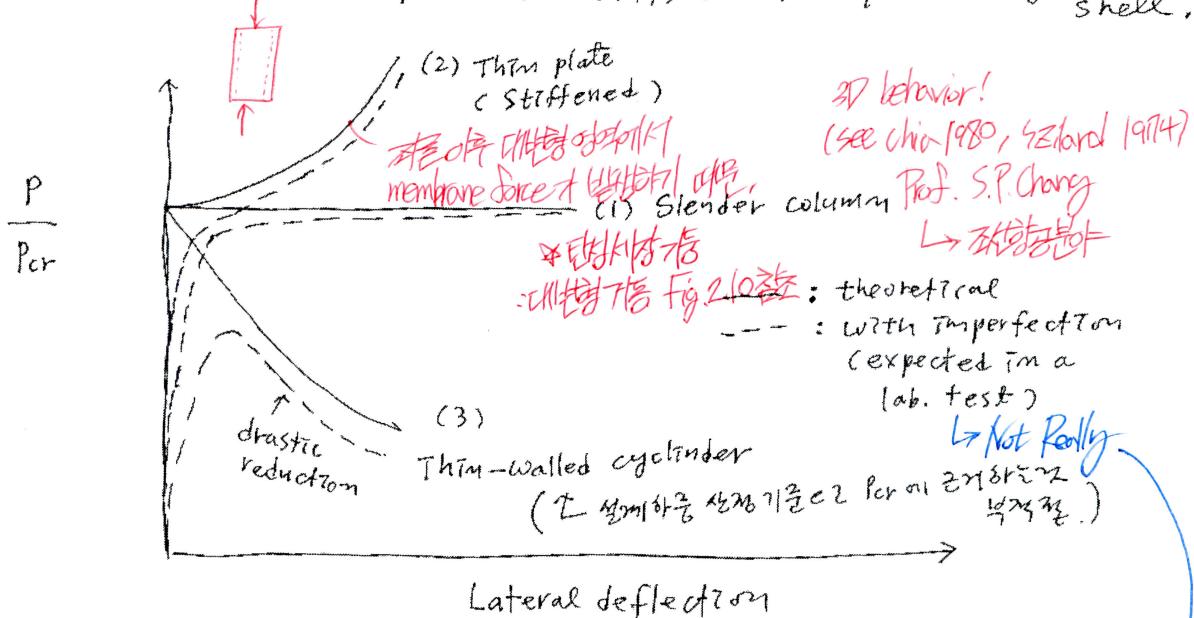


Postbuckling Behavior

- (1) Load-deflection relationship in the postbuckling range have an important bearing on the structural design significance of the "critical load".
- (2) For the idealized "perfect" compression element (one that is perfectly elastic, devoid of imperfection), three different types of postbuckling behavior are typified by
- the column,
 - the stiffened plate, and
 - cylindrical shell.

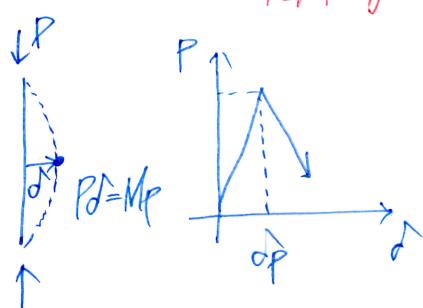


: "Elastic" postbuckling curves for compressed members

↑ postbuckling curve of 71% of 2/2

(2/2 is 2/2)

Not Really!



20128 2주차 (Stability theory)

(2)

1. Introduction

- "Instability": a condition wherein a compression member loses the ability to resist increasing loads and exhibits instead a decrease in load-carrying capacity; instability occurs at the maximum point on the load-deflection curve.
- Categories in instability problem

(i) bifurcation of equilibrium:

axially compressed columns, plates, and cylindrical shells.
critical load may deformation mode 을 갖는 부정형을 찾을 수 있다

(ii) limit-load Instability (without bifurcation):

shallow arches / spherical caps subjected to uniform external pressure

↳ 단면에 대한 최대荷重을 초과한荷重을 받을 때

2. Bifurcation buckling

2.1 Initially Perfect Systems

1) "real" imperfect members 실제 하중에 대한 완벽한 부재

2) initial imperfections 부재의 초기 불완전한 모양 부재가 부재로 부재가 부재로 부재가 부재로 부재가 부재로

3) Simple model 부재가 부재로 부재가 부재로 부재가 부재로 부재가 부재로 부재가 부재로

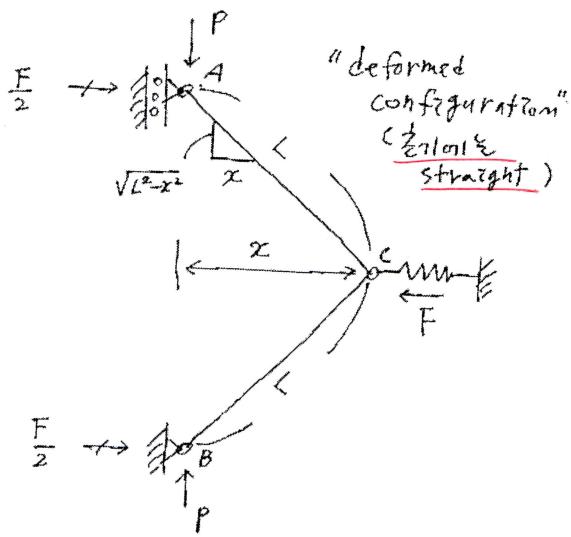
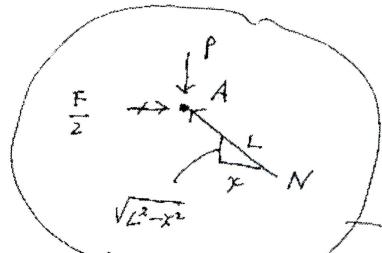


Fig. 1 : Bifurcation-buckling model
(two rigid bars hinged and laterally supported by a nonlinear spring)
↑ classical model

$$\text{Hinge } \Delta \approx \frac{\theta}{2} \approx \frac{x}{L} \quad (3)$$

$$F = k_1 \varepsilon - k_2 \varepsilon^2 + k_3 \varepsilon^3 \quad (1)$$

$$\text{where } \varepsilon = \frac{x}{L}$$



$$\sum Y_C = \frac{F}{2} - \left(\frac{x}{L}\right)N = 0 ; F = 2\varepsilon N \quad (2)$$

$$\sum Y_C = P - \frac{\sqrt{L^2-x^2}}{L}N = 0,$$

$$P - \sqrt{1-\varepsilon^2}N = 0 ; N = \frac{P}{\sqrt{1-\varepsilon^2}}$$

$$F = 2\varepsilon \times \frac{P}{\sqrt{1-\varepsilon^2}}$$

Truth deformed configuration에서 $P - F$ 와의 차이가 0

$$PE = \frac{F}{2} \times \sqrt{1-\varepsilon^2} = \frac{1}{2} (k_1 \varepsilon - k_2 \varepsilon^2 + k_3 \varepsilon^3) \underbrace{\sqrt{1-\varepsilon^2}}_{\text{忽略}}^{1/2} \quad (2)$$

of ε 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3.0 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 4.0 4.1 4.2 4.3 4.4 4.5 4.6 4.7 4.8 4.9 5.0 5.1 5.2 5.3 5.4 5.5 5.6 5.7 5.8 5.9 6.0 6.1 6.2 6.3 6.4 6.5 6.6 6.7 6.8 6.9 7.0 7.1 7.2 7.3 7.4 7.5 7.6 7.7 7.8 7.9 8.0 8.1 8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 9.0 9.1 9.2 9.3 9.4 9.5 9.6 9.7 9.8 9.9 10.0 10.1 10.2 10.3 10.4 10.5 10.6 10.7 10.8 10.9 11.0 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 11.9 12.0 12.1 12.2 12.3 12.4 12.5 12.6 12.7 12.8 12.9 13.0 13.1 13.2 13.3 13.4 13.5 13.6 13.7 13.8 13.9 14.0 14.1 14.2 14.3 14.4 14.5 14.6 14.7 14.8 14.9 15.0 15.1 15.2 15.3 15.4 15.5 15.6 15.7 15.8 15.9 16.0 16.1 16.2 16.3 16.4 16.5 16.6 16.7 16.8 16.9 17.0 17.1 17.2 17.3 17.4 17.5 17.6 17.7 17.8 17.9 18.0 18.1 18.2 18.3 18.4 18.5 18.6 18.7 18.8 18.9 19.0 19.1 19.2 19.3 19.4 19.5 19.6 19.7 19.8 19.9 20.0 20.1 20.2 20.3 20.4 20.5 20.6 20.7 20.8 20.9 21.0 21.1 21.2 21.3 21.4 21.5 21.6 21.7 21.8 21.9 22.0 22.1 22.2 22.3 22.4 22.5 22.6 22.7 22.8 22.9 23.0 23.1 23.2 23.3 23.4 23.5 23.6 23.7 23.8 23.9 24.0 24.1 24.2 24.3 24.4 24.5 24.6 24.7 24.8 24.9 25.0 25.1 25.2 25.3 25.4 25.5 25.6 25.7 25.8 25.9 26.0 26.1 26.2 26.3 26.4 26.5 26.6 26.7 26.8 26.9 27.0 27.1 27.2 27.3 27.4 27.5 27.6 27.7 27.8 27.9 28.0 28.1 28.2 28.3 28.4 28.5 28.6 28.7 28.8 28.9 29.0 29.1 29.2 29.3 29.4 29.5 29.6 29.7 29.8 29.9 30.0 30.1 30.2 30.3 30.4 30.5 30.6 30.7 30.8 30.9 31.0 31.1 31.2 31.3 31.4 31.5 31.6 31.7 31.8 31.9 32.0 32.1 32.2 32.3 32.4 32.5 32.6 32.7 32.8 32.9 33.0 33.1 33.2 33.3 33.4 33.5 33.6 33.7 33.8 33.9 34.0 34.1 34.2 34.3 34.4 34.5 34.6 34.7 34.8 34.9 35.0 35.1 35.2 35.3 35.4 35.5 35.6 35.7 35.8 35.9 36.0 36.1 36.2 36.3 36.4 36.5 36.6 36.7 36.8 36.9 37.0 37.1 37.2 37.3 37.4 37.5 37.6 37.7 37.8 37.9 38.0 38.1 38.2 38.3 38.4 38.5 38.6 38.7 38.8 38.9 39.0 39.1 39.2 39.3 39.4 39.5 39.6 39.7 39.8 39.9 40.0 40.1 40.2 40.3 40.4 40.5 40.6 40.7 40.8 40.9 41.0 41.1 41.2 41.3 41.4 41.5 41.6 41.7 41.8 41.9 42.0 42.1 42.2 42.3 42.4 42.5 42.6 42.7 42.8 42.9 43.0 43.1 43.2 43.3 43.4 43.5 43.6 43.7 43.8 43.9 44.0 44.1 44.2 44.3 44.4 44.5 44.6 44.7 44.8 44.9 45.0 45.1 45.2 45.3 45.4 45.5 45.6 45.7 45.8 45.9 46.0 46.1 46.2 46.3 46.4 46.5 46.6 46.7 46.8 46.9 47.0 47.1 47.2 47.3 47.4 47.5 47.6 47.7 47.8 47.9 48.0 48.1 48.2 48.3 48.4 48.5 48.6 48.7 48.8 48.9 49.0 49.1 49.2 49.3 49.4 49.5 49.6 49.7 49.8 49.9 50.0 50.1 50.2 50.3 50.4 50.5 50.6 50.7 50.8 50.9 51.0 51.1 51.2 51.3 51.4 51.5 51.6 51.7 51.8 51.9 52.0 52.1 52.2 52.3 52.4 52.5 52.6 52.7 52.8 52.9 53.0 53.1 53.2 53.3 53.4 53.5 53.6 53.7 53.8 53.9 54.0 54.1 54.2 54.3 54.4 54.5 54.6 54.7 54.8 54.9 55.0 55.1 55.2 55.3 55.4 55.5 55.6 55.7 55.8 55.9 56.0 56.1 56.2 56.3 56.4 56.5 56.6 56.7 56.8 56.9 57.0 57.1 57.2 57.3 57.4 57.5 57.6 57.7 57.8 57.9 58.0 58.1 58.2 58.3 58.4 58.5 58.6 58.7 58.8 58.9 59.0 59.1 59.2 59.3 59.4 59.5 59.6 59.7 59.8 59.9 60.0 60.1 60.2 60.3 60.4 60.5 60.6 60.7 60.8 60.9 61.0 61.1 61.2 61.3 61.4 61.5 61.6 61.7 61.8 61.9 62.0 62.1 62.2 62.3 62.4 62.5 62.6 62.7 62.8 62.9 63.0 63.1 63.2 63.3 63.4 63.5 63.6 63.7 63.8 63.9 64.0 64.1 64.2 64.3 64.4 64.5 64.6 64.7 64.8 64.9 65.0 65.1 65.2 65.3 65.4 65.5 65.6 65.7 65.8 65.9 66.0 66.1 66.2 66.3 66.4 66.5 66.6 66.7 66.8 66.9 67.0 67.1 67.2 67.3 67.4 67.5 67.6 67.7 67.8 67.9 68.0 68.1 68.2 68.3 68.4 68.5 68.6 68.7 68.8 68.9 69.0 69.1 69.2 69.3 69.4 69.5 69.6 69.7 69.8 69.9 70.0 70.1 70.2 70.3 70.4 70.5 70.6 70.7 70.8 70.9 71.0 71.1 71.2 71.3 71.4 71.5 71.6 71.7 71.8 71.9 72.0 72.1 72.2 72.3 72.4 72.5 72.6 72.7 72.8 72.9 73.0 73.1 73.2 73.3 73.4 73.5 73.6 73.7 73.8 73.9 74.0 74.1 74.2 74.3 74.4 74.5 74.6 74.7 74.8 74.9 75.0 75.1 75.2 75.3 75.4 75.5 75.6 75.7 75.8 75.9 76.0 76.1 76.2 76.3 76.4 76.5 76.6 76.7 76.8 76.9 77.0 77.1 77.2 77.3 77.4 77.5 77.6 77.7 77.8 77.9 78.0 78.1 78.2 78.3 78.4 78.5 78.6 78.7 78.8 78.9 79.0 79.1 79.2 79.3 79.4 79.5 79.6 79.7 79.8 79.9 80.0 80.1 80.2 80.3 80.4 80.5 80.6 80.7 80.8 80.9 81.0 81.1 81.2 81.3 81.4 81.5 81.6 81.7 81.8 81.9 82.0 82.1 82.2 82.3 82.4 82.5 82.6 82.7 82.8 82.9 83.0 83.1 83.2 83.3 83.4 83.5 83.6 83.7 83.8 83.9 84.0 84.1 84.2 84.3 84.4 84.5 84.6 84.7 84.8 84.9 85.0 85.1 85.2 85.3 85.4 85.5 85.6 85.7 85.8 85.9 86.0 86.1 86.2 86.3 86.4 86.5 86.6 86.7 86.8 86.9 87.0 87.1 87.2 87.3 87.4 87.5 87.6 87.7 87.8 87.9 88.0 88.1 88.2 88.3 88.4 88.5 88.6 88.7 88.8 88.9 89.0 89.1 89.2 89.3 89.4 89.5 89.6 89.7 89.8 89.9 90.0 90.1 90.2 90.3 90.4 90.5 90.6 90.7 90.8 90.9 91.0 91.1 91.2 91.3 91.4 91.5 91.6 91.7 91.8 91.9 92.0 92.1 92.2 92.3 92.4 92.5 92.6 92.7 92.8 92.9 93.0 93.1 93.2 93.3 93.4 93.5 93.6 93.7 93.8 93.9 94.0 94.1 94.2 94.3 94.4 94.5 94.6 94.7 94.8 94.9 95.0 95.1 95.2 95.3 95.4 95.5 95.6 95.7 95.8 95.9 96.0 96.1 96.2 96.3 96.4 96.5 96.6 96.7 96.8 96.9 97.0 97.1 97.2 97.3 97.4 97.5 97.6 97.7 97.8 97.9 98.0 98.1 98.2 98.3 98.4 98.5 98.6 98.7 98.8 98.9 99.0 99.1 99.2 99.3 99.4 99.5 99.6 99.7 99.8 99.9 100.0 100.1 100.2 100.3 100.4 100.5 100.6 100.7 100.8 100.9 100.10 100.11 100.12 100.13 100.14 100.15 100.16 100.17 100.18 100.19 100.20 100.21 100.22 100.23 100.24 100.25 100.26 100.27 100.28 100.29 100.30 100.31 100.32 100.33 100.34 100.35 100.36 100.37 100.38 100.39 100.40 100.41 100.42 100.43 100.44 100.45 100.46 100.47 100.48 100.49 100.50 100.51 100.52 100.53 100.54 100.55 100.56 100.57 100.58 100.59 100.60 100.61 100.62 100.63 100.64 100.65 100.66 100.67 100.68 100.69 100.70 100.71 100.72 100.73 100.74 100.75 100.76 100.77 100.78 100.79 100.80 100.81 100.82 100.83 100.84 100.85 100.86 100.87 100.88 100.89 100.90 100.91 100.92 100.93 100.94 100.95 100.96 100.97 100.98 100.99 100.100 100.101 100.102 100.103 100.104 100.105 100.106 100.107 100.108 100.109 100.110 100.111 100.112 100.113 100.114 100.115 100.116 100.117 100.118 100.119 100.120 100.121 100.122 100.123 100.124 100.125 100.126 100.127 100.128 100.129 100.130 100.131 100.132 100.133 100.134 100.135 100.136 100.137 100.138 100.139 100.140 100.141 100.142 100.143 100.144 100.145 100.146 100.147 100.148 100.149 100.150 100.151 100.152 100.153 100.154 100.155 100.156 100.157 100.158 100.159 100.160 100.161 100.162 100.163 100.164 100.165 100.166 100.167 100.168 100.169 100.170 100.171 100.172 100.173 100.174 100.175 100.176 100.177 100.178 100.179 100.180 100.181 100.182 100.183 100.184 100.185 100.186 100.187 100.188 100.189 100.190 100.191 100.192 100.193 100.194 100.195 100.196 100.197 100.198 100.199 100.200 100.201 100.202 100.203 100.204 100.205 100.206 100.207 100.208 100.209 100.210 100.211 100.212 100.213 100.214 100.215 100.216 100.217 100.218 100.219 100.220 100.221 100.222 100.223 100.224 100.225 100.226 100.227 100.228 100.229 100.230 100.231 100.232 100.233 100.234 100.235 100.236 100.237 100.238 100.239 100.240 100.241 100.242 100.243 100.244 100.245 100.246 100.247 100.248 100.249 100.250 100.251 100.252 100.253 100.254 100.255 100.256 100.257 100.258 100.259 100.260 100.261 100.262 100.263 100.264 100.265 100.266 100.267 100.268 100.269 100.270 100.271 100.272 100.273 100.274 100.275 100.276 100.277 100.278 100.279 100.280 100.281 100.282 100.283 100.284 100.285 100.286 100.287 100.288 100.289 100.290 100.291 100.292 100.293 100.294 100.295 100.296 100.297 100.298 100.299 100.300 100.301 100.302 100.303 100.304 100.305 100.306 100.307 100.308 100.309 100.310 100.311 100.312 100.313 100.314 100.315 100.316 100.317 100.318 100.319 100.320 100.321 100.322 100.323 100.324 100.325 100.326 100.327 100.328 100.329 100.330 100.331 100.332 100.333 100.334 100.335 100.336 100.337 100.338 100.339 100.340 100.341 100.342 100.343 100.344 100.345 100.346 100.347 100.348 100.349 100.350 100.351 100.352 100.353 100.354 100.355 100.356 100.357 100.358 100.359 100.360 100.361 100.362 100.363 100.364 100.365 100.366 100.367 100.368 100.369 100.370 100.371 100.372 100.373 100.374 100.375 100.376 100.377 100.378 100.379 100.380 100.381 100.382 100.383 100.384 100.385 100.386 100.387 100.388 100.389 100.390 100.391 100.392 100.393 100.394 100.395 100.396 100.397 100.398 100.399 100.400 100.401 100.402 100.403 100.404 100.405 100.406 100.407 100.408 100.409 100.410 100.411 100.412 100.413 100.414 100.415 100.416 100.417 100.418 100.419 100.420 100.421 100.422 100.423 100.424 100.425 100.426 100.427 100.428 100.429 100.430 100.431 100.432 100.433 100.434 100.435 100.436 100.437 100.438 100.439 100.440 100.441 100.442 100.443 100.444 100.445 100.446 100.447 100.448 100.449 100.450 100.451 100.452 100.453 100.454 100.455 100.456 100.457 100.458 100.459 100.460 100.461 100.462 100.463 100.464 100.465 100.466 100.467 100.468 100.469 100.470 100.471 100.472 100.473 100.474 100.475 100.476 100.477 100.478 100.479 100.480 100.481 100.482 100.483 100.484 100.485 100.486 100.487 100.488 100.489 100.490 100.491 100.492 100.493 100.494 100.495 100.496 100.497 100.498 100.499 100.500 100.501 100.502 100.503 100.504 100.505 100.506 100.507 100.508 100.509 100.510 100.511 100.512 100.513 100.514 100.515 100.516 100.517 100.518 100.519 100.520 100.521 100.522 100.523 100.524 100.525 100.526 100.527 100.528 100.529 100.530 100.531 100.532 100.533 100.534 100.535 100.536 100.537 100.538 100.539 100.540 100.541 100.542 100.543 100.544 100.545 100.546 100.547 100.548 100.549 100.550 100.551 100.552 100.553 100.554 100.555 100.556 100.557 100.558 100.559 100.560 100.561 100.562 100.563 100.564 100.565 100.566 100.567 100.568 100.569 100.570 100.571 100.572 100.573 100.574 100.575 100.576 100.577 100.578 100.579 100.580 100.581 100.582 100.583 100.584 100.585 100.586 100.587 100.588 100.589 100.590 100.591 100.592 100.593 100.594 100.595 100.596 100.597 100.598 100.599 100.600 100.601 100.602 100.603 100.604 100.605 100.606 100.607 100.608 100.609 100.610 100.611 100.612 100.613 100.614 100.615 100.616 100.617 100.618 100.619 100.620 100.621 100.622 100.623 100.624 100.625 100.626 100.627 100.628 100.629 100.630 100.631 100.632 100.633 100.634 100.635 100.636 100.637 100.638 100.639 100.640 100.641 100.642 100.643 100.644 100.645 100.646 100.647 100.648 100.649 100.650 100.651 100.652 100.653 100.654 100.655 100.656 100.657 100.658 100.659 100.660 100.661 100.662 100.663 100.664 100.665 100.666 100.667 100.668 100.669 100.670 100.671 100.672 100.673 100.674 100.675 100.676 100.677 100.678 100.679 100.680 100.681 100.682 100.683 100.684 100.685 100.686 100.687 100.688 100.689 100.690 100.691 100.692 100.693 100.694 100.695 100.696 100.697 100.698 100.699

