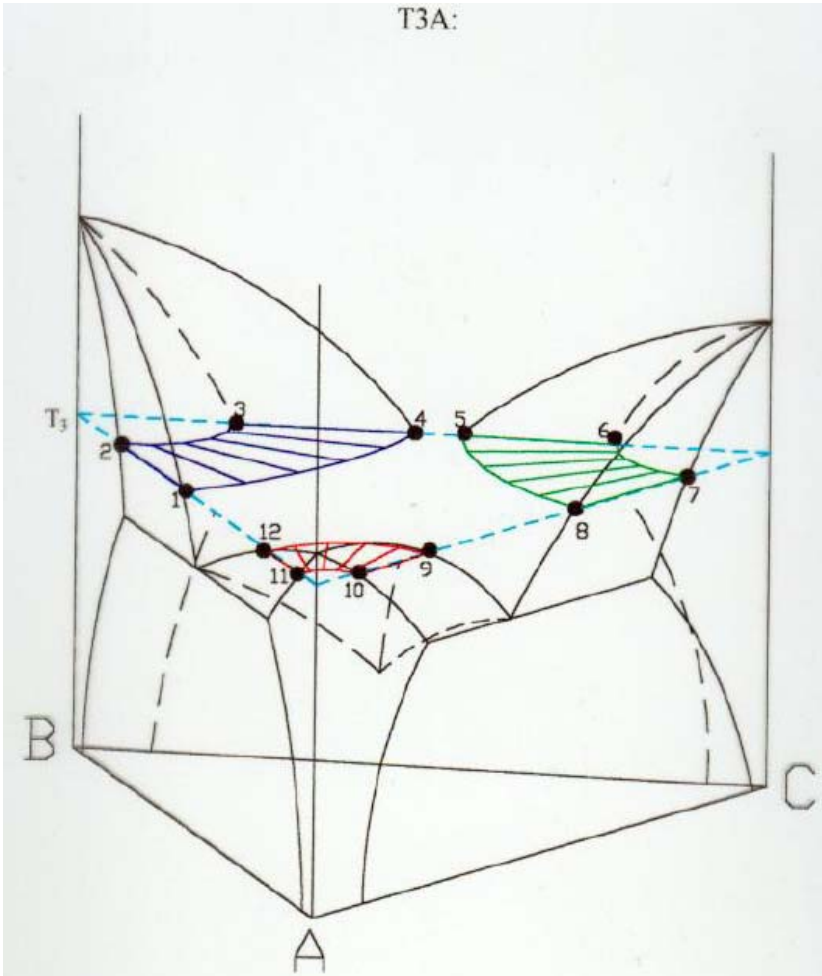


Isothermal Section At  $T=T_2$



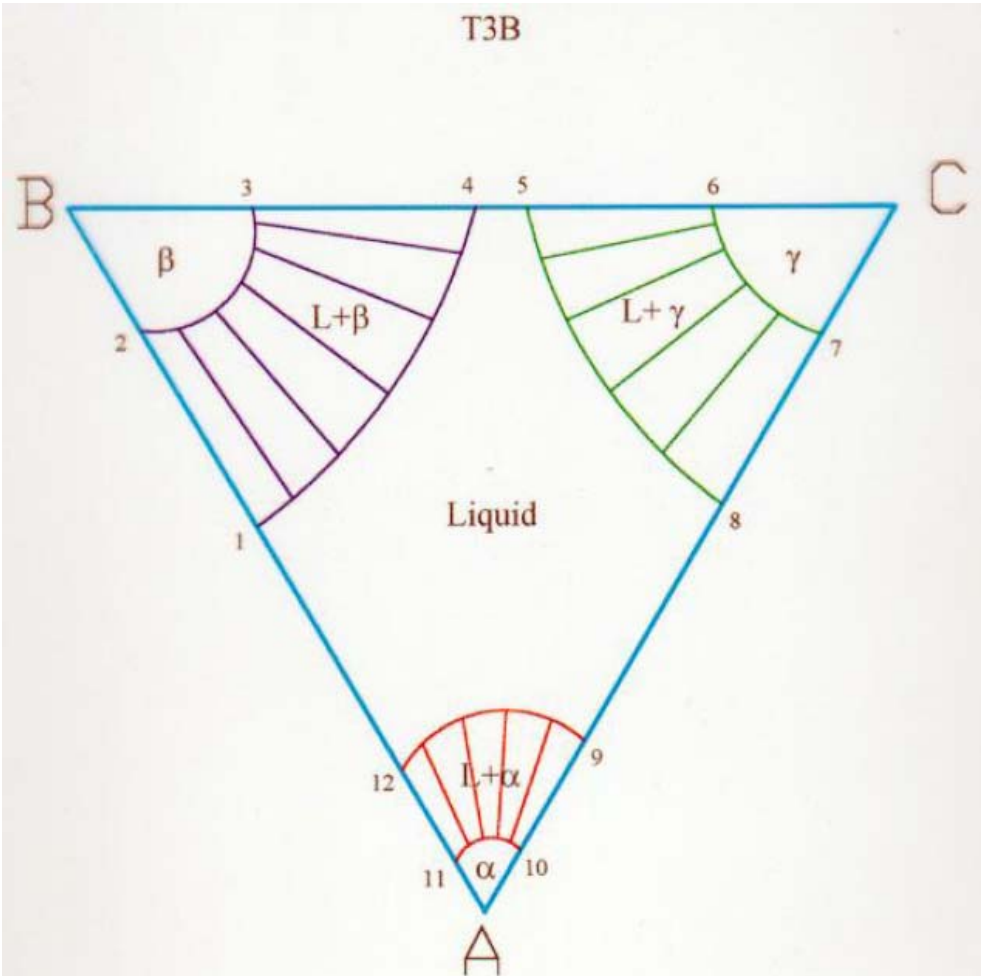
# Ternary Eutectic System (with Solid Solubility)

T3A:

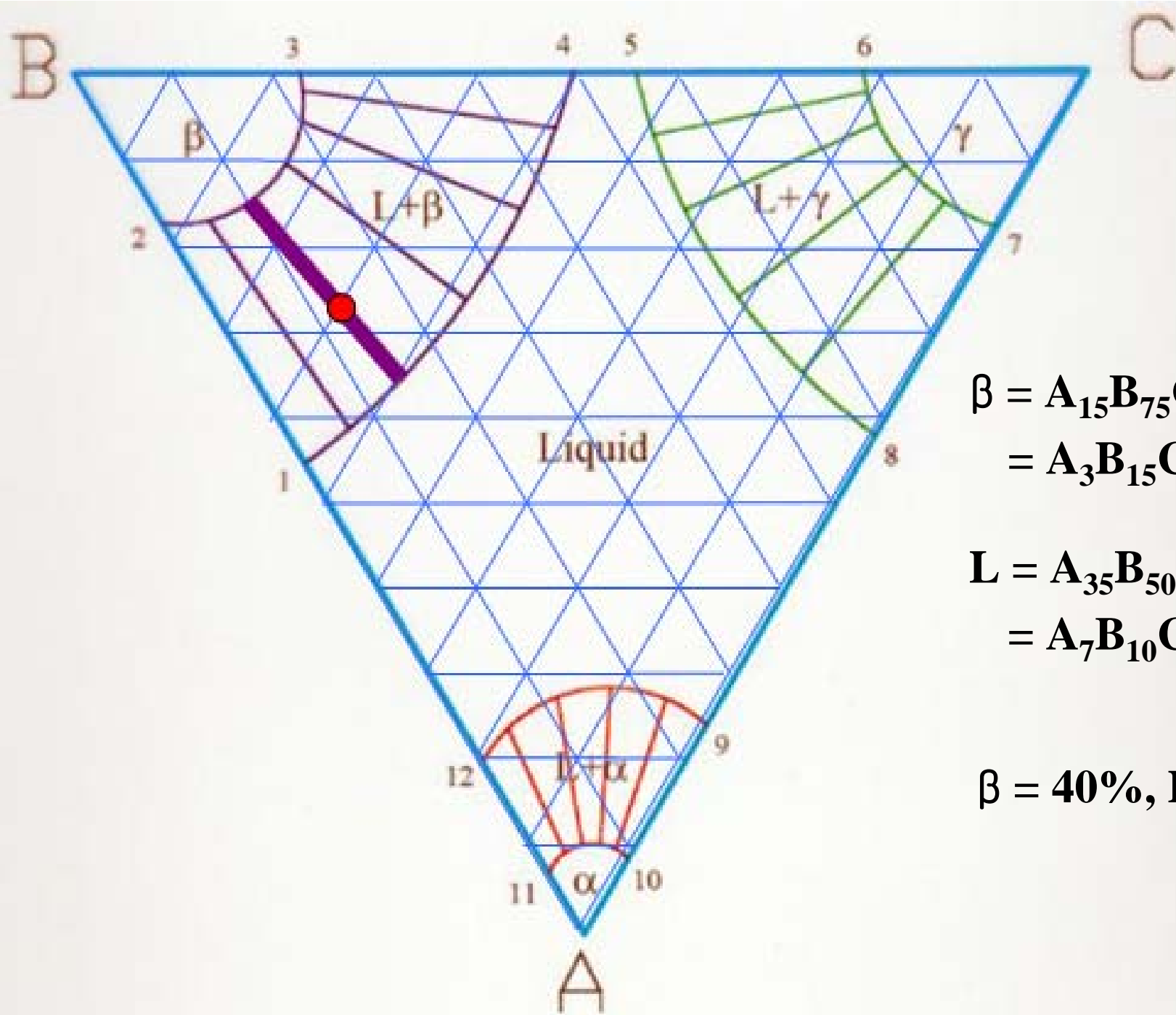


Temperature Slice At  $T_3 < T_A, T_B, T_C$ , But  $T_3 > T_{E1}, T_{E2}, T_{E3}$

