15 Linear Matrix Inequality (LMI)

A linear matrix inequality is an expression of the form

$$F(x) \triangleq F_0 + x_1 F_1 + \cdots + x_m F_m > 0 \tag{1}$$

where

- $x = (x_1, \cdots, x_m)$ is a vector of real numbers,
- $F_0, \cdots, F_m$ are real symmetric matrices, i.e., $F_i = F_i^T \in \mathbb{R}^{n\times n}, i = 0, \cdots, m$ for some $n \in \mathbb{Z}_+$, and
- the inequality > 0 in (1) means positive definite, i.e., $u^T F(x) u > 0$ for all $u \in \mathbb{R}^n$, $u \neq 0$. Equivalently, the smallest eigenvalue of $F(x)$ is positive.

Definition 15.1 (Linear matrix inequality(LMI)) *A linear matrix inequality is*

$$F(x) > 0 \tag{2}$$

where F is an affine function mapping a finite dimensional vector space to the set $\mathbb{S}^n \triangleq \{M : M = M^T \in \mathbb{R}^{n\times n}\}$, $n > 0$, of real matrices.

"Affine matrix inequality" would have been a better name. Note that the constraints $F(x) < 0$ and $A(x) < B(x)$ are special cases of (2).

Remark. Recall, from definition, that an affine mapping $F : \mathbb{V} \to \mathbb{S}^n$ necessarily takes the form $F(x) = F_0 + T(x)$ where $F_0 \in \mathbb{S}^n$ and $T : \mathbb{V} \to \mathbb{S}^n$ is a linear transformation. Thus if $\mathbb{V}$ is finite dimensional, say of dimension m, and $\{e_1, \cdots, e_m\}$ constitutes a basis for $\mathbb{V}$, then we can write

$$T(x) = \sum_{j=1}^{m} x_j F_j$$

where the elements $\{x_1, \cdots, x_m\}$ are such that $x = \sum_{j=1}^{m} x_j e_j$ and $F_j = T(e_j)$ for $j = 1, \cdots, m$. Hence we obtain (1) as a special case.

Remark. The same remark applies to mappings $F : \mathbb{R}^{m_1 \times m_2} \to \mathbb{S}^n$ where $m_1, m_2 \in \mathbb{Z}^+$. A simple example where $m_1 = m_2$ is the Lyapunov inequality

$$F(X) = A^T X + XA + Q > 0 \; .$$

Here, $A, Q \in \mathbb{R}^{m_1 \times m_2}$ are assumed to be given and $X \in \mathbb{R}^{m \times m}$ is the unknown. The unknown variable is therefore a matrix. Note that this defines an LMI only if Q is symmetric. In this case, the domain $\mathbb{V}$ of F in definition 15.1 is equal to $\mathbb{S}^m$. We can view this LMI as a special case of (1) by defining a basis $E_1, \cdots, E_m$ of $\mathbb{S}^m$ and writing $X = \sum_{j=1}^{m} x_j E_j$:

$$F(X) = F\left(\sum_{j=1}^{m} x_j E_j\right) = F_0 + \sum_{j=1}^{m} x_j F(E_j) = F_0 + \sum_{j=1}^{m} x_j F_j$$

which is of the form (1).

The LMI

$$F(x) = F_0 + xF_1 + \cdots + x_m F_m$$

defines a *convex constraint* on $x = (x_1, \cdots, x_m)$. i.e., the set

$$\mathcal{F} \triangleq \{x : F(x) > 0\}$$

is convex. Indeed, if $x_1, x_2 \in \mathcal{F}$ and $\alpha \in (0,1)$ then

$$F(\alpha x_1 + (1-\alpha)x_2) = \alpha F(x_1) + (1-\alpha) F(x_2) > 0$$

Convexity has an important consequence: even though the LMI has no analytical solution in general, it can be solved numerically with guarantees of fining a solution when one exists. Although the LMI may seem special, it turns out that many convex sets can be represented in this way.