$$P = P_{cr} (1 - a\epsilon + b\epsilon^2) \quad \dots \quad (5)$$

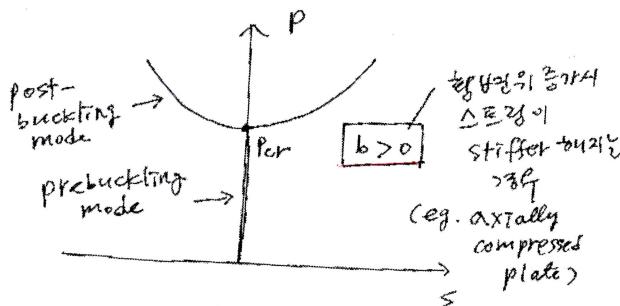
"자기 틀성이 범위에서 무게로 인해 탄성을 놓지 않는 구조는 주로 그 자체로 안정성을
(5) 아니면 $a = 0$ 인 경우에만 안정성이 가능." ↗

"symmetric buckling"

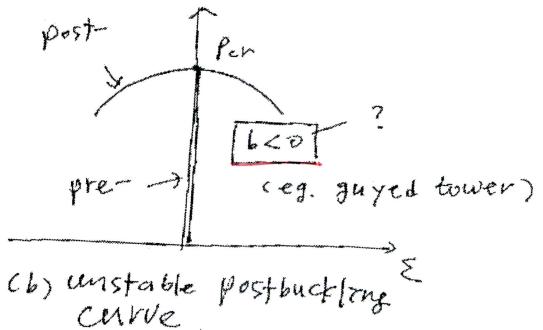
$$P = P_{cr} (1 + b\epsilon^2) \quad \dots \quad (6)$$

(4)

가장 안정성을
얻는 경우.



(a) Stable postbuckling curve



(b) unstable postbuckling curve

Fig. 2 : Symmetric buckling of a bifurcation model
(based on Eq. 6)

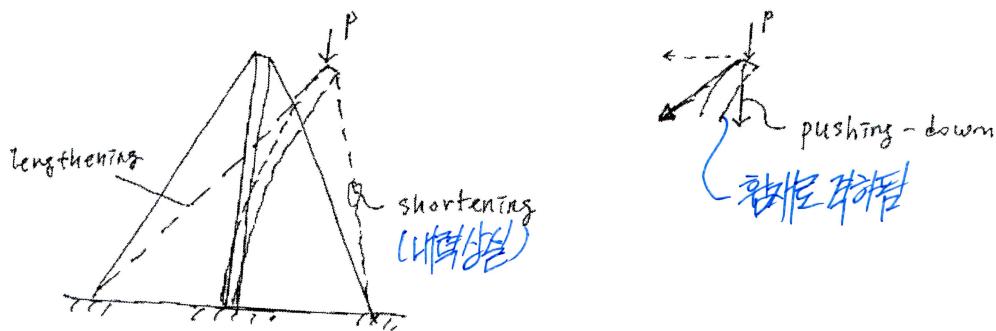


Fig. 3 : Guyed tower (unstable post-buckling example)

"자기 틀성이 범위에서 무게로 인해 탄성을 놓지 않는 구조는 주로 그 자체로 안정성을

Asymmetric buckling ↗ $\hookrightarrow b = 0$ 인 경우에만 안정성이 가능

$$P = P_{cr} \times (1 - a\epsilon) \quad \dots \quad (7)$$

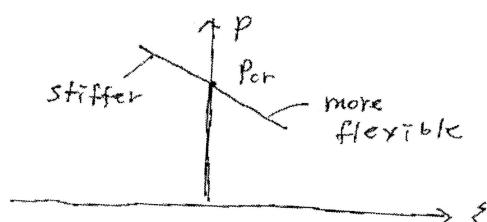
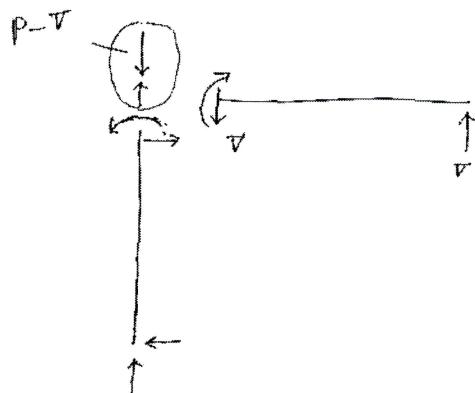
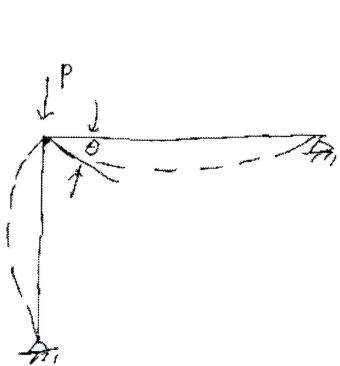
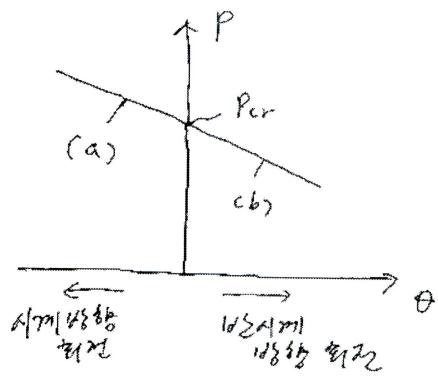
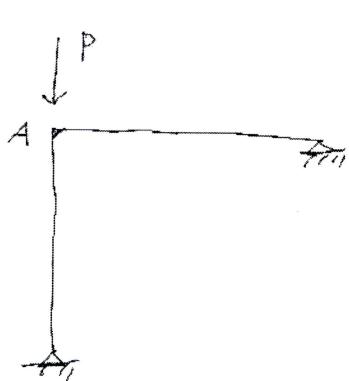
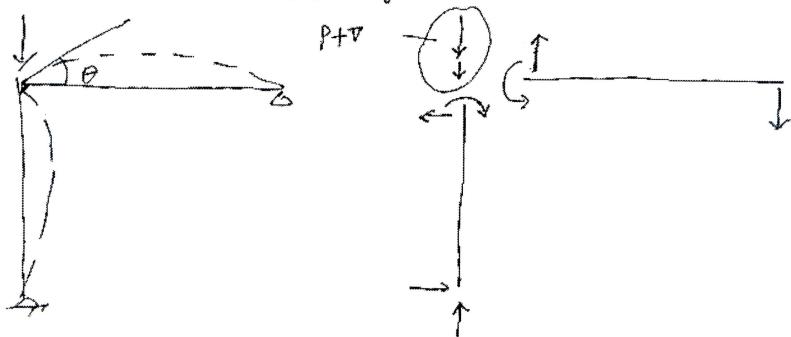


Fig. 4 : Asymmetric buckling of a bifurcation model



(a) becoming stiffer (각각 A가 A에서 더 빠르게 변함)



(b) becoming more flexible (각각 A가 A에서 더 빠르게 변함)

Fig. 6 : Buckling of an L-shaped frame

(L형기둥의 부수기작용 예시)