T3B



Isothermal Section At  $T = T_3$



$$\beta = A_{15}B_{75}C_{10}$$

$$= A_3B_{15}C_2$$

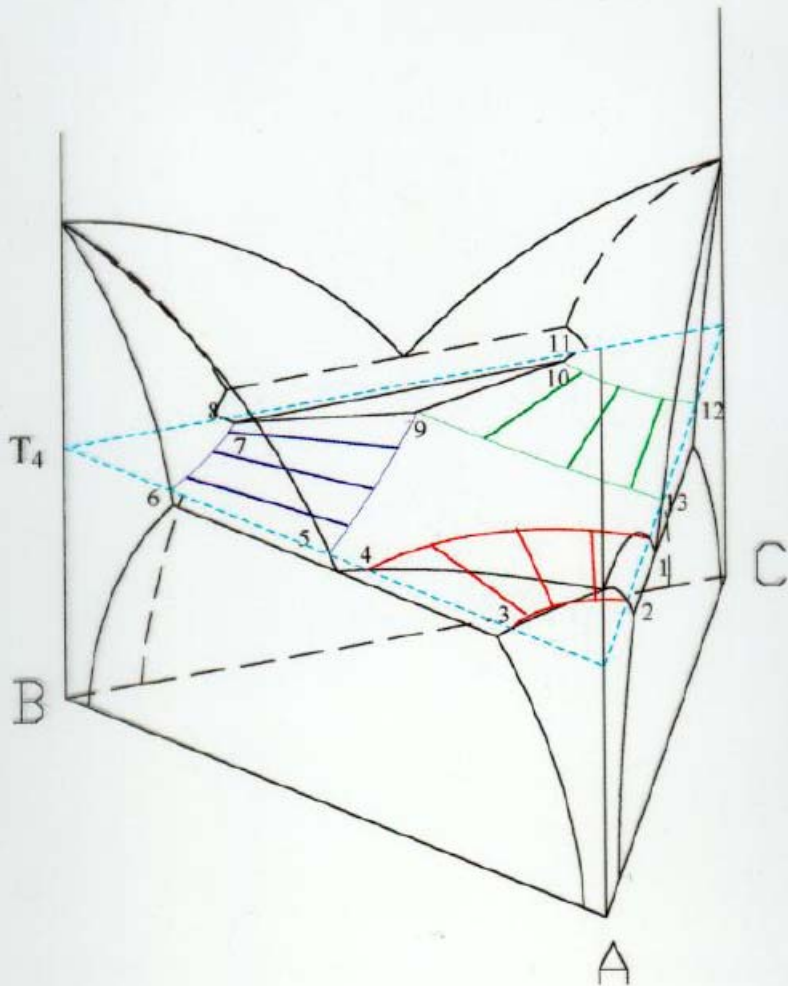
$$L = A_{35}B_{50}C_{15}$$

$$= A_7B_{10}C_3$$

$$\beta = 40\%, L=60\%$$

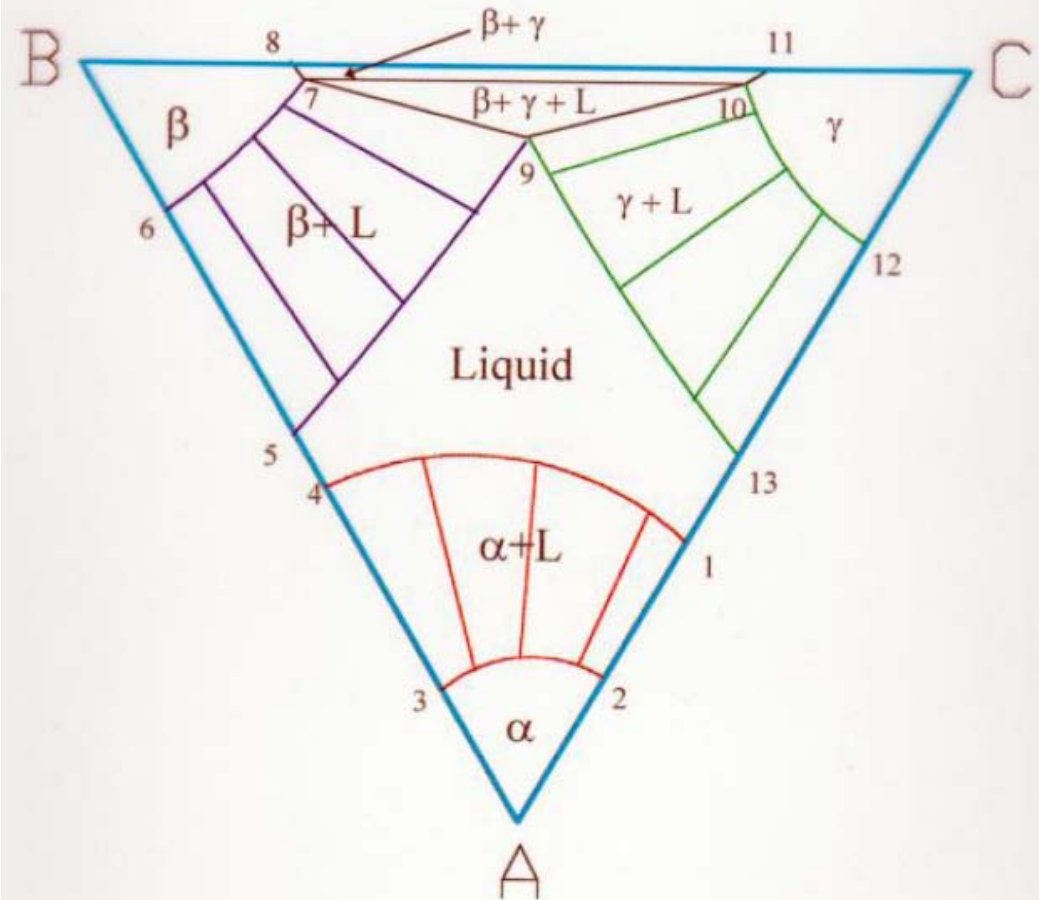
# Ternary Eutectic System (with Solid Solubility)

T4A:



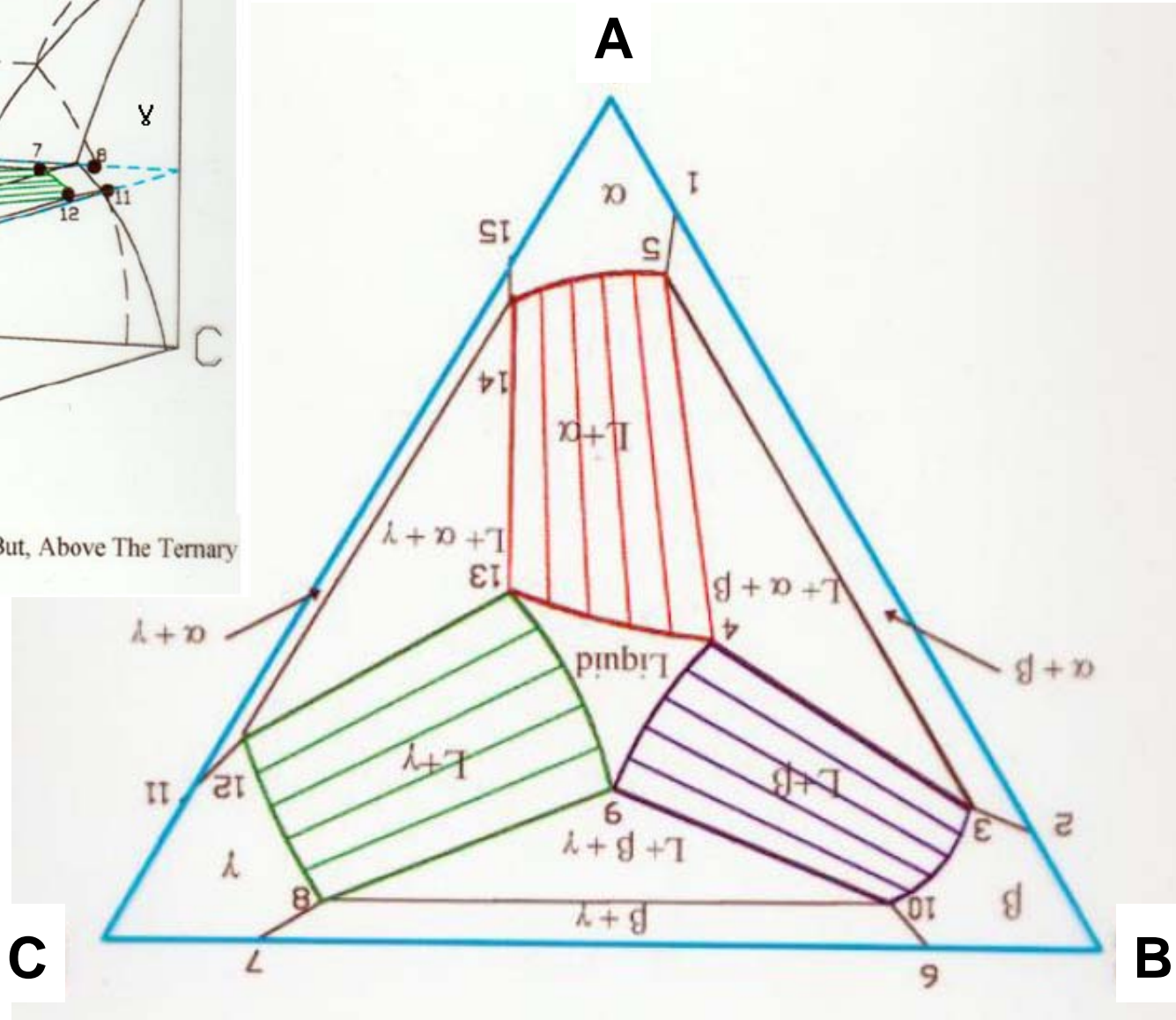
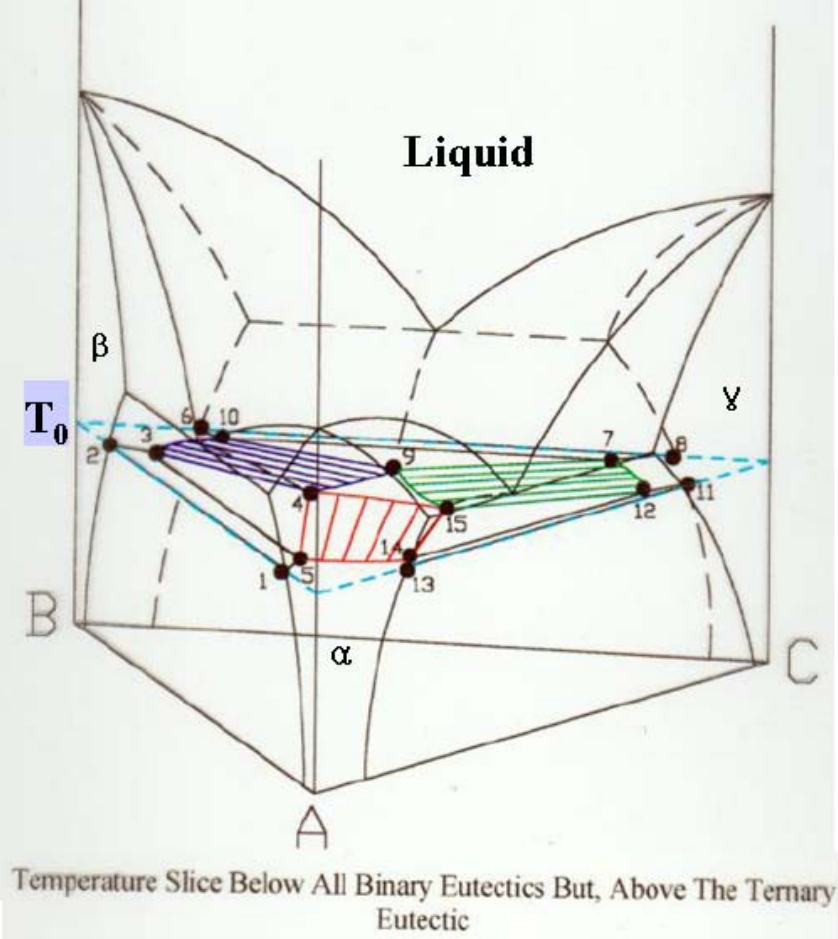
Temperature Slice At  $T_4 < T_{E2}$  And  $T_4 > T_{E1}, T_{E3}$

T4B:



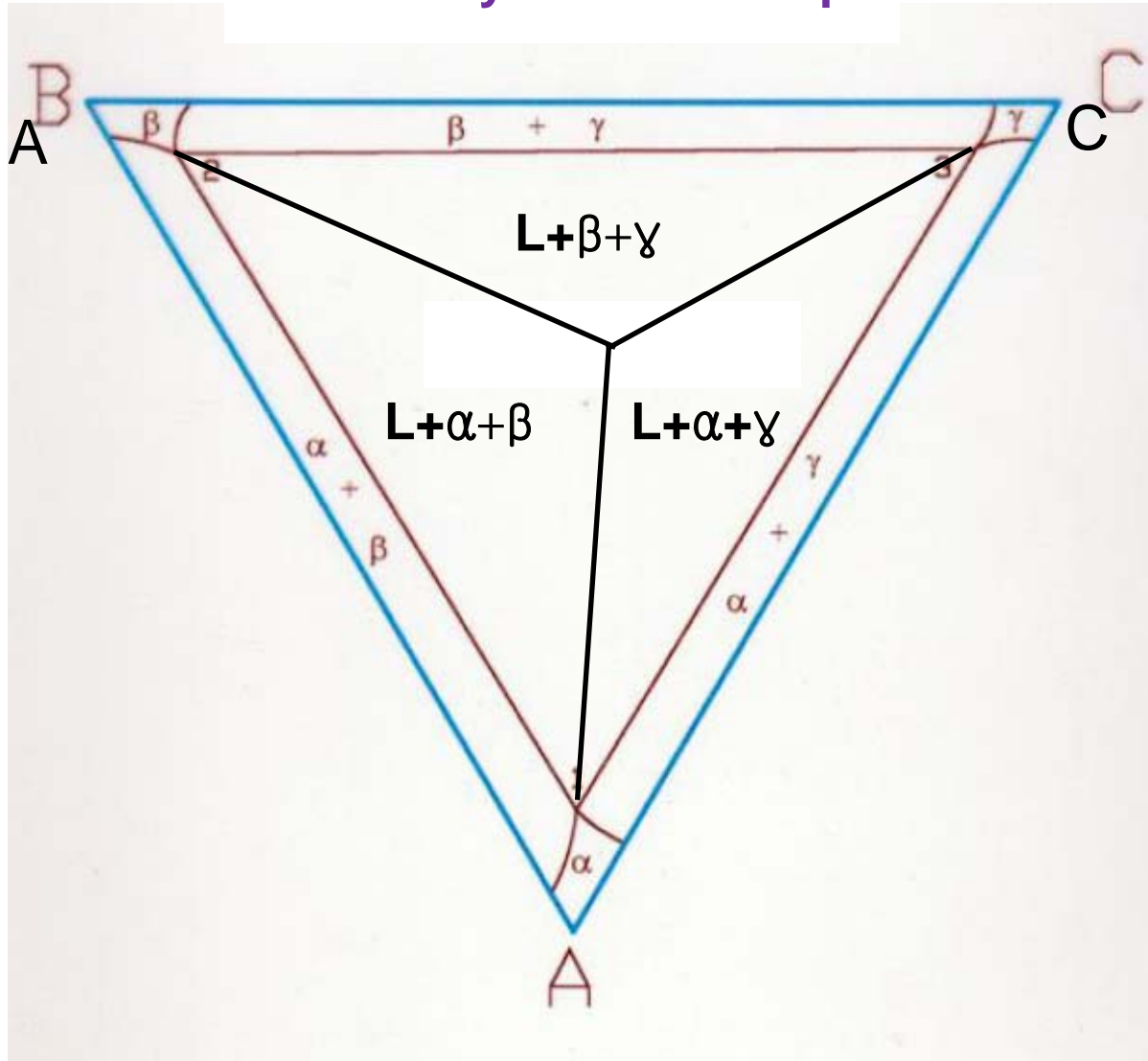
Isothermal Section At  $T = T_4$

# Ternary Eutectic System (with Solid Solubility)

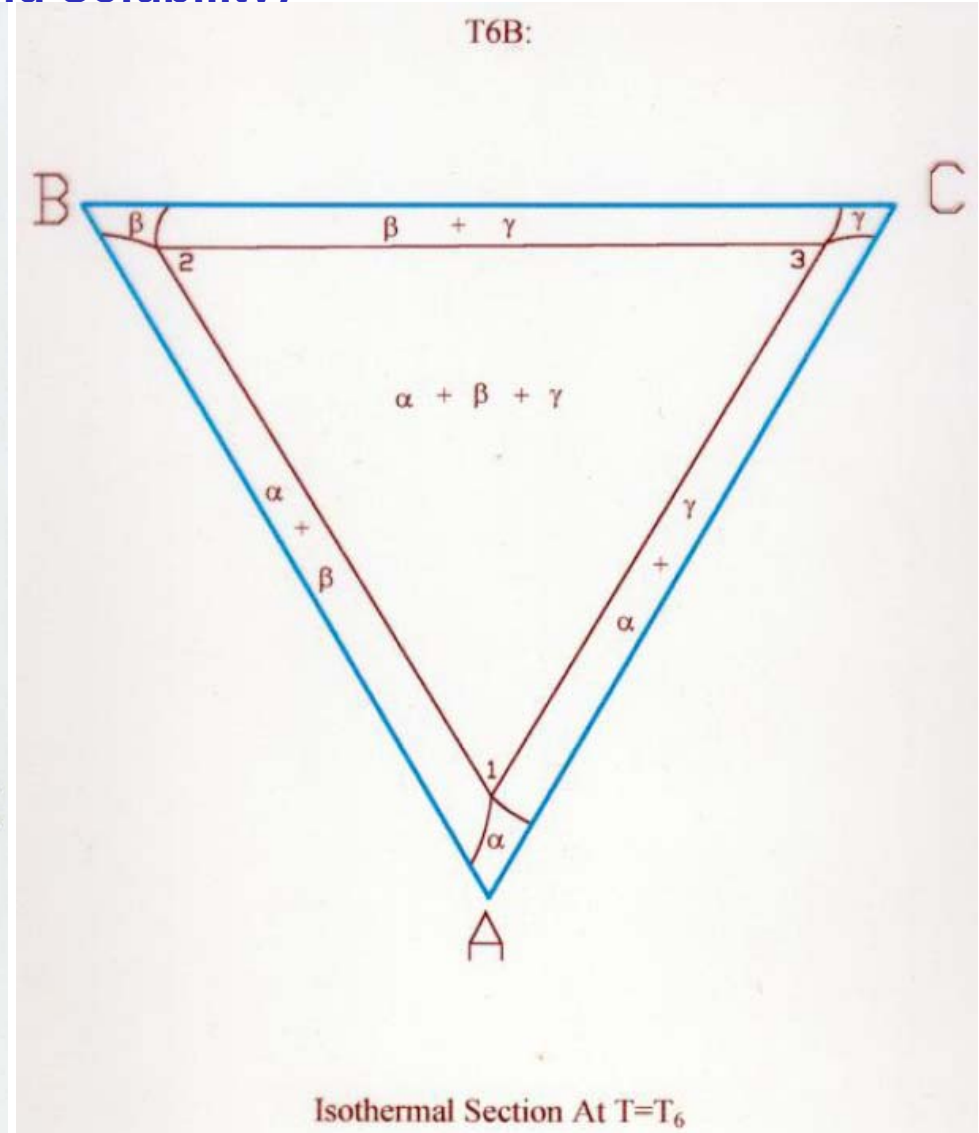
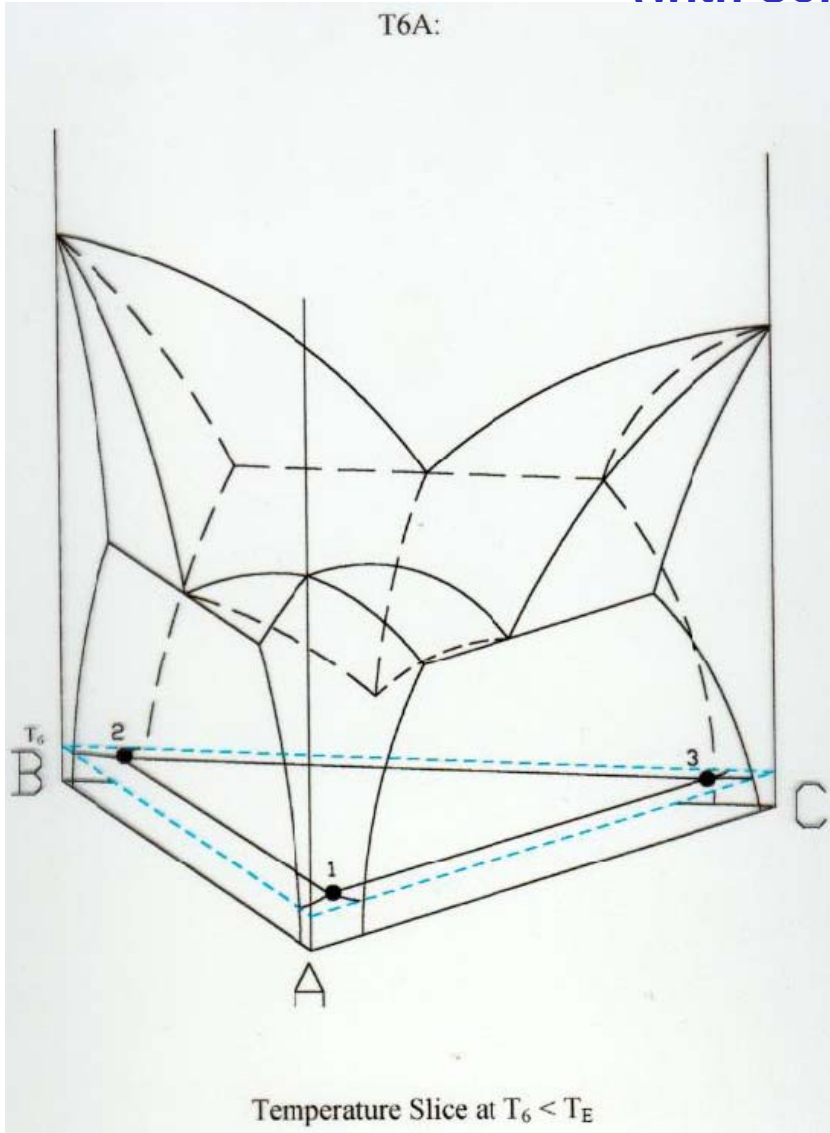


# Ternary Eutectic System (with Solid Solubility)

T = ternary eutectic temp.