1. Note that a system of LMIs (i.e. a finite set of LMIs) can be written as a single LMI since

$$\left\{ \begin{array}{c} F_1(x) < 0 \\ \vdots \\ F_K(x) < 0 \end{array} \right\} \text{ is equivalent to } F(x) \triangleq \mathrm{diag}[F_1(x), \cdots, F_K(x)] < 0$$

2. Combined constraints (in the unknown x) of the form

$$\left\{ \begin{array}{l} F(x) > 0 \\ Ax = b \end{array} \right. \quad \text{or} \quad \left\{ \begin{array}{l} F(x) > 0 \\ x = Ay + b \;\; \text{for some } y \end{array} \right.$$

where the affine function $F : \mathbb{R}^m \to \mathbb{S}^n$ and matrices $A \in \mathbb{R}^{n\times m}$ and $b \in \mathbb{R}^n$ are given can be lumped into one LMI. More generally, the combined equations

$$\begin{cases} F(x) > 0 \\ x \in \mathcal{M} \end{cases} \tag{3}$$

where $\mathcal{M}$ is an affine subset of $\mathbb{R}^n$, i.e.

$$\mathcal{M} = x_0 + \mathcal{M}_0 = \{x_0 + m \mid m \in \mathcal{M}_0\}$$

with $x_0 \in \mathbb{R}^n$ and $\mathcal{M}_0$ a linear subspace of $\mathbb{R}^n$, can be written in the form of one single LMI. In order to see this, let $e_1, \cdots, e_k \in \mathbb{R}^n$ be a basis of $\mathcal{M}_0$ and let $F(x) = F_0 + T(x)$ be decomposed as in remark. Then (3) can be rewritten as

$$\begin{aligned} 0 < F(x) &= F_0 + T(x_0 + \sum_{j=1}^{k} x_j e_j) = \underbrace{F_0 + T(x_0)}_{\text{constant part}} + \underbrace{\sum_{j=1}^{k} x_j T(e_j)}_{\text{linear part}} \\ &= \bar{F}_0 + x_1\bar{F}_1 + ... + x_k\bar{F}_k \\ &\triangleq \bar{F}(\bar{x}) \end{aligned}$$

where $\bar{F}_0 = F_0 + T(x_0)$, $\bar{F}_j = T(e_j)$ and $x = (x_1, \cdots, x_k)$. This implies that $x \in \mathbb{R}^n$ satisfies (3) if and only if $F(x) > 0$. Note that the dimension of $\bar{x}$ is smaller than the dimension of x.

3. (Schur Complement) Let $F : \mathbb{V} \to \mathbb{S}^n$ be an affine function partitioned to

$$F(x) = \begin{bmatrix} F_{11}(x) & F_{12}(x) \\ F_{21}(x) & F_{22}(x) \end{bmatrix}$$

where $F_{11}(x)$ is square. Then

$$F(x) > 0 \quad \text{iff} \quad \begin{cases} F_{11}(x) > 0 \\ F_{22}(x) - F_{21}(x)F_{11}^{-1}(x)F_{12}(x) > 0 \end{cases} \tag{4}$$

Note that the second inequality in (4) is a nonlinear matrix inequality in x. It follows that nonlinear matrix inequalities of the form (4) can be converted to LMIs, and nonlinear inequalities (4) define a convex constraint on x.

15.1 Types of LMI problems

Suppose that $F, G : \mathbb{V} \to \mathbb{S}^{n_1}$ and $H : \mathbb{V} \to \mathbb{S}^{n_2}$ are affine functions. There are three generic problems related to the study of linear matrix inequalities:

Feasibility: The test whether or not there exist solutions x of $F(x) > 0$ is called a feasibility problem. The LMI is called non-feasible if no solutions exist.

Optimization: Let $f : \mathcal{S} \to \mathbb{R}$ and suppose that $\mathcal{S} = \{x | F(x) > 0\}$. The problem to determine $V_{\text{opt}} = \inf_{x \in \mathcal{S}} f(x)$ is called an optimization problem with an LMI constraint.

Generalized eigenvalue problem: Minimize a scalar $\lambda \in \mathbb{R}$ subject to

$$\begin{cases} \lambda F(x) - G(x) > 0 \\ F(x) > 0 \\ H(x) > 0 \end{cases}$$

15.2 What are LMIs good for?

Many optimization problems in control design, identification, and signal processing can be formulated using LMIs.

