16.242

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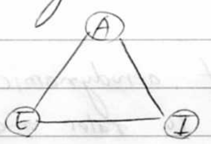
Course Outline

1. Introduction / Overview

2. Static Aeroelasticity* (divergence, control reversal ...)

Aeroelasticity --- mutual interaction of elastic, aerodynamic and inertial forces in a structure (airplane, rocket, engine, building, bridge, etc ...)

* Collar's triangle



E + I = Structural Dynamics

E + A = Static Aeroelasticity

A + I = Dynamic stability (Rigid Body Dynamics)

E + I + A = Aeroelasticity (Dynamic aeroelasticity)

Drela/Harris → 3. Unsteady Aerodynamics

mid-term → 4. Dynamic Aeroelasticity (flutter, A/C response)

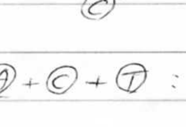
Hall → 5. Turbomachinery Aeroelasticity

6. Helicopter Aeroelasticity

Crawley → 7. Control Aeroelasticity

E + I + A + C = Aeroservoelasticity

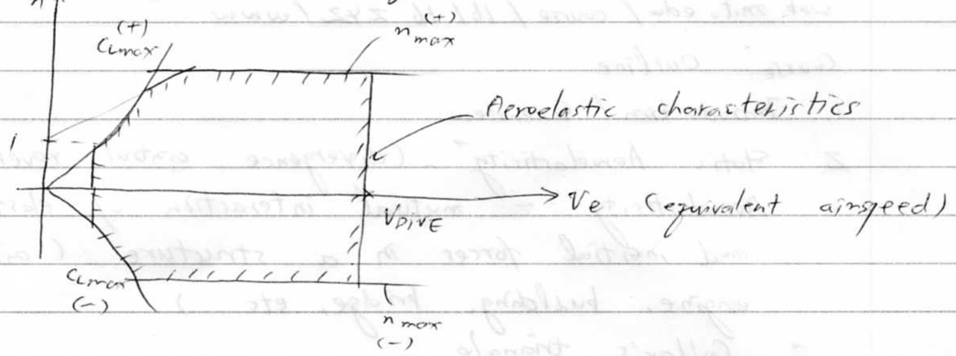
Generalized Collar's diagram



T : thermal effect

E + I + A + C + T : Aerothermo servoelasity

- Flight Envelope (V-n diagram)

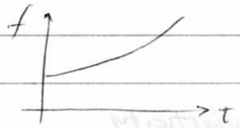


II. Static Aeroelasticity

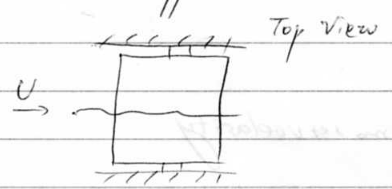
- Definition --- interaction of aerodynamic and elastic forces
  - insensitive to rates and accelerations of the structural deflection
- Two classes of problems
  1. Effect of elastic deformation on the airloads — normal operating condition --- performance, handling qualities, structural load distribution
  2. Static instabilities --- divergence

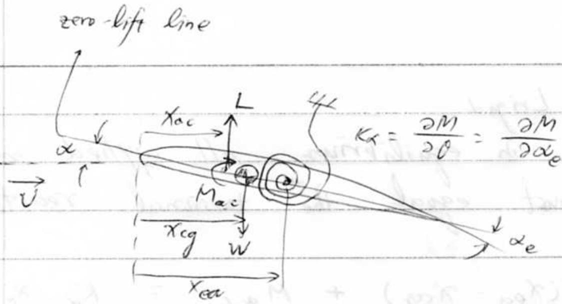
[Note] instability --- tendency to move away from equilibrium

static $\rightarrow$ $e^{(a+i\omega)t}$: $a > 0$, $\omega = 0$



- Typical section model of an airfoil
  --- Consider a rigid airfoil model mounted on an elastic support (in a wind tunnel)





α_r = rigid angle of attack
α_e : elastic ″
$\alpha = \alpha_r + \alpha_e$
a.c = aerodynamic center
--> $M_{a.c.} \neq fn(\alpha)$

subsonic ~ 1/4 chord, supersonic ~ 1/2 chord

e.a. = elastic axis (center) ⇒ decouples flexure and torsion problems

α_i, α : ↻ (+)
L : lift ↑ (+) $L = \frac{1}{2}\rho U^2 S C_L$ where U: flow speed
W : weight ρ : ″ density
q : $\frac{1}{2}\rho U^2$: dynamic pressure
S : reference area : c · 1
C_L : lift coefficient
∘ $M_{a.c.} = q \cdot S \cdot c \cdot C_{Mac}$
where C_{Mac} : moment coefficient at a.c.