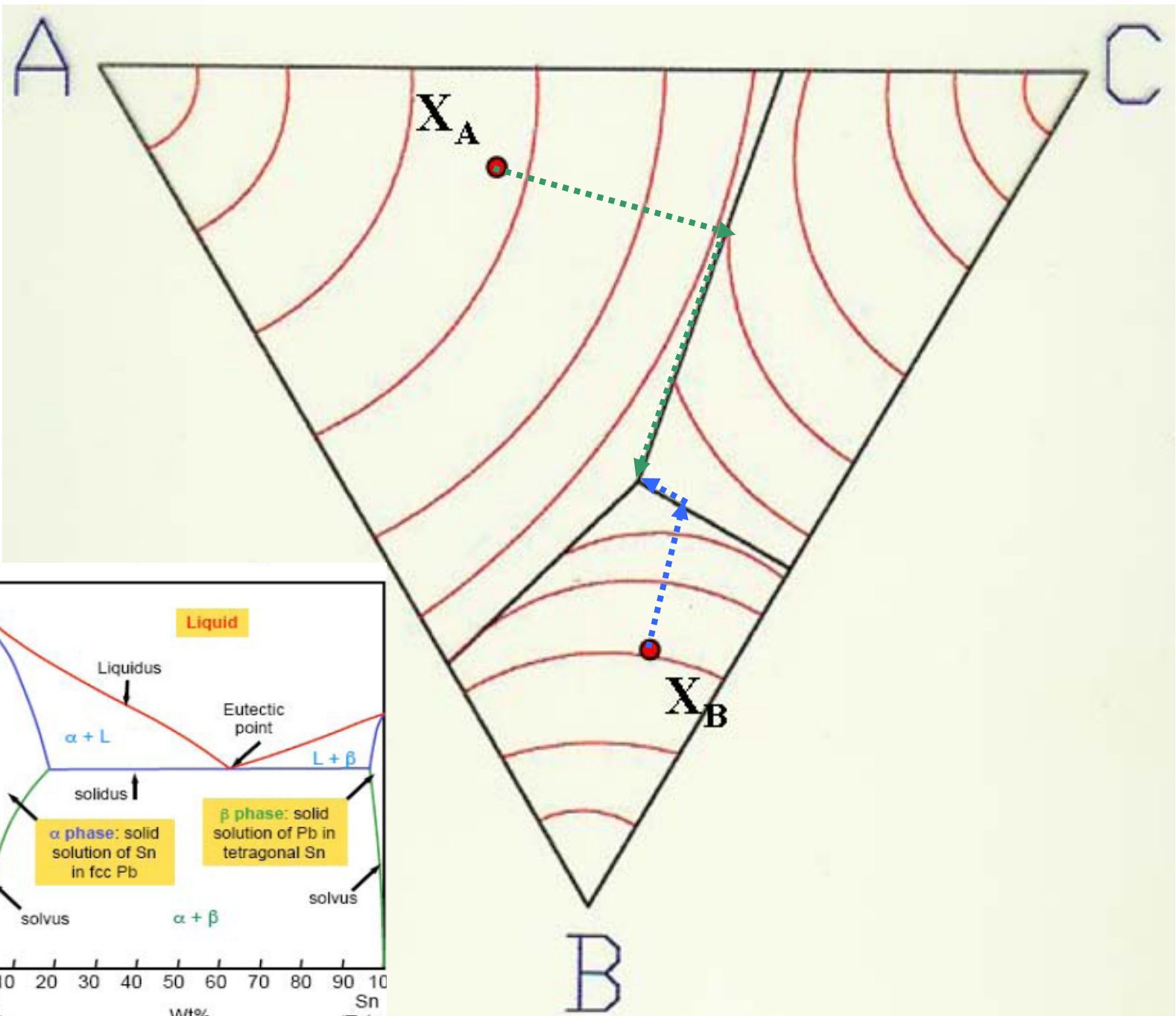


# Ternary Eutectic System (with Solid Solubility)



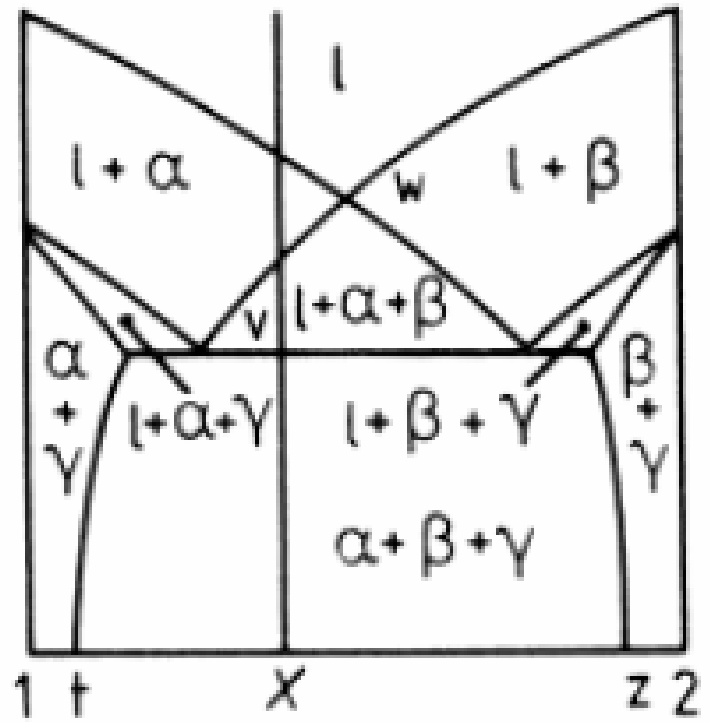
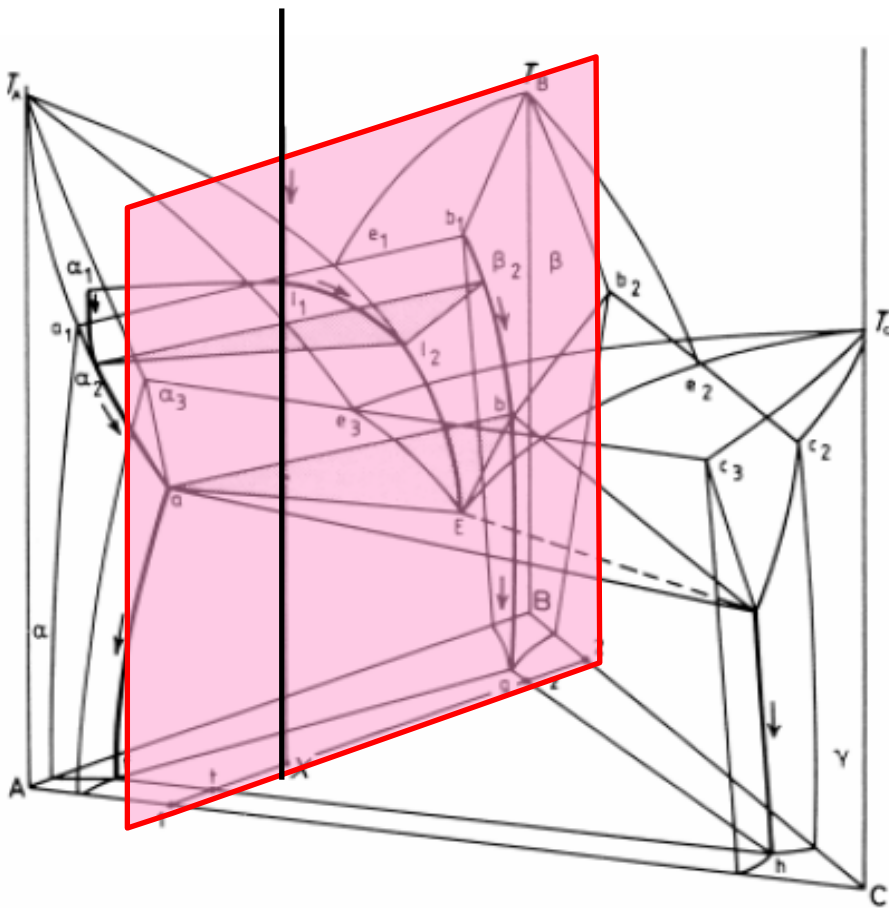
정해솔 학생 제공 자료 참조: 실제 isothermal section의 온도에 따른 변화

<http://www.youtube.com/watch?v=yzhVomAdetM>



# Ternary Eutectic System

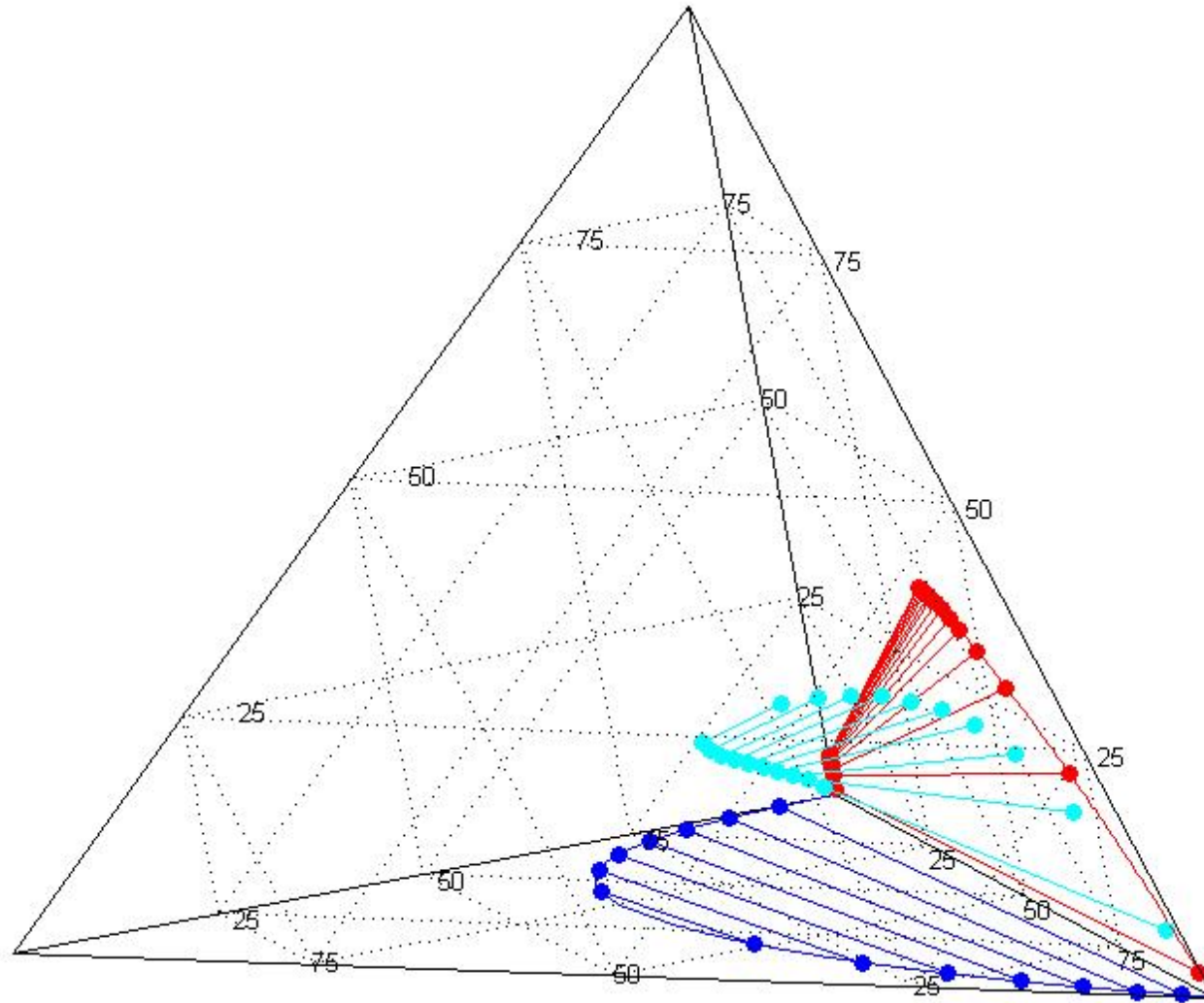
## Solidification Sequence



2 상영역에서 수직 단면이 tie line과 불일치하므로 다른 온도에서 평형상만 나타내고 조성은 표시할 수 없음.



## Quarternary의 평형상태를 기하학적으로 표시한 그림.



4성분원소들 가운데서 임의의 3성분의 농도가 독립적으로 변할 수 있는 함수이므로 여러가지 조성의 Quarternary alloy은 공간적으로 표시된다. 3원계의 조성은 정4면체의 면상에, 그리고 4원계 합금의 조성은 정4면체의 내부공간에 표시된다. 합금의 조성은 정4면체의 기하학적성질에 의하여 결정된다. 4원계에서 상조성을 결정하기 위하여 lever rule을 이용한다. 4원합금의 변태행정을 고찰할 때 정4면체안의 추상적인 4차원투영을 이용한다.

