457.562 Special Issue on River Mechanics (Sediment Transport)
.04 Sediment Properties



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Suspended particles

1. Introduction

- As noted in the previous lecture, the solid phase of the pr oblem embodied in sediment transport can be any granul ar substances.
- In terms of geophysical application, however, the granula r substance in question typically consists of fragments ult imately derived from rocks – hence the name “ sediment transport
- Important properties of particles,
  - Types
  - Size distributions
  - Porosity
  - Shape

2. Rock Types

- The most common rock type (in the river or coastal envir onment) is quartz
  - Highly resistant rock, can travel long distances or remain i n place for long periods of time without losing its integrity
  - Another highly resistant one is feldspar
- Limesotone
  - Not a resistant, tends to abrade to silt rather easily, silt-size d limestone particles are susceptible to solution.
  - It cannot be found far from the source (mountain area)
- Basaltic rocks: Volcanic, heavier.
  - Much less common
  - Granite

3. Model Laboratory Sediments

- How about Han-river? slope, bankfull depth, width, and t he size of sediment?
- Can you reduce size of it? What based on?
  - Froude Number and Reynolds number
- What do you think about sediment? Do you think that it c an move?
- 5μm sediment (reduced size of boulder (0.5m in real fiel d)) cannot move
  - It is almost cohesive, viscous effects can be expected to b e greatly exaggerated due to the small size.
  - The net result is a model sediment that is much less mobil e that it ought to be and, in addition, behaves in ways radic ally different from the prototype

3. Model Laboratory Sediment

- Artificial sediment with a low specific gravity
  - Let ρ denote the density of water, and ρ_s denote the specif ic gravity of the material in question.
  - The weight W of a particle of volume V_p is given by

$$W = \rho_s g V_p$$

  - Where g is the acceleration of gravity.
- Example
  - Coal (γ:=1.3) for quartz (γ=2.65)
  - The coal grain would be only 1.3/2.65=0.49 times the weig ht of the quartz grain. (so almost twice mobile)

3. Model Laboratory Sediment

- But, the effective weight determining the mobility of a grain is the submerged weight W_s, actual weight minus the buoy ant force associated with the hydrostatic pressure distributi on about particle.
- R is the submerged specific gravity.

$$W_s = (\rho_s - \rho) g V_p = \rho R g V_p$$

$$\text{Where } R = \left(\frac{\rho_s}{\rho} - 1 \right)$$

- Comparing coal and quartz,

$$\frac{(W_s)_{coal}}{(W_s)_{quartz}} = \frac{(R)_{coal}}{(R)_{quartz}} = \frac{0.30}{1.65} = 0.18$$

- It follows that under water, the coal grain is 1/0.18=5.5 time s lighter than a quartz grain of the same size.

- But, still hard to realize the sediment in the laboratory.

4. Specific Gravity

- Some typical specific gravities for various natural and arti ficial sediments.

Rock type or material	Specific gravity ρ_s/ρ
quartz	2.60 ~ 2.70
limestone	2.60 ~ 2.80
basalt	2.70 ~ 2.90
magnetite	3.20 ~ 3.50
plastic	1.00 ~ 1.50
coal	1.30 ~ 1.50
walnut shells	1.30 ~ 1.40

5. Size

- D in this course denotes sediment size, the typical units of which are millimeters or microns.
- Another standard way of classifying grain sizes is the se dimentological Φ scale, according to which

