

# Force and Moment Coefficients for Vibrating Aerofoils in Cascade

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*Summary.* This Report gives a method for calculating the aerodynamic forces and moments acting on unstalled vibrating cascade blades. The forces and moments due to both bending and torsional vibration are calculated, and also due to wakes from periodic obstructions far upstream which move relative to the cascade in question. The calculation has been programmed for EDSAC II, the electronic computer at the Cambridge University Mathematical Laboratory, and the essential formulae are summarized in an Appendix, in order to facilitate the preparation of programmes for other machines. The Report includes skeleton tables of the force and moment coefficients for two space/chord ratios. The coefficients can be used to predict the occurrence of pure torsional or coupled flutter and the vibration induced by periodic disturbance in the flow.

1. *Introduction.* Blade vibration in axial compressors and turbines has been one of the most troublesome problems in the development of gas-turbine engines. In practice the trouble usually occurs when the blade row in question is stalled, but in order to achieve an understanding and control of blade vibration under all conditions it appears to be essential to start with a study of the problem under the much simpler flow conditions when the blades are not stalled. The basic problem of the theory of blade vibration is then to calculate the aerodynamic forces and moments acting on the blades, if they are presupposed to be vibrating in a given manner. When this has been done, the conditions under which self-excited vibration of the blades will occur can be predicted, and the related problem of predicting the vibration which will be forced by given pulsating disturbances in the air flow can also be solved.

In this Report, therefore, the aerodynamic forces and moments for a cascade of vibrating blades will be calculated, subject to the following simplifying assumptions:

- (a) The system considered is two-dimensional, so that the bending modes of actual blades are represented by a translational motion of the two-dimensional aerofoils, and the torsional modes of actual blades are represented by rotation of the two-dimensional aerofoils about a known axis. Only translational motion perpendicular to the chord line has been considered, since motion parallel to the chord line gives rise to second order effects only.
- (b) The fluid is assumed to be incompressible and inviscid.

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- (c) It is assumed that the blades do not stall, so that the flow always follows the blade surface.
- (d) Effects due to blade camber and thickness are neglected so that the blades are assumed to be flat plates.
- (e) It is assumed that the blades operate at zero mean incidence, so that the mean deflection is zero.
- (f) The amplitude of vibration is assumed to be small. It follows that the wakes of the blades, which are vortex sheets in which the strength varies sinusoidally with distance from the trailing edge, can be taken to be straight. It also follows that the theory becomes linear, so that results for any two types of vibration can be superposed to give a third type of vibration.
- (g) It is assumed that all blades vibrate with the same amplitude, and with a constant phase angle between one blade and the next. This involves no loss of generality, since any required motion of the blades can be obtained by superposing solutions of the type considered.

The theory of vibrating flow subject to the above assumptions has been gradually built up by several authors. Theodorsen<sup>16</sup> gave the solution for a single aerofoil, which is the limit for cascades as the spacing of the blades becomes large. The case of zero stagger cascades in which adjacent blades vibrate out of phase has been studied by Reissner<sup>12</sup>, Tinman<sup>17</sup> and Lilley<sup>9</sup>. In this case the stream lines halfway between the blades are lines of symmetry and therefore do not change position during the vibration so that they may be replaced by solid walls. This therefore corresponds to the case of a single aerofoil vibrating in a wind tunnel and was studied for this reason.

The case of zero stagger cascades with all blades vibrating in phase has been studied by Billington<sup>1</sup>, Mendelson and Carroll<sup>11</sup>, Woods<sup>21</sup> and by Chang and Chu<sup>2</sup>.

Sisto<sup>14</sup> has given solutions for zero stagger cascades with a general phase angle between adjacent blades. Lane and Wang<sup>7</sup> have given a general method of calculation applicable to cascades of any stagger and any phasing angle, and give numerical results for a cascade with 45 deg stagger. Legendre<sup>8</sup> has given a method for cascades of any stagger when adjacent blades vibrate out of phase, and calculations for a 45 deg stagger cascade have been carried out by this method by Eichelbrenner<sup>3</sup>. Jones<sup>5</sup> has considered the case of cascades with large spacing and very high stagger (nearly 90 deg), this being the case of interest in helicopter blade vibration.

The case of zero stagger with adjacent blades vibrating out of phase has been extended to subsonic compressible flow by Woolston and Runyan<sup>22</sup> and by Runyan, Woolston, and Rainey<sup>13</sup>. A method for subsonic compressible flow with general stagger angle and phasing angle has been given by Lane and Friedman<sup>6</sup>. The results show a resonance phenomenon, similar to an acoustic resonance.

The method of calculation to be used in this Report owes much to the previous reports of Lilley<sup>9</sup>, Lane and Wang<sup>7</sup> and Lane and Friedman<sup>6</sup>.

*2. General Method of Calculation. 2.1. Vorticity.* Since the blades are assumed to be flat plates, and the wakes from the blades can be taken to be straight lines extending from the trailing edges of the blades, both the blades and their wakes can be regarded as vortex sheets. The basic approach will be to find the distribution of vorticity along the blade which induces the correct upwash velocities normal to the blade surface along the chord of the blade.

Consider therefore an element of vorticity  $\gamma dx e^{i\omega t}$  on the reference blade at a distance  $x$  from the origin at the leading edge, as shown in Fig. 1. On the next blade above this everything is advanced

in phase by an angle  $\beta$ , and so this blade will carry a corresponding element of vorticity  $\gamma e^{i\omega t - i\beta}$  at the point  $(x + s \sin \xi, s \cos \xi)$ . In general, the  $m$ th blade will carry an element of vorticity

$$\gamma dx e^{i\omega t - i\beta}$$

at the point

$$x + ms \sin \xi, ms \cos \xi.$$

Since the strength of these elements of vorticity varies sinusoidally with time, associated with each there will be a vortex sheet shed from the element, and extending a great distance downstream. The strength of this vortex sheet will be denoted by  $\epsilon e^{i\omega t}$ . Following Lilley<sup>9</sup>, the vorticity  $\gamma$  will be referred to as the bound vorticity, and the vorticity  $\epsilon$  will be referred to as the free vorticity. The free vorticity is being continually washed downstream at the speed of the mainstream,  $U$ . Considering then the reference blade and its wake, the strength of the free vorticity due to an element  $\gamma dx e^{i\omega t}$  of bound vorticity is given at a point  $(x_1, 0)$  by an expression of the form

$$\epsilon e^{i\omega t} = \text{Constant } e^{i\omega t - x_1 U} \quad (1)$$

During a small time interval  $\delta t$  the strength of the element of bound vorticity changes by an amount

$$\gamma dx e^{i\omega t} i\omega \delta t.$$

This is equal in magnitude and opposite in sign to the free vorticity created in the time interval  $\delta t$ . Also during this time interval the sheet of free vorticity moves back a distance  $U \delta t$ . Hence the strength of the sheet of free vorticity just behind the element of bound vorticity at  $(x, 0)$  is given by

$$-\frac{\gamma dx e^{i\omega t} i\omega \delta t}{U \delta t}$$

determines the constant in (1), and the free vorticity is given by

$$\epsilon = -\gamma dx \frac{i\omega}{U} e^{i\omega(x - x_1) U} \quad (2)$$

The total free vorticity at  $(x_1, 0)$  is obtained by summing up the contributions from all the elements of bound vorticity from  $x = 0$  to  $x = x_1$ . This gives

$$\epsilon = -\frac{i\omega}{U} \int_0^{x_1} \gamma e^{i\omega(x - x_1) U} dx$$

The total vorticity is therefore  $(\gamma + \epsilon)$  on the reference blade, with corresponding vorticity on the other blades. In the wakes,  $\gamma$  is zero.

A useful differential relation between  $\gamma$  and  $\epsilon$  can be obtained by writing this

$$\epsilon e^{i\omega x_1 U} = -\frac{i\omega}{U} \int_0^{x_1} \gamma e^{i\omega x U} dx$$

Then differentiation with respect to  $x_1$  gives

$$\frac{d\epsilon}{dx_1} e^{i\omega x_1 U} + \frac{i\omega}{U} \epsilon e^{i\omega x_1 U} = -\frac{i\omega}{U} \gamma(x_1) e^{i\omega x_1 U}$$

Or, writing  $x$  as the independent variable

$$\frac{d\epsilon}{dx} + \frac{i\omega}{U} (\epsilon + \gamma) = 0 \quad (3)$$

Next it is necessary to relate the bound vorticity to the pressure difference across the blade, and since the theory is to be used in cases when the whole flow contains vorticity, the velocity potential cannot be used in the usual way, since when there is vorticity it does not exist. The equation of motion in the  $x$  direction, linearized by neglecting small terms of second order is

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x}\right) (ue^{i\omega t}) = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$

Writing this for a point just below the blade, denoted by a suffix  $-$ , and for a point just above the blade, denoted by a suffix  $+$ , and subtracting the two equations gives

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x}\right) \left\{ (u_- - u_+) e^{i\omega t} \right\} = -\frac{1}{\rho} \frac{\partial}{\partial x} (p_- - p_+)$$

But  $u_- - u_+ = \gamma + \epsilon$ , being the total vorticity on the blade. Hence

$$\begin{aligned} -\frac{1}{\rho} \frac{\partial}{\partial x} (p_- - p_+) &= \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x}\right) \left\{ (\gamma + \epsilon) e^{i\omega t} \right\} \\ &= \left\{ i\omega(\gamma + \epsilon) + U \frac{d\gamma}{dx} + U \frac{d\epsilon}{dx} \right\} e^{i\omega t} \\ &= U \frac{d\gamma}{dx} e^{i\omega t} \end{aligned}$$

from (3). This can be integrated directly, and the constant of integration vanishes since both  $\gamma$  and  $(p_- - p_+)$  are zero off the blade. Hence

$$p_- - p_+ = -\rho U \gamma e^{i\omega t} \quad (4)$$

If the aerodynamic force acting upwards on the blade is  $F e^{i\omega t}$ , this is given by

$$F e^{i\omega t} = \int_0^c (p_- - p_+) dx$$

Hence

$$F = -\rho U \int_0^c \gamma dx \quad (5)$$

Similarly the aerodynamic moment  $M e^{i\omega t}$  acting anticlockwise about the leading edge is given by

$$M = -\rho U \int_0^c \gamma x dx \quad (6)$$

It is also necessary to consider conditions at the trailing edge of the blade. Corresponding to the Joukowski condition for steady flow, the required condition is that  $(\gamma + \epsilon)$  must be finite at the trailing edge. Then Equation (3) shows that  $\epsilon$  must be finite at the trailing edge, and therefore  $\gamma$  will also be finite at the trailing edge. At the leading edge, however,  $\gamma$  will be infinite, as in normal thin aerofoil theory for single aerofoils.