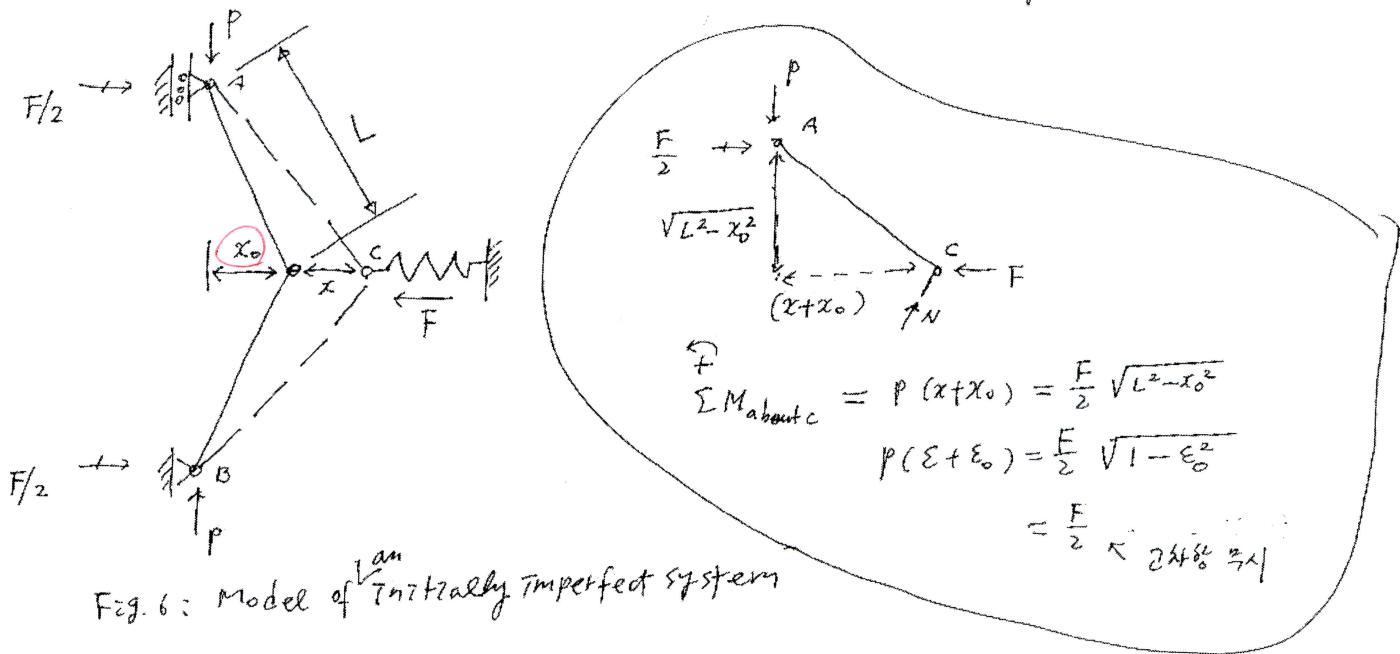
↑ 부수기작용 예시로 G2 가정

설명은 다음 참조 (Roorda 1965)

2.2 Initially Imperfect Systems

- ① The postbuckling curve of an "initially perfect" system does not by itself give sufficient information to allow one to determine when failure takes place.

In all "real" structures; Initial imperfection & loading eccentricities



$$P(\varepsilon + \varepsilon_0) = \frac{1}{2} (k_1 \varepsilon - k_2 \varepsilon^2 + k_3 \varepsilon^3) \quad \dots \quad (8)$$

where $\varepsilon_0 = x_0/L$.

$$P = \frac{P_{cr} (\varepsilon - a \varepsilon^2 + b \varepsilon^3)}{\varepsilon + \varepsilon_0} \quad \dots \quad (9)$$

For "symm." behavior $a=0$ and

$$P = \frac{P_{cr} (\varepsilon + b \varepsilon^3)}{\varepsilon + \varepsilon_0} \quad \dots \quad (10)$$

For "asymm." behavior $b=0$, and

$$P = \frac{P_{cr} (\varepsilon - a \varepsilon^2)}{\varepsilon + \varepsilon_0} \quad \dots \quad (11)$$

$b > 0$: stable post-buckling

$b < 0$: unstable post-buckling

(7)

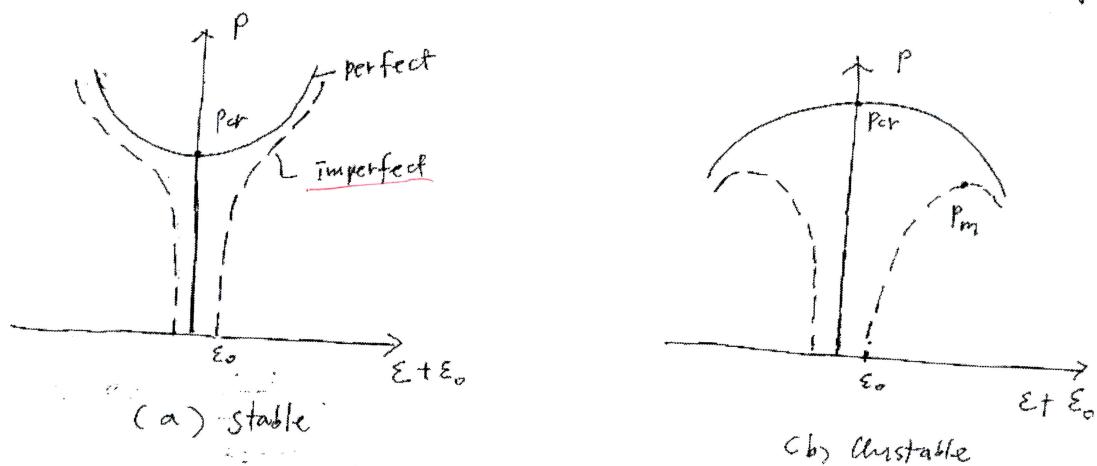


Fig. 7: Postbuckling behavior of initially imperfect system

Notes: 1) Stable postbuckling systems: 초기 불완전성이 크지 않을 때
불완전성이 작을 때 (P_m이 P_cr보다 크거나 같은 경우)

(후단을 거친 후 선이 가로가 Steep \Rightarrow P_cr > P_m) postbuckling strength 증가

↳ axially compressed plates : steep slope
axially loaded columns : extremely small slope

2) Unstable postbuckling systems: 초기 불완전성이 크거나
불완전성이 크거나 초기 불완전성이 크거나 $P_m \ll P_cr$

↳ Imperfection-sensitive structures

HW #

{ (1) Fig. 7(b)-의 P_m 의 대비를 $\frac{P_m}{P_cr}$ 의 일정한 관계를, 2) (10)식을
온다. 이는 관계를 구하면 된다.

$$\frac{P_m}{P_cr} = 1 - 3 \left(-\frac{b}{4} \right)^{1/3} \varepsilon_0^{2/3} \quad \text{--- (*)}$$

(2) (*) 식의 시사하는 바는 관계를 찾는 것이다.

In conclusion, the behavior of real Imperfection members can be predicted from the shape of the post-buckling curve for perfect systems. Members with stable postbuckling curves will fail at loads equal to or above the critical load, whereas members with unstable postbuckling curves will fail at loads below the critical load.

2.3 Limit-load buckling \leftrightarrow a bifurcation of equilibrium model
 ↓
 a second type of instability
 (near shallow arch)

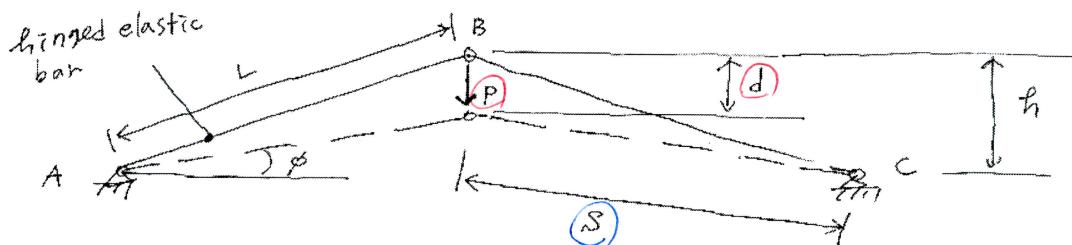
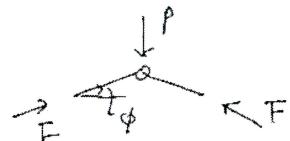


Fig. 8: Limit-load buckling model

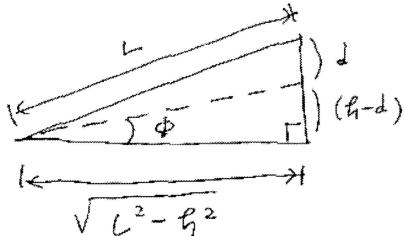
\hookrightarrow Prs. d \neq 0



$$2 \times F \sin \phi = P, \text{ or}$$

$$F = \frac{P}{2 \sin \phi} = \frac{P S}{2(h-d)} \quad \dots \quad (12)$$

shortening $\Rightarrow \Delta = \frac{F}{K} = \frac{P S}{2 K (h-d)} \quad \dots \quad (13)$



$$\begin{aligned} \Delta &= \sqrt{(\sqrt{L^2 - d^2})^2 + (h-d)^2} \\ &= \sqrt{L^2 + d^2 - 2dL} \end{aligned}$$

$$\underline{\Delta = L - S} \quad (\text{the axial shortening of each bar})$$

(9)

$$\Delta = L - \delta = \frac{P\delta}{2K(h-d)}$$

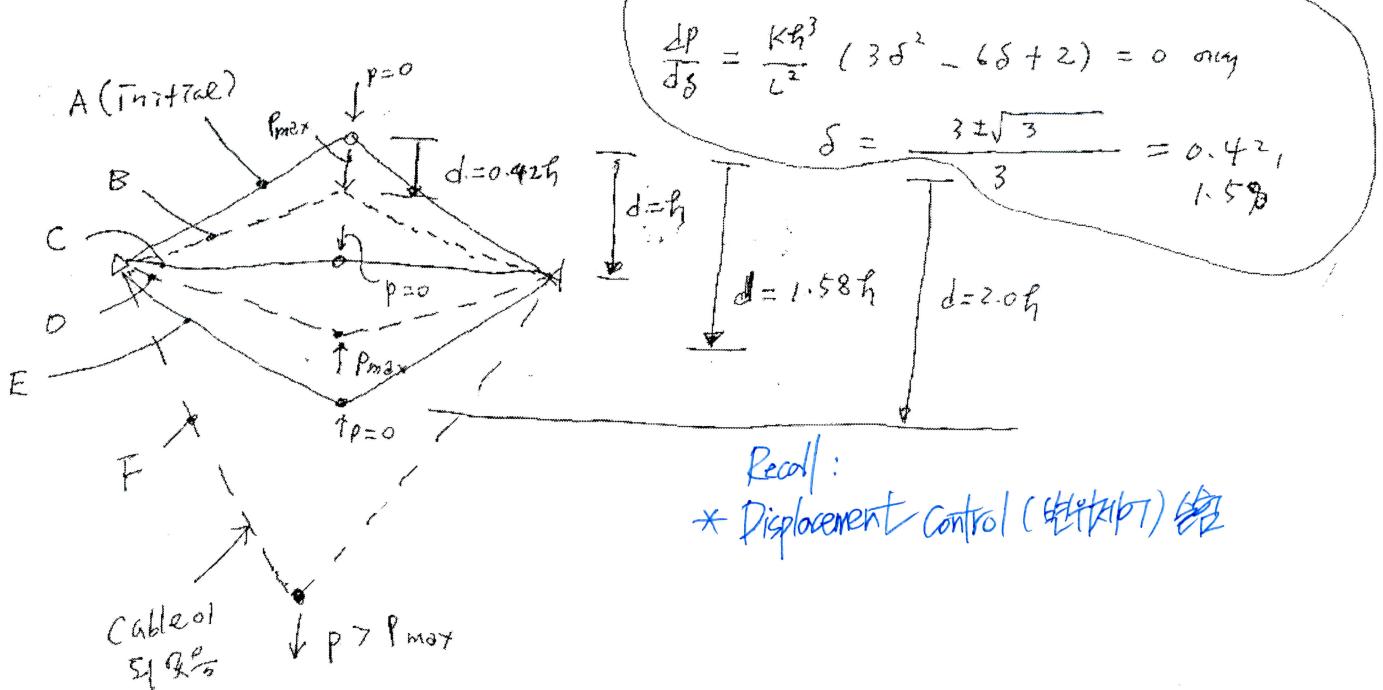
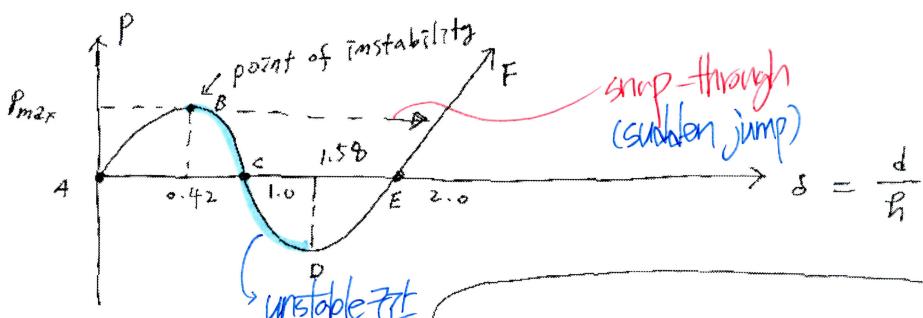
$$L - \sqrt{L^2 + d^2 - 2d\delta} = \frac{P(\sqrt{L^2 + d^2 - 2d\delta})}{2K(h-d)} \quad \dots (14)$$

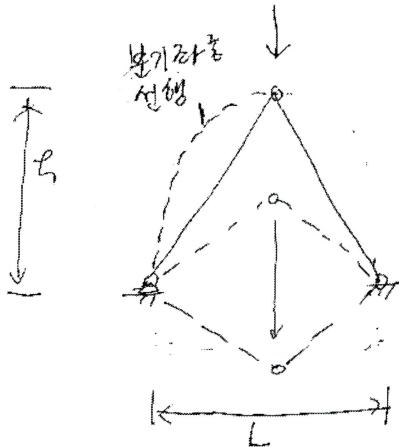
If $\frac{h}{L}$ is small (shallow arch), Eq. (14) reduces to

$$P = \frac{Kh^3}{L^2} \frac{(\delta^3 - 3\delta^2 + 2\delta)}{\delta(\delta-1)(\delta-2)} \quad \dots (15)$$

$$\text{where } \delta = \frac{d}{h}$$

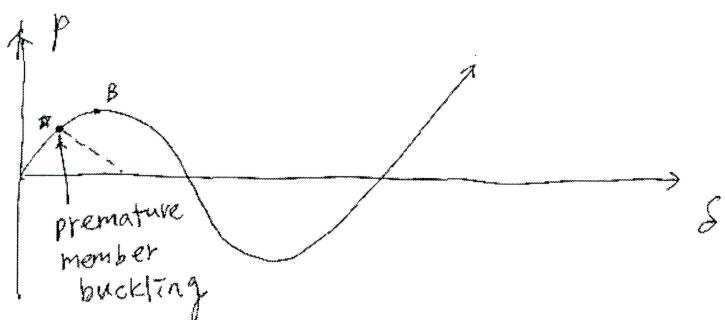
"Homework"
(πΣεργή)



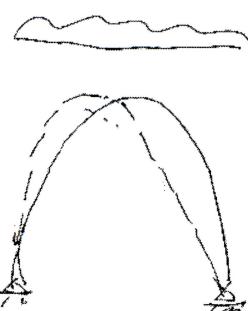


: "High" rise arch

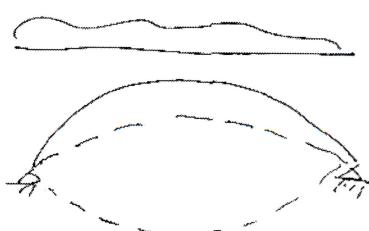
If the rise, h , of the model is large enough compared to L , the axial forces in the legs may reach their critical loads, causing the legs to buckle as hinged-hinged columns before the entire system reaches its limit load at point B. In that case buckling occurs as a result of a bifurcation of equilibrium at point \star . (16)



Note : high rise vs. shallow arches (and spherical caps) under uniform external pressure.