[Note] $C_L = fn(\alpha, M, \text{airfoil shape})$
$C_{Mac} = fn(M, \text{airfoil shape})$

From a Taylor series,

$$C_L = C_{L_0} + \frac{\partial C_L}{\partial \alpha}\alpha + h.o.t.$$
$$= \cancelto{0}{C_{L_0}} + C_{L_\alpha} \cdot \alpha$$
$$C_{Mac} = C_{Mac_0} + \cancelto{0}{\frac{\partial C_{Mac}}{\partial \alpha}}\alpha + h.o.t.$$

[Note] For a flat plate

$$C_{L_\alpha} = 2\pi, \quad C_{Mac} = 0$$

For a rigid support

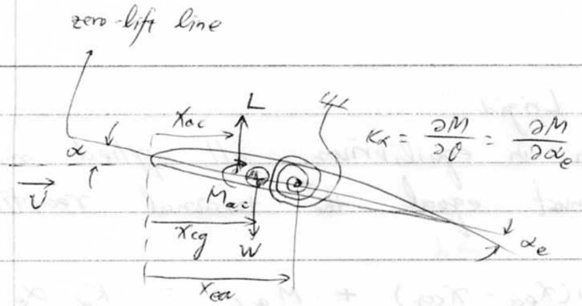
$$L_{rigid} = \frac{1}{2}\rho U^2 S \, C_{L_\alpha} \cdot \alpha_r$$

However, for elastic support

$$L_{elastic} = L = q S \, C_{L_\alpha} \underset{\alpha_r + \alpha_e}{\alpha}$$

Note that $L_{elastic} \neq L_{rigid}$

Katz & Plotkin
"Low-Speed Aerodynamics"
5.4 Aerodynamic Forces and Moments on a Thin Airfoil



α_r = rigid angle of attack

α_e : elastic "

$\alpha = \alpha_r + \alpha_e$

a.c = aerodynamic center

$\dashrightarrow$ $M_{a.c.} \neq fn(\alpha)$

subsonic ~ 1/4 chord, supersonic ~ 1/2 chord

e.a. = elastic axis (center) $\Rightarrow$ decouples flexure and torsion problems

α_i, α : $\circlearrowright$ (+)

L : lift $\uparrow$ (+) $\quad L = \frac{1}{2}\rho U^2 S C_L$ where U : flow speed

W : weight $\qquad\qquad\qquad\qquad\qquad\qquad$ ρ : " density

q : $\frac{1}{2}\rho U^2$: dynamic pressure

S : reference area : $c \cdot 1$

C_L : lift coefficient

- $M_{a.c.} = q \cdot S \cdot c \cdot C_{M_{ac}}$

  where $C_{M_{ac}}$: moment coefficient at a.c.

[Note] $C_L = fn(\alpha, M, \text{airfoil shape})$

$C_{M_{ac}} = fn(M, \text{airfoil shape})$

From a Taylor series,

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$$= \cancelto{0}{C_{L_0}} + C_{L_\alpha} \cdot \alpha$$

$$C_{M_{ac}} = C_{M_{ac_0}} + \cancelto{0}{\frac{\partial C_{M_{ac}}}{\partial \alpha}}\alpha + h.o.t.$$

[Note] For a flat plate

$$C_{L_\alpha} = 2\pi, \quad C_{M_{ac}} = 0$$

For a rigid support

$$L_{rigid} = \frac{1}{2}\rho U^2 S \, C_{L_\alpha} \cdot \alpha_r$$

However, for elastic support

$$L_{elastic} = L = qS\,C_{L_\alpha}\,\underset{\alpha_r + \alpha_e}{\alpha}$$

Note that $L_{elastic} \neq L_{rigid}$

Katz & Plotkin "Low-Speed Aerodynamics"