# Contents in Phase Transformation

상변태를  
이해하는데  
필요한 배경

(Ch1) 열역학과 상태도: **Thermodynamics**

(Ch2) 확산론: **Kinetics**

(Ch3) 결정계면과 미세조직

대표적인 상변태

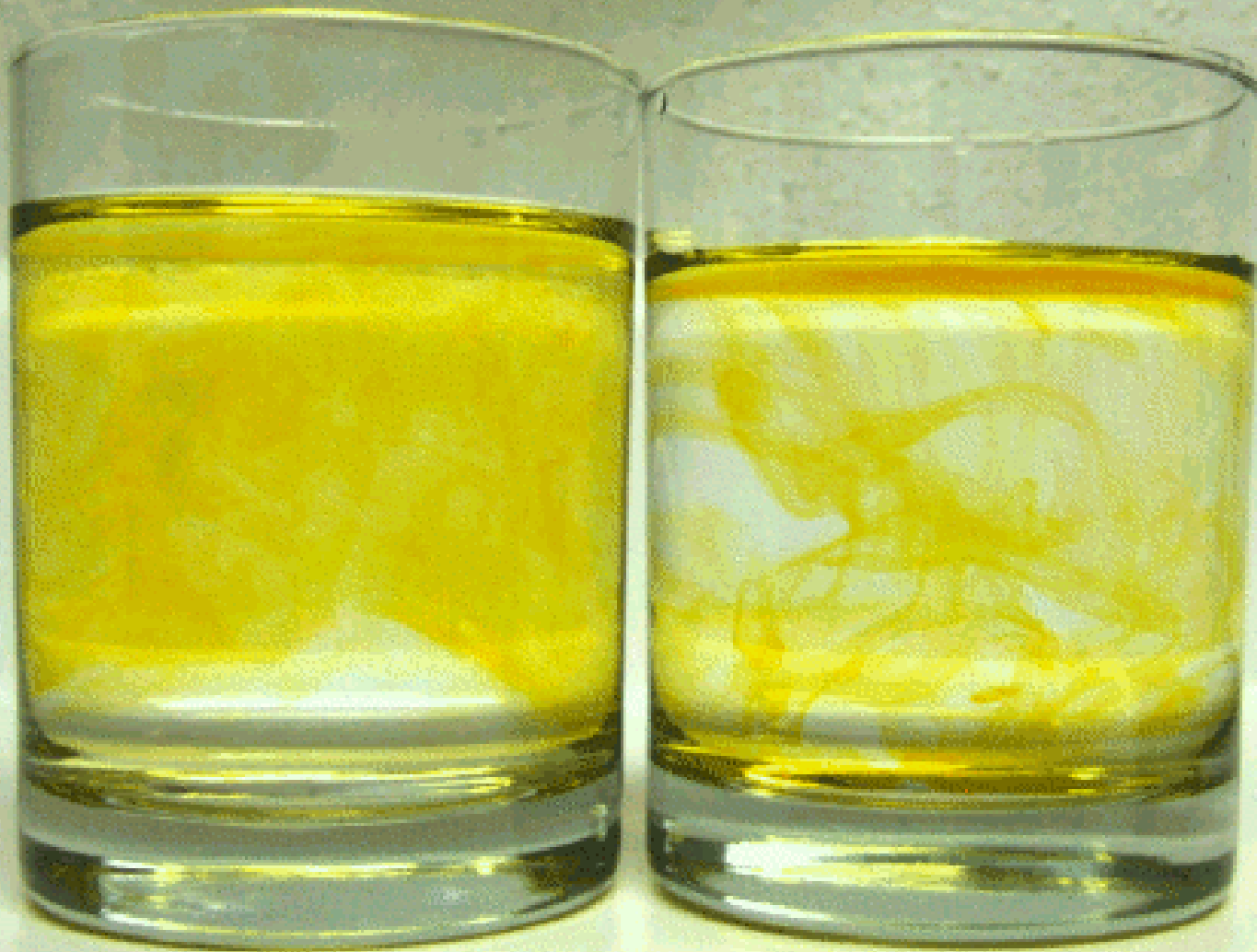
(Ch4) 응고: **Liquid → Solid**

(Ch5) 고체에서의 확산 변태: **Solid → Solid (Diffusional)**

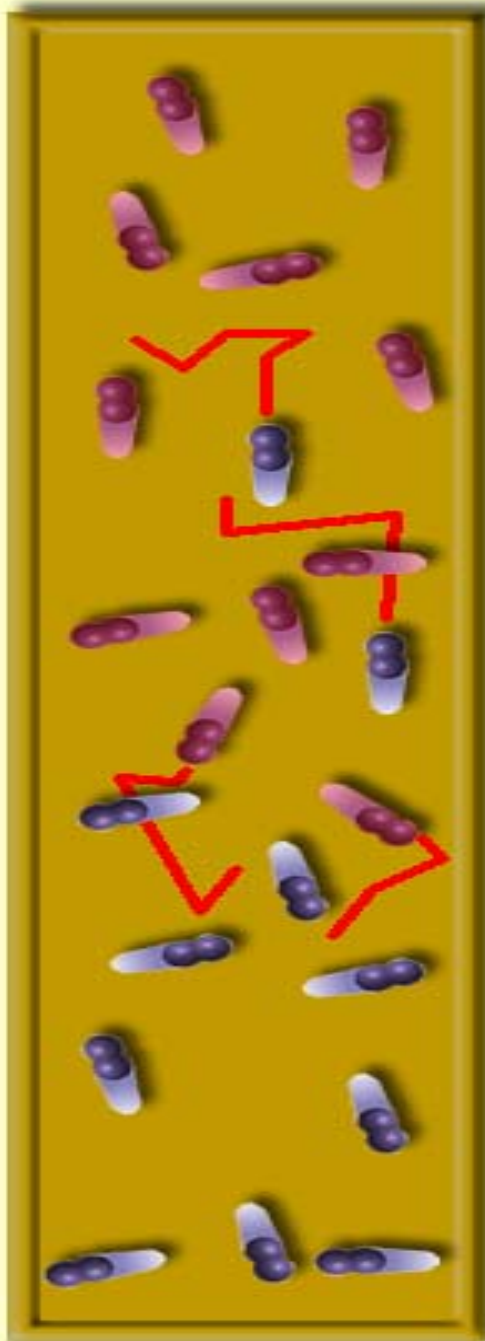
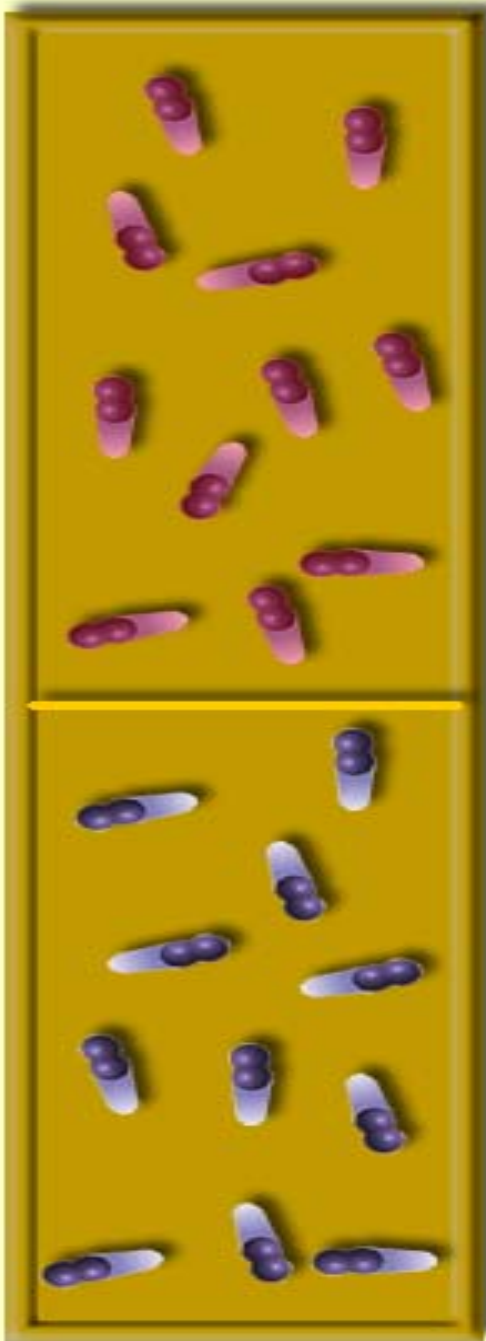
(Ch6) 고체에서의 무확산 변태: **Solid → Solid (Diffusionless)**

# Contents for today's class

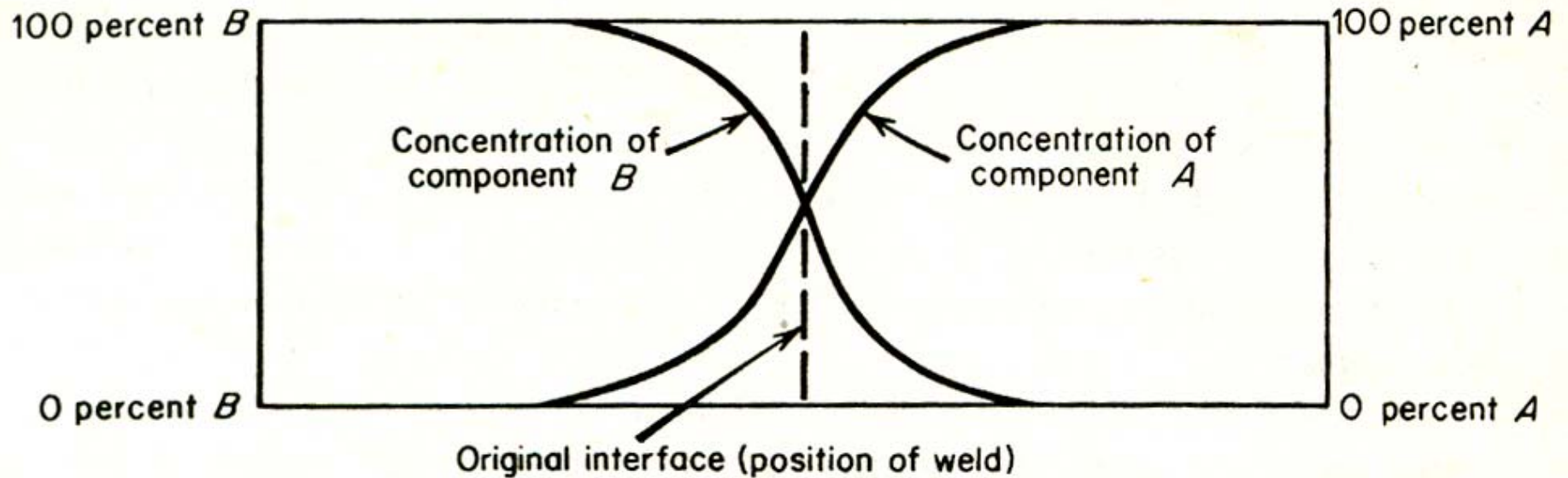
- **Diffusion**
- **Interstitial Diffusion – Fick's First Law**
- **Effect of Temperature on Diffusivity**
- **Nonsteady-state diffusion – Fick's Second Law**



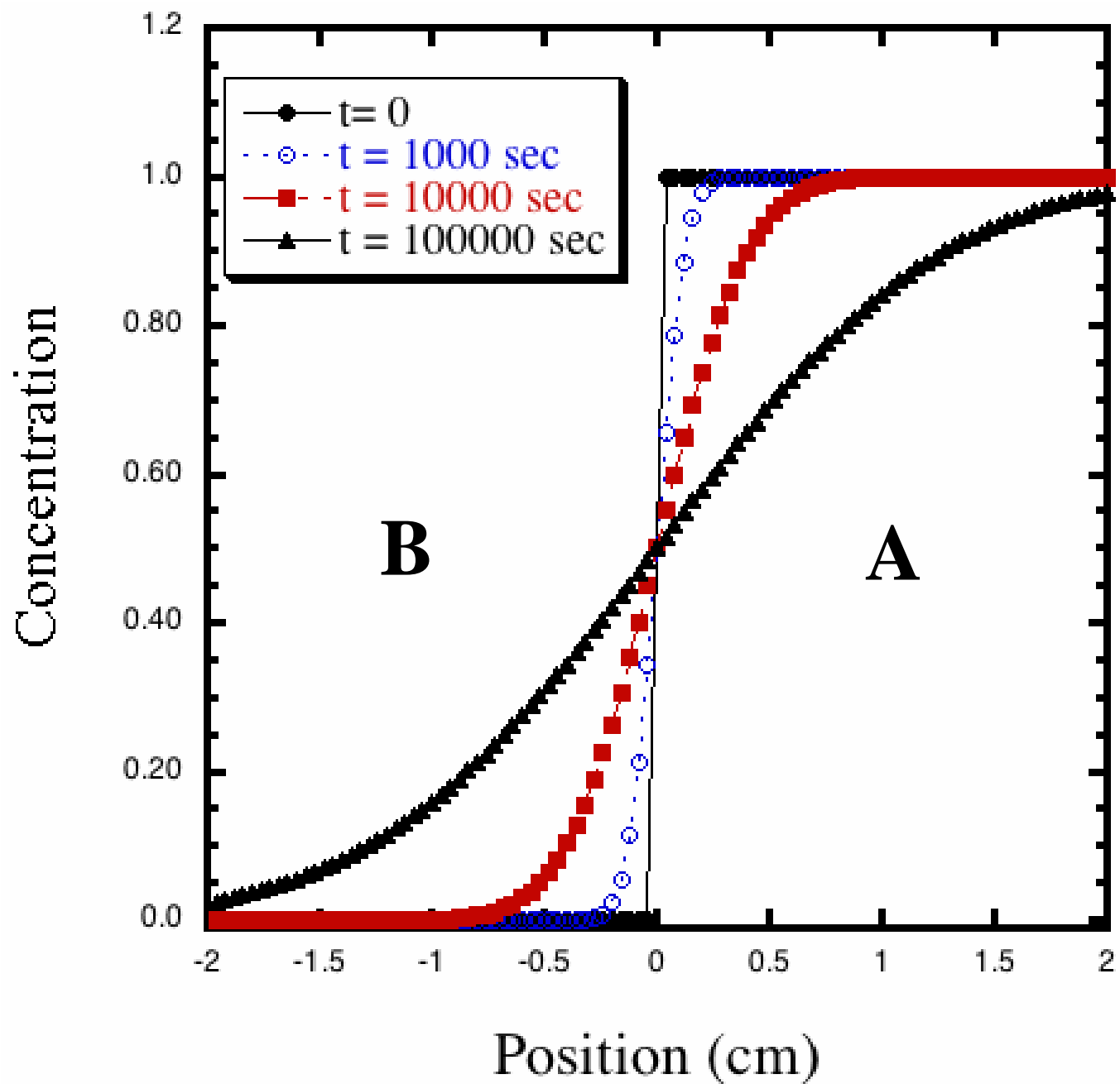
**Diffusion**



# Mechanism of Solid-State Diffusion



**Fig. 10.6** Curves showing concentration as a function of distance along a diffusion couple. Curves of this type are usually called *penetration curves*.



# Diffusion

Movement of atoms to reduce its chemical potential  $\mu$ .

Driving force: Reduction of  $G$

## Down-hill diffusion

movement of atoms from a high concentration region to low concentration region.

## Up-hill diffusion

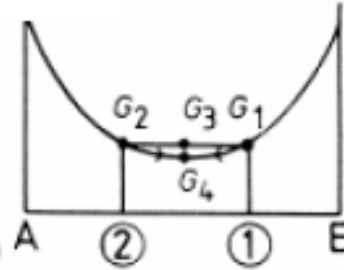
movement of atoms from a low concentration region to high concentration region

'down-hill' diffusion,



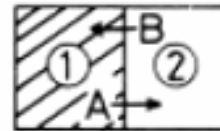
B-rich A-rich

(a)



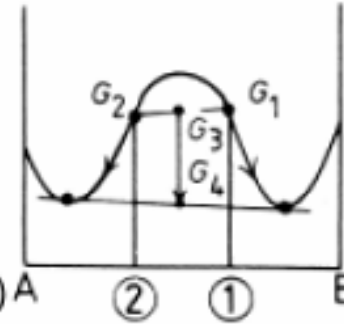
(b) A ② ① B

'up hill' diffusion.