: Bifurcation buckling



: Limit-load buckling
(symmetric)

1장 보통 차로 (ローク モード) - 1.2 ~ 1.4%

(1)

1. 1 see 교과서

1. 2 Types of stability \rightarrow 보통 차로 충돌

1. 3 Methods of analysis in stability

* Bifurcation approach
* Energy
* Dynamic

beyond the scope
of this book (불요)
(간략한 설명, p. 12 1/2)

Bifurcation approach

1) Geometrically perfect system on 적용 가능

2) eigenvalue 해석을 통하여 2개 이상의 서로 다른 경계상태가
존재하는 가능점 (bifurcation point)을 찾으면서 그에 따라.

3) procedure

i) 초기화중에서 구조시스템이 가장 수 있는 모든
경계상태 탐색 (buckling shape 고려, 종종 自明)

ii) Deformed configuration in case of 경계조건에서 적용
(Buckled)

iii) 초기화중에서 구조물의 절연강성이 zero인 태세
를 찾는다. 절연강성이 zero인 태세는 zero
로드를 갖는 system의 critical condition은
그것의 load (the lowest eigenvalue is the critical
load of the system)

(0이거나 그보다 작은 경우 태세)

A geometrically imperfect systems on 적용 불가능

기준을 기준으로
증가하는 경계상태
를 찾는 방법

The problem of a load-deflection rather than
a bifurcation problem

일정한 단계별로
증가하는 경계상태
를 찾는 방법

ex) Initial Imperfection $\Sigma \frac{1}{T}$

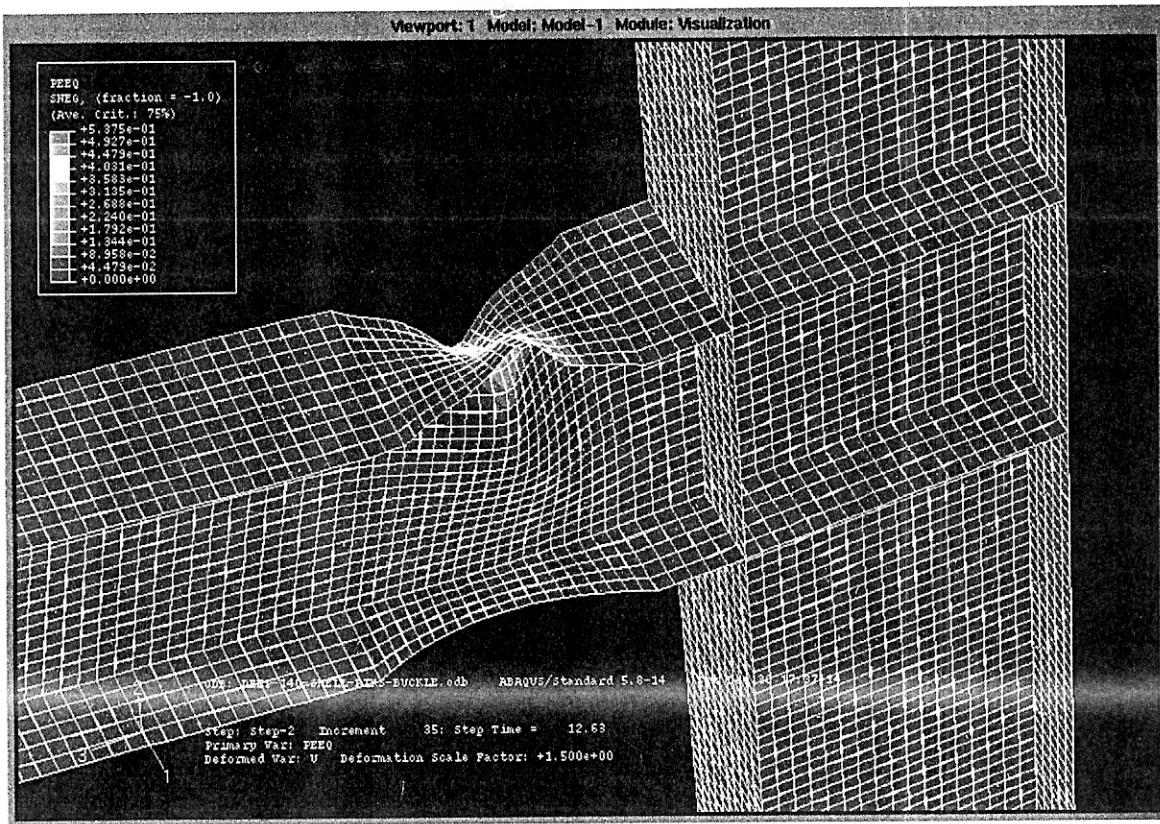
증가하는 경계상태를 찾는 방법

\hookrightarrow C.H. Lee et al. steel moment connection
FEM cyclic analysis study

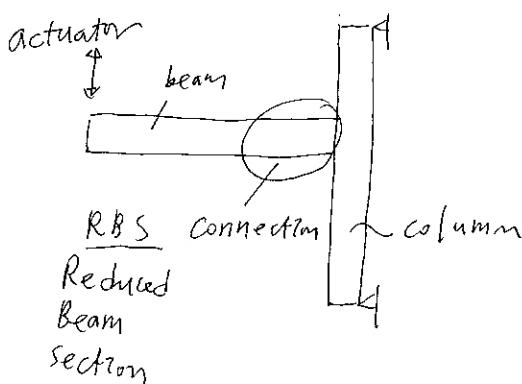
* "Real" plastic Hinge *

①-1

DEEP-140-BUCKLE-PEEQ-CASE4 (919x651x16 png)



"Numerical WLB (Web Local buckling)
simulation FLB (Flange " ")
of Deep LTB (Lateral Torsional buckling)
Column Reduced Beam Section
connection "



: Initial imperfection Σ_0^0
plus geometric/material
nonlinear postbuckling analysis
(RK5 algorithm)

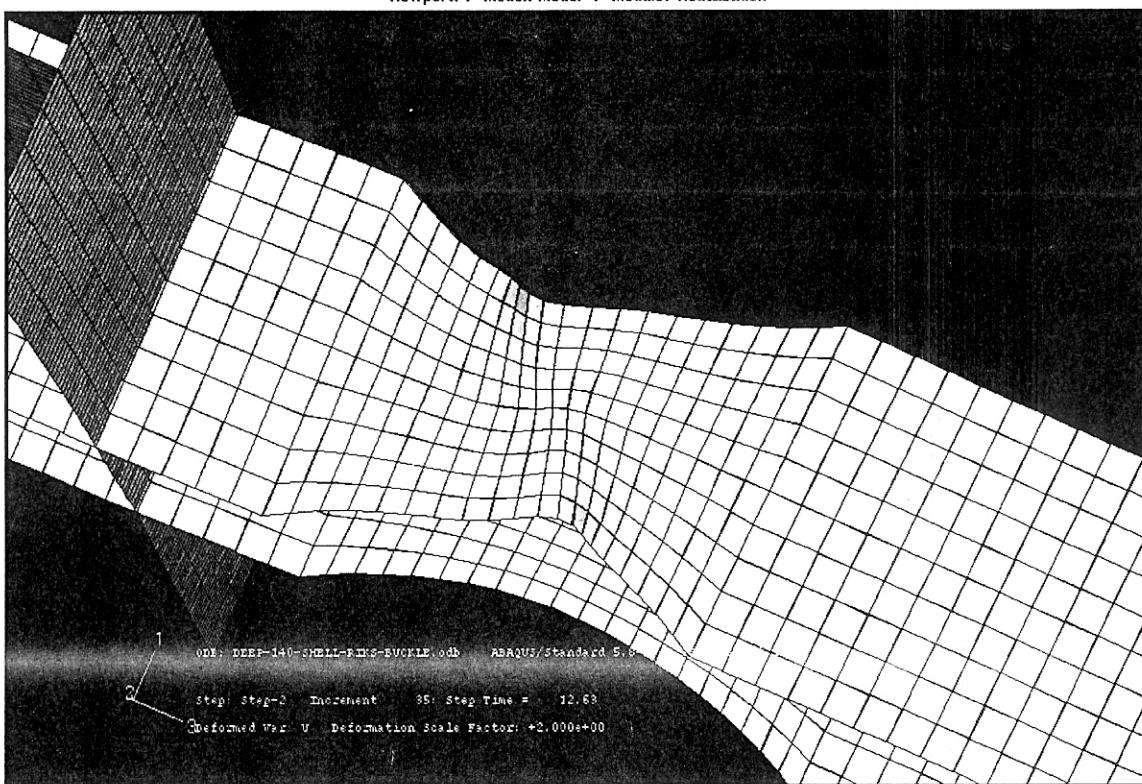
WLB FLB LTB
FLB, WLB, LTB 2/2
Simulation 2%!
($\frac{2}{7}$ compact section of 27695
Cantilever 0.03 mm
2% local, global
buckling of $12 \times 8 \frac{1}{2}$)

$12 \frac{1}{2} \times 8 \frac{1}{2}$

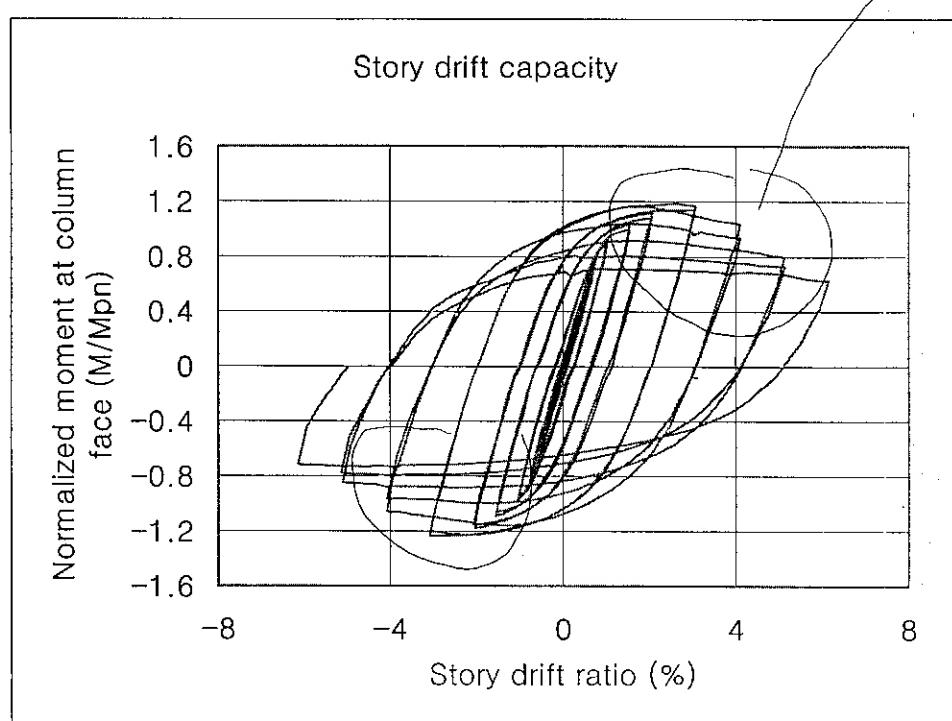
①-2

DEEP-140-BUCKLE-LTB1-CASE4 (919x651x256 png)

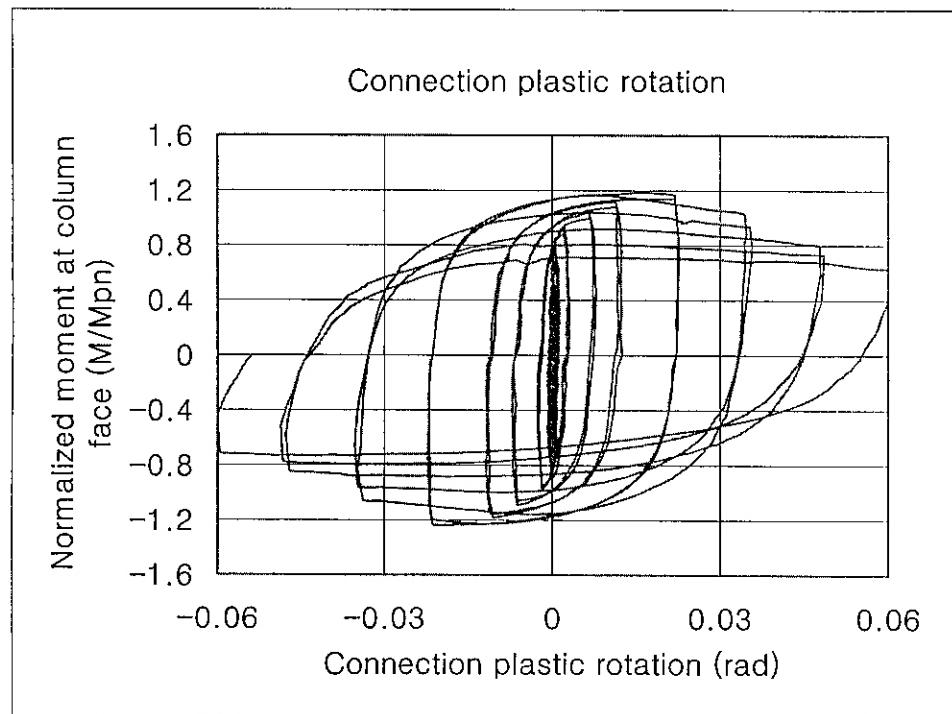
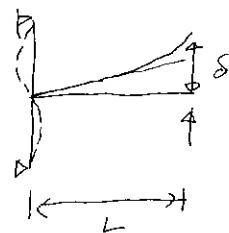
Viewport: 1 Model: Model-1 Module: Visualization



①-3



$\frac{\delta}{L}$ = equivalent story drift Rat 70



(2)

Energy approach (for elastic system under conservative forces)

Using "the principle of stationary value of total potential energy"



Equilibrium condition

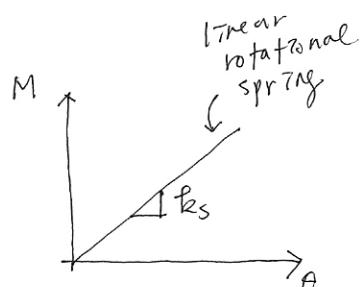
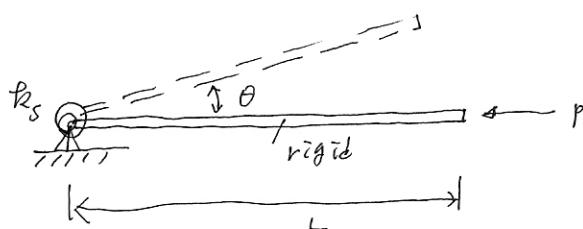
(2) procedure