5.4 Aerodynamic Forces and Moments on a Thin Airfoil

$L_{elastic} > L_{rigid}$

Considering the system in equilibrium, all applied moments at support point must equal the torsional reaction at the point

$$L(x_{ea} - x_{ac}) - W(x_{ea} - x_{cg}) + M_{a.c} = K_\alpha \alpha_e$$

$$qSC_{L\alpha}(\alpha_r + \alpha_e)(x_{ea} - x_{ac}) - W(x_{ea} - x_{cg}) + M_{a.c}$$
$$= K_\alpha \alpha_e$$

$$\Rightarrow \alpha_e = \frac{-W(x_{ea} - x_{cg}) + qSc\, C_{Mac_0} + qS\, C_{L\alpha} \alpha_r (x_{ea} - x_{ac})}{K_\alpha - qSC_{L\alpha}(x_{ea} - x_{ac})}$$

For given q, and α_r, we can evaluate L

Note : Consider equilibrium equation

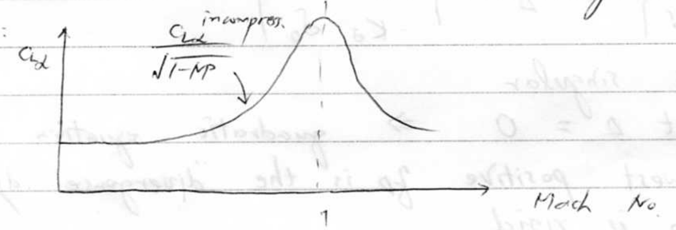
$$\underbrace{(K - qSC_{L\alpha} e)}_{A} \underbrace{\alpha_e}_{x} = \underbrace{qScC_{mac} + qSC_{L\alpha}\alpha_r e - W \cdot d}_{B}$$

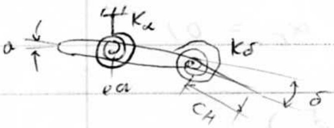
$$Ax = B \quad \rightarrow \quad Kx = F$$

The stability is associated with the homgeneous part of the equation : $Ax = 0$

$$A \overset{\Downarrow}{=} 0 \quad \rightarrow \quad \text{divergence condition}$$

Note :





δ_0 : commanded deflection

δ_e $\quad \delta - \delta_0$

- Typical section with control surface

By adding a control surface.

$$L = q\,c\,C_L$$

$$C_L = C_{L0} + \frac{\partial C_L}{\partial \alpha} \cdot \alpha + \frac{1}{2}\frac{\partial^2 C_L}{\partial \alpha^2}\alpha^2 + \cdots$$

$$+ \frac{\partial C_L}{\partial \delta}\delta + h.o.t$$

$$= C_{L0} + C_{L\alpha}\,\alpha + C_{L\delta}\,\delta$$

$$C_{Ma.c.} = C_{Ma.c.\,0} + \frac{\partial C_{Ma.c}}{\partial \delta}\delta$$

$$= C_{Ma.c.\,0} + C_{Ma.c\,\delta} \cdot \delta$$

The equilibrium equations are :

$$\underbrace{\begin{bmatrix} eqC_{L\alpha} - K_\alpha & eqSC_{L\delta} + qScC_{Mac\delta} \\ qS_H c_H C_{H\alpha} & qS_H c_H C_{H\delta} - K_\delta \end{bmatrix}}_{A} \begin{Bmatrix} \alpha \\ \delta \end{Bmatrix} = \begin{Bmatrix} 0 \\ -K_\delta \delta_0 \end{Bmatrix}$$

where

H : moment about the hinge line of control surface

$$= q\,S_H\,c_H\,C_H$$

$= q S_H C_H (C_{H\alpha} \alpha + C_{H\delta} \delta)$

Assumptions $\alpha_r = 0$

$C_{m_{a.c.0}} = 0$

$C_{L_0} = 0$

If $|A| = 0 \Rightarrow q_{D1}, q_{D2}$ --- divergence q = lowest + $\{q_{D1}, q_{D2}\}$

If A is non-singular,

$$\begin{Bmatrix} \alpha \\ \delta \end{Bmatrix} = A^{-1} \begin{Bmatrix} 0 \\ -K_\delta \delta_0 \end{Bmatrix}$$