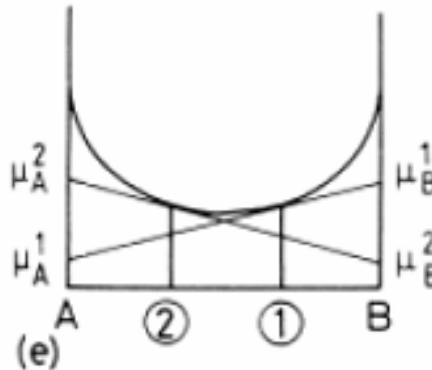
B-rich A-rich

(c)

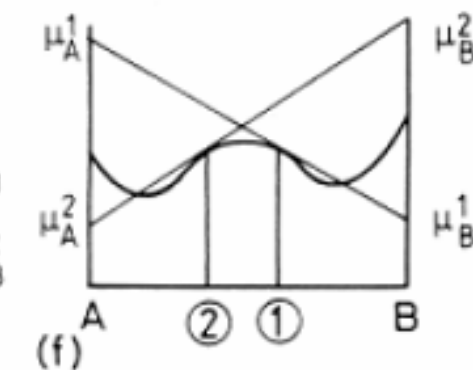


(d) A ② ① B

$$\begin{aligned} \mu_A^1 &< \mu_A^2 \\ \mu_B^1 &> \mu_B^2 \end{aligned}$$



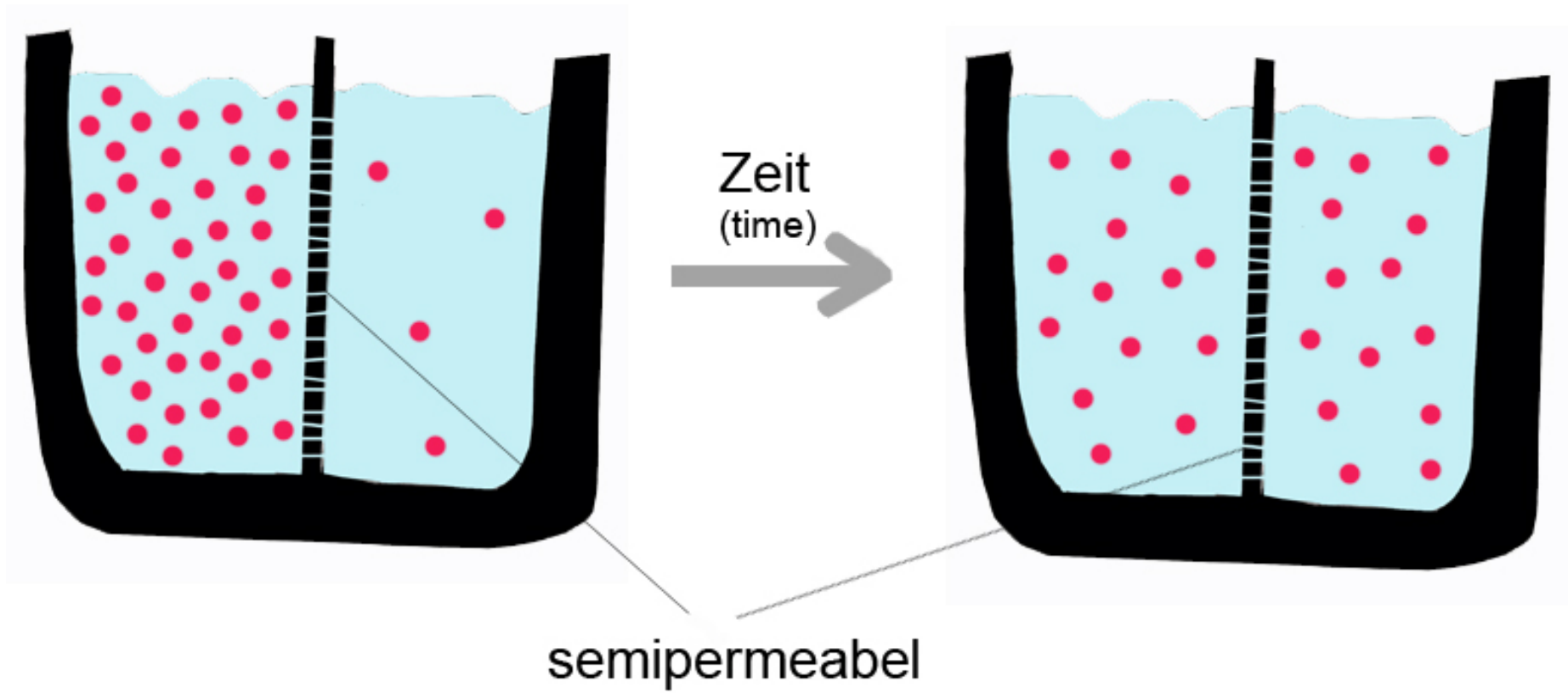
(e)



(f)

$$\begin{aligned} \mu_A^1 &> \mu_A^2 \\ \mu_B^1 &< \mu_B^2 \end{aligned}$$





# Diffusion

Movement of atoms to reduce its chemical potential  $\mu$ .

Driving force: Reduction of G

## Down-hill diffusion

movement of atoms from a high concentration region to low concentration region.

## Up-hill diffusion

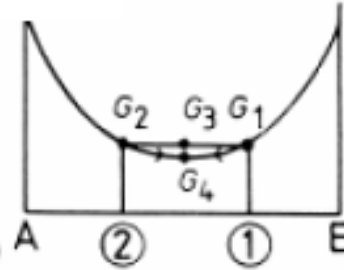
movement of atoms from a low concentration region to high concentration region

'down-hill' diffusion,



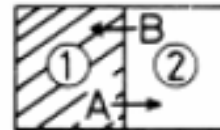
B-rich A-rich

(a)



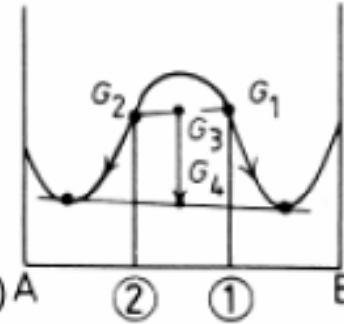
(b) A ② ① B

'up hill' diffusion.



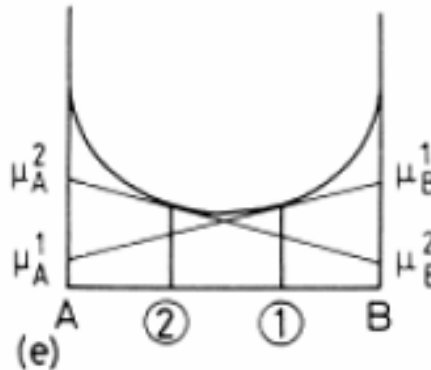
B-rich A-rich

(c)

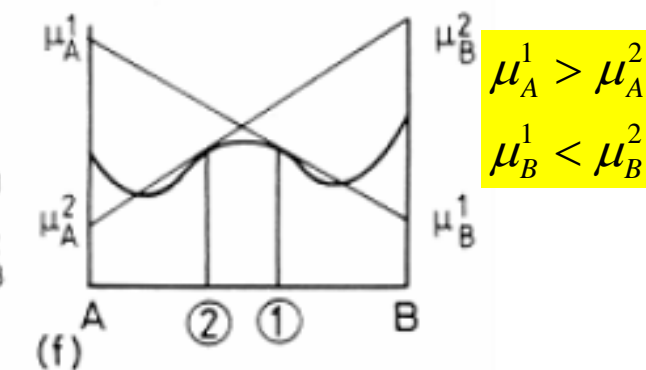


(d) A ② ① B

$$\begin{aligned} \mu_A^1 &< \mu_A^2 \\ \mu_B^1 &> \mu_B^2 \end{aligned}$$



(e)



(f)

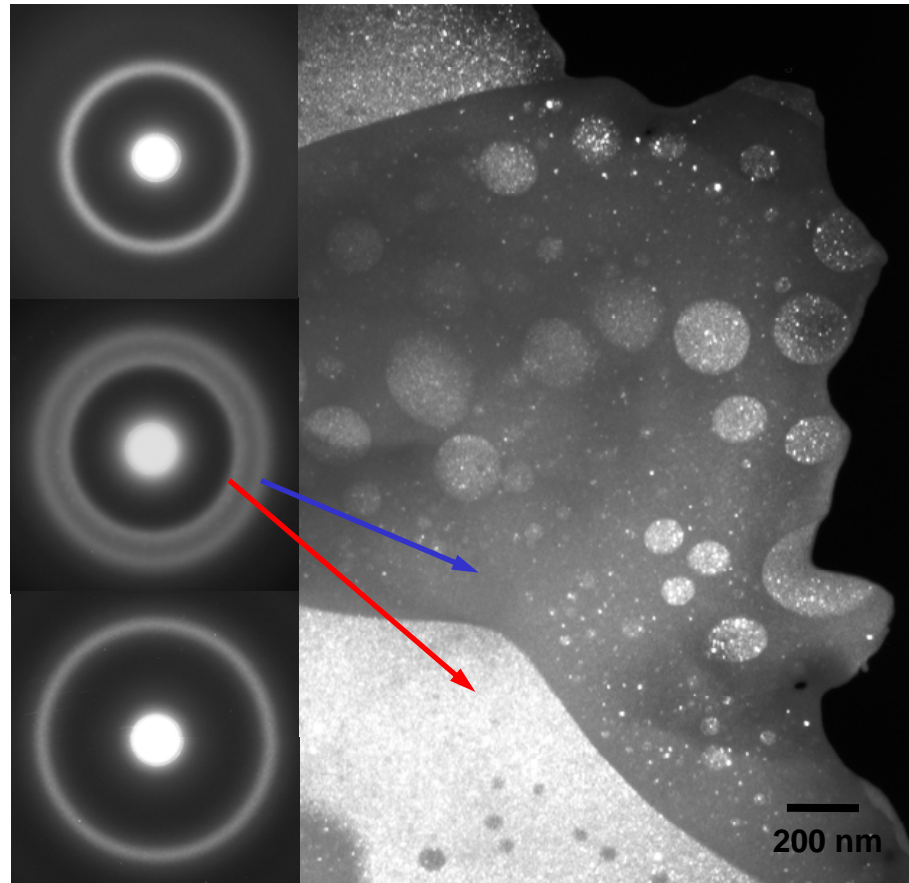
$$\begin{aligned} \mu_A^1 &> \mu_A^2 \\ \mu_B^1 &< \mu_B^2 \end{aligned}$$

# Phase separation in metallic glasses

$\text{Nd}_{60}\text{Al}_{10}\text{Co}_{30}$   
2.91 Å

$\text{Nd}_{30}\text{Zr}_{30}\text{Al}_{10}\text{Co}_{30}$   
2.37 Å, 2.99 Å

$\text{Zr}_{60}\text{Al}_{10}\text{Co}_{30}$   
2.40 Å



TEM results for  $\text{Nd}_{30}\text{Zr}_{30}\text{Al}_{10}\text{Co}_{30}$  alloy

# Diffusion

**Diffusion** : Mechanism by which matter transported through matter

**What is the driving force for diffusion?**

⇒ a concentration gradient ( $x$ )

⇒ a chemical potential ( $\mu$ )

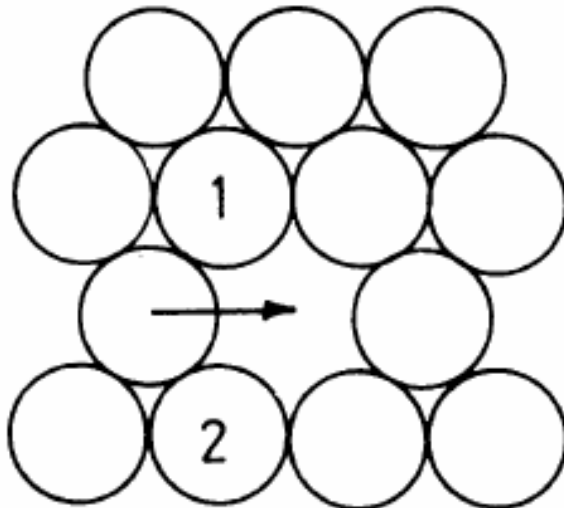
**But this chapter will explain with concentration gradients for a convenience.**

# Atomic mechanisms of diffusion

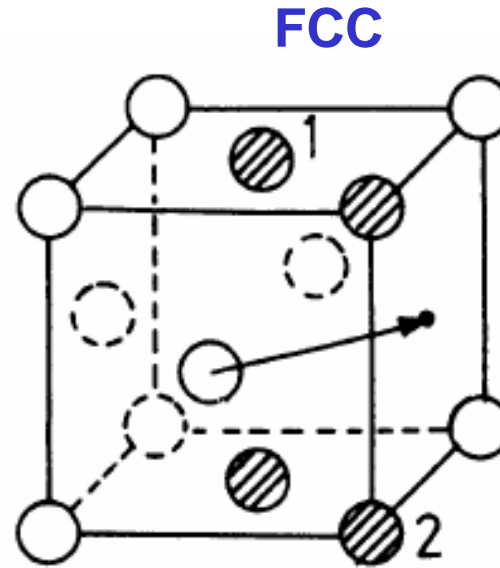
## Substitutional vs. Interstitial diffusion

The mean vibrational energy is  $3kT$ , therefore the amplitude of thermal oscillation of an atom increases markedly with temperature. If a neighboring atom site is vacant, then the oscillation atom can change site with a certain probability (atom jumping). The probability consists of creation of the vacant site and movement of a neighboring atom into the site.