**2.2. Induced Velocity.** In Appendix I it is shown that the velocity normal to the blade surface ( $v e^{i\omega t}$ ) induced at a point  $(\eta, 0)$  by a row of vortices with the spacing ( $s$ ) stagger angle ( $\xi$ ) and phasing angle ( $\beta$ ) of the cascade in question (see Fig. 2) is given by an expression of the form

$$v = \frac{\Gamma_0}{c} V \left( \frac{\eta}{c} - \frac{x}{c} \right)$$

where  $\Gamma_0 e^{i\omega t}$  is the strength of the vortex on the reference blade or in its wake at  $(x, 0)$  and  $V(z)$  is a rather complicated non-dimensional complex function of  $z$  and also of  $s/c$ ,  $\xi$  and  $\beta$ .

om this point onwards, in order to simplify the equations slightly, it will be supposed that  $y$  is measured in units of chordal length, so that  $x = \eta = 1$  at the trailing edge of the end blade.

the normal velocity induced at a point  $(\eta, 0)$  by an element of bound vorticity  $\gamma dx e^{i\omega t}$  at  $(x, 0)$  on the reference blade, and corresponding elements on the other blades, is therefore given by

$$v(\eta) = \gamma(x) dx V(\eta - x) \quad (7)$$

associated with this is the free vorticity given by (2) which induces a velocity given by

$$v(\eta) = \int_x^\infty \epsilon(x_1) V(\eta - x_1) dx_1 \quad (8)$$

where the total effect is obtained by integrating over all the elements of free vorticity from  $x_1 = x$  where it starts at the position of the bound vorticity) to infinity. Special consideration of the case  $\omega = 0$  is required, since for this case  $V(z)$  does not tend to zero as  $z \rightarrow -\infty$  and the above integral does not converge but oscillates. To resolve this it is necessary to note that, assuming the system started from rest, the total vorticity on each blade and in its wake is zero

$$\int_0^\infty (\gamma + \epsilon) dx = 0$$

for an element of bound vorticity and its associated free vorticity this gives

$$\gamma(x) dx + \int_x^\infty \epsilon(x_1) dx_1 = 0$$

this equation is multiplied by the constant  $V(-\infty)$  and subtracted from the sum of Equations (7) and (8), the velocity induced by an element of bound vorticity and its associated free vorticity is given by

$$v(\eta) = \{V(\eta - x) - V(-\infty)\} \gamma dx + \int_x^\infty \epsilon(x_1) \{V(\eta - x_1) - V(-\infty)\} dx_1$$

where the integral will now always converge at the top limit. Substituting for  $\epsilon$  from (2) gives

$$v(\eta) = \{V(\eta - x) - V(-\infty)\} \gamma dx - \gamma dx \frac{i\omega}{U} \int_x^\infty e^{i\omega(x-x_1)/U} \{V(\eta - x_1) - V(-\infty)\} dx_1$$

putting  $z_1 = x_1 - \eta$  and  $\lambda = \omega c/U$  (where  $c = 1$ ) this is

$$v(\eta) = \gamma(x) dx \left[ \{V(\eta - x) - V(-\infty)\} - i\lambda \int_{x-\eta}^\infty e^{i\lambda(x-\eta-z_1)} \{V(-z_1) - V(-\infty)\} dz_1 \right]$$

This may be written

$$v(\eta) = \gamma(x) dx \cdot K(x - \eta) \quad (9)$$

where  $K$  is a function, to be called the Kernel function, of  $z = x - \eta$ , given by

$$K(z) = V(-z) - V(-\infty) - i\lambda e^{i\lambda z} \int_z^\infty e^{-i\lambda z_1} \{V(-z_1) - V(-\infty)\} dz_1 \quad (10)$$

Integrating Equation (9) for the total velocity induced by all the elements of bound vorticity along the blade chords gives

$$v(\eta) = \int_0^1 K(x - \eta) \gamma(x) dx \quad (11)$$

Since the Kernel function is infinite when  $x = \eta$ , it is the principal value of this integral which is required, and principal values are to be understood for all subsequent similar integrals.

This induced velocity must be equated to the actual upwash velocity. The aerodynamic forces and moments will be calculated for three cases. The first case is for pure translational motion of the blades, with velocity  $qe^{i\omega t}$ . In this case the induced velocity must be equal to the blade velocity, so that

$$v = q.$$

The second case is for torsional motion of the blades about an axis at the leading edge, the angular displacement being  $\alpha e^{i\omega t}$ . The air velocity normal to the blade surface then becomes

$$(\dot{\alpha} - \alpha U)e^{i\omega t}$$

This must be equal to the velocity of the blade itself, which at a point distance  $\eta$  from the axis is

$$\eta \frac{d}{dt} (\alpha e^{i\omega t}).$$

Equating these gives for the velocity induced in the second case

$$v = \alpha U(1 + i\lambda\eta).$$

The third case to be calculated is when the blades are operating in the wakes of some kind of periodic obstruction far upstream, which moves steadily parallel to the cascade under consideration. It is supposed that a Fourier analysis of the wake profile has been carried out, and one component is considered. The wake spacing and the relative velocity of motion of the obstruction determine the frequency  $\omega$  and phasing angle  $\beta$  of the disturbance felt by the cascade. Let the velocity which the disturbance would induce normal to the blade surface at the position of the leading edge of the reference blade, if the cascade were removed, be  $w e^{i\omega t}$ . Then since the whole disturbance is washed downstream at a velocity  $U$ , the velocity which the disturbance would induce at a point  $(\eta, 0)$  is given by

$$w e^{i\omega t - \eta U}.$$

Since the total velocity induced normal to the blade surface must be zero, this is equal and opposite to the velocity induced by the vorticity on the blades,  $v e^{i\omega t}$ . Hence

$$v = -w e^{-i\lambda\eta}.$$

If disturbances corresponding to all three cases are present together, the induced velocity is given by

$$v = q + \alpha U(1 + i\lambda\eta) - w e^{-i\lambda\eta} \quad (12)$$

The strength of the sheet of bound vorticity required to induce this is then given by

$$\gamma = q\gamma_q + \alpha U\gamma_z - w\gamma_w \quad (13)$$

Where  $\gamma_q$ ,  $\gamma_z$  and  $\gamma_w$  are the solutions of the following integral equations:

$$\int_0^1 [\gamma_q, \gamma_z, \gamma_w] K(x - \eta) dx = [1, (1 + i\lambda\eta), e^{-i\lambda\eta}] \quad (14)$$

These equations hold for  $0 < \eta < 1$ . Each of the expressions in square brackets is a matrix with one row and three columns. These equations have to be solved subject to the condition that  $\gamma$  is finite when  $x = 1$ . The aerodynamic forces and moments may then be obtained from Equations (5) and (6).

2.3 *Solution of the Integral Equations.* In order to solve the integral equations, a numerical method will be used. A transformation to new independent variables  $\theta$  and  $\phi$  is made, in the same way as in the usual thin aerofoil theory for single aerofoils.

$$\begin{aligned}x &= \frac{1}{2}(1 - \cos \theta) \\ \eta &= \frac{1}{2}(1 - \cos \phi)\end{aligned}$$

Making these substitutions in Equation (14) gives

$$\int_0^\pi K(\frac{1}{2} \cos \phi - \frac{1}{2} \cos \theta) [\gamma_q, \gamma_x, \gamma_w]_{\frac{1}{2}} \sin \theta d\theta = [1, \{1 + \frac{1}{2}i\lambda(1 - \cos \phi)\}, e^{-i\lambda(1 - \cos \phi)^2}] \quad (15)$$

Then the bound vorticity will be specified at  $(n+1)$  points, given by

$$\theta = \pi l/n$$

where  $l$  is an integer taking values from 0 to  $n$ . The value of  $\gamma$  when  $l = n$ , at the trailing edge, is, however, irrelevant since in Equation (15) it is always multiplied by  $\sin \theta$  which is zero. The same argument does not apply at the leading edge, since  $\gamma$  will become infinite there although the product  $\gamma \sin \theta$  remains finite. The  $n$  values of  $\gamma$  can therefore be found to satisfy the Equations (15) at  $n$  points only, and these will be chosen to be given by

$$\phi = \pi(2m+1)/2n$$

where  $m$  is an integer taking values from 0 to  $(n-1)$ . These points have values of  $\phi$  midway between the values of  $\theta$  at which the bound vorticity is specified.

It is next supposed that the integrals in Equations (15) can be evaluated by the trapezoidal rule. (This approximation is re-examined later and is found to be remarkably accurate, although a correction is needed to allow for the logarithmic singularity in  $K$ .) Then Equations (15) become

$$\begin{aligned}\sum_{l=0}^n K(\frac{1}{2} \cos \pi \overline{2m+1}/2n - \frac{1}{2} \cos \pi l/n) [\gamma_q, \gamma_x, \gamma_w]_{\frac{1}{2}} \sin \pi l/n \cdot \pi/n \\ = [1, \{1 + \frac{1}{2}i\lambda(1 - \cos \pi \overline{2m+1}/2n)\}, e^{-i\lambda(1 - \cos \pi \overline{2m+1}/2n)^2}]\end{aligned} \quad (16)$$

where the notation  $\sum_{l=0}^n$  indicates a summation over  $(n+1)$  terms, in which the first and last terms are to be taken with half weight. In this case the last term in the summation drops out, and the Equations (16) represent three sets of  $n$  simultaneous equations for the  $n$  unknowns,  $\gamma$ . Equation (16) may conveniently be written in matrix notation as

$$A\Gamma = B \quad (17)$$

where  $A$  is a  $n \times n$  square matrix in which the element in the  $l$ th column and  $m$ th row is

$$K(\frac{1}{2} \cos \pi \overline{2m+1}/2n - \frac{1}{2} \cos \pi l/n) \quad (18)$$

$\Gamma$  is a  $n \times 3$  matrix in which the  $l$ th row is

$$[\gamma_q, \gamma_x, \gamma_w] \times \frac{\pi}{2n} \sin \pi l/n \quad (19)$$

except for the first row which has half weight.

$B$  is also a  $n \times 3$  matrix in which the  $m$ th row is given by the right-hand side of Equation (16).

The formal solution of (17) is then

$$\Gamma = A^{-1}B \quad (20)$$

2.4. *Calculation of Force and Moment.* If Equation (13) for  $\gamma$  is substituted into Equation (5) the blade force is given by

$$F = -\rho U \int_0^c (q\gamma_q + xU\gamma_x - w\gamma_w) dx$$

This may be written in terms of non-dimensional force coefficients as follows

$$F = \pi\rho Uc(qC_{Fq} + xUC_{Fx} - wC_{Fw}) \quad (21)$$

where the coefficients are given by

$$[C_{Fq}, C_{Fx}, C_{Fw}] = -\frac{1}{\pi} \int_0^1 [\gamma_q, \gamma_x, \gamma_w] dx = -\frac{1}{\pi} \int_0^\pi [\gamma_q, \gamma_x, \gamma_w] \frac{1}{2} \sin \theta d\theta$$

This integral may be evaluated by the trapezoidal rule, like the integral for the induced velocity, giving

$$[C_{Fq}, C_{Fx}, C_{Fw}] = -\frac{1}{\pi} \sum_{l=0}^{n'} [\gamma_q, \gamma_x, \gamma_w] \frac{1}{2} \sin \pi l/n \cdot \pi/n \quad (22)$$

The aerodynamic moment may be treated in exactly the same way, being expressed as

$$M = \pi\rho Uc^2(qC_{Mq} + xUC_{Mx} - wC_{Mw}) \quad (23)$$

where

$$\begin{aligned} [C_{Mq}, C_{Mx}, C_{Mw}] &= -\frac{1}{\pi} \int_0^1 [\gamma_q, \gamma_x, \gamma_w] x dx = -\frac{1}{\pi} \sum_{l=0}^{n'} [\gamma_q, \gamma_x, \gamma_w] \cdot \times \\ &\times \frac{1}{2}(1 - \cos \pi l/n) \frac{1}{2} \sin \pi l/n \cdot \pi/n \end{aligned} \quad (24)$$

Equations (22) and (24) may be combined into a single matrix equation

$$C = -\frac{1}{\pi} X\Gamma \quad (25)$$

where

$$C = \begin{bmatrix} C_{Fq} & C_{Fx} & C_{Fw} \\ C_{Mq} & C_{Mx} & C_{Mw} \end{bmatrix} \quad (26)$$

and  $X$  is a  $2 \times n$  matrix whose  $l$ th column is given by

$$\begin{bmatrix} 1 \\ \frac{1}{2}(1 - \cos \pi l/n) \end{bmatrix} \quad (27)$$

Eliminating  $\Gamma$  from Equations (20) and (25) gives

$$C = -\frac{1}{\pi} XA^{-1}B \quad (28)$$

This is the required result for the matrix of force and moment coefficients, since  $X$ ,  $A$  and  $B$  are matrices with known elements.