If A is singular

$\Rightarrow$ det $A = 0$ $\Rightarrow$ quadratic equation in q_D

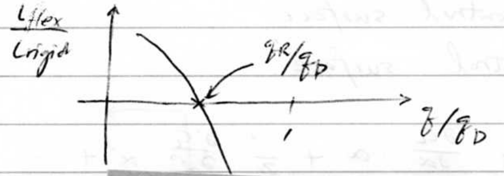
$\Rightarrow$ lowest positive q_D is the "divergence dynamic pressure"

If system is rigid,

$L_{rigid} = q S C_{L\delta} \delta_0 \quad (\alpha_r = 0)$

However, with flexible attachment

$L_{flexible} = q S (C_{L\alpha} \alpha + C_{L\delta} \delta)$



q_R ... "... mic pressure"

... t $q = q_R$

$\Rightarrow$

Fro...

(... mac δ) $\delta = 0$

(...) $= 0$

$\Rightarrow$

$$qS \left\{ C_{L\alpha} (-) \frac{eqSC_{L\delta} + qScC_{mac\delta}}{eqC_{L\alpha} - K_\alpha} + C_{L\delta} \right\} = 0$$

$$qS \left\{ C_{L\alpha} (-) \frac{eqSC_{L\delta} + qScC_{mac\delta}}{eqSC_{L\alpha} - K_\alpha} + \frac{eqSC_{L\alpha}C_{L\delta} - C_{L\delta}K_\alpha}{eqSC_{L\alpha} - K_\alpha} \right\} = 0$$

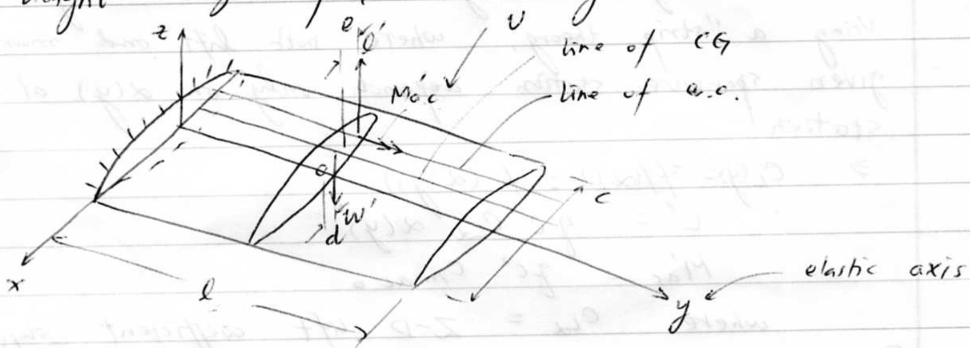
$$-C_{L\alpha} qSc\, C_{mac\delta} - C_{L\delta} K_\alpha = 0$$

$$q = -\frac{K_\alpha}{S \cdot c} \frac{C_{L\delta}/C_{L\alpha}}{C_{mac\delta}}$$

For $\frac{dL}{d\alpha} = 0$

$$\boxed{q_R = -\frac{K_\alpha}{S \cdot c} \cdot \frac{C_{L\delta}/C_{L\alpha}}{C_{mac\delta}}}$$

- Straight high-aspect-ratio wing



l' : lift per unit span

$e = x_{ea} - x_{ac}$

$d = x_{ea} - x_{cg}$

$\Lambda = \frac{b^2}{S_w} = \frac{l}{c}$

clamped - free BC

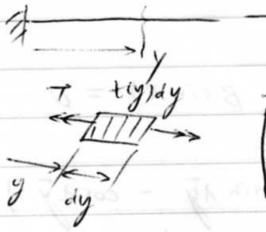
$N m g = W'$, N : normal load factor

$= 1 + \frac{\ddot{z}}{g}$

The total moment applied at the elastic axis :

$M_{ea}' = L'e + M_{ac}' - w'd = t(y)$: applied torque/span

For the equilibrium equations, let's consider a rod in torsion.