$$D = 2^{-\Phi}$$

$$\Phi = -log_2\left(D\right) = -\frac{\ln(D)}{\ln(2)}$$

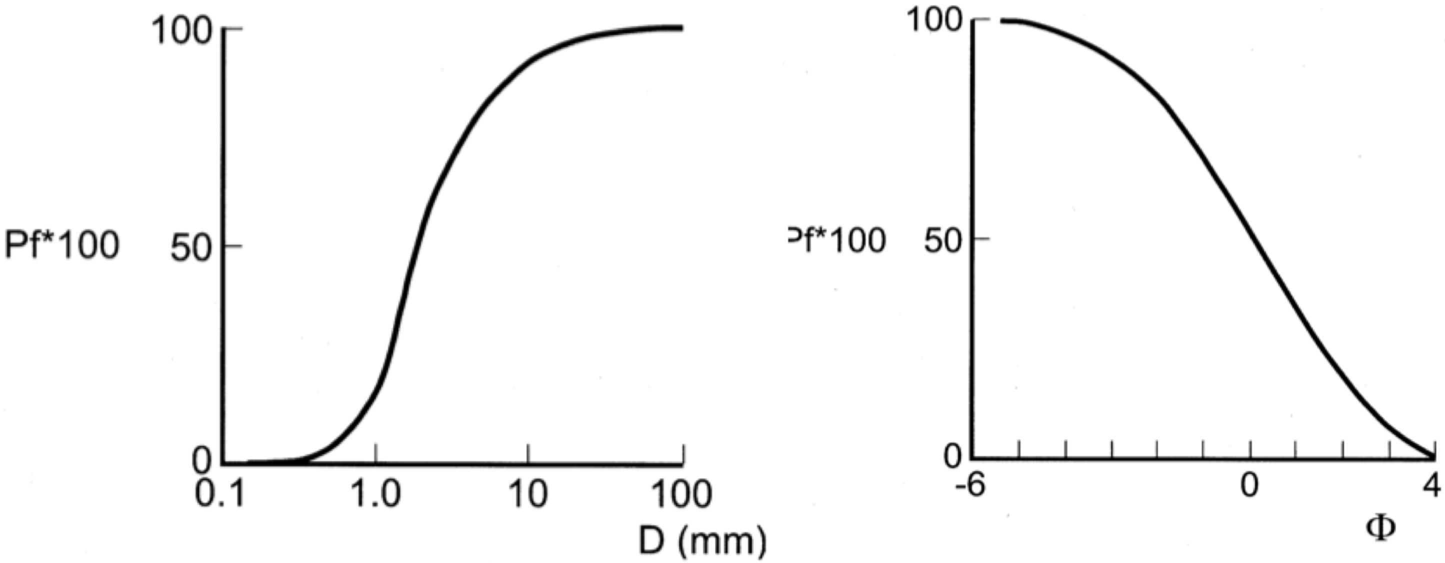
- Note that Φ=0 corresponding to D=1mm.
- It is convenient for the finer material description.
- Negative means larger.

Φ scale

Class Name	Size Range				Approximate Sieve Mesh Openings per Inch	
	Millimeters	Φ	Microns	Inches	Tyler	U.S. standard
Very large boulders	4,096 ~ 2,048			160 ~ 80		
Large boulders	2,048 ~ 1,024			80 ~ 40		
Medium boulders	1,024 ~ 512			40 ~ 20		
Small boulders	512 ~ 256	−9 ~ −8		20 ~ 10		
Large cobbles	256 ~ 128	−8 ~ −7		10 ~ 5		
Small cobbles	128 ~ 64	−7 ~ −6		5 ~ 2.5		
Very coarse gravel	64 ~ 32	−6 ~ −5		2.5 ~ 1.3		
Coarse gravel	32 ~ 16	−5 ~ −4		1.3 ~ 0.6		
Medium gravel	16 ~ 8	−4 ~ −3		0.6 ~ 0.3	2 ~ 1/2	
Fine gravel	8 ~ 4	−3 ~ −2		0.3 ~ 0.16	5	5
Very fine gravel	4 ~ 2	−2 ~ −1		0.16 ~ 0.08	9	10
Very coarse sand	2.000 ~ 1.000	−1 ~ 0	2,000 ~ 1,000		16	18
Coarse sand	1.000 ~ 0.500	0 ~ 1	1,000 ~ 500		32	35
Medium sand	0.500 ~ 0.250	1 ~ 2	500 ~ 250		60	60
Fine sand	0.250 ~ 0.125	2 ~ 3	250 ~ 125		115	120
Very fine sand	0.125 ~ 0.062	3 ~ 4	125 ~ 62		250	230
Coarse silt	0.062 ~ 0.031	4 ~ 5	62 ~ 31			
Medium silt	0.031 ~ 0.016	5 ~ 6	31 ~ 16			
Fine silt	0.016 ~ 0.008	6 ~ 7	16 ~ 8			
Very fine silt	0.008 ~ 0.004	7 ~ 8	8 ~ 4			
Coarse clay	0.004 ~ 0.0020	8 ~ 9	4 ~ 2			
Medium clay	0.0020 ~ 0.0010		2 ~ 1			
Fine clay	0.0010 ~ 0.0005		1 ~ 0.5			
Very fine clay	0.0005 ~ 0.00024		0.5 ~ 0.24			