- Express the total potential energy as a function of a set of generalized displacements and the external applied forces
- By setting the first derivative of the total potential energy function w.r.t. each generalized displacement equal to zero, identify the equilibrium conditions of the system.
- Determine whether the equilibrium is stable or unstable by investigating higher order derivatives of the total potential energy function.

$$(0 = \sum M_{eq} \frac{d^2\theta}{dt^2} + M_{ext} \ddot{\theta})$$

1.4 Illustrative Examples - "Small" deflection analysis

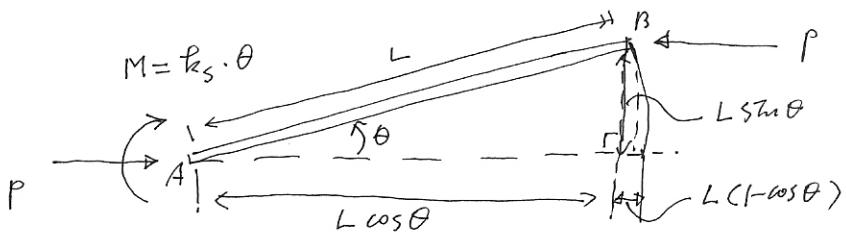
1.4.1 Rigid bar supported by a rotational spring



e: 1 dof system ($I \ddot{\theta} + k_s \theta = 0$)

(3)

Bifurcation approach solution



Deformed (or buckled) configuration

$$\begin{cases} \text{Eq. 1: } \sum \text{moments} \\ \text{Eq. 2: } \sum F_x = 0 \\ \text{Eq. 3: } \sum M = 0 \end{cases}$$

free body diagram of
beam configuration

Free body diagram of the beam configuration.

$$\begin{aligned} \sum M_{about A} &= k_s \cdot \theta - P \times (L \sin \theta) \\ &\doteq k_s \cdot \theta - PL \cdot \theta = 0 \quad (\text{for small } \theta, \sin \theta \approx \theta) \end{aligned}$$

$$(k_s - PL) \times \theta = 0 \quad \leftarrow \text{Eigenvalue problem}$$

For a nontrivial solution,

$$k_s - PL = 0 \text{ or } \text{Eigenf.} \quad (\theta = \frac{0}{0} \leftarrow \text{不定型})$$

$$\therefore P = P_{cr} = \frac{k_s}{L}$$

Note: ① $k_{eff} = k_s - PL = \cancel{k_s} + \cancel{-k_g} \rightarrow \text{zero of the system}$
 (Efficiency) original geometric stiffness

(2) $P = P_{cr} = \frac{k_s}{L}$ original horizontal position
 slightly deflected shape position \Rightarrow on curve
 略微弯曲的形状。

(4)

Energy approach solution (0.2 22 26 3)

Referred by minimum total potential energy

$$\Pi = U + V$$

$$\left\{ \begin{array}{l} U = \frac{1}{2} k_s \theta^2 ; \quad V = - PL(1 - \cos \theta) \\ \text{Lagrangian "function of" } \theta \end{array} \right.$$

$$\therefore \Pi = \frac{1}{2} k_s \theta^2 - PL(1 - \cos \theta) \quad \cdots (1.4.5)$$

\nwarrow 1 dof - discrete system
of 2 functional of
one θ a function of
 θ

for equilibrium,

$$\frac{d\Pi}{d\theta} = 0 \quad (\text{equilibrium}) \quad \leftarrow \begin{array}{l} \text{first variation of} \\ \text{by } \delta \end{array}$$

$$\text{or } \frac{d\Pi}{d\theta} = k_s \theta - PL(0 + \sin \theta) = 0$$

for small θ , $\sin \theta \approx \theta$

$$"k_s \theta - PL \theta = 0"$$

Brackets
are
written
below!

$$\therefore P_{cr} = \frac{k_s}{L} \quad \cdots (1.4.8) \quad \text{by}$$

Note: Energy approach of eqg, similar to Pcr being one of the eigenvalues

특성 (eigenvalue or characteristic value) of the system $\frac{\partial^2 \Pi}{\partial \theta^2}$. "Academic interest"

from flat position ($\theta=0$) of eigenvalue of $\frac{\partial^2 \Pi}{\partial \theta^2} > 0$ (extremum theory of extrema)

$$\underbrace{\Pi(\theta) - \Pi(0)}_{\text{variation}} = \left. \frac{d\Pi}{d\theta} \right|_{\theta=0} \times \theta + \frac{1}{2!} \left. \frac{d^2\Pi}{d\theta^2} \right|_{\theta=0} \times \theta^2 + O(\theta^3)$$

note: $\frac{d\Pi}{d\theta} = k_s \theta - PL \sin \theta \approx k_s \theta - PL \theta$ for "small" θ
 $\frac{d^2\Pi}{d\theta^2} = k_s - PL \cos \theta \approx k_s - PL$

(5)

or

$$\pi(\theta) - \pi(0) = (k_s \theta - p\theta) \Big|_{\theta=0} + \frac{1}{2!} (k_s - pL) \times \theta^2 + \theta(\theta^3)$$

~~θ~~
zero

"dominant term"

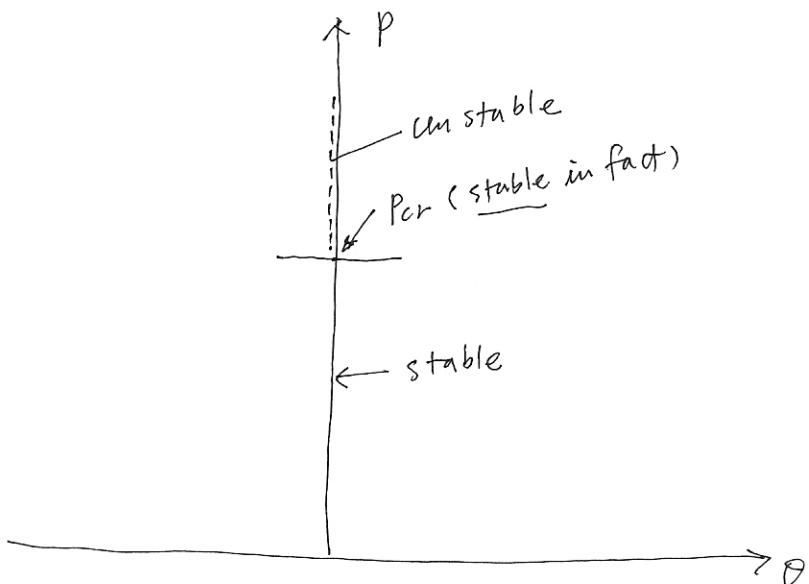
when

$$\begin{cases} p < \frac{k_s}{L} \text{ or } \text{if } \theta \text{ is small, } \pi(\theta) - \pi(0) > 0 ; \pi(0) \text{ is local min.} \\ \text{(stable equilibrium)} \\ p > \frac{k_s}{L} \text{ or } \theta \text{ is large, } \pi(\theta) - \pi(0) < 0 ; \pi(0) \text{ is local max.} \\ \text{(unstable equilibrium)} \end{cases}$$

* $p = p_{cr}$ ~~or θ is small~~, "차도는"은 zero이 되는 θ 를 찾는다
 $= \frac{k_s}{L}$ 작은 틀림은 무시할 수 있다 (small deflection)

↑
 허용한 틀림 θ 로 $p = p_{cr} + \Delta p$
 stable한 틀림 θ 는 θ_{cr}

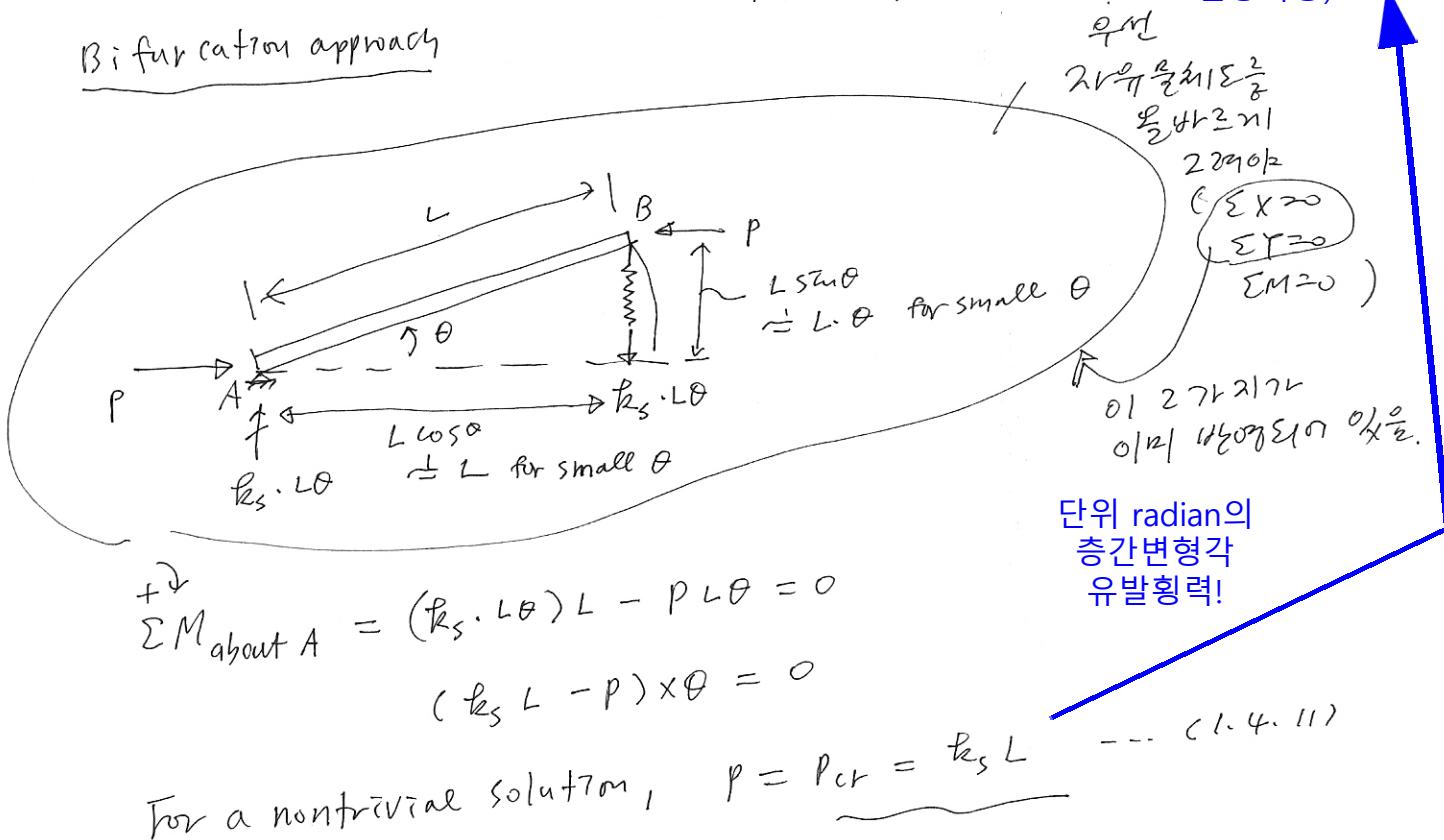
↳ $1.5 \pi \frac{L}{k_s}$



*골조의
sidesway Buckling
강도 산정에
활용가능)

1.4.2 Rigid bar supported by a translational spring (단지 1 dof model)

Bifurcation approach



단위 radian의
증간변형각
유발횡력!

Energy approach

$$\Pi = U + V$$

$$= \frac{1}{2} k_s (L\theta)^2 - P(L - L \cos \theta) \quad \sin \theta \approx \theta \text{ for small } \theta$$

$$\frac{d\Pi}{d\theta} = k_s (L\theta) \cdot L - PL(\sin \theta) = 0 \text{ on } \gamma$$

$$(k_s L - P) \times \theta = 0 \rightarrow \therefore P_{cr} = k_s L$$

note: flat position ($\theta = 0$)에서 증간변형각이 0인 경우 유발 횡력은 0이다.

$$\frac{d\Pi}{d\theta} = k_s L^2 \theta - PL \sin \theta = k_s L^2 \theta - PL \cdot \theta \quad \text{for small } \theta$$

$$\frac{d^2\Pi}{d\theta^2} = k_s L^2 - PL \cos \theta = k_s L^2 - PL \quad \text{Dominant term}$$

$$\frac{\Pi(\theta) - \Pi(0)}{\theta^2} = \left. \frac{(k_s L^2 \theta - PL \theta)}{\theta} \right|_{\theta=0} + \left. \frac{L}{2!} (k_s L - PL) \theta^2 + O(\theta^3) \right|_{\theta=0}$$

$\frac{\Pi(\theta) - \Pi(0)}{\theta^2}$ + flat position

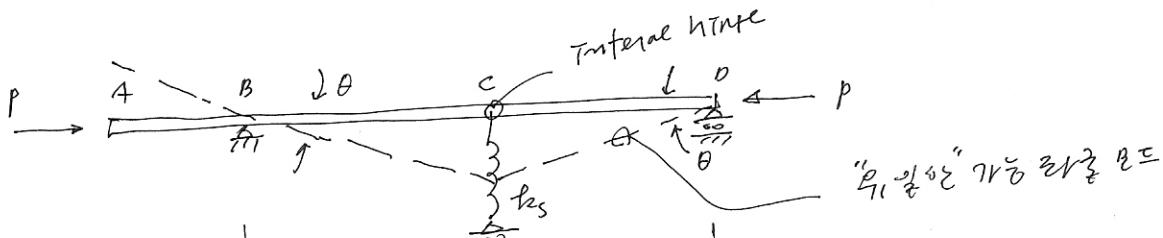
$\begin{cases} P < k_s L \text{ 일 때}, \Pi(\theta) - \Pi(0) > 0 \\ \text{local min} \end{cases} \rightarrow \text{stable}$

$P > k_s L \text{ 일 때} \rightarrow \text{unstable}$

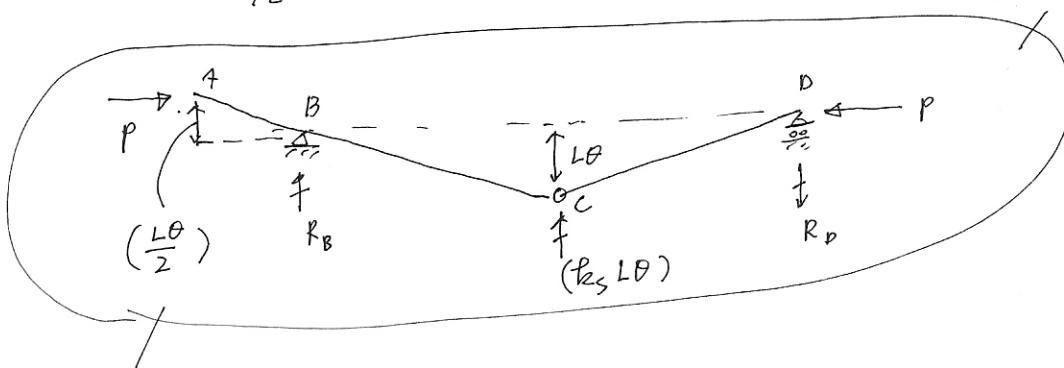
(7)