$T + dT = T + \frac{\partial T}{\partial y} dy$

$$\boxed{\frac{\partial T}{\partial y} = -t(y)}$$

Constitutive law for a uncoupled torsion rod is

$$\boxed{T = GJ \frac{d\theta}{dy}}$$ where GJ : torsional stiffness

J : function of (geometry of airfoil)

$J = I_p$ = polar moment of inertia for circular (11)

$\leftarrow$ p. 17 (Book)

Combining the two equations

$$GJ\frac{d^2\theta}{dy^2} = -t(y) = -L'e - M'_{ac} + Nmgd$$

Using a "strip theory," where both lift and moment at a given spanwise station depend only on $\alpha(y)$ at given station!

$\Rightarrow \quad C_L(y) = f(\alpha) = f(\alpha(y))$

$\therefore \quad L' = qc\,C_{L\alpha}\,\alpha(y)$

$M'_{ac} = qc^2\,C_{mac\,0}$

where $C_{L\alpha}$ = 2-D lift coefficient slope

[Note] Assume $C_{L\alpha}, C_{mac\,0} \neq f(y)$

constant airfoil, chord

Therefore,

$$\frac{d^2\theta}{d\bar{y}^2} + \lambda^2\theta = K \quad \text{--- governing equation}$$

where, $\bar{y} = y/l$

$\lambda^2 = l^2 qec\,C_{L\alpha}/GJ$

$K = -l^2/GJ\,(qcC_{L\alpha}e\alpha_r + qc^2 C_{mac\,0} - Nmgd)$

The B.C. : $\theta(0) = 0$ --- cantilevered

$T(1) = 0 = \frac{d\theta}{dy}(1)$ --- free end

The general solution is :

$$\theta(\bar{y}) = A\sin\lambda\bar{y} + B\cos\lambda\bar{y} + \frac{K}{\lambda^2}$$

Applying B.C. :

$\theta(0) = 0 \rightarrow B = -\frac{K}{\lambda^2}$

$\frac{d\theta}{d\bar{y}}(1) = 0 \rightarrow \lambda(A\cos\lambda - B\sin\lambda) = 0$

$\Rightarrow A = B\tan\lambda$

Finally, $\theta(\bar{y}) = \frac{K}{\lambda^2}[1 - \tan\lambda\sin\lambda\bar{y} - \cos\lambda\bar{y}]$

Divergence occurs when $\theta(\bar{y}) \rightarrow \infty$, and this happens when

$\lambda_n \rightarrow (2n-1)\frac{\pi}{2} \quad (n = 1, 2, \ldots)$

The torsional divergence will occur for the lowest n

$(n=1)$, yielding $\boxed{q_D = \left(\frac{\pi}{2l}\right)^2 \frac{GJ}{ecC_{l\alpha}}}$

[Note] comparing with typical section

$$K_\alpha = \frac{GJ}{l}\left(\frac{\pi}{2}\right)^2$$

Using the homogeneous part of the governing equation

$$\theta(\bar{y}) = A\sin\lambda\bar{y} + B\cos\lambda\bar{y}$$

Applying B.C.

$$\begin{bmatrix} 0 & 1 \\ \lambda\cos\lambda & -\lambda\sin\lambda \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

For nontrivial solution,

$$\det(A) = 0 \quad \Rightarrow \quad \lambda\cos\lambda = 0$$

$$\lambda_n = (2n-1)\cdot\frac{\pi}{2}$$

If $e < 0$, the characteristic equation becomes

$$\cosh|\lambda| = 0$$

$\nexists\, \lambda \in \mathbb{R}$, $\cosh|\lambda| = 0 \Rightarrow$ no divergence !

- Airload Distribution

The spanwise lift distribution (L') as,

$$L'(\bar{y}) = qcC_{l\alpha}(\alpha_r + \theta(\bar{y}))$$

where, $\theta(\bar{y}) = K/\lambda^2(1 - \tan\lambda\sin\lambda\bar{y} - \cos\lambda\bar{y})$

and $K = K(\alpha_r, N)$

For a given $\alpha_r \Rightarrow$ corresponding elastic twist distribution

$\Rightarrow$ particular airload distribution

$\Rightarrow$ integrate over the span : total lift (L)

But also $L = NW$

Therefore, only α_r or N may be specified and the other is determined from total lift