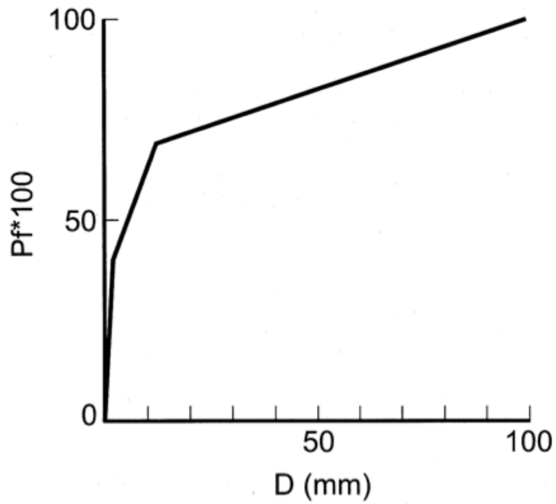
SOURCE: Adapted from Vanoni, 1975.

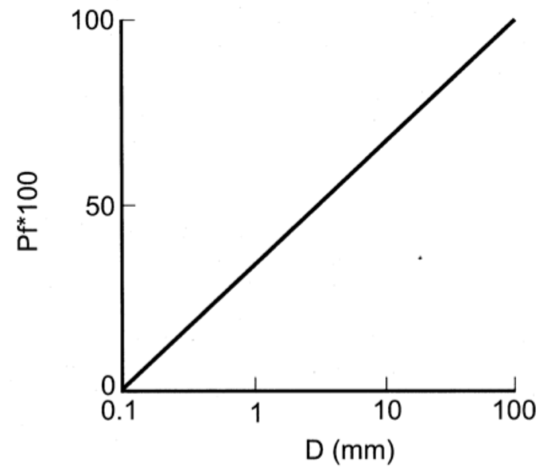
6. Size distribution



- The utility of a logarithmic scale (phi) for grain size now b ecomes apparent. Consider a sediment sample in which one-third of the material lies in the range 0.1~1.0mm, on e-third lies in the range 1.0~10 mm, and one-third lies in the range 10~100 mm.

6. Size distribution





- Left one is linear and light one is logarithmic.
- Left one is virtually unreadable as finest two ranges are c rowded off the scale.
- Right one provides a useful and consistent characterizati on of the distribution.
- Now, in the statistics, Φ is rather than D.

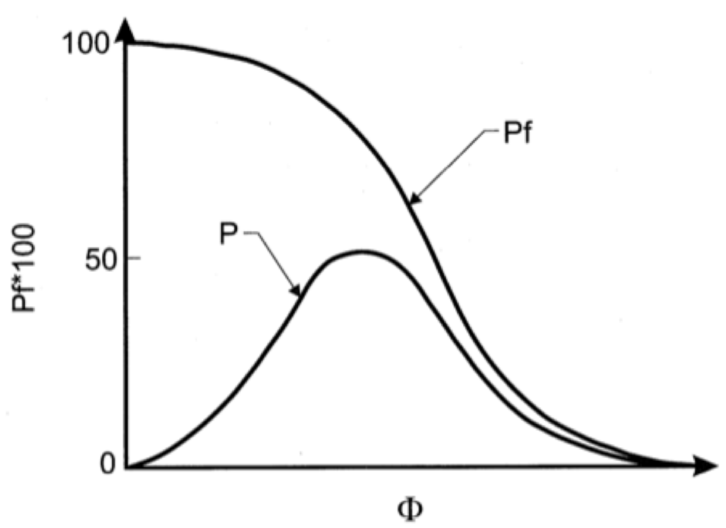
6. Size distribution

- The size density of a heterogeneous sample $p(\Phi)$ is defined such $p(\Phi)d\Phi$ denotes the fraction of material in the size range $(\Phi, \Phi + d\Phi)$

$$p(\Phi)d\Phi = p_f(\Phi + d\Phi) - p_f(\Phi)$$

$$p(\Phi) = \frac{dp_f}{d\Phi}$$

- The size distribution $p_f(\Phi)$ and size density $p(\Phi)$ by weight can be used to extract useful statistics concerning the sediment in question



6. Size distribution

- X denotes some percentage, (e.g., 50%); the grain size Φ_x denotes the size such that x% of the weight of the sample is composed of finer grains.
- That is

$$p_f(\Phi_x) = \frac{x}{100}$$

- The corresponding grain size

$$D_x = 2^{-\phi_x}$$

- Common grain size for using is D_{50}, and D_{90} (the sample by weight consists of finer grains) used for characterizing bed roughness.