1.4.3 Two-bar system

↳ two-bar system օւալու օրակ 1 dof միջու ցուց



deformed shape on axis
 $\sqrt{2k_s \frac{\theta}{2} \sin \frac{\theta}{2}}$
 ըստ 2 մոկ
 $(2k_s \frac{\theta}{2})$
 \downarrow
 $R_B, R_D \frac{\theta}{2}$ պահ
 P_3 ուժական



$$\sum X = 0, \text{ o.k.}$$

$$\sum Y = R_B + (k_s L \theta) - R_D = 0$$

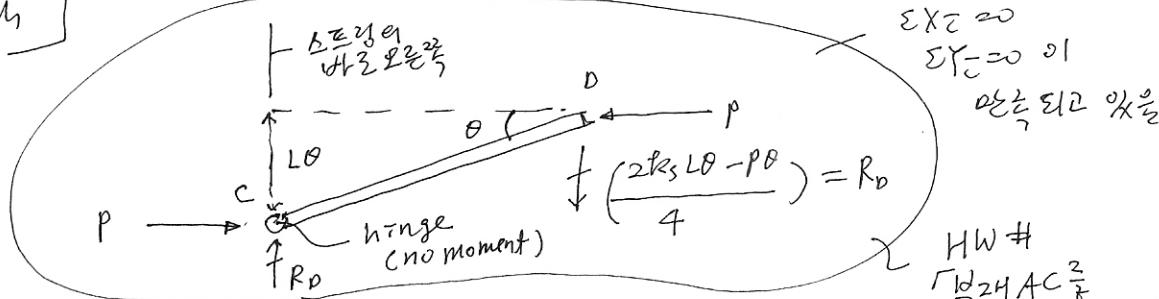
$$\sum M_{\text{about } B} = P \left(\frac{L \theta}{2}\right) - (k_s L \theta)(L) + R_D(2 \cdot \theta L) = 0$$

$$\therefore R_D = \frac{(k_s L \theta)(L) - P(\frac{L \theta}{2})}{2 \cdot \theta L} = \frac{2 k_s L \theta - P \theta}{4}$$

$$\text{ուշադրություն } R_B = R_D - (k_s L \theta) = \boxed{\quad}$$

overall equilibrium condition
 $\sum F_x = 0$
 $\sum M_C = 0$

Bifurcation approach



$$\therefore \text{կառավագություն}$$

$$\sum M_{\text{about } C} = R_D \times L - P L \theta = \frac{2 k_s L^2 \theta - P L \theta}{4} - P L \theta = 0$$

$$\frac{1}{2} k_s L \theta - \frac{5}{4} P \theta = (\frac{1}{2} k_s L - \frac{5}{4} P) \times \theta = 0$$

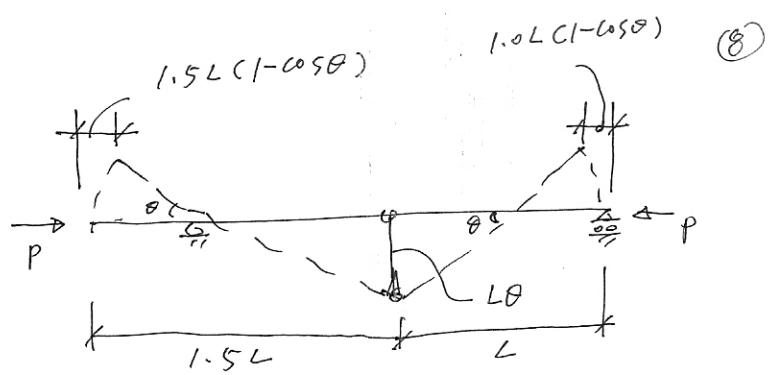
$$\therefore P = P_{cr} = \frac{1}{2} k_s L \times \frac{4}{5} = \frac{2}{5} k_s L$$

HW #
 $\Gamma D 24 AC \frac{3}{8}$
 բախում $P_{cr} \frac{5}{4}$
 չեղական պահ

Energy approach

$$\Pi = U + V$$

$$= \frac{1}{2} k_s (L\theta)^2 - p (2.5L) (1 - \cos \theta)$$



For equilibrium,

$$\begin{aligned} \frac{d\Pi}{d\theta} &= k_s (L\theta) - 2.5 p L (\sin \theta) \\ &= (k_s L^2 - 2.5 p L) \times \theta = 0 \quad (\text{for small } \theta) \end{aligned}$$

$$\therefore P = P_{cr} = \frac{k_s L}{2.5} = \underline{\underline{\frac{2}{5} k_s L}}$$

* $\theta = 0$ position is a local minima of energy

$$\begin{aligned} \hookrightarrow \frac{d^2\Pi}{d\theta^2} &= k_s L^2 - 2.5 p L \cos \theta \\ &\stackrel{\approx}{=} k_s L^2 - 2.5 p L \quad (\text{for small } \theta) \end{aligned}$$

For $p < P_{cr}$ $\rightarrow \frac{d^2\Pi}{d\theta^2} > 0 \rightarrow \theta = 0$ is local min. point (stable)

For $p > P_{cr}$ $\rightarrow \frac{d^2\Pi}{d\theta^2} < 0 \rightarrow \theta = 0$ is local max. point (unstable)

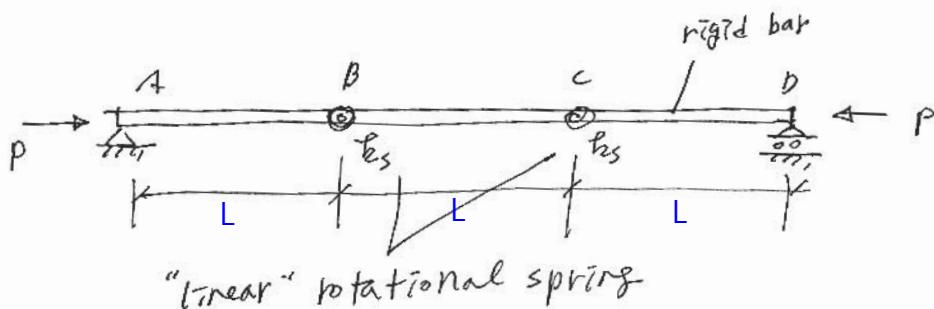
1. 4. 4 Three-bar system

(Q&A 24 Q&A 2)

↳ oral upload

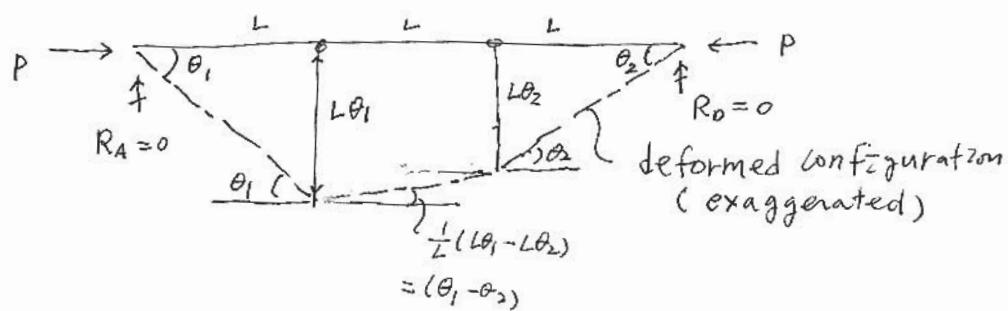
①

1.4.4 Three-Bar System



Bifurcation approach ← slightly deformed configuration 이경우는 각각의 각도를 미지수로 두면

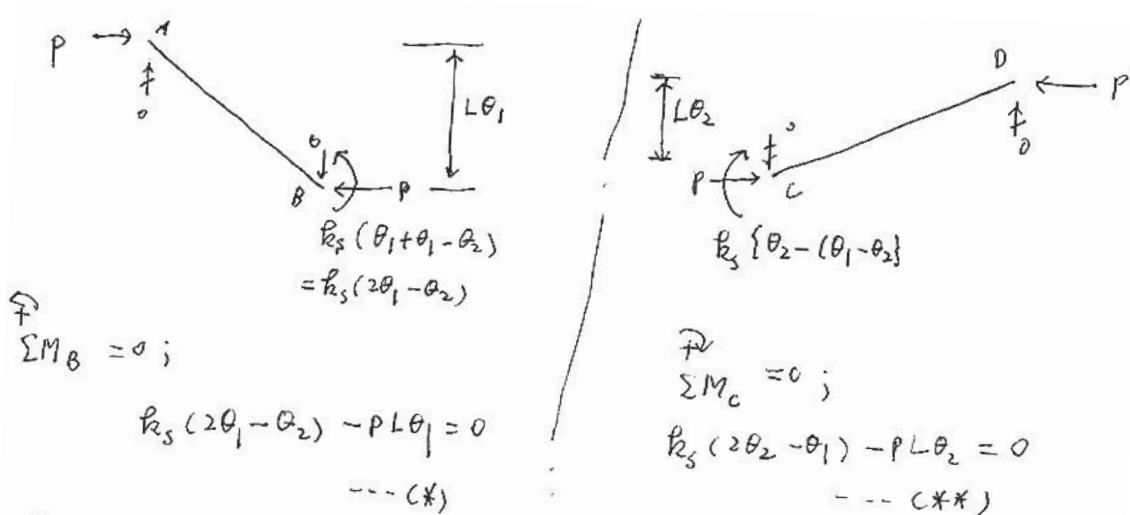
- ① 2dof system; 점점 A, D의 위치가变了 변화한 kinematics 을 가짐
→ 2개의 방정식을 갖도록 (미지수는 θ_1, θ_2 만 존재해야함)



- ② 구조를 정체에 대상으로 대수법을 사용하되 $\sum M = 0, \sum Y = 0$ 을 사용함,

$$R_A = 0 ; R_D = 0$$

- ③ 점 A, B 및 C 의 'cut' 을 하여 풀어진 2개의 free-body



In matrix form,

$$\begin{bmatrix} 2k_s - PL & -k_s \\ -k_s & 2k_s - PL \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For a nontrivial solution (by Cramer's rule),

(2)

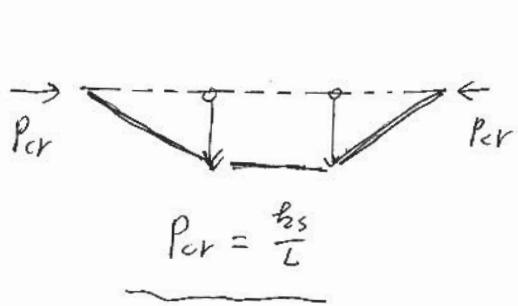
$$\det \begin{bmatrix} 2k_s - PL & -k_s \\ -k_s & 2k_s - PL \end{bmatrix} = 0$$

$$(2k_s - PL)^2 - k_s^2 = 0 ; 2k_s - PL = \pm k_s$$

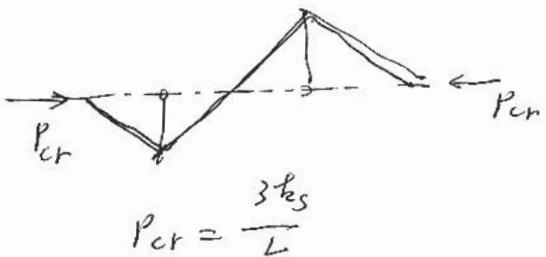
$$\therefore \left. \begin{array}{l} P = \frac{k_s}{L} \\ P = \frac{3k_s}{L} \end{array} \right\} \text{on eigenvalues}$$

\therefore eigenvectors

$$\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$



(the lowest value is
the critical load of the system)



Effect of bracing



2차 mode를
강제할 것임.

(좌굴강도 3배로 증대)

Energy Approach

By 2nd total potential energy $\Pi = V + T$ ④

$$U = \frac{1}{2} k_s (2\theta_1 - \theta_2)^2 + \frac{1}{2} k_s (2\theta_2 - \theta_1)^2$$

$$T = -PL [(1 - \cos \theta_1) + \{ (1 - \cos(\theta_1 - \theta_2)) + (1 - \cos \theta_2) \}]$$

$$\Pi = U + T = \frac{1}{2} k_s (2\theta_1 - \theta_2)^2 + \frac{1}{2} k_s (2\theta_2 - \theta_1)^2$$

$$-PL \{ 3 - \cos \theta_1 - \cos \theta_2 - \cos(\theta_1 - \theta_2) \}$$

w

(3)

For equilibrium, Π must be in stationary. In mathematical terms, this requires

$$\text{평온한 } \rightarrow \left\{ \begin{array}{l} \frac{\partial \Pi}{\partial \theta_1} = 2k_s(2\theta_1 - \theta_2) - k_s(2\theta_2 - \theta_1) - PL \left[\frac{\sin \theta_1 + \sin(\theta_1 - \theta_2)}{2\theta_1 - \theta_2} \right] = 0 \\ \frac{\partial \Pi}{\partial \theta_2} = -k_s(2\theta_1 - \theta_2) + 2k_s(2\theta_2 - \theta_1) - PL \left[\frac{\sin \theta_2 - \sin(\theta_1 - \theta_2)}{2\theta_2 - \theta_1} \right] = 0 \end{array} \right\} \dots (4)$$

Upon simplification and using small angle approximation, $\sin \theta \approx \theta$,

$$\left\{ \sin \theta_1 \approx \theta_1 ; \sin(\theta_1 - \theta_2) \approx \theta_1 - \theta_2 \right.$$

$$\begin{bmatrix} 5k_s - 2PL & -4k_s + PL \\ -4k_s + PL & 5k_s - 2PL \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For nontrivial solution,

$$\det \begin{vmatrix} & & \end{vmatrix} = 0,$$

$$P_1 = \frac{k_s}{L} ; P_2 = \frac{3k_s}{L}$$

$$\therefore P_{\text{or}} = P_1 = \frac{k_s}{L}$$

Academic interest.