Example. Asymptotic stability of the LTI system

$$\dot{x} = Ax \quad , A \in \mathbb{R}^{n \times n} \tag{5}$$

Lyapunov said, asymptotically stable iff there exists $X \in \mathbb{S}^n$ such that $X > 0$ and $A^T X + XA < 0$. i.e. equivalent to feasibility of the LMI

$$\begin{bmatrix} X & 0 \\ 0 & -A^T X - XA \end{bmatrix} > 0$$

Example. Determine a diagonal matrix D such that $||DMD^{-1}|| < 1$ where M is some given matrix. Since

$$\begin{aligned} ||DMD^{-1}|| < 1 &\iff D^{-T} M^T D^T D M D^{-1} < I \\ &\iff M^T D^T D M < D^T D \\ &\iff X - M^T X M > 0 \end{aligned}$$

where $X := D^T D > 0$ we see that the existence of such a matrix means the feasibility of LMI.

Example. Let F be an affine function and consider the problem of minimizing $f(x) \triangleq \lambda_{\max}(F(x))$ over x.

$$\begin{aligned} \lambda_{\max}(F^T(x)F(x)) < \gamma \quad &\Longleftrightarrow \quad \gamma I - F^T(x)F(x) > 0 \\ &\Longleftrightarrow \quad \begin{bmatrix} \gamma I & F^T(x) \\ F(x) & I \end{bmatrix} > 0 \end{aligned}$$

if we define

$$\bar{x} \triangleq \begin{bmatrix} x \\ \gamma \end{bmatrix}, \quad \bar{F}(\bar{x}) \triangleq \begin{bmatrix} \gamma I & F^T(x) \\ F(x) & I \end{bmatrix}, \quad \bar{f}(\bar{x}) \triangleq \gamma \, ,$$

then $\bar{F}$ is an affine function of $\bar{x}$ and the problem to minimize the maximum eigenvalue of $F(x)$ is equivalent to determining $\inf \bar{f}(\bar{x})$ subject to the LMI $\bar{F}(\bar{x}) > 0$. Hence, this is an optimization problem with a linear objective function $\bar{f}$ and an LMI constraint.

Example (Simultaneous stabilization)
Consider k LTI systems with n-dim state space and m-dim input space:

$$\dot{x} = A_i x + B_i u$$

where $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$, $i \in 1, \cdots, k$. We'd like to find a state feedback law $u = Fx$, $F \in \mathbb{R}^{m\times n}$ such that the eigenvalues $\lambda(A_i + B_i F)$ lie on the LHP for $i \in 1, \cdots, k$. From the example above, this is solved when we find matrices F and X_i, $i \in 1, \cdots, k$ such that for $i \in 1, \cdots, k$,

$$\begin{cases} X_i > 0 \\ (A_i + B_i F)^T X_i + X_i (A_i + B_i F) < 0 \end{cases} \tag{6}$$

Note that this is *not* a system of LMIs in X_i and F. If we introduce $Y_i = X_i^{-1}$ and $K = FY_i$, then (6) becomes

$$\begin{cases} Y_i > 0 \\ A_i Y_i + Y_i A_i^T + B_i K + K_i^T B_i < 0 \end{cases} ,$$

which can be further simplified by assuming the existence of a joint Lyapunov function, i.e. $X_i = \cdots = X_k = X$. The joint stabilization problem has a solution if this system of LMIs is feasible.

15.3 Nominal Performance

Consider

$$x = Ax + Bu \quad (7)$$
$$y = Cx + Du \quad (8)$$

with state space $X = \mathbb{R}^n$, input space $U = \mathbb{R}^m$ and output space $Y = \mathbb{R}^p$.

15.3.1 H_∞ nominal performance

Proposition 15.2 *If the system* (7) *is asymptotically stable then* $||G||_\infty < \gamma$ *whenever there exists a solution* $K = K^T > 0$ *to the LMI*

$$\begin{bmatrix} A^TK + KA + C^TC & KB + C^TD \\ B^TK + D^TC & D^TD - \gamma^2 I \end{bmatrix} < 0. \quad (9)$$

Proof requires discussion on linear dissipative systems. See Scherer, if interested.

Therefore, we can compute the smallest possible upper bound of the L2-induced gain of the system (which is the H_∞ norm of the transfer function) by minimizing $\gamma > 0$ over all variables γ and $K > 0$ that satisfy the LMI (9).