6. Size distribution

- Size density can be used to extract statistical moments.
- 1st moment, 2n moment…..

$$\phi_m = \int \phi p(\phi) d\phi \qquad \sigma^2 = \int (\phi - \phi_m)^2 p(\phi) d\phi$$

- The corresponding geometric mean diameter and geome tric standard deviation are given as

$$D_g = 2^{-2\phi_m} \qquad \sigma_g = 2^{\sigma}$$

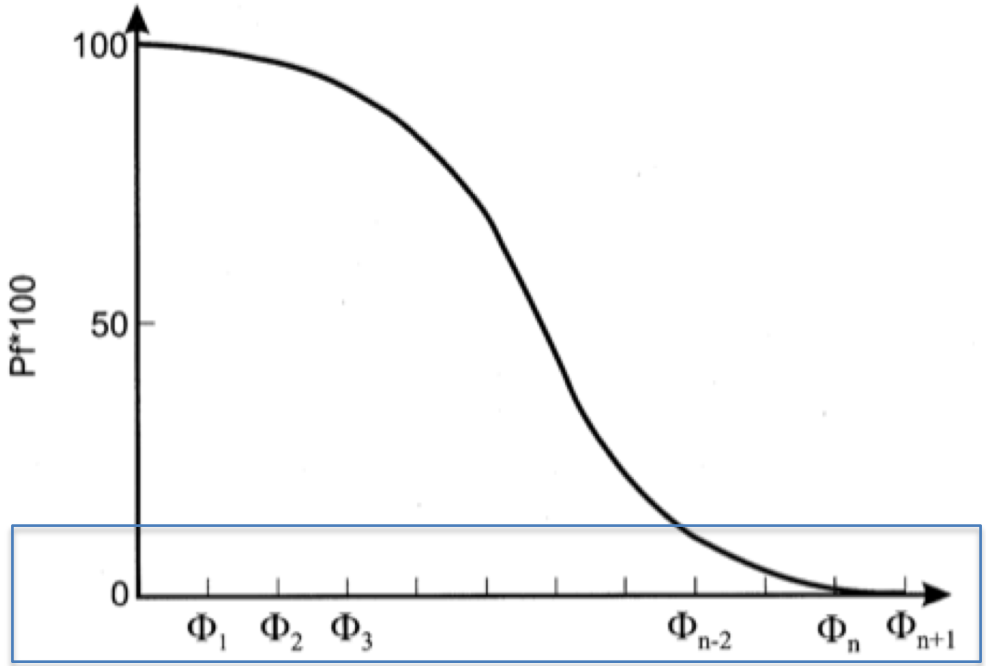
- A perfectly uniform material, σ=0 and σ_g=1.

$\sigma_g < 1.3$ Can be treated as "uniform material"

$\sigma_g > 1.6$ Can said to be "poorly sorted"

6. Size distribution

- The continuous function must be fascinate, but we canno t get this function by measurement or sampling.
- Generally all data should be discretized



6. Size distribution

- From *i* to *n*: $\bar{\phi}_i = \frac{1}{2}\left(\phi_i + \phi_{i+1}\right)$

  $$P_i = P_f\left(\phi_i\right) - P_f\left(\phi_{i+1}\right)$$

- Discretize: $\phi_m = \int \phi p(\phi) d\phi \qquad \sigma^2 = \int \left(\phi - \phi_m\right)^2 p\left(\phi\right) d\phi$

  $$\phi_m = \sum_{i=1}^{n} \bar{\phi}_i p_i \qquad \sigma^2 = \sum_{i=1}^{n} \left(\bar{\phi}_i - \phi_m\right)^2 p_i$$

- For a perfectly Gaussian distribution, the mean and medi an sizes coincide:

  $$\phi_m = \phi_{50} = \frac{\phi_{84} + \phi_{16}}{2} \qquad \phi_m = \frac{\phi_{84} + \phi_{16}}{2} \quad and \quad \sigma = \frac{\phi_{84} - \phi_{16}}{2}$$

- Why? 84 and 16?

6. Size distribution

- Rearranging the previous relations with

$$D_x = 2^{-\phi_x} \qquad D_g = 2^{-2\phi_m} \qquad \sigma_g = 2^{\sigma}$$

$$\sigma_g = \left(\frac{D_{84}}{D_{16}} \right)^{1/2} \qquad D_g = \left(D_{84} D_{16} \right)^{1/2}$$

- Keep in mind that the above equations only for the Gaus sian distribution not works for natural field.