In the preceding presentation the integrals occurring have been evaluated by the trapezoidal rule. In Appendix II it is found that this is correct for the force and moment integrals, but that a correction is required in the expression for the induced velocity integral, which has the effect of modifying the matrix  $A$ . This correction arises due to a logarithmic singularity in the Kernel function  $K$ .



A complication arises in the evaluation of the infinite integral which occurs in Equation (10). The series method used is given in Appendix III.

The calculation involves a very large amount of arithmetic, and this would only be practicable on a fast digital computer. In order to assist in the possible preparation of programmes, the required formulae are collected in Appendix IV.

3. *Accuracy and Convergence.* The method of calculation has been programmed for EDSAC II, a fast electronic digital computer at the Cambridge University Mathematical Laboratory. With  $n = 8$  the machine takes about 25 sec. to calculate a set of coefficients from given values of  $s/c$ ,  $\xi$ ,  $\beta$  and  $\lambda$ .

The accuracy obtained in the calculation depends on the value taken for  $n$ . The programme is written so that the value of  $n$  can be changed by merely altering the value of a 'preset parameter' at the head of the programme tape. It is therefore possible to check the accuracy obtained by running tests with the same data and increasing values of  $n$ , and examining the convergence of the coefficients so calculated to a limit. The convergence obtained is illustrated in the following table, which shows the value of  $n$  required to give four decimals correct in all coefficients.

$s/c$	$\xi$	$\beta/2\pi$	$\lambda$	$n$
1.0	0	$2^{-30}$	0	4
1.0	0	0.5	1.0	4
1.0	75 deg	0.5	2.0	6
0.5	0	0.5	0	6

The convergence is extremely rapid in these cases, but tests on a cascade with  $s/c = 0.5$  and  $\xi = 75$  deg showed poor convergence up to  $n = 8$ , and the method is evidently unsatisfactory for cascades in which the blades are very close together. For this case a theory due to Sohngen<sup>15</sup> is applicable.

The correctness of the programme has been checked by comparing solutions with the following previously known solutions:

- (a) A single aerofoil. (This is the only case in which a check on  $C_{Fw}$  and  $C_{Mw}$  is available.)
- (b) Solutions obtained from actuator disc theory (Ref. 19) for cases when  $\beta$  and  $\lambda$  are both small (but not both zero).
- (c) Results calculated from Mendelson and Carroll's solution<sup>11</sup> for  $\xi = 0$  and  $\beta = 0$ .
- (d) Some of Sisto's results<sup>14</sup> for  $\xi = 0$  and various values of  $\beta$ .
- (e) Some of Lane and Wang's results<sup>7</sup> for  $\xi = 45$  deg.

In all these cases agreement is obtained to within the accuracy of the comparison solutions. In making these comparisons it is necessary to allow for the fact that different authors refer their coefficients to different axis positions. Formulae for converting from the axis position at the leading edge used here to other axis positions are given in Appendix V.

4. *Properties of the Coefficients.* In this Section the main properties of the coefficients will be listed. Some of these properties account for the definitions of the coefficients which have been used, which are slightly unusual compared with single aerofoil practice.

When  $\lambda \rightarrow 0$  the coefficients tend to a finite limit which is in general not zero. This is the reason for basing  $C_{Fq}$  and  $C_{Mq}$  on the velocity of vibration, in the translational mode, instead of the displacement. Also when  $\lambda = 0$ , all the elements of the matrix  $B$  are 1, so that

$$\text{and } \left. \begin{aligned} C_{Fq} &= C_{Fz} = C_{Fw} (= C_F) \\ C_{Mq} &= C_{Mz} = C_{Mw} (= C_M) \end{aligned} \right\} \quad (29)$$

If  $\beta$  is also zero  $C_F$  and  $C_M$  depend on the ratio  $\beta/\lambda$  as both  $\beta$  and  $\lambda$  tend to zero, and the solution is indeterminate. This case can be treated by actuator disc methods (Ref. 19).

A further property when  $\lambda = 0$  can be obtained by noting that when  $\beta$  is replaced by  $(2\pi - \beta)$ , Equation A1 shows that  $V(z)$  is replaced by its complex conjugate. When  $\lambda = 0$  this is the only point where complex numbers enter the calculation. Hence, denoting by  $C(s/c, \xi, \beta, \lambda)$  the fact that the matrix of coefficients depends on the four variables  $s/c$ ,  $\xi$ ,  $\beta$  and  $\lambda$ , it follows that

$$C(s/c, \xi, \beta, 0) = \bar{C}(s/c, -\xi, 2\pi - \beta, 0) \quad (30)$$

where  $\bar{C}$  denotes the complex conjugate of  $C$ .

If the cascade is drawn upside down,  $\xi$  is replaced by  $-\xi$  and  $\beta$  is replaced by  $(2\pi - \beta)$ . Hence

$$C(s/c, \xi, \beta, \lambda) = C(s/c, -\xi, 2\pi - \beta, \lambda) \quad (31)$$

It can be deduced from Equations (30) and (31) that  $C(s/c, \xi, \pi, 0)$  and  $C(s/c, 0, \beta, 0)$  are real. If  $s/c \rightarrow \infty$  the coefficients are given by single aerofoil theory (Ref. 16). It is found that

$$\begin{aligned} C_{Fq} &= -C(\lambda) - i\lambda/4 \\ C_{Fz} &= -(1 + \frac{3}{4}i\lambda)C(\lambda) + \frac{\lambda}{4} \left( \frac{\lambda}{2} - i \right) \\ C_{Fw} &= -[J_0(\lambda/2) - iJ_1(\lambda/2)]C(\lambda) + iJ_1(\lambda/2)e^{-i\lambda^2} \\ C_{Mq} &= \frac{1}{4}C_{Fq} - i\frac{\lambda}{16} \\ C_{Mz} &= \frac{1}{4}C_{Fz} + \frac{\lambda}{16}(\frac{3}{8}\lambda - 2i) \\ C_{Mw} &= \frac{1}{4}C_{Fw} \end{aligned}$$

where  $J_0$  and  $J_1$  are the usual Bessel functions, and  $C(\lambda)$  is Theodorsen's Circulation function, which has been tabulated, for instance, by Luke and Dengler<sup>10</sup> as a complex function of  $k = \lambda/2$ . The factor of  $\pi$  was introduced into the definitions of the coefficients, since it occurs naturally in these single aerofoil results.

Mendelson and Carroll<sup>11</sup> have shown that solutions for 180 deg phase angle between adjacent blades and zero stagger also apply to the case of 90 deg phase angle for a cascade with half the spacing, since two of the former systems can be superimposed without interfering with each other to give the latter. This relationship is

$$C(s/c, 0, \pi/2, \lambda) = C(2s/c, 0, \pi, \lambda).$$

**5. Tabulation of Coefficients.** In order to provide some data on the values of the coefficients, skeleton tables are included in this Report for space/chord ratios of 1 and 2. Table 1 gives some values for  $\lambda = 0$ , and in this case on account of Equations (29) it is only necessary to tabulate  $C_F$  and  $C_M$ . Table 2 gives values for  $\lambda = 0.2, 0.5, 1$  and 2.

The tables were calculated with  $n = 6$  and checked by calculations with  $n = 4, 5$  or  $7$ . These checks show that the computational error does not exceed one unit in the fourth decimal place.

6. *Use of the Coefficients.* In this Report it is proposed to discuss only briefly the use of the coefficients and the conclusions to be drawn from them.

Bending flutter of unstalled blades is controlled by the real part of  $C_{Fq}$ , since this determines the component of the blade force which is in phase with the velocity of motion. If the real part of  $C_{Fq}$  is positive, and there is no mechanical damping present, the blades will flutter. In fact, the tables show that the real part of  $C_{Fq}$  is always negative, so that pure bending vibration is always damped.

Torsional flutter of unstalled blades is similarly controlled by the imaginary part of  $(C_{Ms})_\eta$ , where the additional suffix  $\eta$  indicates that the coefficient is referred to an axis corresponding to the torsional axis of the actual blades. It has been shown (Ref. 20) that this can occur in high stagger cascades, the position of the torsional axis being of great importance in this connection.

The coefficients can be used for the calculation of both bending and torsional vibration forced by flow fluctuations from moving upstream obstructions. For bending vibration, it is found that the velocity of the blade in its forced vibration at resonance is roughly equal to the velocity of the fluctuations in the incident airflow.

The coefficients can also be used for the calculation of two degrees of freedom coupled flutter of unstalled blades. For normal compressor blades this gives rise to only a small modification of the torsional flutter referred to above, but for plastic blades the coupling effect may be important.

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## NOTATION

Some of the notation used is shown in Fig. 1.

$a$	$= (c/s) \cos \xi$
$b$	$= (c/s) \sin \xi$
$c$	Blade chord
$i$	$= \sqrt{-1}$ Indicates component leading 90 deg in phase
$k, l, m$	Integers
$n$	Integer giving order of approximation used
$p$	Static pressure
$qe^{i\omega t}$	Translational velocity of blade due to vibration
$r$	Integer
$s$	Cascade spacing
$t$	Time
$ve^{i\omega t}$	Velocity induced by vorticity on blades and their wakes
$we^{i\omega t}$	Velocity of disturbance due to wakes from upstream obstructions
$x, y$	Rectangular co-ordinates
$A$	Square $n \times n$ matrix, given initially by Equation (18) and later modified
$B$	Matrix defined by Equation (A14)
$C$	Matrix of force and moment coefficients given by Equation (26)
$Fe^{i\omega t}$	Aerodynamic force on blade (positive upwards)
$G, H, I, J$	Coefficients in series, whose value is immaterial
$K$	Kernel function defined by Equation (10)
$Me^{i\omega t}$	Aerodynamic moment on blade (positive anticlockwise)
$P, Q, R$	Coefficients for evaluation of integrals
$U$	Mainstream velocity.
$V$	Induced velocity function given by Equation (A1)
$X$	Matrix defined by Equation (27)
$\alpha e^{i\omega t}$	Torsional displacement of blade (positive anticlockwise)
$\beta$	Phase angle between adjacent blades
$\gamma e^{i\omega t}$	Bound vorticity
$\epsilon e^{i\omega t}$	Free vorticity
$\eta$	Co-ordinate for induced velocity
$\theta$	Variable defined by $x = \frac{1}{2}(1 - \cos \theta)$

NOTATION—*continued*

$\lambda$	Frequency parameter
$= \omega c/U$	
$\xi$	Stagger angle
$\rho$	Air density
$\phi$	Variable defined by $\eta = \frac{1}{2}(1 - \cos \phi)$
$\omega$	Angular frequency of vibration
$\Gamma$	Matrix defined by Equation (19)
$\Gamma_m$	Strength of $m$ th vortex in a row
$\Sigma$	Indicates summation
$\Sigma'$	Indicates summation in which the 0 and $n$ th terms have half weight

The force coefficients  $C_{Fq}$ ,  $C_{Fx}$  and  $C_{Fw}$ , give the aerodynamic forces arising due to  $q$ ,  $\alpha$  and  $w$  respectively. They are defined by Equation (21).