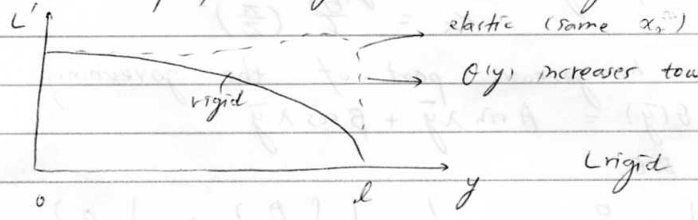
$$L = \underbrace{2}_{\text{two wings}} l \int_0^1 L'\, d\bar{y}$$

$$\Rightarrow L = 2qcC_{l\alpha}\, l\left[\alpha_r + \frac{K(\alpha_r, N)}{\lambda^2}\left(1 - \frac{\tan\lambda}{\lambda}\right)\right] = NW$$

Generally, in industry:

a) α_r is specified by aerodynamicist or performance engineer

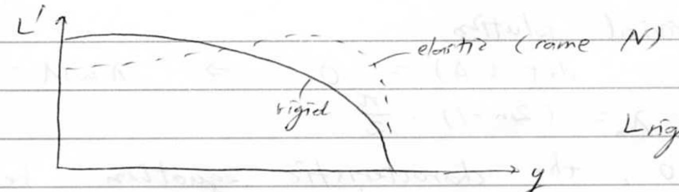
b) N is specified by structural engineer



$\theta(y)$ increases towards the tip

$L_{rigid} < L_{elastic}$

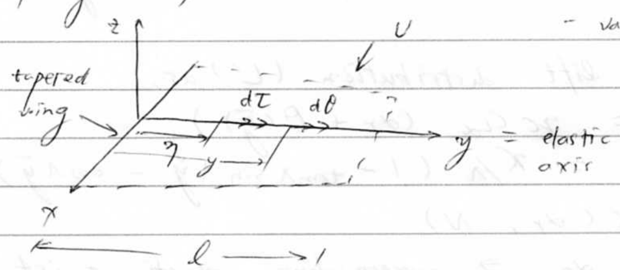
additional load outboard



$L_{rigid} = L_{elastic}$

increase of inboard moment

- Non-Uniform Lifting Surface



- variable cross section properties

- Structural (Flexibility) Influence Function

For a linear elastic structure,

$$C^{\theta\theta}(y,\eta) = \frac{d\theta(y)}{d\tau(\eta)}$$: resulting twist at point y due to torque at point η

(Notation in textbook : $C^{\alpha\alpha}$)

$t(y)$ = applied torque / span

$d\tau(\eta) = t(\eta)\, d\eta$

$$\Rightarrow \theta(y) = \int_0^l C^{\theta\theta}(y,\eta)\, t(\eta)\, d\eta$$

- Numerical Integration by Weighting Numbers

$$I = \int_a^b f(y)\,dy \simeq \sum_{i=1}^{N} f_i\, w_i$$

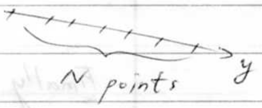
$$I = \lfloor W \rfloor \{f\} = \lfloor 1 \rfloor [{}^{\backslash}W_{\backslash}] \{f\}$$

↑ integration matrix of weight numbers

[Note] Ref. B.A.H. p. 809–812

Therefore, the elastic twist can be expressed as

$$\underset{N\times 1}{\{\theta\}} = \underset{N\times N}{[C^{\theta\theta}]}\,\underset{N\times N}{[{}^{\backslash}W_{\backslash}]}\,\underset{N\times 1}{\{t\}}$$



- Structural (Flexibility) Influence Coefficients

Discretized version of the Structural Influence Function

If $\Delta\tau_j$ = variation of torque at station j

$\Delta\theta_i$ = corresponding variation of twist at station i

Then, for linear elastic structure, $C_{ij} = \dfrac{\Delta\theta_i}{\Delta\tau_j}$

$$\Rightarrow \theta_i = \sum_{j=1}^{N} C_{ij}\,\tau_j = \lfloor C_{ij} \rfloor \{\tau\}$$

$$\therefore \{\theta\} = [C]\{\tau\}$$

[Note] $\{\tau\} = [C]^{-1}\{\theta\}$

↑ stiffness matrix

- Static Aeroelastic Equilibrium

$$\theta(y) = \int_0^l C^{\theta\theta}(y,\eta)\, t(\eta)\, d\eta$$

$$= \int_0^l C^{\theta\theta}(y,\eta) \{ q\,[e(\eta)\, c c_l(\eta) + c^2(\eta)\, c_{mac}(\eta)] + - N g\, m(\eta)\, d(\eta) \}\, d\eta$$