15.3.2 H_2 nominal performance

Suppose that we are interested only in the impulse responses of this system. This means, that we take impulsive inputs of the form $u(t) = \delta(t)e_i$ with e_i the i_{th} basis vector in the standard basis of the input space $\mathbb{R}^m$. (i runs from 1 till m). With zero initial conditions, the corresponding output y^i belongs to $\mathcal{L}_2$ and is given by

$$y^i(t) = \begin{cases} C\exp(At)Be_i & \text{for } t > 0 \\ De_i\delta(t) & \text{for } t = 0 \\ 0 & \text{for } t < 0. \end{cases} .$$

Only if $D = 0$, the sum of the squared norms of all such impulse responses $\sum_{i=1}^m ||y_i||_2^2$ is well defined and given by

$$\begin{aligned} \sum_{i=1}^m ||y_i||_2^2 &= \text{trace} \int_0^\infty B^T \exp(A^t)C^TC\exp(At)B\; dt \\ &= \text{trace} \int_0^\infty C\exp(At)BB^T\exp(A^Tt)C^T\; dt \\ &= \text{trace} \int_{-\infty}^\infty G(j\omega)G^*(j\omega)\; d\omega \end{aligned}$$

where G is the transfer function of the system.

Proposition 15.3 *Suppose that the system* (7) *is asymptotically stable (and* $D = 0$*), then the following statements are equivalent.*

(a) $||G||_2 < \gamma$

(b) there exists $K = K^T > 0$ *and* Z *such that*

$$\begin{bmatrix} A^TK + KA & KB \\ B^TK & -I \end{bmatrix} < 0; \quad \begin{bmatrix} K & C^T \\ C & Z \end{bmatrix} > 0; \quad trace(Z) < \gamma^2 \quad (10)$$

(c) there exists $K = K^T > 0$ *and* Z *such that*

$$\begin{bmatrix} AK + KA^T & KC^T \\ CK & -I \end{bmatrix} < 0; \quad \begin{bmatrix} K & B \\ B^T & Z \end{bmatrix} > 0; trace(Z) < \gamma^2 \quad (11)$$

Proof. . note that $||G||_2 < \gamma$ is equivalent to requiring that the controllability gramian $W_c := \int_0^\infty \exp(At)BB^T exp(A^Tt)\, dt$ satisfies trace$(CWC^T) < \gamma^2$. Since the controllability gramian is the unique positive definite solution to the Lyapunov equation $AW + WA^T + BB^T = 0$ this is equivalent to saying that there exists $X > 0$ such that

$$AX + XA^T + BB^T < 0; \quad trace(CXC^T) < \gamma^2.$$

With a change of variables $K := X^{-1}$, this is equivalent to the existence of $K > 0$ and Z such that

$$A^TK + KA + KBB^TK < 0; \quad CK^{-1}C^T < Z; \quad \text{trace}(Z) < \gamma^2.$$

Now, using Schur complements for the first two inequalities yields that $||G||_2 < \gamma$ is equivalent to the existence of $K > 0$ and Z such that

$$\begin{bmatrix} A^TK + KA & KB \\ B^TK & I \end{bmatrix} < 0; \quad \begin{bmatrix} K & C^T \\ C & Z \end{bmatrix} > 0; \quad \text{trace}(Z) < \gamma^2\ .$$

The equivalence with (11) is obtained by a direct dualization and the observation that $||G||_2 = ||G^T||_2$.

Therefore, the smallest possible upper bound of the H2-norm of the transfer function can be calculated by minimizing the criterion trace(Z) over the variables $K > 0$ and Z that satisfy the LMIs defined by the first two inequalities in (10) or (11).

15.4 Controller Synthesis

Let

$$\begin{aligned}\dot{x} &= Ax + B_1 w + B_2 u\\ z_\infty &= C_\infty x + D_{\infty 1} w + D_{\infty 2} u\\ z_2 &= C_2 x + D_{21} w + D_{22} u\\ y &= C_y x + D_{y1} w\end{aligned}$$

and

$$\begin{aligned}\dot{x}_K &= A_K x_K + B_K y\\ u &= C_K x_K + D_K y\end{aligned}$$

be state-space realizations of the plant $P(s)$ and the controller $K(s)$ respectively.