The moment coefficients  $C_{Mq}$ ,  $C_{Mx}$  and  $C_{Mw}$ , similarly give the moments, and are defined by Equation (23).

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## APPENDIX I

### *The Velocity induced by a Row of Vortices*

It is required to calculate the velocity normal to the blade surface induced by a row of vortices with the spacing, stagger, and phasing of the cascade in question (see Fig. 2). For one vortex of strength  $\Gamma_m$  at  $(x_m, y_m)$  this velocity at  $(\eta, 0)$  is

$$v = \frac{\Gamma_m}{2\pi} \frac{(\eta - x_m)}{(\eta - x_m)^2 + y_m^2}$$

(The factor  $e^{im\beta}$  indicating sinusoidal variation will not be written but is implied.)

If the centre vortex at  $(x, 0)$  has strength  $\Gamma_0$  then the strength of a vortex on the  $m$ th blade, or in its wake, is given by

$$\Gamma_m = \Gamma_0 e^{im\beta}$$

and its position is given by

$$x_m = x + ms \sin \xi$$

$$y_m = ms \cos \xi$$

Summing the effect for all blades gives

$$v = \frac{\Gamma_0}{2\pi} \sum_{m=-\infty}^{\infty} \frac{e^{im\beta} (\eta - x - ms \sin \xi)}{(\eta - x - ms \sin \xi)^2 + (ms \cos \xi)^2}$$

This will be written

$$v = \Gamma_0/c \cdot V(\eta/c - x/c)$$

where  $V(z)$  is a non-dimensional function given by

$$V(z) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \frac{e^{im\beta} \left( z - m \frac{s}{c} \sin \xi \right)}{\left( z - m \frac{s}{c} \sin \xi \right)^2 + \left( m \frac{s}{c} \cos \xi \right)^2}$$

This series can be summed by standard methods. The result for  $0 < \beta < 2\pi$  is

$$V(z) = \frac{1}{4} (a+ib) \frac{\exp\{- (\pi-\beta)(a+ib)z\}}{\sinh\{\pi(a+ib)z\}} + \frac{1}{4} (a-ib) \frac{\exp\{+ (\pi-\beta)(a-ib)z\}}{\sinh\{\pi(a-ib)z\}} \quad (A1)$$

where

$$a = c/s \cdot \cos \xi$$

and

$$b = c/s \cdot \sin \xi$$

If however  $\beta$  is put equal to zero and the series then summed, it is found that

$$\begin{aligned} V(z) &= \frac{1}{4} (a+ib) \coth\{\pi z(a+ib)\} + \frac{1}{4} (a-ib) \coth\{\pi z(a-ib)\} \\ &= \frac{1}{4} (a+ib) \frac{\exp\{-\pi z(a+ib)\}}{\sinh\{\pi z(a+ib)\}} + \frac{1}{4} (a-ib) \frac{\exp\{\pi z(a-ib)\}}{\sinh\{\pi z(a-ib)\}} + \frac{1}{2} ib \end{aligned} \quad (A2)$$

It will be seen that this differs from the result of putting  $\beta = 0$  in Equation (A1) by the term  $\frac{1}{2}ib$ . The difference between the two cases arises from the vorticity existing at points very remote from the origin. This vorticity can be considered as inducing a velocity which is constant over the plane.

From Equation (10) it can be seen that only the difference between  $V(-z)$  and  $V(-\infty)$  is of significance. The constant  $\frac{1}{2}ib$  can therefore be absorbed into the constant term  $V(-\infty)$  and Equation (A1) will be used in both cases. With this understanding, it is found from Equation (A1) that

$$V(-\infty) = 0 \text{ if } 0 < \beta < 2\pi$$

$$V(-\infty) = -\frac{1}{2}(a+ib) \text{ if } \beta = 0.$$



## APPENDIX II

### *Evaluation of the Integrals for Lift, Moment and Induced Velocity*

In this Appendix accurate numerical evaluation of the integrals occurring in Equations (5), (6) and (11) is considered. In terms of the variable  $\theta$  these are:

$$F = -\frac{1}{2}\rho U c \int_0^\pi \gamma(\theta) \sin \theta d\theta \quad (\text{A3})$$

$$M = -\frac{1}{4}\rho U c^2 \int_0^\pi \gamma(\theta)(1 - \cos \theta) \sin \theta d\theta \quad (\text{A4})$$

$$v = \frac{1}{2} \int_0^\pi K(\frac{1}{2} \cos \phi - \frac{1}{2} \cos \theta) \gamma(\theta) \sin \theta d\theta \quad (\text{A5})$$

As in normal thin aerofoil theory it is assumed that the bound vorticity may be expanded as a series in terms of the variable  $\theta$  as follows:

$$\gamma = G_0 \cot \theta/2 + \sum_{r=1}^{\infty} G_r \sin r\theta$$

where the first term gives the vorticity which occurs for steady flow over a single aerofoil, the vorticity being infinite at the leading edge, where  $\theta = 0$ . Then

$$\gamma(\theta) \sin \theta = G_0(1 + \cos \theta) + \frac{1}{2} \sum_{r=1}^{\infty} G_r \{\cos(r-1)\theta - \cos(r+1)\theta\}$$

so that  $\gamma \sin \theta$  can be expanded as a cosine series.

Considering next the  $V$  function given by Equation (A1) and noting that the exponential and hyperbolic functions can be expanded in their usual power series, it is found that the  $V$  function can be expanded as a convergent series as follows

$$V(z) = \frac{1}{2\pi z} + \sum_{r=0}^{\infty} H_r z^r$$

where the first term represents the velocity induced by a single vortex on the reference blade. When this is substituted into Equation (10) it is found that the  $K$  function can be expanded as follows:

$$K(z) = -\frac{1}{2\pi z} - \frac{i\lambda}{2\pi} e^{i\lambda z} \log |z| + \sum_{r=0}^{\infty} I_r z^r \quad (\text{A6})$$

This gives:

$$K(\frac{1}{2} \cos \phi - \frac{1}{2} \cos \theta) = -\frac{1}{\pi(\cos \phi - \cos \theta)} - \frac{i\lambda}{2\pi} \exp \frac{1}{2} i\lambda(\cos \phi - \cos \theta) \times \\ \times \log \frac{1}{2} |\cos \phi - \cos \theta| + \sum_{r=0}^{\infty} J_r(\phi) \cos r\theta$$

where the power series has been converted to a Fourier series.

When these expressions are substituted into Equations (A3), (A4) and (A5) it is found that integrals of the following three types have to be evaluated:

$$\int_0^\pi f(\theta) d\theta$$

$$\int_0^\pi \frac{f(\theta) d\theta}{\cos \phi - \cos \theta}$$

$$\int_0^\pi f(\theta) \log \frac{1}{2} |\cos \phi - \cos \theta| d\theta$$

where in each case  $f(\theta)$  can be expanded as a cosine series.

For the method to be numerically practical, the series must in each case converge rapidly. This is to be expected, since the singularities have been separately considered and  $f(\theta)$  represents a smooth function. In each case  $f(\pi) = 0$ , since the bound vorticity is finite at the trailing edge.  $f(\theta)$  is then specified at  $(n+1)$  points given by

$$\theta = \pi l/n \quad 0 \leq l \leq n$$

and the last two integrals have to be evaluated for values of  $\phi$  given by

$$\phi = \pi(2m+1)/2n \quad 0 \leq m \leq n-1$$

For a given value of  $n$  the integrals must be evaluated by sums of the following form:

$$\int_0^\pi f(\theta) d\theta = \sum'_{l=0} f(\pi l/n) P_l$$

$$\int_0^\pi \frac{f(\theta) d\theta}{\cos \phi - \cos \theta} = \sum'_{l=0} f(\pi l/n) Q_{l,m}$$

$$\int_0^\pi f(\theta) \log \frac{1}{2} |\cos \phi - \cos \theta| d\theta = \sum'_{l=0} f(\pi l/n) R_{l,m}$$

Where  $\sum'$  indicates a summation in which the first and last terms have half weight, and the zero last term has been included in order to improve the symmetry of the equations. The coefficients,  $P$ ,  $Q$  and  $R$  will be chosen to make these equations correct if  $f(\theta) = \cos r\theta$  where  $r$  is an integer ( $0 \leq r \leq n$ ). Then  $P$ ,  $Q$  and  $R$  are determined by the equation

$$\sum'_{l=0} P_l \cos(\pi r l/n) = \int_0^\pi \cos r\theta d\theta = \begin{cases} \pi & (r=0) \\ 0 & (r \geq 1) \end{cases}$$

$$\sum'_{l=0} Q_{l,m} \cos(\pi r l/n) = \int_0^\pi \frac{\cos r\theta d\theta}{\cos \phi - \cos \theta} = -\pi \frac{\sin r\phi}{\sin \phi}$$

$$\sum'_{l=0} R_{l,m} \cos(\pi r l/n) = \int_0^\pi \cos r\theta \cdot \log \frac{1}{2} |\cos \phi - \cos \theta| \cdot d\theta = \begin{cases} -2\pi \log_e 2 & (r=0) \\ -\frac{\pi}{r} \cos r\phi & (r \geq 1) \end{cases} \quad (A7)$$

The second integral above is well known, and its evaluation has been given by Glauert<sup>1</sup>. The third integral when  $r \geq 1$  can be integrated by parts to give integrals of the Glauert type. When  $r = 0$  it can be shown that

$$\int_0^\pi \log \frac{1}{2} |\cos \phi - \cos \theta| d\theta = \int_{-\pi}^\pi \log |\sin \frac{1}{2}(\theta + \phi)| d\theta$$

The integral is then a periodic function of  $(\theta + \phi)$  and the integral is therefore independent of  $\phi$ . When  $\phi = 0$  it reduces to a standard form.