Discretizing the span along N points and using an appropriate integration scheme

$$\{\theta\} = q\,[E]\{c c_l\} + q\,[F]\{c_{mac}\} - Ng\,[G]\{m\}$$

where,

$$[E] = [C^{\theta\theta}][{}^{\backslash}W_{\backslash}][{}^{\backslash}e_{\backslash}]$$

$$[F] = [C^{\theta\theta}][{}^{\backslash}W_{\backslash}][{}^{\backslash}c^2{}_{\backslash}]$$

$$[G] = [\;^{\theta\theta}][{}^{\backslash}W_{\backslash}][{}^{\backslash}d_{\backslash}]$$

- Aerodynamic Influence Coefficients

If the spanwise airload distribution can be linearly related to the angle of attack distribution, then :

$$A_{ij} = \frac{(cc_l)_i}{\alpha_j}$$

[Note] From 'strip theory' $\rightarrow$ $[A]$: diagonal

Therefore, $\{cc_l\} = [A]\{\alpha\}$

where $\{\alpha\} = \{\alpha_r\} + \{\theta\}$

$\{\alpha_r\} = \alpha_r(0)\{1\} + \{\alpha_t\}$

wing root angle of attack $\quad$ built-in geometric twist

Finally, $[A]^{-1}\{cc_l\} - \{\alpha_r\} \Rightarrow \{\theta\}$

- Aeroelastic Load Distribution

$$[A]^{-1}\{cc_l\} = \alpha_r(0)\{1\} + \{\alpha_t\} + q[E]\{cc_l\} + q[F]\{c_{mac}\} - Ng[G]\{m\}$$

We can solve by specifying $\alpha_r(0)$ or N, and use the normal load eqn.

$$L/W = N$$

· Airload Distribution

$$[A]^{-1}\{cc_l\} = \alpha_r(0)\{1\} + \{\alpha_t\} + q[E]\{cc_l\} + q[F]\{c_{mac}\} - Ng[G]\{m\}$$

As before, $\alpha_r(0)$ and N are related through total lift

$$N = L/W$$

$$\Rightarrow L = NW = 2q\int_0^l cc_l\, dy = 2q\lfloor 1 \rfloor [\,\grave{}W\,\grave{}\,]\{cc_l\}$$

$$0 = NW - 2q\lfloor 1 \rfloor [\,\grave{}W\,\grave{}\,]\{cc_l\}$$

Specify $\alpha_r(0)$

The two sets of equations together

$$\begin{bmatrix} [A]^{-1} - q[E] & q[G]\{m\} \\ 2q\lfloor 1 \rfloor [\,\grave{}W\,\grave{}\,] & -W \end{bmatrix} \begin{Bmatrix} \{cc_l\} \\ N \end{Bmatrix} = \begin{Bmatrix} \alpha_r(0)\{1\} + \{\alpha_t\} + q[F]\{c_{mac}\} \\ 0 \end{Bmatrix}$$

· Torsional Divergence

What dynamic pressure will yield a finite load distribution when there are no external disturbance?

$$\left([A]^{-1} - q[E] \right) \{c c_e\} = \{0\}$$
$$\overset{\shortparallel}{\Delta}$$

divergence occurs when $\det \Delta = 0$

$\rightarrow$ q_D is lowest root

We could also write:

$$[A][E]\{c c_e\} = \left(\frac{1}{q}\right)\{c c_e\}$$
$$A \quad x \quad = \quad \lambda \quad x$$

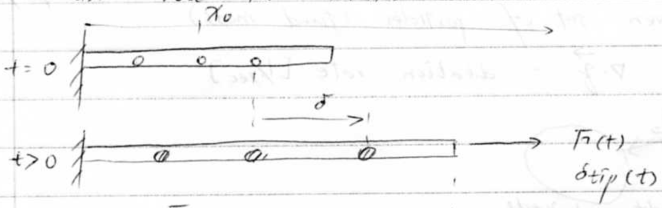
Ref: Dowell's § 2.4 (2-D wing representation)