Denoting by $T_\infty(s)$ and $T_2(s)$ the CL TF from w to z_∞ and z_2, respectively, we consider the following multi-objective synthesis problem:

Design an output feedback controller $u = K(s)y$ such that

- H_∞ performance: maintains the H_∞ norm of T_∞ below γ_0.
- H_2 performance: maintains the H_2 norm of T_2 below ν_0.
- Multi-objective H_2/H_∞ controller design: minimizes the trade-off criterion of the form $\alpha||T_\infty||_\infty^2 + \beta||T_2||_2^2$ with some $\alpha, \beta \geq 0$.
- Pole placement: places the CL poles in some prescribed LMI region $\mathcal{D}$.

Let the following denote the corresponding CL state-space eqns,

$$\begin{aligned}\dot{x_{cl}} &= A_{cl} x_{cl} + B_{cl} w\\ z_\infty &= C_{cl1} x_{cl} + D_{cl1} w\\ z_2 &= C_{cl2} x_{cl} + D_{cl2} w\end{aligned}$$

then our design objectives can be expressed as follows:

- H_∞ performance: the CL RMS gain from w to z_∞ does not exceed γ iff there exists a symmetric matrix X_∞ such that

$$\begin{bmatrix} A_{cl}X_\infty + X_\infty A_{cl}^T & B_{cl} & X_\infty C_{cl1}^T \\ B_{cl}^T & -I & D_{cl1}^T \\ C_{cl1}X_\infty & D_{cl1} & -\gamma^2 I \end{bmatrix} < 0$$

$$X_\infty > 0$$

- H_2 performance: the LQG cost from w to z_2 does not exceed ν iff $D_{cl2} = 0$ and there exists a symmetric matrices X_2 and Q such that

$$\begin{bmatrix} A_{cl}X_2 + X_2 A_{cl}^T & B_{cl} \\ B_{cl}^T & -I \end{bmatrix} < 0$$
$$\begin{bmatrix} Q & C_{cl2}^T X_2 \\ X_2 C_{cl2}^T & X_2 \end{bmatrix} > 0$$
$$\text{trace}(Q) < \nu^2$$

- Pole placement: the CL poles lie in the LMI region $\mathcal{D} := \{z \in \mathbb{C} : L + Mz + M^T \bar{z} < 0\}$ with $L = L^T = [\lambda_{ij}]_{1\le i,j\le m}$ and $M = [\mu_{ij}]_{1\le i,j\le m}$ iff there exists a symmetric matrix X_{pol} such that

$$\begin{aligned} [\lambda_{ij} X_{pol} + \mu_{ij} A_{cl} X_{pol} + \mu_{ji} X_{pol} A_{cl}^T]_{1\le i,j\le m} &< 0 \\ X_{pol} &> 0\,. \end{aligned}$$

For tractability, we seek a single Lyapunov matrix $X := X_\infty = X_2 = X_{pol}$ that enforces all three sets of constraints. Factorizing X as

$$X = \begin{bmatrix} R & I \\ M^T & 0 \end{bmatrix} \begin{bmatrix} 0 & S \\ I & N^T \end{bmatrix}^{-1}$$

and introducing the transformed controller variables:

$$\begin{aligned} \mathcal{B}_K &:= NB_K + SB_2 D_K \\ \mathcal{C}_K &:= C_K M^T + D_K C_y R \\ \mathcal{A}_K &:= NA_K M^T + NB_K C_y R + SB_2 C_K M^T + S(A + B_2 D_K C_y)R\,, \end{aligned}$$

the inequality constraints on X are turned into LMI constraints in the variables $R, S, Q, \mathcal{A}_K, \mathcal{B}_K, \mathcal{C}_K$ and D_K. And we have the following suboptimal LMI formulation of our multi-objective synthesis problem:

Minimize $\alpha\gamma^2 + \beta\text{trace}(Q)$ over $R, S, Q, \mathcal{A}_K, \mathcal{B}_K, \mathcal{C}_K, D_K$ and γ^2 satisfying:

$$\begin{bmatrix} AR + RA^T + B_2\mathcal{C}_K + \mathcal{C}_K^T B_2^T & \mathcal{A}_K + A + B_2 D_K C_y & B_1 + B_2 D_K D_{y1} & \star \\ \star & A^T S + SA + \mathcal{B}_K C_y + C_y^T \mathcal{B}_K^T & SB_1 + \mathcal{B}_K D_{y1} & \star \\ \star & \star & -I & \star \\ C_\infty R + D_{\infty 2}\mathcal{C}_K & C_\infty + D_{\infty 2} D_K C_y & D_{\infty 1} + D_{\infty 2} D_K D_{y1} & -\gamma^2 I \end{bmatrix} < 0$$