Equations (A7) represent sets of  $(n+1)$  simultaneous equations for the unknown coefficients  $P$ ,  $Q$  and  $R$ . They may be solved by multiplying the  $r$ th equation by  $\cos \pi rk/n$  (where  $k$  is another integer  $0 \leq k \leq n$ ) and the first and last equations by  $\frac{1}{2}$  and adding. Then since

$$\begin{aligned} \sum_{r=0}^n \cos(\pi r l/n) \cos(\pi r k/n) &= 0 \quad \text{if } l \neq k \\ &= n \quad \text{if } l = k = 0 \text{ or } n \\ &= \frac{1}{2}n \quad \text{if } l = k \neq 0 \text{ or } n \end{aligned}$$

it is found that

$$\begin{aligned} P_l &= \pi/n \\ Q_{l,m} &= -\frac{2\pi}{n} \sum_{r=0}^n \frac{\sin r\phi}{\sin \phi} \cos \pi r l/n \\ R_{l,m} &= -\frac{2\pi}{n} \left\{ \log_e 2 + \sum_{r=1}^n \frac{1}{r} \cos r\phi \cos \pi r l/n \right\} \end{aligned}$$

The series for the  $Q$  coefficients can be summed. Provided that  $\phi$  is of the form

$$\phi = \pi(2m+1)/2n$$

it is found that

$$Q_{l,m} = \frac{\pi}{n} \frac{1}{\cos \pi(2m+1)/2n - \cos \pi l/n}$$

The series for the  $R$  coefficients cannot be summed.

It is therefore found that the three types of integrals can be evaluated by sums as follows:

$$\begin{aligned} \int_0^\pi f(\theta) d\theta &= \frac{\pi}{n} \sum_{l=0}^n f(\pi l/n) \\ \int_0^\pi \frac{f(\theta) d\theta}{\cos \phi - \cos \theta} &= \frac{\pi}{n} \sum_{l=0}^n \frac{f(\pi l/n)}{\cos \pi(2m+1)/2n - \cos \pi l/n} \\ \int_0^\pi f(\theta) \log \frac{1}{2} |\cos \phi - \cos \theta| d\theta &= -\frac{2\pi}{n} \sum_{l=0}^n f(\pi l/n) \left\{ \log_e 2 + \sum_{r=1}^n \frac{1}{r} \cos \pi r(2m+1)/2n \cos \pi r l/n \right\} \end{aligned}$$

It is observed that the first two integrals are given by the trapezoidal rule, but that the third integral is not. The lift and moment integrals have therefore been correctly evaluated in the main body of the report but the induced velocity integral requires correction. This method of evaluating the integrals is very similar to that used by Watson<sup>18</sup>.

A modified kernel function  $K'$  which does not contain the troublesome logarithmic singularity may be defined from Equation (A6) by

$$K(z) = K'(z) - \frac{i\lambda}{2\pi} e^{i\lambda z} \log |z| \quad (\text{A8})$$

Then Equation (A5) may be evaluated as

$$\begin{aligned} v(\phi) &= \sum_{l=0}^n \left[ K'(\frac{1}{2} \cos \pi \overline{2m+1}/2n - \frac{1}{2} \cos \pi l/n) + \frac{i\lambda}{\pi} \exp i\lambda(\frac{1}{2} \cos \pi \overline{2m+1}/2n - \right. \\ &\quad \left. - \frac{1}{2} \cos \pi l/n) \left\{ \log_e 2 + \sum_{r=1}^n \frac{1}{r} \cos \pi r(2m+1)/2n \cdot \cos \frac{\pi r l}{n} \right\} \right] \times \left[ \frac{\pi}{2n} \sin \pi l/n \cdot \gamma(\pi l/n) \right] \end{aligned}$$

The matrix  $A$  therefore requires modification, its elements being given by the first factor in square brackets in the above equation.

### APPENDIX III

#### *Evaluation of the Kernel Integral*

From Equations (A8) and (10) the modified Kernel function is given by

$$K'(z) = V(-z) - V(-\infty) - i\lambda e^{i\lambda z} \int_{-\infty}^z e^{-i\lambda z_1} \{V(-z_1) - V(-\infty)\} dz_1 + \frac{i\lambda}{2\pi} e^{i\lambda z} \log |z|$$

Dividing up the range of integration, this may be written

$$K'(z) = \{V(-z) - V(-\infty)\} - i\lambda e^{i\lambda z} \left[ \int_{-\infty}^1 \left\{ e^{-i\lambda z_1} (V(-z_1) - V(-\infty)) + \frac{1}{2\pi z_1} \right\} dz_1 + \int_1^z e^{-i\lambda z_1} \{V(-z_1) - V(-\infty)\} dz_1 \right] \quad (A10)$$

The first integral has a finite range and the integrand does not contain any singularity, so that it may be evaluated by numerical quadrature. The second integral is evaluated by a series. The sinh function may be expanded as follows:

$$\frac{1}{\sinh z} = -2 \sum_{r=0}^{\infty} e^{(2r+1)z}$$

the series being absolutely convergent provided the real part of  $z$  is negative. Hence from Equation (A1)

$$V(z) = -\frac{1}{2}(a+ib) \sum_{r=0}^{\infty} \exp\{(2\pi r + \beta)(a+ib)z\} - \frac{1}{2}(a-ib) \sum_{r=1}^{\infty} \exp\{(2\pi r - \beta)(a-ib)z\}$$

When  $0 < \beta < 2\pi$ ,  $V(-\infty) = 0$ , so that

$$e^{-i\lambda z_1} \{V(-z_1) - V(-\infty)\} = -\frac{1}{2}(a+ib) \sum_{r=0}^{\infty} \exp\{-(2\pi r + \beta)(a+ib)z_1 - i\lambda z_1\} - \frac{1}{2}(a-ib) \sum_{r=1}^{\infty} \exp\{-(2\pi r - \beta)(a-ib)z_1 - i\lambda z_1\} \quad (A11)$$

This may be integrated term by term from 1 to  $\infty$ , and all terms vanish at the top limit. Hence

$$\int_1^z e^{-i\lambda z_1} \{V(-z_1) - V(-\infty)\} dz_1 = -\frac{1}{2} \sum_{r=0}^{\infty} \frac{\exp\{-(2\pi r + \beta)(a+ib) - i\lambda\}}{(2\pi r + \beta) + \frac{i\lambda}{a+ib}} - \frac{1}{2} \sum_{r=1}^{\infty} \frac{\exp\{-(2\pi r - \beta)(a-ib) - i\lambda\}}{(2\pi r - \beta) + \frac{i\lambda}{a-ib}} \quad (A12)$$

When  $\beta = 0$ ,  $V(-\infty) = -\frac{1}{2}(a+ib)$  and this case requires special consideration. It is found that Equations (A11) and (A12) still apply if  $\beta$  is put equal to zero, and if the first term corresponding

to  $r = 0$  in the first summation is omitted. However, when these expressions are substituted into Equation (A10) it is found that whether  $\beta$  is zero or not,  $K'(z)$  is given by

$$\begin{aligned}
 K'(z) = & V(-z) - i\lambda e^{i\lambda z} \left[ \int_z^1 e^{-i\lambda z_1} V(-z_1) + \frac{1}{2\pi z_1} \right] dz_1 - \\
 & - \frac{1}{2} \sum_{r=0}^{\infty} \frac{\exp \left\{ - (2\pi r + \beta)(a + ib) - i\lambda \right\}}{(2\pi r + \beta) + \frac{i\lambda}{a + ib}} - \\
 & - \frac{1}{2} \sum_{r=1}^{\infty} \frac{\exp \left\{ - (2\pi r - \beta)(a - ib) - i\lambda \right\}}{(2\pi r - \beta) + \frac{i\lambda}{a - ib}} \quad (A13)
 \end{aligned}$$

The convergence of these series depends on  $a$  ( $= c/s \cdot \cos \xi$ ) and is very rapid for all normal values.

It is seen that when  $\beta = 0$  and  $\lambda = 0$ , the first term in the first series in Equation (A13) becomes infinite, and  $K'(z)$  is indeterminate. The programme therefore fails for this case. However, this case can be treated by actuator disc methods (Ref. 19).

## APPENDIX IV

### Summary of Computational Method

In order to facilitate the construction of programmes for other digital computers, the essential equations are collected in this Appendix.

The matrix of force and moment coefficients is given by

$$C = -\frac{1}{\pi} X A^{-1} B \quad (28)$$

where

$$C = \begin{bmatrix} C_{Fq} & C_{Fz} & C_{Fv} \\ C_{Mq} & C_{Mz} & C_{Mv} \end{bmatrix} \quad (26)$$

$X$  is a  $2 \times n$  matrix whose  $l$ th column ( $0 \leq l \leq n-1$ ) is given by

$$\begin{bmatrix} 1 \\ \frac{1}{2}(1 - \cos \pi l/n) \end{bmatrix} \quad (27)$$

$B$  is a  $n \times 3$  matrix whose  $m$ th row ( $0 \leq m \leq n-1$ ) is given by

$$[1, \{1 + \frac{1}{2}i\lambda(1 - \cos \pi \overline{2m+1}/2n)\}, \exp \{-\frac{1}{2}i\lambda(1 - \cos \pi \overline{2m+1}/2n)\}] \quad (A14)$$

$A$  is a  $n \times n$  square matrix whose element in the  $l$ th column and  $m$ th row is given by

$$\begin{aligned} V(-z) + i\lambda e^{i\lambda z} & \left[ - \int_z^1 \left\{ e^{-i\lambda z_1} V(-z_1) + \frac{1}{2\pi z_1} \right\} dz_1 + \right. \\ & + \frac{1}{2} \sum_{r=0}^{\infty} \frac{\exp \{-(2\pi r + \beta)(a+ib) - i\lambda\}}{(2\pi r + \beta) + \frac{i\lambda}{a+ib}} + \frac{1}{2} \sum_{r=1}^{\infty} \frac{\exp \{-(2\pi r - \beta)(a-ib) - i\lambda\}}{(2\pi r - \beta) + \frac{i\lambda}{a-ib}} + \\ & \left. + \frac{1}{\pi} \log 2 + \frac{1}{\pi} \sum'_{r=1}^n \frac{1}{r} \cos \pi r(2m+1)/2n \cdot \cos \pi r l/n \right] \quad (A15) \end{aligned}$$

where

$$z = \frac{1}{2}(\cos \pi \overline{2m+1}/2n - \cos \pi l/n)$$

and

$$V(z) = \frac{1}{4}(a+ib) \frac{\exp \{-(\pi-\beta)(a+ib)z\}}{\sinh \{\pi(a+ib)z\}} + \frac{1}{4}(a-ib) \frac{\exp \{(\pi-\beta)(a-ib)z\}}{\sinh \{\pi(a-ib)z\}} \quad (A1)$$

Also

$$a = c/s \cdot \cos \xi$$

$$b = c/s \cdot \sin \xi$$

The notation  $\sum'_{r=1}^n$  indicates that the  $n$ th term in the summation has half weight.