- (*) To study the nature of equilibrium for the system in its undelected position ($\theta_1 = \theta_2 = 0$), we need to investigate higher order derivatives of Π .

$$\left(\begin{array}{c} \frac{\partial^2 \Pi}{\partial \theta_1^2} = 5k_s - PL[\cos \theta_1 + \cos(\theta_1 - \theta_2)] \approx 5k_s - 2PL \text{ (for small angle approx.)} \\ \frac{\partial^2 \Pi}{\partial \theta_2^2} = \approx 5k_s - 2PL \text{ (" ")} \\ \frac{\partial^2 \Pi}{\partial \theta_1 \partial \theta_2} = \approx -4k_s + PL \text{ (" ")} \end{array} \right)$$

" 0% 2nd order term is 0 "

(4)

Note: 차원수 확장과 Taylor 확장법

“formalism”

$$f(\underline{P} + \underline{H}) = f(\underline{P}) + \frac{(\underline{H} \cdot \nabla) f(\underline{P})}{1!} + \frac{(\underline{H} \cdot \nabla)^2 f(\underline{P})}{2!} + \dots$$

가장 2변수 경우에 미분
 $\nabla = \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right)$ ← 편미분 연산자

$$\underline{P} = (a, b), \quad \underline{H} \cdot \nabla = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}$$

$$\underline{H} = (x, y), \quad (\underline{H} \cdot \nabla)^2 = x^2 \frac{\partial^2}{\partial x^2} + 2xy \frac{\partial^2}{\partial x \partial y} + y^2 \frac{\partial^2}{\partial y^2} \Big|_{\substack{x=a \\ x=b}}$$

$$\therefore f(\underline{P} + \underline{H}) = f(a + \underline{x}, b + \underline{y})$$

$$\begin{aligned} &= f(a, b) + \left\{ x \cdot \frac{\partial f(a, b)}{\partial x} + y \frac{\partial f(a, b)}{\partial y} \right\} \\ &\quad + \frac{1}{2!} \left[x^2 \frac{\partial^2 f}{\partial x^2} \Big|_{\substack{x=a \\ y=b}} + 2xy \frac{\partial^2 f(a, b)}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} \Big|_{\substack{x=a \\ y=b}} \right] + \dots \end{aligned}$$

Back to the problem

$$\underline{P} = (0, 0), \quad \underline{H} = (\theta_1, \theta_2) \text{ on } \frac{1}{2}\theta^2$$

$$\begin{aligned} \pi(\theta_1, \theta_2) &= \pi(0, 0) + \left\{ \frac{\partial \pi(0, 0)}{\partial \theta_1} \times \theta_1 + \frac{\partial \pi(0, 0)}{\partial \theta_2} \times \theta_2 \right\} \\ &\quad + \frac{1}{2!} \left\{ \frac{\partial^2 \pi(0, 0)}{\partial \theta_1^2} \times \theta_1^2 + 2\theta_1 \theta_2 \cdot \frac{\partial^2 \pi(0, 0)}{\partial \theta_1 \partial \theta_2} + \frac{\partial^2 \pi(0, 0)}{\partial \theta_2^2} \right\} + O(\theta^3) \end{aligned}$$

$$\begin{aligned} \overbrace{\pi(\theta_1, \theta_2) - \pi(0, 0)}^{\text{Variet path}} &= \left\{ \frac{\partial \pi(0, 0)}{\partial \theta_1} \times \theta_1 + \frac{\partial \pi(0, 0)}{\partial \theta_2} \times \theta_2 \right\} \xrightarrow{\text{zero } \theta_1, \theta_2 \text{ are fixed}} + O(\theta^2) + O(\theta^3) \\ &\quad \downarrow \text{dominant term} \end{aligned}$$

따라서 $\pi(0, 0)$ 은 stable equilibrium state이거나 균형

$\pi(0, 0)$ 은 local minimum이거나 $\pi(0, 0)$ 은 $\pi(0, 0)$ 을 주변에서 $\pi(0, 0)$ 의

근방에서 $\pi(0, 0)$ 에 가까워 θ_1, θ_2 에 $O(\theta^2) > 0$ 이므로 균형.

(3) 22번의 (***) 틀림

그림 64번에는
 θ_1, θ_2 의 조건을
인수로 놓았는데
조건을 조건으로
놓았을 때

$$\left(\frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_1^2} \times \theta_1^2 + \frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_2^2} \times \theta_2^2 + \frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_1 \partial \theta_2} (\theta_1, \theta_2) > 0 \right) \quad \text{--- (***)}$$

(i) $\theta_2 = 0 \neq \theta_1 \neq 0$ 일 때 (perturb θ_1 only)

$$\frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_1^2} > 0 \quad \therefore \quad (3) 22번의 (***) 가 맞음$$

$$(5k_s - 2PL) > 0 \quad \text{--- (a)}$$

(ii) $\theta_2 \neq 0$ and $\theta_1 = 0$ 일 때 (perturb θ_2 only)

$$\frac{\partial^2 \pi(0, \theta_2)}{\partial \theta_2^2} > 0, \text{つまり}$$

$$(5k_s - 2PL) > 0 \quad \text{--- (b)}$$

(iii) $\theta_1 \neq 0 \neq \theta_2 \neq 0$ 일 때 (perturb θ_1 and θ_2 simultaneously)

(***) 가 맞음

상수판정

$$\left[\frac{\frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_1^2} \theta_1^2 + \frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_2^2} \theta_2^2}{2} \right] + \frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_1 \partial \theta_2} (\theta_1, \theta_2)$$

$$\geq \sqrt{\frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_1^2} \theta_1^2 \times \frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_2^2} \theta_2^2} + \frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_1 \partial \theta_2} (\theta_1, \theta_2) > 0$$

기하학적 관점

$$\sqrt{\dots} > -\frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_1 \partial \theta_2} (\theta_1, \theta_2)$$

$$\left\{ \frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_1^2} \right\} \times \left\{ \frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_2^2} \right\} (\theta_1^2 \cdot \theta_2^2) > \left[\frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_1 \partial \theta_2} \right]^2 (\theta_1^2 \cdot \theta_2^2)$$

$$\text{or } \left[\frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_1^2} \right] \left[\frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_2^2} \right] > \left[\frac{\partial^2 \pi(\theta_1, 0)}{\partial \theta_1 \partial \theta_2} \right]^2, \text{つまり}$$

$$\text{or } (5k_s - 2PL)^2 - (4k_s - PL)^2 > 0$$

$$(9k_s - 3PL)(k_s - PL) > 0 \quad \text{--- (c)}$$

(6)

$$(5k_s - 2PL) > 0 \quad \dots (a)$$

$$(5k_s - 2PL) > 0 \quad \dots (b)$$

$$(P_s - PL)(3k_s - PL) > 0 \quad \dots (c)$$

0/3719322

영화 <기마관>

Undeflected position of
($\theta_1 = \theta_2 = 0$)

$P < \frac{k_s}{L}$ 일 때 : 위의 (a), (b), (c) 조건을 모두 만족 \rightarrow stable equilibrium
상태의 일 때.

$P > \frac{k_s}{L}$ 일 때 : (c)의 조건에 $\pi(0, 0)$ 은 local maximum of π 이
unstable equilibrium state의 일 때.

and general

* For more rigorous mathematical treatment,
refer to "Elastic stability of structures", pp. 8~14
by G. J. Simitses (1976)

$$1.28 \frac{m}{V} \cdot 24 \frac{V}{s} = 1.5 \frac{m}{s} \sim \underline{\underline{1.6 \frac{m}{s}}}$$

1

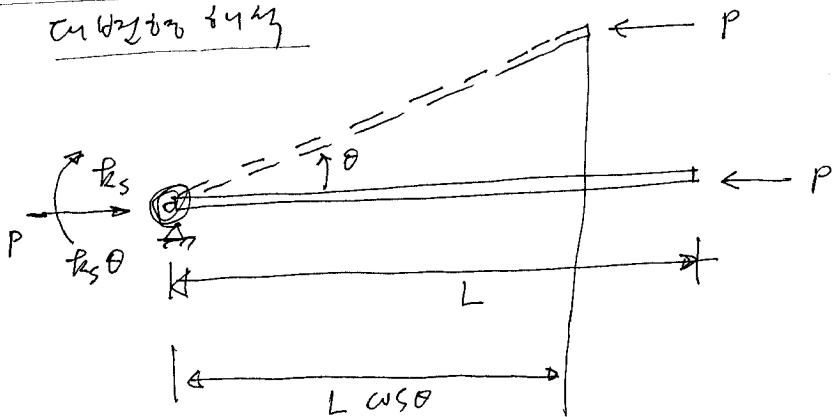
1.5 Illustrative examples - large deflection analysis

$$(1) \quad \text{Deflection} = \frac{P_{cr} L^2}{32 E I} \rightarrow \text{small deflection numbers}$$

(2) Small deflection theory of elasto-plastic systems
Postbuckling behavior → large displacement theory

1.5.1 Rigid bar supported by a rotational spring

Curly top fly



: Fig. 1.16

Note : Fig. 1. 11 は (2 つの異なる 1 つ目を除く 2 つ)

\rightarrow stable or unstable when $p = p_{cr}$?

- ① stable or unstable behavior
- ② no information about postbuckling behavior

Energy approach on the crystallizing energy (Fig. 1.16 計算)

제작되는 가능성이 있는 제품을 찾는 것을 목표로 하는 설계 방법

$$U = \frac{1}{2} k_s \theta^2$$

$$T = -pL(1-\cos\theta)$$

$$\pi = U + V = \frac{1}{2} k_s \theta^2 - p L (1 - \cos \theta)$$

$$\text{For equilibrium, } \frac{d\pi}{d\theta} = k_s \theta - PL \sin \theta = 0 \quad \leftarrow \begin{array}{l} \theta = 0 \text{ or } \theta = \arctan \frac{PL}{k_s} \\ (\text{trivial equilibrium path}) \\ \text{fundamental} \end{array}$$

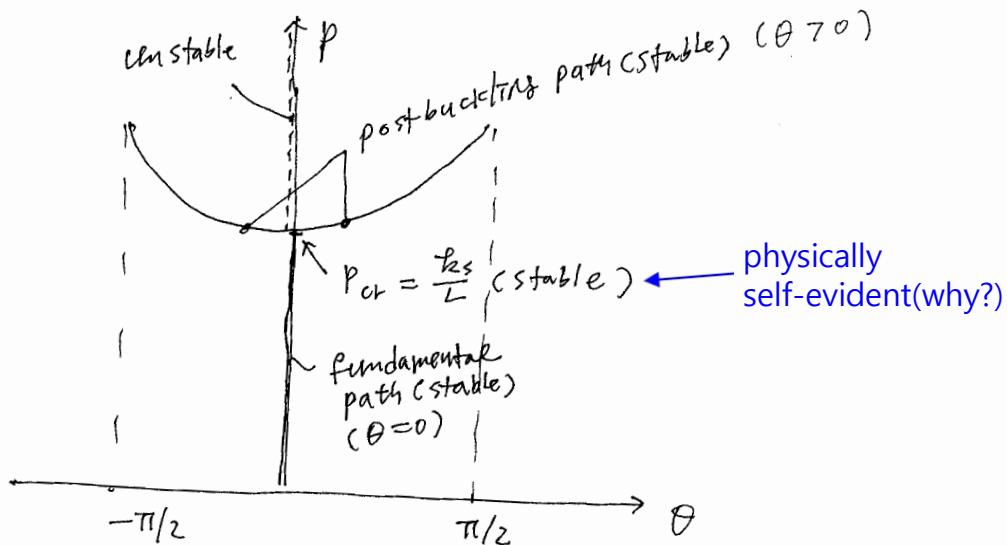
2

The postbuckling path ($\underline{\theta > 0}$) is

$$\frac{d\theta}{dt} = K_s \theta - p L \sin \theta = 0 \text{ only}$$

$$P = \frac{Ks\theta}{Ls_2\theta} \quad \dots \quad (1.5.5)$$

- even function for the range ($-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$)



Study of the stability of the equilibrium paths

$$\frac{d\tau}{d\theta} = \rho_s \theta - \rho L \sin \theta \quad \dots (1.5.4)$$

$$\frac{d^2\pi}{dg^2} = k_s - \rho L \cos \theta \quad \dots \quad (1.5.6)$$

① For the fundamental path (or $\theta = 0$)

$$\left. \frac{d^2\pi}{d\theta^2} \right|_{\theta=0} = k_s - pL \cos 0 = k_s - pL \quad \dots \quad (1.5.7)$$

$$\pi(\theta) - \pi(0) = 0 + \frac{1}{2!} (\ell_{2s} - pL) \times \theta^2$$

$$\begin{aligned} P &< \frac{k_s}{L} \text{ or } \text{stable?} & (\text{stable}) \\ P &> \frac{k_s}{L} \text{ or } \text{unstable?} & (\text{unstable}) \end{aligned}$$

(226 263)

(3)

(2) For the postbuckling path,

$$P = \frac{k_s \theta}{L \sin \theta} \quad \text{when } \theta > 0$$

(1.5.4) when $\theta > 0$
($\frac{1}{2} k_s L^2 \theta^2 + \frac{1}{2} PL^2 \cos \theta$)

return (1.5.6) when $\theta < 0$

$$\frac{d^2\pi}{d\theta^2} = k_s - \left(\frac{k_s \theta}{L \sin \theta} \right) L \cos \theta$$

$$= k_s \left(1 - \frac{\theta}{\tan \theta} \right) < 1.0$$

$$= k_s \left(1 - \underbrace{\left(\theta + \frac{1}{3} \theta^3 + \frac{2}{15} \theta^5 + \dots \right)}_{\text{π function}} \right)$$

π function

is always +%; thus, the postbuckling path is
always stable.
(when $\theta > 0$, $\frac{d^2\pi}{d\theta^2} > 0$,
when local minimum) ($\frac{1}{2} k_s L^2 \theta^2 + \frac{1}{2} PL^2 \cos \theta$)

(3) At the critical point, $P = P_{cr} = \frac{k_s}{L}$

$$(1.5.7) \text{ when } P = P_{cr} = \frac{k_s}{L} \quad \frac{d^2\pi}{d\theta^2} = 0 \text{ or } 0$$

obviously zero at $\theta = 0$ is why it's dominant term is $\frac{1}{2} k_s L^2 \theta^2$
zero at $\theta = 0$: zero

$$\pi(\theta) - \pi(0) = \left. \frac{d\pi}{d\theta} \right|_{\theta=0} \times \theta + \frac{1}{2} \left. \frac{d^2\pi}{d\theta^2} \right|_{\theta=0} \times \theta^2$$

zero (1.5.4) +

$$+ \frac{1}{6} \left. \frac{d^3\pi}{d\theta^3} \right|_{\theta=0} \times \theta^3 + \frac{1}{24} \left. \frac{d^4\pi}{d\theta^4} \right|_{\theta=0} \times \theta^4 + O(\theta^5)$$

$\left. PL \sin \theta \right|_{\theta=0} = \text{zero}$ $\left. PL \cos \theta \right|_{\theta=0} = PL > 0$

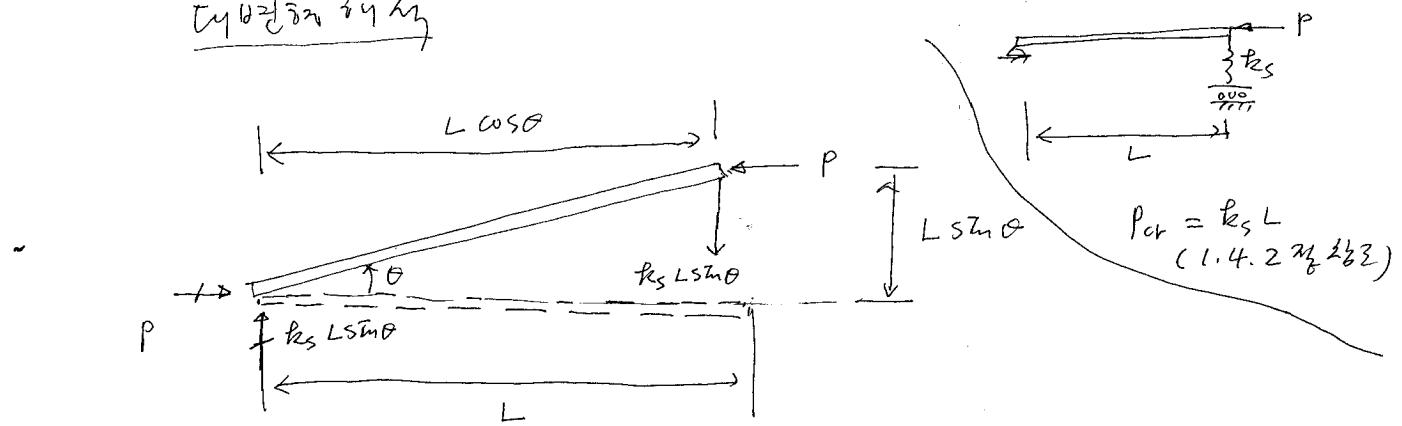
Note: $P_c = \frac{k_s}{L}$
 $\text{when } \theta = 0$

return $\pi(0) = \frac{1}{2} k_s L^2$
(return stable!)