- UNSTEADY AERODYNAMICS — Review

A. Field Description

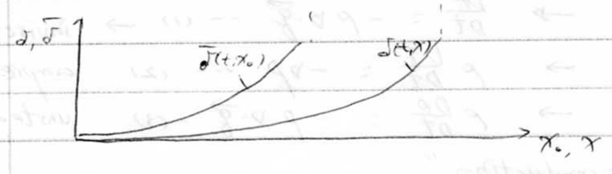
B. Governing Equations & Simplifications

- Elastic rod in extension (1-D)



a) $\bar{\delta}(t, x_0)$ --- Lagrangian description, material based $\rightarrow$ structure

b) $\delta(t, x)$ --- Eulerian description, spatially-based, fluids



- Velocity seen by particle (given x_0)

a) $\bar{u}(t, x_0) = \left.\frac{\partial \bar{\delta}}{\partial t}\right|_{\text{fixed } x_0}$

b) $u(t, x) = \lim\limits_{\substack{\Delta t \to 0 \\ \Delta x \to u\Delta t}} \dfrac{\delta(t+\Delta t, x+\Delta x) - \delta(t, x)}{\Delta t} = \dfrac{\partial \delta}{\partial t} + u \dfrac{\partial \delta}{\partial x}$

--- not useful for defining u

--- definition for $\delta(t, x)$ given $u(t, x)$

- Acceleration seen by particle

a) $\bar{a}(t, x_0) = \frac{\partial \bar{u}}{\partial t} = \frac{\partial^2 u}{\partial t^2}\Big|_{\text{fixed } x_0}$

b) $a(t, x) = \lim_{\substack{\Delta t \to 0 \\ \delta x \to u \delta t}} \frac{u(t+\delta t, x+\delta x) - u(t, x)}{\delta t} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \frac{Du}{Dt}$

$\frac{Du}{Dt}$: total acceleration

$\frac{\partial u}{\partial t}$: field acceleration

$u\frac{\partial u}{\partial x}$: convective acceleration

$\frac{Du}{Dt} = \frac{\partial \bar{u}}{\partial t}$ --- "invariant" same value in any inertial frame x, t

$\frac{D}{Dt}$: total rate of change of any quantity

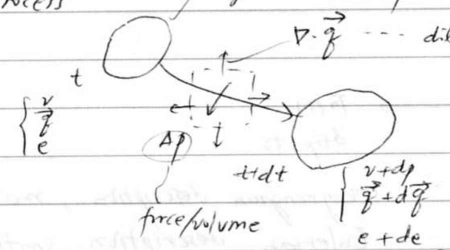
- Governing Equations

$v(x, y, z, t)$: volume / mass $\equiv 1/\rho(x, y, z, t)$

$\vec{q}(x, y, z, t)$: momentum / mass $\equiv$ velocity

$e(x, y, z, t)$: internal-energy / mass $= \int C_v dT = \underbrace{C_v T}_{\text{perfect gas}}$

Process



$\frac{Dv}{Dt} = v\nabla\cdot\vec{q} \rightarrow \frac{D\rho}{Dt} = -\rho\nabla\cdot\vec{q}$ -- (1) → inviscid,

$\frac{D\vec{q}}{Dt} = -v\nabla p \rightarrow \rho\frac{D\vec{q}}{Dt} = -\nabla p$ -- (2) compressible,

$\frac{De}{Dt} = -p\frac{Dv}{Dt} \rightarrow \rho\frac{De}{Dt} = -p\nabla\cdot\vec{q}$ -- (3) unsteady

"Inviscid & non-conducting"

$+ \quad \underbrace{p = p(\rho, e)}_{\text{General}} = \underbrace{(\gamma-1)\rho e}_{\text{Ideal Gas}}$

$\frac{1}{\rho}(3) - \frac{p}{\rho^2}(1)$:

$\frac{De}{Dt} - p\frac{D(1/\rho)}{Dt} = 0$

definition of enthalpy $T ds = de - p\, d\left(\frac{1}{\rho}\right)$