## Substitutional diffusion



(a)



(b)

## Interstitial diffusion

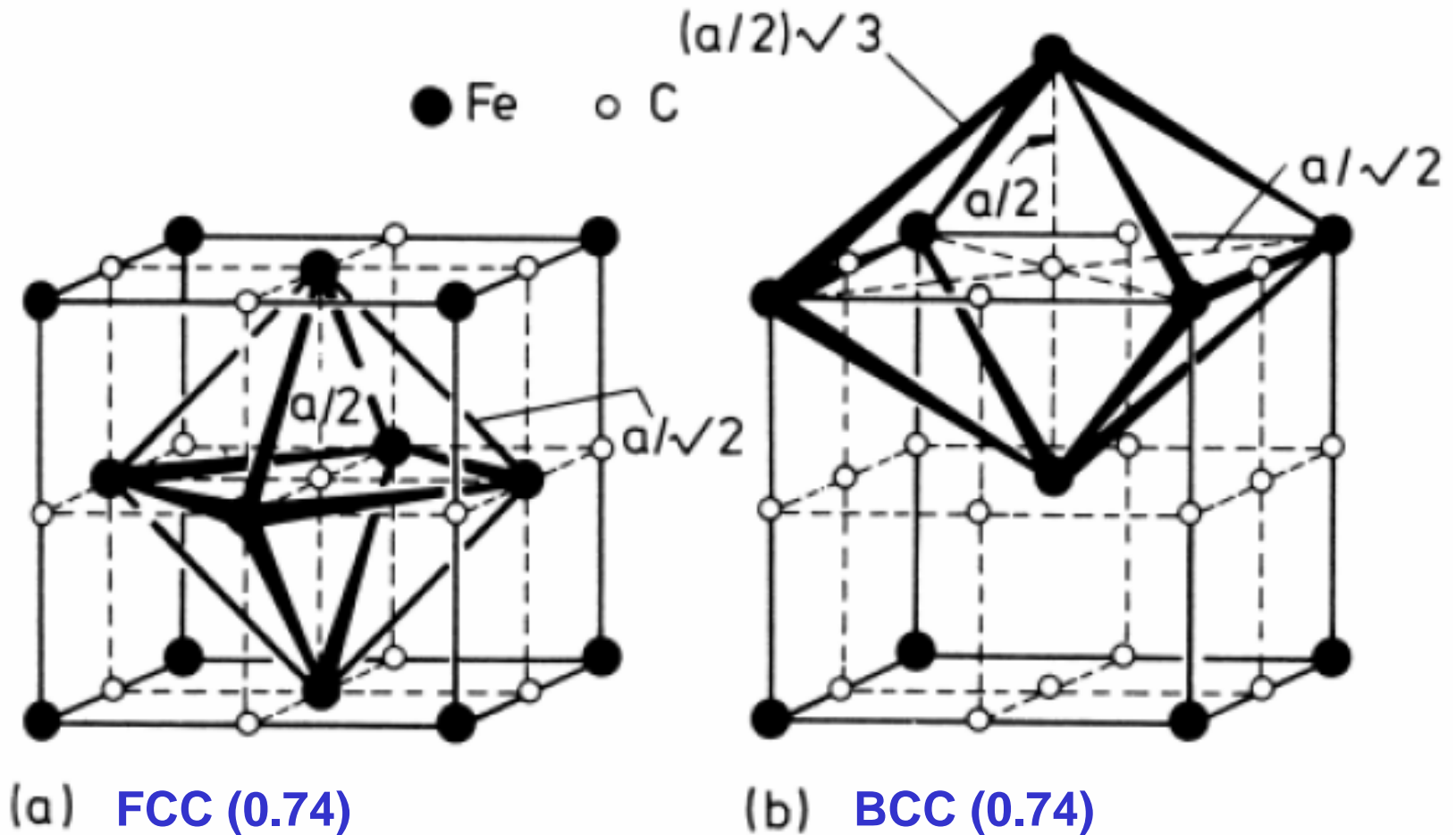
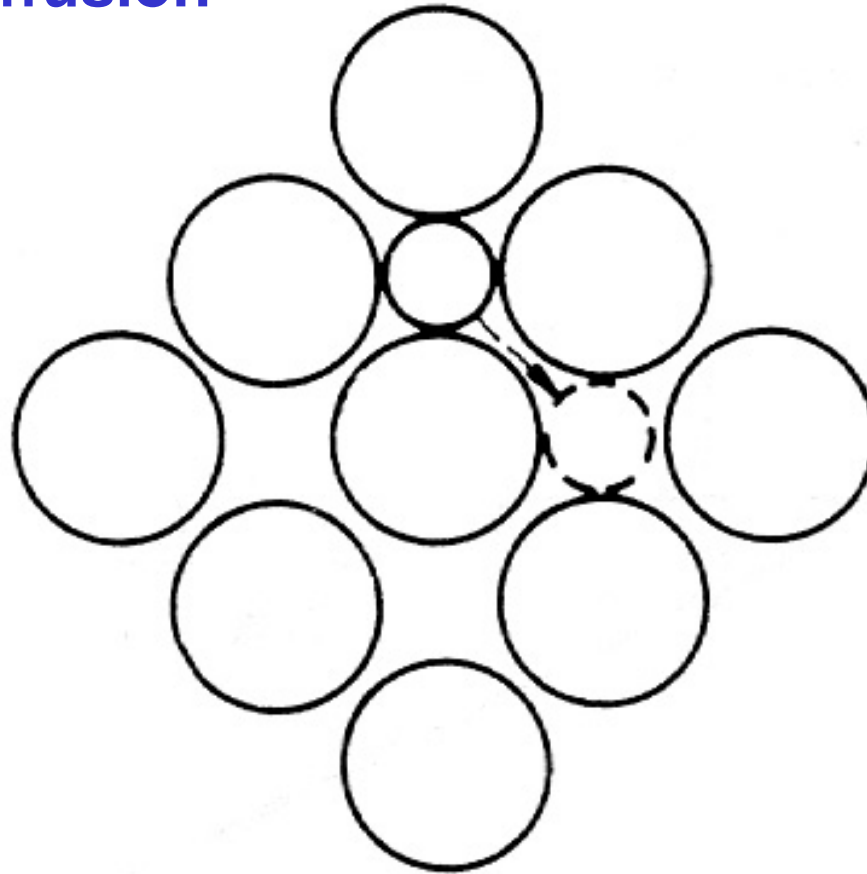


Fig. 2.3 (a) Octahedral interstices (0) in an fcc crystal. (b) Octahedral interstices in a bcc crystal. (After P. Haasen, *Physical Metallurgy*, Cambridge University Press, Cambridge, 1978.)

## Interstitial diffusion



**How interstitial diffusion differs from substitutional diffusion?**

# Interstitial diffusion

Random jump of solute B atoms in a dilute solid solution of a simple cubic lattice

Assume that there is no lattice distortion and also that there are always six vacant sites around the diffusion atom.

$$J_B = \frac{1}{6} \Gamma_B n_1 - \frac{1}{6} \Gamma_B n_2$$

$J_B$  : Flux of B atom

$\Gamma_B$  : Average jump rate of B atoms

$n_1$  : # of atoms per unit area of plane 1

$n_2$  : # of atoms per unit area of plane 2

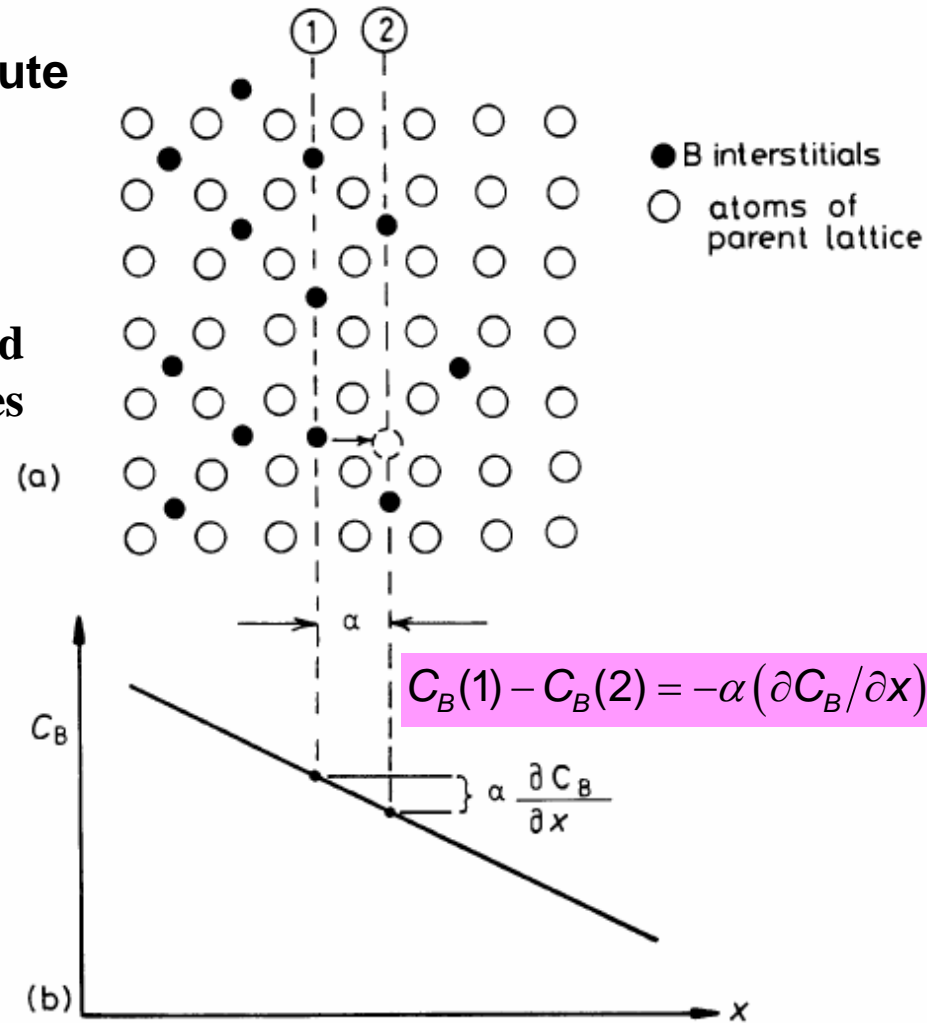


Fig. 2.5 Interstitial diffusion by random jumps in a concentration gradient.

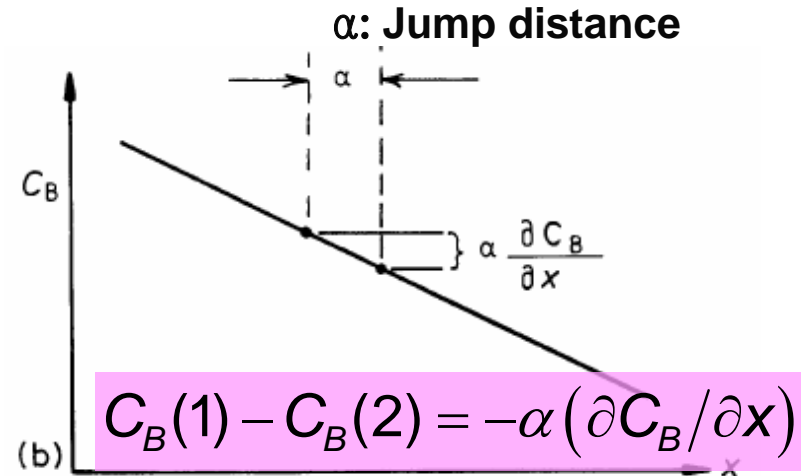


# Fick's First Law of Diffusion

$$J_B = \frac{1}{6} \Gamma_B n_1 - \frac{1}{6} \Gamma_B n_2 = \frac{1}{6} \Gamma_B (n_1 - n_2)$$

$$C_B(1) = n_1/\alpha, \quad C_B(2) = n_2/\alpha$$

$$(n_1 - n_2) = \alpha (C_B(1) - C_B(2))$$



$$J_B = \frac{1}{6} \Gamma_B (n_1 - n_2) = - \left( \frac{1}{6} \Gamma_B \alpha^2 \right) \frac{\partial C_B}{\partial x} = -D_B \frac{\partial C_B}{\partial x}$$

(atoms  $\text{m}^{-2} \text{s}^{-1}$ )