6.0 Grain sizes

- The log normal probability density function is given by *f(ϕ)*

$$f(\phi)=\frac{1}{\sigma_\phi\sqrt{2\pi}}\exp\left[-\frac{(\phi-\mu_\phi)^2}{2\sigma_\phi^2}\right]$$

  - When μ_ϕ is the mean grain size in phi unit
  - σ_ϕ is the standard deviation in size

$$p[\phi<\phi_g]=\int_{-\infty}^{\phi_g} f(\phi)d\phi$$

    - The probability that a sand size is coarser than a give size ϕ_g

- Statistics of the sand sizes
  1. Mean
  2. Standard deviation (STD)
  3. Skewness
  4. Kurtosis

6.0 Grain sizes (mean)

- One common measure of sand sample is d_{50} or ϕ_{50}
  - Directly from the cumulative distribution curve
  - According to normal probability theory, 68 % of all sizes will be within plus or minus one STD of the mean
  - Therefore phi sizes of ϕ_{84} and ϕ_{16} are important (= $\phi_{50+64/2}$)

$$M_{d\phi} = \frac{(\phi_{84} + \phi_{16})}{2} \qquad (mean \quad diameter)$$

$$M_{d\phi} = \frac{(\phi_{84} + \phi_{50} + \phi_{16})}{3} \qquad (bi-\mathrm{mod}\,al \quad diameter)$$

6.0 Grain sizes (STD)

- Sorting
  - Poorly sorted sample -> wide range of size
  - Well sorted sample -> all most same size
  - Numerical measure of sorting is STD (σ_ϕ)

$$\sigma_\phi = \frac{(\phi_{84} - \phi_{16})}{2}$$

    - Poorly graded -> well sorted
    - Well graded -> Poorly sorted

  - Perfectly sorted sand : Homogeneous in size

$$\phi_{84}=\phi_{16}, \quad \sigma_\phi=0$$

  - In realistic σ_ϕ<0.5 is well sorted and >1 is poorly sorted

6.0 Grain sizes (Skewness)

- Sediment size distribution, symmetry
  - Negative skewness:
    - Small phi size -> larger grain sizes
    - Indicator of an erosive environment
  - Positive skewness
    - Depositional environment

$$\alpha_{d\phi} = \frac{M_{d\phi} - \phi_{50}}{\sigma_{\phi}}$$

$$M_{d\phi} = \frac{(\phi_{84} + \phi_{16})}{2}$$

6.0 Grain sizes (Kurtosis)

- Kurtosis - Peakedness

$$\beta_\phi = \frac{(\phi_{16} - \phi_5) + (\phi_{95} - \phi_{84})}{2\sigma_\phi}$$

  - For normal distribution β_ϕ=0.65
  - If spread out wider than the normal distribution, the Kurtosis will be less than 0.65

6.0 Grain sizes (Example)

%	d(mm)	ϕ
16	0.4	1.32
50	0.32	1.64
84	0.27	1.89

- D_{50} = 0.32mm or 1.64 ϕ (표 참조)
- Mean grain size $$M_{d\phi} = \frac{\phi_{16} + \phi_{84}}{2} = \frac{1.89 + 1.32}{2} = 1.61\phi$$
- Sorting (well sorted) $$\sigma_{\phi} = \frac{\phi_{84} - \phi_{16}}{2} = 0.285\phi$$
- Skewness $$\alpha_{\phi} = \frac{M_{\alpha\phi} - \phi_{50}}{\sigma_{\phi}} = \frac{1.61\phi - 1.64\phi}{0.285\phi} = -0.081$$
  - Larger grain sizes are slightly more prevalent