## APPENDIX V

### *Force and Moment Coefficients for General Axis Position*

The coefficients given in Equations (21) and (23) are referred to an axis position at the leading edge of the blade. It is necessary to be able to find from these coefficients corresponding to an axis position distant  $\eta$  from the leading edge. These two cases will be distinguished by the suffices 0 and  $\eta$  respectively. The new translational velocity of the blade is then given by

$$q_{\eta} e^{i\eta x} = q_0 e^{i\eta x} + \eta \frac{d}{dt} (x e^{i\eta x})$$

or

$$q_{\eta} = q_0 + i\lambda\eta U x$$

where  $c$  has been put equal to 1 so that  $\eta = 1$  at the trailing edge. The angular displacement of the blade ( $x$ ) is not affected by the change of axis. The velocity induced by the wakes at the point  $\eta$  is given by

$$w_{\eta} = w_0 e^{-i\lambda\eta}$$

The blade force  $F$  is not affected by the change of axis and the moment about the new axis is given by

$$M_{\eta} = M_0 - \eta F$$

Putting these in Equations (21) and (23) and identifying terms the following conversion formulae are found:

$$(C_{Fq})_{\eta} = (C_{Fq})_0$$

$$(C_{Fz})_{\eta} = (C_{Fz})_0 - i\lambda\eta(C_{Fq})_0$$

$$(C_{Fw})_{\eta} = e^{i\lambda\eta}(C_{Fw})_0$$

$$(C_{Mq})_{\eta} = (C_{Mq})_0 - \eta(C_{Fq})_0$$

$$(C_{Mz})_{\eta} = (C_{Mz})_0 - \eta(C_{Fz})_0 - i\lambda\eta(C_{Mq})_0 + i\lambda\eta^2(C_{Fq})_0$$

$$(C_{Mw})_{\eta} = e^{i\lambda\eta}(C_{Mw})_0 - \eta e^{i\lambda\eta}(C_{Fw})_0$$

TABLE 1  
 $\lambda = 0$  (if also  $\beta = 0, B/\lambda \rightarrow \infty$ )

$\rho/2\pi$	$C_F$	$C_M$	$\rho/2\pi$	$C_F$	$C_M$
$s/c = 1.0$			$s/c = 2.0$		
$\xi = 0.0 \text{ deg}$			$\xi = 0.0 \text{ deg}$		
0.0	-0.5839	0.0000	0.0	-0.8350	0.0000
0.1	-0.8009	0.0000	0.1	-0.9227	0.0000
0.2	-1.0054	0.0000	0.2	-0.9955	0.0000
0.3	-1.1729	0.0000	0.3	-1.0500	0.0000
0.4	-1.2827	0.0000	0.4	-1.0837	0.0000
0.5	-1.3209	0.0000	0.5	-1.0952	0.0000
0.6	-1.2827	0.0000	0.6	-1.0837	0.0000
0.7	-1.1729	0.0000	0.7	-1.0500	0.0000
0.8	-1.0054	0.0000	0.8	-0.9955	0.0000
0.9	-0.8009	0.0000	0.9	-0.9227	0.0000
$s/c = 1.0$			$s/c = 2.0$		
$\xi = 15.0 \text{ deg}$			$\xi = 15.0 \text{ deg}$		
0.0	-0.5692	-0.1397	0.0	-0.8259	-0.1428
0.1	-0.7861	-0.1260	0.1	-0.9136	-0.1197
0.2	-0.9913	-0.1038	0.2	-0.9864	-0.0930
0.3	-1.1598	-0.0740	0.3	-1.0410	-0.0637
0.4	-1.2704	-0.0386	0.4	-1.0748	-0.0323
0.5	-1.3089	0.0000	0.5	-1.0862	0.0000
0.6	-1.2704	0.0386	0.6	-1.0748	0.0323
0.7	-1.1598	0.0740	0.7	-1.0410	0.0637
0.8	-0.9913	0.1038	0.8	-0.9864	0.0930
0.9	-0.7861	0.1260	0.9	-0.9136	0.1197
$s/c = 1.0$			$s/c = 2.0$		
$\xi = 30.0 \text{ deg}$			$\xi = 30.0 \text{ deg}$		
0.0	-0.5246	-0.2760	0.0	-0.7992	-0.2821
0.1	-0.7406	-0.2502	0.1	-0.8867	-0.2367
0.2	-0.9473	-0.2071	0.2	-0.9595	-0.1842
0.3	-1.1184	-0.1482	0.3	-1.0142	-0.1261
0.4	-1.2313	-0.0773	0.4	-1.0481	-0.0641
0.5	-1.2707	0.0000	0.5	-1.0595	0.0000
0.6	-1.2313	0.0773	0.6	-1.0481	0.0641
0.7	-1.1184	0.1482	0.7	-1.0142	0.1261
0.8	-0.9473	0.2071	0.8	-0.9595	0.1842
0.9	-0.7406	0.2502	0.9	-0.8867	0.2367
$s/c = 1.0$			$s/c = 2.0$		
$\xi = 45.0 \text{ deg}$			$\xi = 45.0 \text{ deg}$		
0.0	-0.4478	-0.4043	0.0	-0.7568	-0.4126
0.1	-0.6596	-0.3700	0.1	-0.8437	-0.3469
0.2	-0.8670	-0.3085	0.2	-0.9163	-0.2704
0.3	-1.0413	-0.2220	0.3	-0.9709	-0.1853
0.4	-1.1573	-0.1162	0.4	-1.0048	-0.0942
0.5	-1.1980	0.0000	0.5	-1.0162	0.0000
0.6	-1.1573	0.1162	0.6	-1.0048	0.0942
0.7	-1.0413	0.2220	0.7	-0.9709	0.1853
0.8	-0.8670	0.3085	0.8	-0.9163	0.2704
0.9	-0.6596	0.3700	0.9	-0.8437	0.3469
$s/c = 1.0$			$s/c = 2.0$		
$\xi = 60.0 \text{ deg}$			$\xi = 60.0 \text{ deg}$		
0.0	-0.3357	-0.5171	0.0	-0.7046	-0.5256
0.1	-0.5343	-0.4800	0.1	-0.7903	-0.4430
0.2	-0.7366	-0.4049	0.2	-0.8621	-0.3460
0.3	-0.9109	-0.2938	0.3	-0.9163	-0.2374
0.4	-1.0287	-0.1546	0.4	-0.9499	-0.1208
0.5	-1.0704	0.0000	0.5	-0.9614	0.0000
0.6	-1.0287	0.1546	0.6	-0.9499	0.1208
0.7	-0.9109	0.2938	0.7	-0.9163	0.2374
0.8	-0.7366	0.4049	0.8	-0.8621	0.3460
0.9	-0.5343	0.4800	0.9	-0.7903	0.4430
$s/c = 1.0$			$s/c = 2.0$		
$\xi = 75.0 \text{ deg}$			$\xi = 75.0 \text{ deg}$		
0.0	-0.1858	-0.6017	0.0	-0.6570	-0.6064
0.1	-0.3460	-0.5693	0.1	-0.7410	-0.5122
0.2	-0.5217	-0.4882	0.2	-0.8116	-0.4008
0.3	-0.6801	-0.3586	0.3	-0.8650	-0.2754
0.4	-0.7901	-0.1901	0.4	-0.8983	-0.1402
0.5	-0.8295	0.0000	0.5	-0.9096	0.0000
0.6	-0.7901	0.1901	0.6	-0.8983	0.1402
0.7	-0.6801	0.3586	0.7	-0.8650	0.2754
0.8	-0.5217	0.4882	0.8	-0.8116	0.4008
0.9	-0.3460	0.5693	0.9	-0.7410	0.5122



TABLE 2a

$$s/c = 1, (\quad) = 0 \text{ deg}$$

$\beta/2\pi$	$C_{F\alpha}$	$C_{F\beta}$	$C_{F\gamma}$	$C_{F\delta}$	$C_{F\epsilon}$	$C_{F\zeta}$	$C_{F\eta}$	$C_{F\theta}$	$C_{F\iota}$	
$\lambda = 0.2$										
0.0	-0.3045	-0.0365	-0.0864	0.3038	0.0134	-0.0550	0.0185	-0.0529	-0.0394	0.0024
0.1	-0.6667	0.1411	0.0378	-0.6338	0.2417	-0.1504	0.0198	-0.1543	0.0156	-0.1429
0.2	-0.9289	0.1289	-0.0101	-0.8957	0.2653	-0.2339	0.0201	-0.2378	0.0273	-0.2255
0.3	-1.1140	0.1343	-0.0423	-1.0804	0.2817	-0.2977	0.0196	-0.3016	0.0367	-0.2888
0.4	-1.2303	0.1178	-0.0605	-1.1963	0.2935	-0.3390	0.0195	-0.3428	0.0425	-0.3296
0.5	-1.2703	0.1170	-0.0664	-1.2360	0.2978	-0.3533	0.0195	-0.3571	0.0445	-0.3437
0.6	-1.2303	0.1178	-0.0605	-1.1963	0.2935	-0.3390	0.0195	-0.3428	0.0425	-0.3296
0.7	-1.1140	0.1210	-0.0423	-1.0804	0.2817	-0.2977	0.0196	-0.3016	0.0367	-0.2888
0.8	-0.9289	0.1289	-0.0101	-0.8957	0.2653	-0.2339	0.0201	-0.2378	0.0273	-0.2255
0.9	-0.6667	0.1411	0.0378	-0.6338	0.2417	-0.1504	0.0198	-0.1543	0.0156	-0.1429
$\lambda = 0.5$										
0.0	-0.3042	-0.0913	-0.2159	0.2997	0.0328	0.0550	0.0463	0.0416	0.0985	0.0059
0.1	-0.5032	0.0555	0.1393	-0.4274	0.2352	-0.1135	0.0177	-0.1123	0.0918	-0.0964
0.2	-0.7421	0.1222	-0.1554	-0.6263	0.3774	-0.1869	0.0002	-0.1926	0.1011	-0.1577
0.3	-0.9327	0.1502	-0.1915	-0.7951	0.4658	-0.2493	0.0084	-0.2583	0.1147	-0.2125
0.4	-1.0539	0.1628	-0.2190	-0.9046	0.5166	-0.2903	0.0125	-0.3010	0.1250	-0.2492
0.5	-1.0955	0.1665	-0.2289	-0.9424	0.5334	-0.3046	0.0138	-0.3157	0.1287	-0.2621
0.6	-1.0539	0.1628	-0.2190	-0.9046	0.5166	-0.2903	0.0125	-0.3010	0.1250	-0.2492
0.7	-0.9327	0.1502	-0.1915	-0.7951	0.4658	-0.2493	0.0084	-0.2583	0.1147	-0.2125
0.8	-0.7421	0.1222	-0.1554	-0.6263	0.3774	-0.1869	0.0084	-0.1926	0.1011	-0.1577
0.9	-0.5032	0.0555	0.1393	-0.4274	0.2352	-0.1135	0.0177	-0.1123	0.0918	-0.0964
$\lambda = 1.0$										
0.0	-0.3034	-0.1828	-0.4313	-0.2863	0.0614	-0.0548	-0.0926	-0.0012	0.1970	-0.0517
0.1	-0.4397	-0.1171	-0.4377	-0.3368	0.1839	-0.0992	-0.0867	-0.0545	0.2239	-0.0759
0.2	-0.6004	-0.0580	-0.5073	-0.4146	0.3204	-0.1512	-0.0766	-0.1171	0.2516	-0.1044
0.3	-0.7439	-0.0163	-0.5614	-0.4940	0.4349	-0.1988	-0.0679	-0.1730	0.2771	-0.1320
0.4	-0.8412	0.0081	-0.6013	-0.5513	0.5096	-0.2318	-0.0624	-0.2109	0.2950	-0.1519
0.5	-0.8755	0.0161	-0.6159	-0.5720	0.5355	-0.2435	-0.0606	-0.2243	0.3014	-0.1591
0.6	-0.8412	0.0081	-0.6013	-0.5513	0.5096	-0.2318	-0.0624	-0.2109	0.2950	-0.1519
0.7	-0.7439	-0.0163	-0.5614	-0.4940	0.4349	-0.1988	-0.0679	-0.1730	0.2771	-0.1320
0.8	-0.6004	-0.0580	-0.5073	-0.4146	0.3204	-0.1512	-0.0766	-0.1171	0.2516	-0.1044
0.9	-0.4397	-0.1171	-0.4377	-0.3368	0.1839	-0.0992	-0.0867	-0.0545	0.2239	-0.0759
$\lambda = 2.0$										
0.0	-0.3009	-0.3671	-0.8602	-0.2470	0.0984	-0.0544	-0.1854	0.1607	0.3935	0.0446
0.1	-0.4171	-0.3814	-1.0275	-0.2732	0.1636	-0.0941	-0.2067	0.1360	0.4731	0.0616
0.2	-0.5358	-0.3854	-1.1873	-0.2941	0.2395	-0.1349	-0.2209	0.1029	0.5467	0.0741
0.3	-0.6382	-0.3846	-1.3198	-0.3105	0.3093	-0.1706	-0.2298	0.0707	0.6069	0.0830
0.4	-0.7074	-0.3826	-1.4076	-0.3211	0.3578	-0.1949	-0.2347	0.0478	0.6464	0.0885
0.5	-0.7318	-0.3817	-1.4382	-0.3249	0.3752	-0.2035	-0.2362	0.0395	0.6602	0.0903
0.6	-0.7074	-0.3826	-1.4076	-0.3211	0.3578	-0.1949	-0.2347	0.0478	0.6464	0.0885
0.7	-0.6382	-0.3846	-1.3198	-0.3105	0.3093	-0.1706	-0.2298	0.0707	0.6069	0.0830
0.8	-0.5358	-0.3854	-1.1873	-0.2941	0.2395	-0.1349	-0.2209	0.1029	0.5467	0.0741
0.9	-0.4171	-0.3814	-1.0275	-0.2732	0.1636	-0.0941	-0.2067	0.1360	0.4731	0.0616