(4)

1.5.2 Rigid bar supported by a translational spring \Rightarrow

Diagram



Energy approach

$$\Pi = \frac{1}{2} k_s (L \sin \theta)^2 - PL(1 - \cos \theta) \quad \dots (1.5.14)$$

For equilibrium,

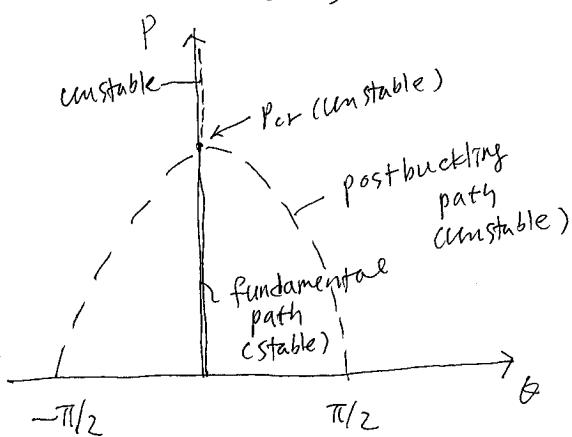
$$\frac{d\Pi}{d\theta} = (k_s L^2 \cos \theta - PL) \sin \theta = 0 \quad \dots (1.5.15)$$

① $\theta = 0$ only, (1.5.15) is valid \Rightarrow fundamental equilibrium path

② postbuckling path (or $\theta > 0$)

(1.5.15) only

$$(k_s L^2 \cos \theta - PL) \sin \theta = 0 ; P = k_s L \cos \theta \quad \dots (1.5.16)$$



= Fig. 1.19

(5)

Study of the stability of the equilibrium paths

$$\hookrightarrow \frac{d^2\pi}{d\theta^2} = k_s L^2 (\cos^2 \theta - \sin^2 \theta) - PL \cos \theta \quad \dots (1.5.17)$$

(1) For the fundamental path ($\theta = 0$)

$$\left. \frac{d^2\pi}{d\theta^2} \right|_{\theta=0} = k_s L^2 - PL = L(k_s L - p)$$

$$\pi(\theta) - \pi(0) = 0 + \underbrace{\frac{L}{2!} (k_s L - p) \times \theta^2}_{\text{from } (1.5.17)} + O(\theta^3)$$

$$\begin{cases} p < k_s L \text{ only} \rightarrow \text{stable} \\ p > k_s L \text{ only} \rightarrow \text{unstable} \end{cases}$$

(2) For the postbuckling path ($\theta > 0$)

$$p = k_s L \cos \theta \leftarrow \text{from (1.8.15)}$$

$$\hookrightarrow (1.5.17) \text{ only on } \theta \neq 0$$

$$\frac{d^2\pi}{d\theta^2} = k_s L^2 (\cos^2 \theta - \sin^2 \theta) - (k_s L \cos \theta) L \cos \theta \quad (1.5.18)$$

$$= -k_s L^2 \sin^2 \theta < 0$$

~~q13~~ $\frac{1}{2} \leq \theta \leq \frac{3}{2} \pi$ (local max. \rightarrow unstable)
 \hookrightarrow ~~q12~~ $\theta = \frac{3}{2} \pi$

(3) At the critical point ($p = p_{cr} = k_s L$),
 $\hookrightarrow \theta = 0$

$$\frac{d^2\pi}{d\theta^2} = k_s L^2 - (k_s L) L = 0 \quad \leftarrow \text{Eq. (1.5.18)}$$

"current value is zero at $\theta = 0$ "

Note: (1.5.17) $\frac{d^2\pi}{d\theta^2} = 1 \frac{1}{2} \geq 0$ \Rightarrow $\sin \theta \neq 0$ at $\theta = 0$
 \hookrightarrow current $\left. \frac{d^3\pi}{d\theta^3} \right|_{\theta=0} = \text{zero} \neq 0$

Fig. 1.19
 \uparrow
 $\left(\frac{1}{2}, \frac{3}{2} \right)$

The first non-zero term in the series,

$$\left. \frac{1}{24} \frac{d^4\pi}{d\theta^4} \right|_{\theta=0} \times \theta^4 = \left. \frac{1}{24} (-4k_s L^2 + PL) \right|_{p=k_s L} \times \theta^4$$

$$= -\frac{1}{8} k_s L^2 \times \theta^4 < 0 \rightarrow \text{current local max.}$$

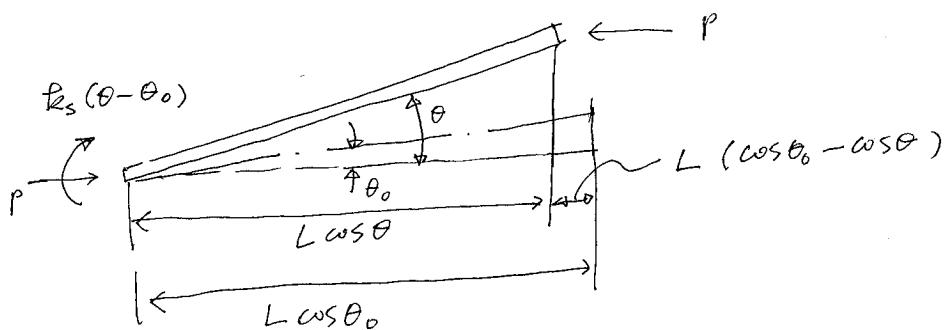
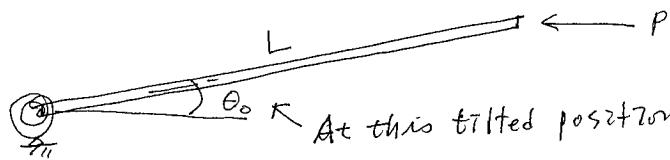
HW #1 ... ol 2nd eqn

unstable!

1.6 Imperfect systems

In the sense that the bars are slightly tilted; the bar will deflect as soon as the load is applied.
(염률하는 경우는 즉시 고정된 위치에서 기울기)

1.6.1 Rigid bar supported by a rotational spring



$$\Pi = \frac{1}{2} k_s (\theta - \theta_0)^2 - p \times L (\cos \theta_0 - \cos \theta)$$

For equilibrium,

$$\frac{d\Pi}{d\theta} = k_s (\theta - \theta_0) - p L \sin \theta = 0 \quad \dots \quad (1.6.4)$$

or $p = \frac{k_s (\theta - \theta_0)}{L \sin \theta}$

"Fig. 1.21 1/2 E" (P. 33)

* Study of the stability of the equilibrium paths of the imperfect system,

$$(1.6.4) \rightarrow \frac{d^2\Pi}{d\theta^2} = k_s - p L \cos \theta \quad (-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2})$$

$$\begin{cases} p < \frac{k_s}{L \cos \theta} & (\text{stable}; \frac{d^2\Pi}{d\theta^2} > 0) \\ p > \frac{k_s}{L \cos \theta} & (\text{unstable}; \frac{d^2\Pi}{d\theta^2} < 0) \end{cases}$$

(1)

The equilibrium paths are stable if

$$P < \frac{k_s}{L \cos \theta} \quad \dots (*)$$

or $\frac{\frac{k_s(\theta - \theta_0)}{L \sin \theta}}{\frac{k_s}{L \cos \theta}} < 1$

(1.6.5) \Rightarrow

$$\cos \theta < \frac{\sin \theta}{\theta - \theta_0} \quad \dots (*)'$$

" $\sin \theta > \cos \theta$ "

$$(자연) = \cos \theta = \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots \right)$$

$$(무한) = \frac{\sin \theta}{\theta - \theta_0} \underset{\text{X}}{\approx} \frac{\sin \theta}{\theta} = \frac{1}{\theta} \left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots \right)$$

$$= \left(1 - \frac{\theta^2}{3!} + \frac{\theta^4}{5!} - \dots \right) > \cos \theta$$

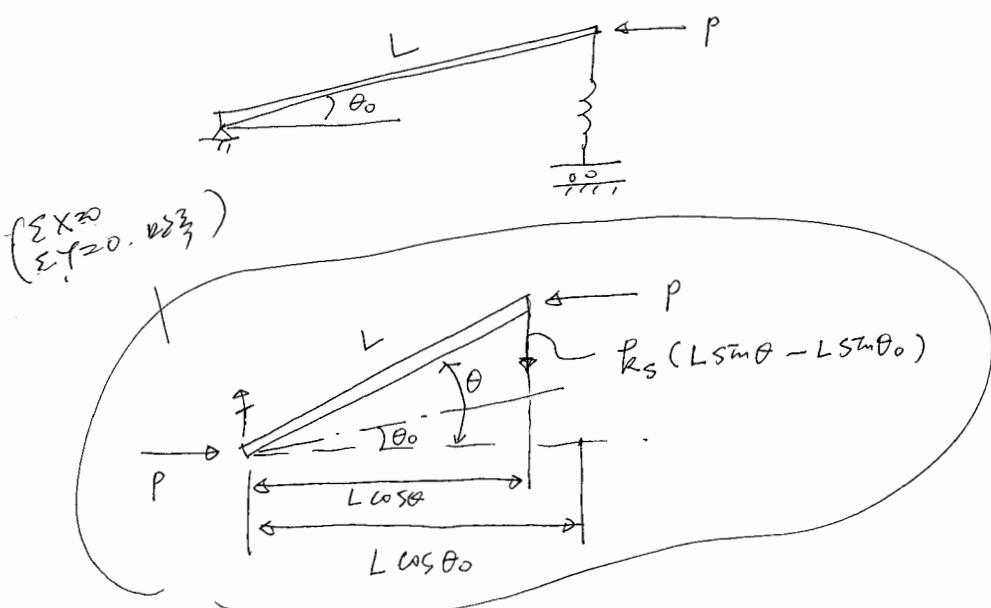
(자연) or (무한)의 흥미로운 비교를 해보면

이는 두 가지로, (*)에서 조건은 항상 만족이 된다.

따라서 Imperfect system의 흥미로운 결과는 항상 stable이다.

Fig. 1.21로도 自明

1.6.2 Rigid bar supported by a translational spring



(8)

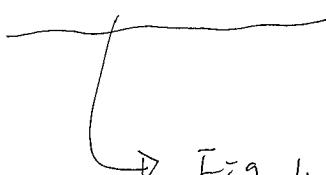
Energy approach

$$\Pi_L = \frac{1}{2} k_s \{ L(\sin\theta - \sin\theta_0) \}^2 - p L (\omega \sin\theta - \cos\theta)$$

For equilibrium,

$$\frac{d\Pi}{d\theta} = k_s L^2 (\sin\theta - \sin\theta_0) \cos\theta - p L \sin\theta = 0 \quad \dots (1.6.12)$$

$$\text{or } p = k_s L \cos\theta \left(1 - \frac{\sin\theta_0}{\sin\theta} \right) \quad \dots (1.6.13)$$



↑ 0.23.1 free body of

$$\sum M = 0 \quad \text{at } \theta_0$$

↑ 0.23.1 free body of

(P.35.1
Fig. 1.23.23)

* Fig. 1.23.1 P_{max} 23.2,

$$\frac{dp}{d\theta} = k_s L \left(-\sin\theta + \frac{\sin\theta_0}{\sin^2\theta} \right) = 0 \quad \text{only}$$

$$\sin\theta_0 = \sin^3\theta \rightarrow (1.6.13) \text{ as in 23.2}$$

$$\therefore P_{\max} = k_s L \cos\theta \left(\frac{1 - \sin^2\theta}{\sin^2\theta} \right)$$

$$= k_s L \cos^3\theta$$

the locus of the max. loads.

∴ θ_{max} =

$$\sin^{-1} \left(\sqrt[3]{\sin\theta_0} \right)$$

* The stability of equilibrium paths

$$(1.6.12) \text{ as 23.2}, \quad \frac{d^2\Pi}{d\theta^2} = \boxed{\text{positive}} \quad \text{P.23.1} \quad (1.6.13) \text{ as 23.2}$$

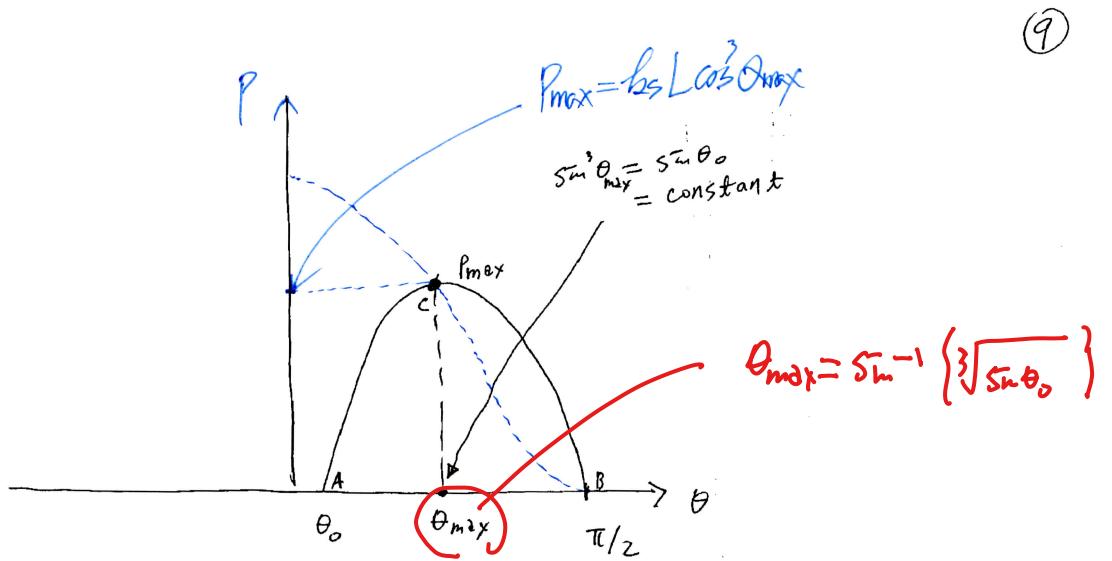
$$= k_s L^2 \left(\frac{\sin\theta_0 - \sin^3\theta}{\sin\theta} \right)$$

when $(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2})$ being $\frac{\sin\theta_0 - \sin^3\theta}{\sin\theta} > 0$ 0.23

equilibrium path is stable for $\theta \neq 0$. why?

↳ (rising equilibrium path is stable)
falling " " unstable)

(P_{max} 23.2
θ_{max} 23.2
θ_{min} 23.2
θ₀ 23.2)



("AC branch (rising branch) only if $\sin^3 \theta < \sin^3 \theta_0$ of which \rightarrow stable
 $\theta_0 \leq \theta \leq \theta_{max}$
 "CB branch (Falling branch) only if $\sin^3 \theta > \sin^3 \theta_0$ of which \rightarrow unstable
 $\theta \geq \theta_{max}$

Also graphically evident!