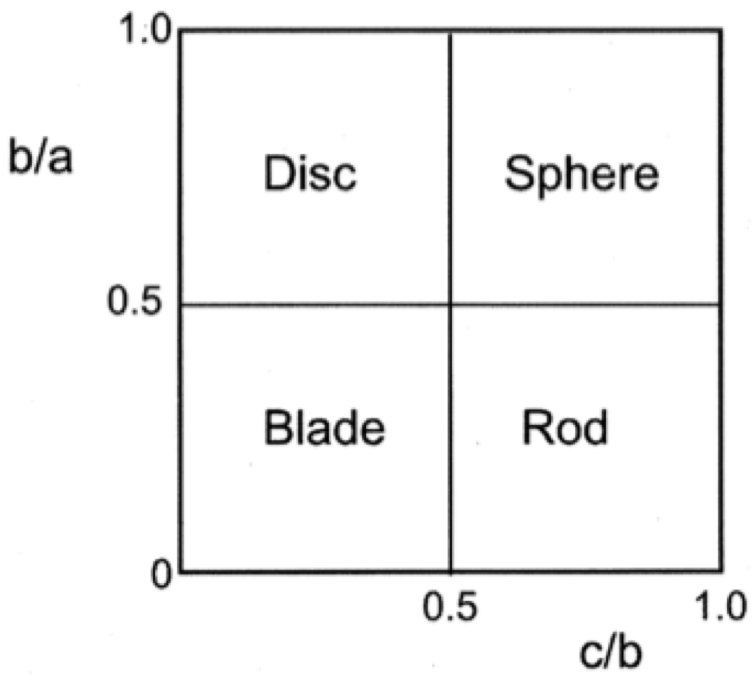
7. Porosity

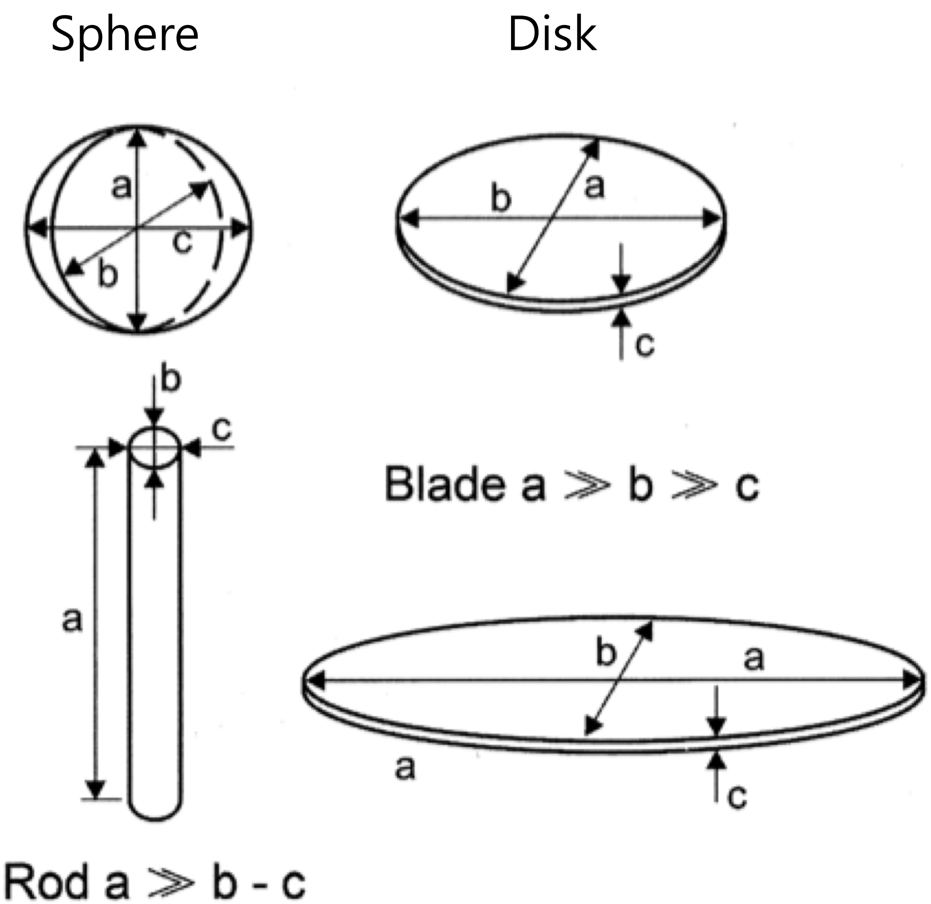
- The porosity quantified the fraction of a given volume of sediment which is composed of void space. That is

$$\lambda_p = \frac{\text{Volume of voids}}{\text{Volume of total space}}$$

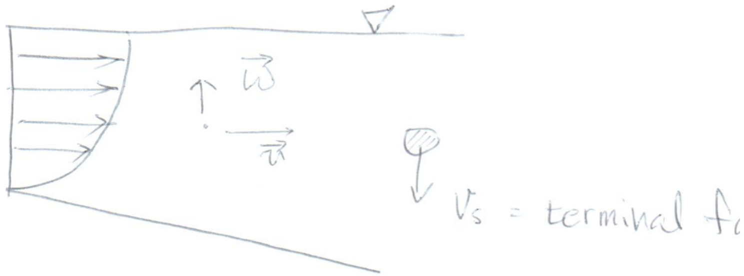
- In the case of well-sorted sand, the porosity can often ta ke values between 0.3 and 0.4. Gravels tend to be more poorly-sorted.
- Consolidation depends on the porosity.
- Porosity is important for ecological issue as like as habit at.