TABLE 2b  
 $s/c = 1, \xi = 15 \text{ deg}$

$\beta/2\pi$	$C_{Fg}$	$C_{F\alpha}$	$C_{Fw}$	$C_{Fq}$	$C_{M\alpha}$	$C_{Mq}$	$C_{M\alpha}$	$C_{Mw}$
$\lambda = 0.2$								
0.0	-0.3150	-0.3113	-0.3142	0.0140	-0.0580	-0.0189	-0.0558	-0.0404
0.1	-0.7215	-0.7292	-0.7043	0.1563	-0.1618	0.0071	-0.1630	-0.0302
0.2	-0.9565	-0.9613	-0.9398	0.1752	-0.2396	0.0089	-0.2408	-0.0394
0.3	-1.1260	-1.1334	-1.1052	0.2126	-0.2998	0.0121	-0.3016	-0.0445
0.4	-1.2310	-1.2435	-1.2040	0.2546	-0.3384	0.0155	-0.3410	-0.0464
0.5	-1.2607	-1.2796	-1.2270	0.2927	-0.3505	0.0186	-0.3541	-0.0449
0.6	-1.2100	-1.2356	-1.1703	0.3215	-0.3342	0.0216	-0.3388	-0.0396
0.7	-1.0818	-1.1134	-1.0373	0.3378	-0.2910	0.0250	-0.2966	-0.0303
0.8	-0.8823	-0.9188	-0.8351	0.3389	-0.2250	0.0287	-0.2313	-0.0173
0.9	-0.6015	-0.6390	0.5564	0.3056	-0.1387	0.0293	0.1448	-0.0043
$\lambda = 0.5$								
0.0	-0.3147	-0.2917	0.3099	0.0345	0.0579	0.0473	0.0443	0.1010
0.1	-0.5613	-0.5803	-0.4976	0.2249	-0.1255	-0.0191	-0.1221	-0.0981
0.2	-0.7939	-0.8290	-0.6989	0.3477	-0.1976	-0.0034	-0.1998	-0.1085
0.3	-0.9668	-1.0148	-0.8485	0.4354	-0.2563	0.0056	-0.2621	-0.1201
0.4	-1.0689	-1.1306	-0.9313	0.4970	-0.2932	0.0108	-0.3019	-0.1276
0.5	-1.0912	-1.1604	-0.9396	0.5293	-0.3034	0.0132	-0.3142	-0.1285
0.6	-1.0303	-1.1162	-0.8726	0.5274	-0.2852	0.0131	-0.2972	-0.1222
0.7	-0.8907	-0.9809	-0.7378	0.4857	-0.2406	0.0099	-0.2522	-0.1097
0.8	-0.6852	-0.7677	-0.5536	0.3949	-0.1755	0.0016	-0.1841	-0.0949
0.9	-0.4482	-0.4968	-0.3656	0.2345	-0.1029	-0.0181	-0.1031	-0.0883
$\lambda = 1.0$								
0.0	-0.3139	-0.2216	-0.2959	0.0646	-0.0578	-0.0946	-0.0030	-0.2020
0.1	-0.4768	-0.4322	-0.3745	0.2045	-0.1074	-0.0838	-0.0621	-0.2273
0.2	-0.6421	-0.6380	-0.4626	0.3405	-0.1604	-0.0725	-0.1252	-0.2542
0.3	-0.7765	-0.8059	-0.5345	0.4493	-0.2061	-0.0641	-0.1791	-0.2782
0.4	-0.8575	-0.9162	-0.5724	0.5171	-0.2353	-0.0597	-0.2140	-0.2943
0.5	-0.8724	-0.9505	-0.5684	0.5357	-0.2425	-0.0593	-0.2240	-0.2989
0.6	-0.8188	-0.9031	-0.5238	0.5019	-0.2265	-0.0628	-0.2072	-0.2911
0.7	-0.7060	-0.7783	-0.4500	0.4185	-0.1901	-0.0700	-0.1661	-0.2728
0.8	-0.5564	-0.5935	-0.3684	0.2966	-0.1413	-0.0804	-0.1082	-0.2486
0.9	-0.4081	-0.3852	-0.3090	0.1609	-0.0923	-0.0917	-0.0469	-0.2245
$\lambda = 2.0$								
0.0	-0.3114	-0.0603	-0.2542	0.1036	-0.0573	-0.1896	0.1622	-0.4036
0.1	-0.4372	-0.0717	-0.2822	0.1879	-0.0999	-0.2031	0.1298	-0.4767
0.2	-0.5577	-0.2101	-0.3033	0.2702	-0.1415	-0.2137	0.0936	-0.5463
0.3	-0.6550	-0.3313	-0.3167	0.3354	-0.1757	-0.2222	0.0618	-0.6036
0.4	-0.7142	-0.4138	-0.3218	0.3717	-0.1970	-0.2287	0.0417	-0.6411
0.5	-0.7262	-0.4425	-0.3192	0.3727	-0.2019	-0.2333	0.0378	-0.6539
0.6	-0.6902	-0.4111	-0.3104	0.3393	-0.1897	-0.2358	0.0510	-0.6402
0.7	-0.6134	-0.3244	-0.2982	0.2797	-0.1630	-0.2339	0.0783	-0.6022
0.8	-0.5109	-0.1981	-0.2851	0.2080	-0.1272	-0.2273	0.1129	-0.5453
0.9	-0.4030	-0.0583	-0.2716	0.1428	-0.0895	-0.2136	0.1448	-0.4767