$D_B$ : Intrinsic diffusivity or  
Diffusion coefficient of B  
⇒ depends on  
microstructure of materials

Magnitude of D in various media

Gas :  $D \approx 10^{-1} \text{ cm}^2/\text{s}$

Liquid :  $D \approx 10^{-4} \sim 10^{-5} \text{ cm}^2/\text{s}$

Solid : Materials near melting temp.  $D \approx 10^{-8} \text{ cm}^2/\text{s}$

Elemental semiconductor (Si, Ge)  $D \approx 10^{-12} \text{ cm}^2/\text{s}$

**Estimate the jump frequency of a carbon atom in  $\gamma$ -Fe at 1000°C.**

$$D_B = \frac{1}{6} \Gamma_B \alpha^2$$

lattice parameter of  $\gamma$ -Fe :  $\sim 0.37$  nm

$$\alpha = 0.37 / \sqrt{2} = 0.26 \text{ nm} \quad D_C = 2.5 \times 10^{-11} \text{ m}^2 \text{ s}^{-1}$$

$$\Gamma = 2 \times 10^9 \text{ jumps s}^{-1}$$

the vibration frequency of carbon :  $\sim 10^{13}$

**If the crystal structure is not cubic, then the probability of jump is anisotropic. For example the probability of jumps along the basal direction and the axial direction of hcp crystal are different.**

**$D_B$ , in general, is concentration-dependent.**

For example, in  $\gamma$ -Fe at 1000°C,

$$D_c = 2.5 \times 10^{-11} \text{ m}^2 \text{ s}^{-1}, \text{ when } C_c = 0.15 \text{ wt\%}$$

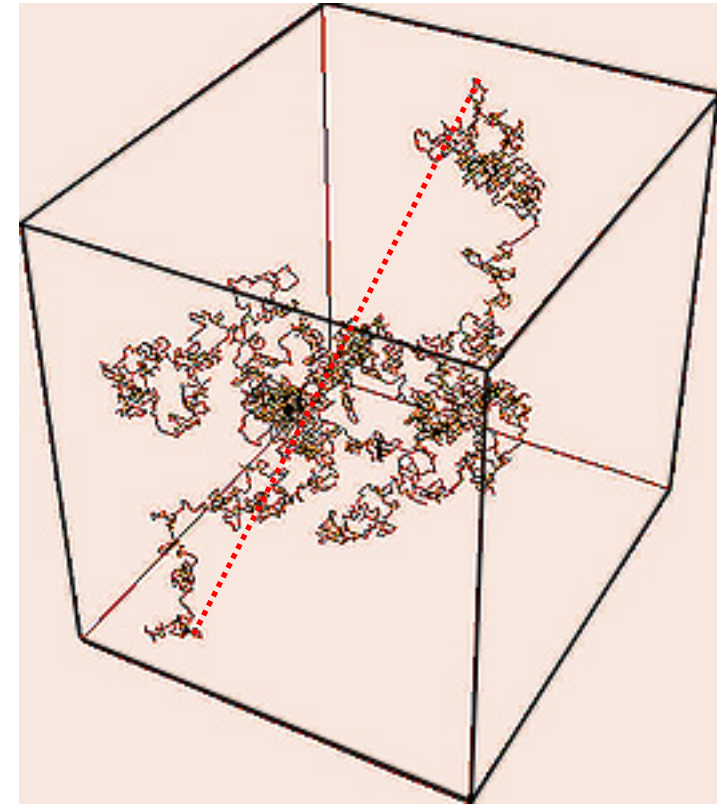
$$D_c = 7.7 \times 10^{-11} \text{ m}^2 \text{ s}^{-1}, \text{ when } C_c = 1.4 \text{ wt\%}$$

$$C_c \uparrow \rightarrow D_c \uparrow$$

For random walk in 3 dimensions,  
after n steps of length  $\alpha$

$$\rightarrow \alpha\sqrt{n}$$

During random walk,  
an atom will move in time (t)  
a radial distance (r) from the origin



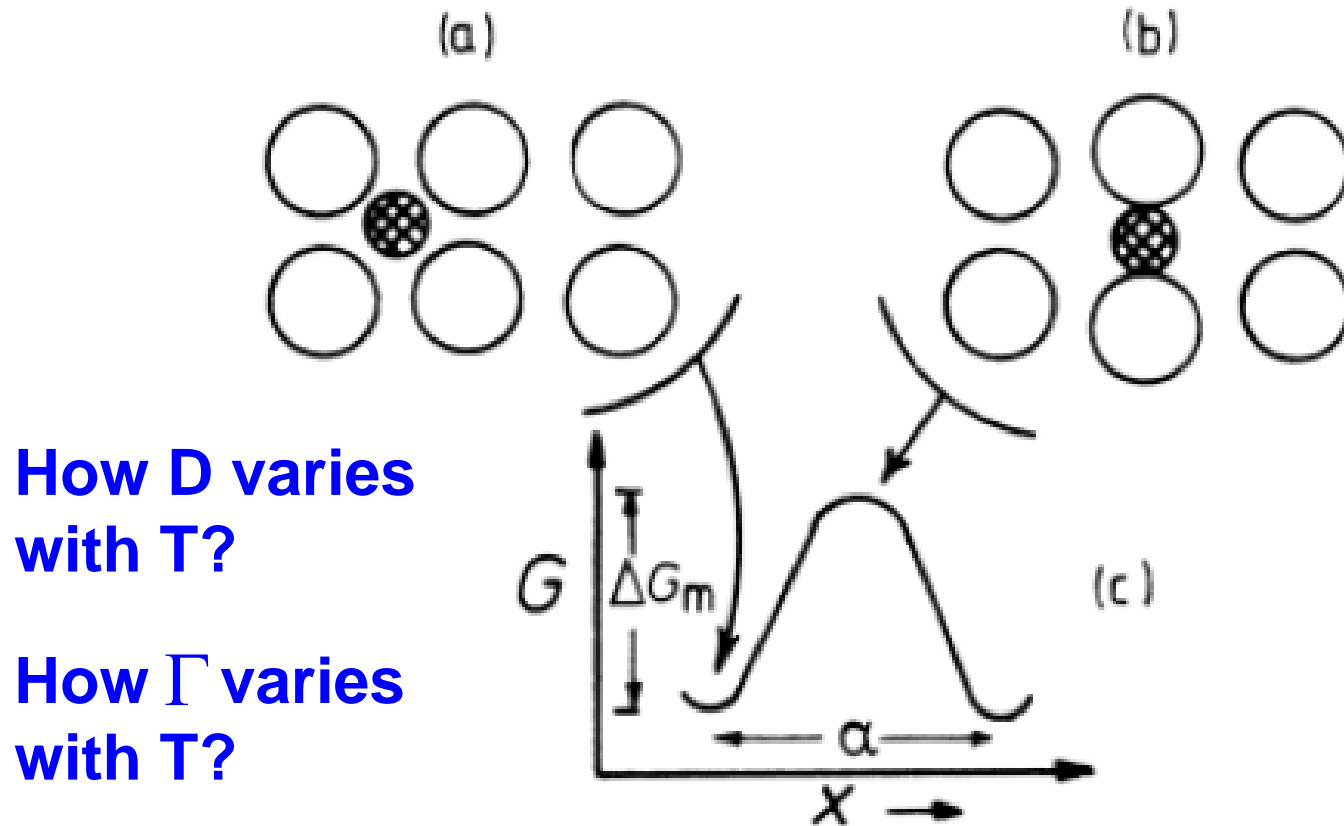
$$r = \alpha\sqrt{(\Gamma t)} = f(D) = 2.4\sqrt{Dt} \quad \because \Gamma = \frac{6D}{\alpha^2} \rightarrow r = \sqrt{6}\sqrt{Dt}$$

$$D_B = \frac{1}{6}\Gamma_B\alpha^2$$

In 1 s, each carbon atom will move a total distance of **~ 0.5 m**  
 $\rightarrow$  a net displacement : **~ 10  $\mu\text{m}$ .**

# EFFECT OF TEMPERATURE on Diffusivity

## Thermal Activation



How  $D$  varies  
with  $T$ ?

How  $\Gamma$  varies  
with  $T$ ?

Fig. 2.6 Interstitial atom, (a) in equilibrium position, (b) at the position of maximum lattice distortion. (c) Variation of the free energy of the lattice as a function of the position of interstitial. (After P.G. Shewmon, in *physical Metallurgy*, 2<sup>nd</sup> edn., R.W. Cahn (Ed.), North-Holland, Amsterdam, 1974.)

## Thermally activated process

jump frequency  $\Gamma_B$  ?

$$\Gamma_B = Z \nu \exp(-\Delta G_m / RT)$$

$Z$  : nearest neighbor sites

$\nu$  : vibration frequency

$\Delta G_m$  : activation energy for moving

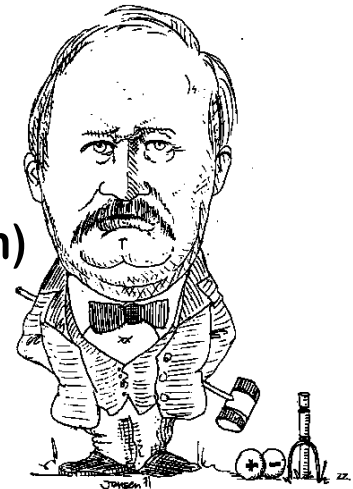
$$\left\{ \Delta G_m = \Delta H_m - T \Delta S_m, D_B = \frac{1}{6} \Gamma_B \alpha^2 \right\}$$

$$D_B = \left[ \frac{1}{6} \alpha^2 Z \nu \exp(\Delta S_m / R) \right] \exp(-\Delta H_m / RT)$$

$$\Delta H_m \equiv Q_{ID}$$

$$D_B = D_{B0} \exp \frac{-Q_{ID}}{RT}$$

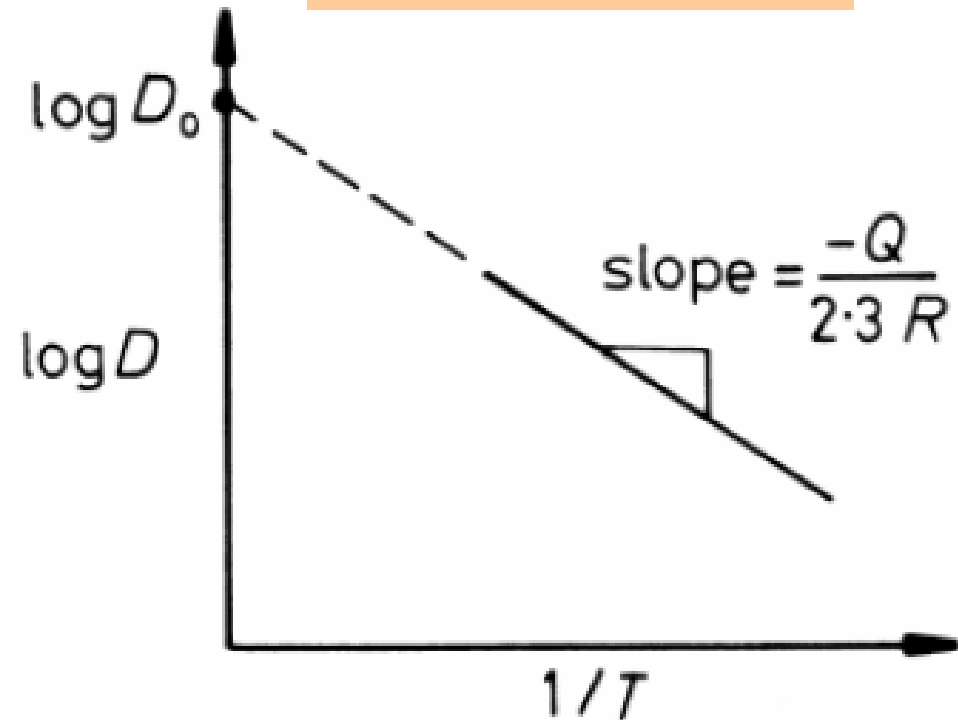
(Arrhenius-type equation)



# Temperature Dependence of Diffusion

$$D_B = D_{B0} \exp \frac{-Q_{ID}}{RT}$$

How to determine  $Q_{ID}$  experimentally?



$$\log D = \log D_0 - \frac{Q}{2.3R} \left( \frac{1}{T} \right)$$

➔ Therefore, from the slope of the  $D$ -curve in an  $\log D$  vs  $1/T$  coordinate, the activation energy may be found.

Fig. 2.7 The slope of  $\log D$  v.  $1/T$  gives the activation energy for diffusion  $Q$ .