$$\begin{bmatrix} Q & C_2 R + D_{22}\mathcal{C}_K & C_2 D_{22} D_K C_y \\ \star & R & I \\ \star & I & S \end{bmatrix} > 0$$

$$\Bigg[\lambda_{ij} \begin{bmatrix} R & I \\ I & S \end{bmatrix} + \mu_{ij} \begin{bmatrix} AR + B_2\mathcal{C}_K & A + B_2 D_K C_y \\ \mathcal{A}_K & SA + \mathcal{B}_K C_y \end{bmatrix} + \mu_{ji} \begin{bmatrix} (AR + B_2\mathcal{C}_K)^T & \mathcal{A}_K^T \\ (A + B_2 D_K C_y)^T & (SA + \mathcal{B}_K C_y)^T \end{bmatrix}\Bigg]_{1 \le i,j \le m} < 0$$

$$\begin{aligned} \mathrm{trace}(Q) &< \nu_0^2 \\ \gamma^2 &< \gamma_0^2 \\ D_{21} + D_{22} D_K D_{y1} &= 0 \, . \end{aligned}$$

Given optimal solutions γ^*, Q^* of this LMI problem, the closed loop performances are bounded by

$$||T||_\infty \le \gamma^*, \quad ||T||_2 \le \sqrt{\mathrm{trace}(Q^*)} \, .$$

This has been implemented by the matlab command "`hinfmix`".

15.5 Affine combinations of linear systems

Often models uncertainty about specific parameters is reflected as uncertainty in specific entries of the state space matrices A, B, C, D. Let $p = (p_1, ..., p_n)$ denote the parameter vector which expresses the uncertain quantities in the system and suppose that this parameter vector belongs to some subset $\mathcal{P} \subset \mathbb{R}^n$. Then the uncertain model can be thought of as being parameterized by $p \in \mathcal{P}$ through its state space representation

$$\begin{aligned} \dot{x} &= A(p)x + B(p)u & (12) \\ y &= C(p)x + D(p)u \, . & (13) \end{aligned}$$

One way to think of equations of this sort is to view them as a set of linear time-invariant systems as parameterized by $p \in \mathcal{P}$. However, if p is time, then (12) defines a linear time varying dynamical system and it can therefore also be viewed as such. If components of p are time varying and coincide with state components then (12) is better viewed as a nonlinear system.

Of particular interest will be those systems in which the system matrices affinely depend on p. This means that

$$A(p) = A_0 + p_1 A_1 + \cdots + p_n A_n \quad (14)$$
$$B(p) = B_0 + p_1 B_1 + \cdots + p_n B_n \quad (15)$$
$$C(p) = C_0 + p_1 C_1 + \cdots + p_n C_n \quad (16)$$
$$D(p) = D_0 + p_1 D_1 + \cdots + p_n D_n \ . \quad (17)$$

Or, written in a more compact form

$$S(p) = S_0 + p_1 S_1 + ... + p_n S_n$$

where

$$S(p) = \begin{bmatrix} A(p) & B(p) \\ C(p) & D(p) \end{bmatrix}$$

is the system matrix associated with (12). We call these models *affine parameter dependent models*. In MATLAB such a system is represented with the routines psys and pvec. For $n = 2$ and a parameter box

$$\mathcal{P} \triangleq \{(p_1, p_2) |\ p_1 \in [p_1^{\min}, p_1^{\max}], p_2 \in [p_2^{\min}, p_2^{\max}]\}$$

the syntax is

```
affsys = psys( p, [s0, s1, s2] );
p = pvec( 'box', [p1min p1max ; p2min p2max])
```

where `p` is the parameter vector whose i-th component ranges between `pimin` and `pimax`. Bounds on the rate of variations, $\dot{p}_i(t)$ can be specified by adding a third argument "`rate`" when calling "`pvec`". See also the following routines:

- `pdsimul` for time simulations of affine parameter models
- `aff2pol` to convert an affine model to an equivalent polytopic model
- `pvinfo` to inquire about the parameter vector

References

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