8. Shape


1.0
b/a
Disc
Sphere
0.5
Blade
Rod
0
0.5
1.0
c/b


Sphere
Disk
a
b
c
Blade a ≫ b ≫ c
Rod a ≫ b - c

1. Fall velocity



$$w_p = \vec{w} - V_s$$

$$\text{Drag}=f\left(v_s, D_s, \nu, \rho_s, \rho\right)$$

Where

v_s = Fall velocity

ρ = Fluid density

ρ_s = Sediment density

ν = Kinematic viscoisty of fluid

D_s = Diameter

- Dimensional analysis (pi theorem):
  - Six parameters – 3 dimensions = 3 dimensionless group

1. Fall velocity

- Choose three repeated variables

$$\left(\rho, D_s, v_s\right) = \left(\left[\frac{M}{L^3}\right], L, \left[\frac{L}{T}\right]\right) \Rightarrow (Mass, Length, Time)$$

- First dimensionless number

$$\pi_1 = f_1\left(\rho, D_s, v_s, f_D(drag\ force)\right)$$

$$M^0 L^0 T^0 = [\rho]^a [D_s]^b [v_s]^c [f_D]^d = \left(M^a L^{-3a}\right) L^b \left(L^c T^{-c}\right)\left(M^d L^d T^{-2d}\right)$$

$$= M^{a+d} L^{-3a+b+c+d} T^{-c-2d}$$

$$a + d = 0, \quad -3a + b + c + d = 0, \quad -c - 2d = 0$$

$$a = -d, \quad b + c = 4a, \quad c = 2a, \quad b = 2a$$

$$\pi_1 = [\rho]^a [D_s]^{2a} [v_s]^{2a} [f_D]^{-a} = f_1\left(\frac{f_D}{\rho v_s^2 D_s^2}\right)$$

1. Fall velocity

- Second dimensionless number

$$\pi_2 = f_2\left(\rho, D_s, v_s, \rho_s\right)$$

$$\pi_2 = f\left(\rho_s / \rho\right)$$

- Third dimensionless number

$$\pi_3 = f_3\left(\rho, D_s, v_s, \nu\right)$$

$$M^0 L^0 T^0 = [\rho]^a [D_s]^b [v_s]^c [\nu]^d = L^b\left(L^c T^{-c}\right)\left(L^{2d} T^{-d}\right)$$

$$= L^{b+c+2d} T^{-c-d} \qquad c = -d, \quad b = -d$$

$$\pi_3 = [D_s]^d [v_s]^d [\nu]^{-d} = f_3\left(\frac{D_s v_s}{\nu}\right) = f_3\left(\mathrm{R}_p\right)$$

1. Fall velocity

- Then,

$$f\left(\frac{f_D}{\rho v_s^2 D_s^2}, \frac{\rho_s}{\rho}, \frac{D_s v_s}{\nu}\right) = 0$$

- We already know that

$$f_D = \frac{1}{2} C_D \rho \frac{\pi D_s^2}{4} v_s^2, \qquad R = \frac{\rho_s - \rho}{\rho}, \qquad \mathrm{R}_p = \frac{D_s V_s}{\nu}$$

- So, $f\left(C_D\left(\mathrm{R}_p\right), R\right) = 0$

1. Fall velocity

- The fall velocity o sediment grains in water is determined by their diameter and density and by the viscosity of the water.
- Terminal velocity: fall velocity becomes constant when th e drag equals the submerged weight of the particle.

$$\frac{1}{2} C_D \rho \frac{\pi D_s^2}{4} v_s = \frac{4}{24} \pi D_s^3 \left(\rho_s - \rho\right) g$$

- Therefore, (Confirmed by dimensional analysis)

$$v_s = \sqrt{\frac{4}{3}\left(\frac{\rho_s - \rho}{\rho}\right)\frac{gD_s}{C_D}} = \sqrt{\frac{4}{3}\frac{gRD_s}{C_D}}$$

$$C_D = f(\mathrm{R}_p) = f\left(\frac{v_s D_s}{\nu}\right)$$

1. Fall velocity

- In laminar flow, $C_D = \dfrac{24}{R_p}$
- For very fine particles

$$v_s = \sqrt{\frac{4}{3} v_s \frac{gR}{24} \frac{D_s^2}{\nu}}$$

$$\frac{v_s^2}{v_s} = v_s = \frac{gRD_s^2}{18\nu}$$ (Stokes' law for settling rate)

$$C_D = \frac{24}{R_p}\left(1 + 0.152 R_p^{1/2} + 0.0151 R_p\right)$$

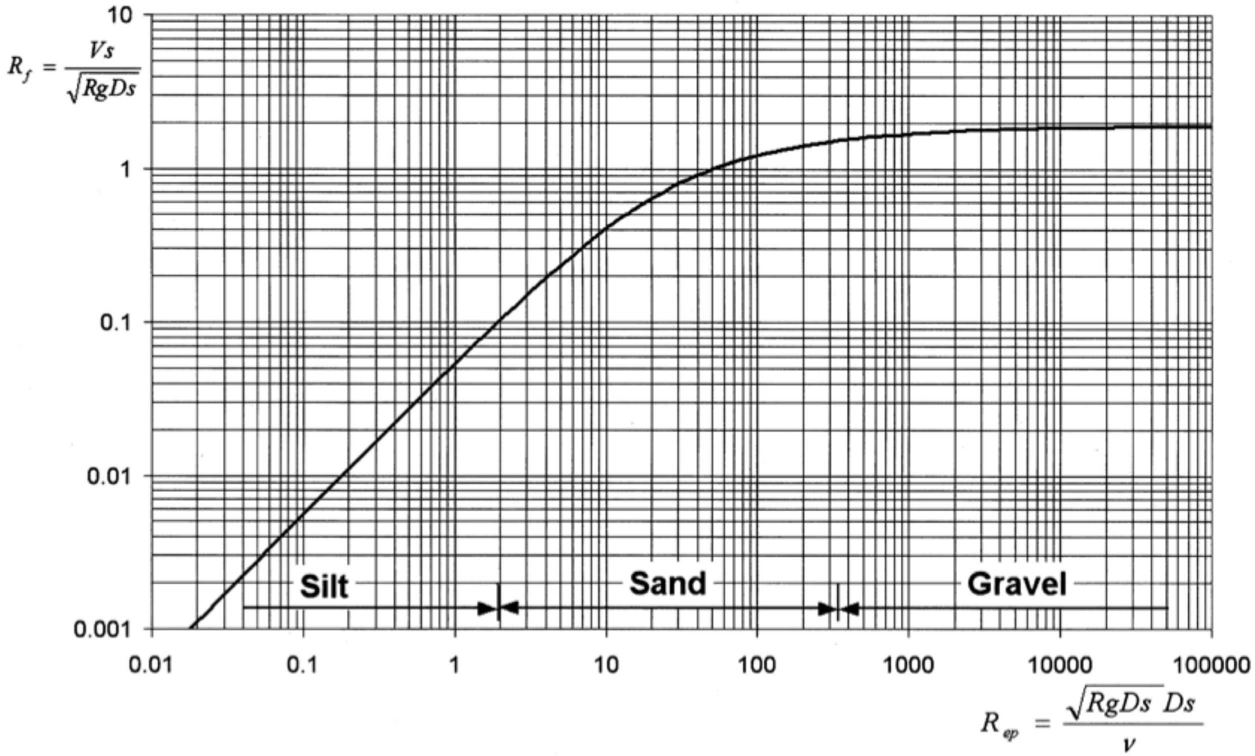
(More general form)

- In general

$$R_p = \frac{v_s D_s}{\nu} = \frac{v_s}{\sqrt{gRD_s}} \cdot \frac{\sqrt{gRD_s} D_s}{\nu}$$

$$= R_f \cdot R_{ep}$$

1. Fall velocity



- Diagram of R_f versus R_{ep} calculated from the drag coefficient for spheres.

1. Fall velocity

- A useful empirical relation to estimate the kinematic visc osity of clear water

$$v = \frac{1.79\times 10^{-6}}{1+0.03368T+0.00021T^2}(m^2/s)$$