$\beta/2\pi$	$C_{F\alpha}$	$C_{F\beta}$	$C_{F\gamma}$	$C_{F\delta}$	$C_{F\epsilon}$	$C_{F\zeta}$	$C_{F\eta}$	$C_{F\theta}$	$C_{F\iota}$	$C_{F\kappa}$	$C_{F\lambda}$
$\lambda = 0.2$											
0.0	-0.3504	-0.0400	-0.0964	-0.3465	-0.0164	-0.0683	0.0203	-0.0659	0.0437	-0.0681	0.0032
0.1	-0.7593	-0.0773	-0.7463	-0.9442	-0.0406	-0.1724	0.0122	-0.1700	-0.0513	-0.1711	0.0269
0.2	-0.9588	-0.0800	-0.9442	-1.1059	-0.0641	-0.2409	0.0071	-0.2387	-0.0556	-0.2386	0.0415
0.3	-1.1132	-0.0354	-1.1059	-1.2125	0.1279	0.2961	0.0011	0.2953	0.0549	-0.2917	0.0569
0.4	-1.2085	-0.0289	-1.2125	-1.2463	0.2032	0.3314	0.0088	0.3325	0.0519	-0.3248	0.0690
0.5	-1.2292	0.1014	-1.2463	-1.1988	0.0762	0.3412	0.0156	0.3444	-0.0463	-0.3325	0.0768
0.6	-1.1686	0.1733	-1.1988	-1.0708	0.0043	0.3234	0.0222	0.3286	-0.0372	-0.3128	0.0807
0.7	-1.0293	0.2367	-1.0708	-0.8671	0.0867	0.2792	0.0292	0.2862	-0.0241	-0.2673	0.0815
0.8	-0.8180	0.2824	-0.8671	-0.5759	0.1613	0.2127	0.0365	0.2210	-0.0076	-0.2002	0.0795
0.9	-0.5278	0.2785	-0.5759	-0.3255	0.1982	0.1263	0.0381	0.1344	0.0064	-0.1150	0.0689
$\lambda = 0.5$											
0.0	-0.3502	-0.1000	-0.3255	-0.6182	-0.2410	-0.0682	-0.0507	-0.0537	-0.1093	-0.0671	0.0078
0.1	-0.6255	-0.0401	-0.6182	-0.8399	0.2827	-0.1404	0.0256	-0.1330	-0.1110	-0.1268	0.0577
0.2	-0.8386	-0.0084	-0.8399	-1.0095	0.3263	-0.2609	0.0110	-0.2048	-0.1201	-0.1853	0.0946
0.3	-0.9901	0.0383	-1.0095	-1.1173	0.2945	-0.8974	0.0005	-0.2622	-0.1277	-0.2306	0.1216
0.4	-1.0729	0.0955	-1.1173	-1.1478	0.2354	-0.9514	0.0067	-0.2929	-0.1310	-0.2563	0.1380
0.5	-1.0767	0.1535	-1.1478	-1.0916	0.1587	-0.9298	0.0112	-0.2989	-0.1282	-0.2581	0.1430
0.6	-0.9974	0.2014	-1.0916	-0.9475	0.0794	-0.8340	0.0130	-0.2771	-0.1187	-0.2350	0.1364
0.7	-0.8403	0.2259	-0.9475	-0.7228	0.0244	-0.6752	0.0115	-0.2438	-0.1036	-0.1898	0.1185
0.8	-0.6221	0.2052	-0.7228	-0.4492	0.0562	-0.4782	0.0038	-0.1629	-0.0880	-0.1302	0.0886
0.9	-0.3927	0.0940	-0.4492	-0.2504	-0.4816	-0.3074	0.0258	-0.0925	-0.0858	-0.0746	0.0438
$\lambda = 1.0$											
0.0	-0.3493	-0.2003	-0.4816	-0.4559	-0.5590	0.3281	0.0756	0.0681	0.1014	0.0639	0.0147
0.1	-0.5245	-0.1530	-0.4559	-0.6503	-0.6197	-0.4286	0.2224	-0.1185	0.0844	-0.0703	0.0602
0.2	-0.6849	-0.1048	-0.6503	-0.8109	0.6487	-0.5185	0.3550	-0.1699	0.0701	-0.1321	0.1024
0.3	-0.8056	-0.0551	-0.8109	-0.9151	0.6404	-0.5759	0.4592	-0.2121	0.0604	-0.1836	0.1340
0.4	-0.8685	0.0089	-0.9151	-0.9446	0.5973	-0.5896	0.5227	-0.2370	0.0556	-0.2158	0.1501
0.5	-0.8629	0.0261	-0.9446	-0.8901	0.5293	-0.5577	0.5359	-0.2395	0.0556	-0.2230	0.1488
0.6	-0.7891	0.0408	-0.8901	-0.7549	0.4557	-0.4879	0.4945	-0.2190	0.0602	-0.2031	0.1305
0.7	-0.6603	0.0253	-0.7549	-0.5599	0.4052	-0.3985	0.4012	-0.1793	0.0693	-0.1588	0.1094
0.8	-0.5061	0.0289	-0.5599	-0.3564	0.4130	-0.3191	0.2695	-0.1297	0.0827	-0.0984	0.0589
0.9	-0.3787	-0.1203	-0.3564	-0.0513	0.9604	-0.2888	0.1340	-0.0860	0.0981	-0.0388	0.0235
$\lambda = 2.0$											
0.0	-0.3467	-0.4021	0.9604	0.0513	0.9604	0.2784	0.1216	0.0676	0.2031	0.1660	0.0237
0.1	-0.4671	-0.4010	0.0729	0.0729	1.1193	0.3003	0.2183	0.1080	0.2038	0.1265	0.0562
0.2	-0.5799	0.3917	0.2099	0.2099	1.2569	0.3136	0.3024	0.1473	0.2067	0.0856	0.0839
0.3	-0.6662	0.3778	0.3334	0.3334	1.3533	0.3172	0.3600	0.1785	0.2114	0.0522	0.1023
0.4	-0.7117	-0.3640	0.4189	0.4189	1.3953	0.3121	0.3818	0.1961	0.2176	0.0332	0.1083
0.5	-0.7092	-0.3549	0.4479	0.4479	1.3795	0.3010	0.3650	0.1969	0.2242	0.0326	0.1013
0.6	-0.6605	-0.3543	0.4120	0.4120	1.3132	0.2884	0.3148	0.1811	0.2301	0.0509	0.0833
0.7	-0.5773	-0.3640	0.3162	0.3162	1.2137	0.2790	0.2445	0.1524	0.2335	0.0840	0.0792
0.8	-0.4795	-0.3820	0.1802	0.1802	1.1056	0.2701	0.1732	0.1178	0.2323	0.1234	0.0360
0.9	-0.3936	-0.4006	0.0400	0.0400	1.0153	0.2791	0.1245	0.0863	0.2238	0.1564	0.0059

TABLE 2d  
 $s/c = 1, \xi = 45 \text{ deg}$

$\beta/2\pi$	$C_{r\epsilon}$	$C_{r\alpha}$	$C_{rW}$	$C_{M1}$	$C_{M\alpha}$	$C_{MW}$
$\lambda = 0.2$						
0.0	-0.4271	-0.0453	-0.4259	0.0217	-0.0894	-0.0917
0.1	-0.7642	-0.2374	-0.7929	-0.1178	-0.1720	-0.1837
0.2	-0.9225	-0.2126	-0.9473	-0.0725	-0.2280	-0.2367
0.3	-1.0641	-0.1334	-1.0747	-0.0241	-0.2790	-0.2821
0.4	-1.1527	-0.0310	-1.1447	0.0155	-0.3135	-0.3101
0.5	-1.1669	0.0808	-1.1391	-0.0018	-0.3244	-0.3144
0.6	-1.0987	0.1892	-1.0524	0.0103	-0.3244	-0.3144
0.7	-0.9509	0.2818	-0.8902	0.0217	-0.3089	-0.2927
0.8	-0.7329	0.3434	-0.6647	0.0335	-0.2680	-0.2467
0.9	-0.4445	0.3296	-0.3831	0.0454	-0.2053	-0.1807
			0.3908	0.0487	-0.1220	-0.0988
$\lambda = 0.5$						
0.0	-0.4269	-0.1133	-0.4194	0.0533	-0.0756	-0.0903
0.1	-0.6960	-0.1417	-0.6882	0.1275	-0.1444	-0.1526
0.2	-0.8674	-0.1185	-0.8443	0.2065	-0.2039	-0.2010
0.3	-0.9926	-0.0498	-0.9356	0.3088	-0.2545	-0.2370
0.4	-1.0569	0.0393	-0.9589	0.4082	-0.2875	-0.2550
0.5	-1.0444	0.1315	-0.9077	0.4843	-0.2961	-0.2505
0.6	-0.9493	0.2104	-0.7844	0.5208	-0.2771	-0.2225
0.7	-0.7777	0.2568	-0.6036	0.5027	-0.2312	-0.1739
0.8	-0.5499	0.2415	-0.3966	0.4103	-0.1624	-0.1131
0.9	-0.3322	0.1023	-0.2479	0.2100	-0.0834	-0.0626
			0.2100	0.0188	-0.0822	-0.0403
$\lambda = 1.0$						
0.0	-0.4261	-0.2267	-0.3976	0.1006	-0.0266	-0.0856
0.1	-0.5918	-0.2035	-0.5136	0.2332	-0.0794	-0.1097
0.2	-0.7318	-0.1558	-0.5923	0.3556	-0.1358	-0.1332
0.3	-0.8317	-0.0913	-0.6245	0.4570	-0.1839	-0.1507
0.4	-0.8734	-0.0232	-0.6062	0.5208	-0.2139	-0.1390
0.5	-0.8458	0.0345	-0.5408	0.5332	-0.2192	-0.1519
0.6	-0.7503	0.0662	-0.4421	0.4869	-0.1974	-0.1493
0.7	-0.6037	0.0559	-0.3358	0.3836	-0.1506	-0.1285
0.8	-0.4446	-0.0118	-0.2621	0.2384	-0.0875	-0.0998
0.9	-0.3484	-0.1368	-0.2763	0.0992	-0.0294	-0.0736
			0.0992	0.1060	-0.2337	-0.0658
$\lambda = 2.0$						
0.0	-0.4234	-0.4547	-0.3293	0.1626	-0.1697	-0.0709
0.1	-0.5162	-0.4344	-0.3344	0.2601	-0.1269	-0.0682
0.2	-0.6079	-0.4034	-0.3289	0.3394	-0.0807	-0.0688
0.3	-0.6750	-0.3687	-0.3131	0.3853	-0.0433	-0.0960
0.4	-0.7011	-0.3376	-0.2904	0.3893	-0.0228	-0.0709
0.5	-0.6796	-0.3180	-0.2671	0.3518	0.0239	-0.0729
0.6	-0.6151	-0.3168	-0.2313	0.2826	0.0468	-0.0971
0.7	-0.5248	-0.3381	-0.2003	0.2005	0.0871	-0.0730
0.8	-0.4370	-0.3804	-0.1464	0.1320	0.0871	-0.0719
0.9	-0.3896	-0.4310	-0.2683	0.1320	0.1346	-0.0712
			0.1086	0.2386	0.1717	-0.0718
			0.3018	0.2386	0.5066	-0.0156

TABLE 2c

 $s/c = \xi = 60 \text{ deg}$ 

$\beta/2\pi$	$C_{Fy}$	$C_{Fz}$	$C_{Fw}$	$C_{My}$	$C_{Mz}$	$C_{Mw}$
$\lambda = 0.2$						
0.0	-0.5993	-0.5938	0.5973	0.0345	0.1475	0.1501
0.1	-0.7014	-0.6329	-0.7608	-0.3301	-0.1596	-0.1876
0.2	-0.8213	-0.7622	-0.8701	-0.4859	-0.1997	-0.2205
0.3	-0.9543	-0.9135	-0.9833	-0.6378	-0.2437	-0.2556
0.4	-1.0401	-1.0232	-1.0443	-0.8503	-0.2749	-0.2772
0.5	-1.0522	-1.0617	-1.0300	-0.2019	-0.2852	-0.2781
0.6	-0.9811	-1.0160	-0.9345	0.3350	-0.2717	-0.2557
0.7	-0.8314	-0.8868	-0.7663	0.4331	-0.2352	-0.2116
0.8	-0.6169	-0.6840	-0.5428	0.4776	-0.1792	-0.1506
0.9	-0.3472	-0.4086	-0.2818	0.4169	-0.1050	-0.0775
$\lambda = 0.5$						
0.0	-0.5991	-0.5649	0.5870	0.0849	0.1311	0.1475
0.1	-0.7561	-0.6360	-0.8141	-0.3303	-0.1503	-0.1920
0.2	-0.8503	-0.7408	-0.8915	-0.6049	-0.1865	-0.2449
0.3	-0.9460	-0.8781	-0.9404	-0.1751	-0.2280	-0.2349
0.4	-0.9957	-0.9832	-0.9334	-0.3104	-0.2576	-0.2425
0.5	-0.9730	-1.0195	-0.8560	0.4222	-0.2664	-0.2311
0.6	-0.8694	-0.9576	-0.7104	0.4896	-0.2500	-0.1990
0.7	-0.6914	-0.8225	-0.5141	0.4937	-0.2087	-0.1491
0.8	-0.4623	-0.5918	-0.3033	0.4098	-0.1451	-0.0898
0.9	-0.2595	-0.3210	-0.1807	0.1863	-0.0703	-0.0389
$\lambda = 1.0$						
0.0	-0.5984	-0.4617	0.5521	0.1609	0.0727	0.1387
0.1	-0.6949	-0.4989	0.6662	0.2135	-0.0877	-0.1507
0.2	-0.7791	-0.6116	-0.7014	-0.3121	-0.1280	-0.1578
0.3	-0.8452	-0.7497	-0.6882	-0.4169	-0.1702	-0.1618
0.4	-0.8626	-0.8567	-0.6255	-0.4922	-0.1990	-0.1569
0.5	-0.8135	-0.8932	-0.5184	0.5152	-0.2054	-0.1400
0.6	-0.6968	-0.8359	-0.3847	0.4735	-0.1853	-0.1119
0.7	-0.5314	-0.6807	-0.2575	0.3651	-0.1392	-0.0788
0.8	-0.3651	-0.4524	-0.1907	0.2027	-0.0744	-0.0416
0.9	-0.3120	-0.2560	-0.0748	0.0464	-0.0173	-0.0658
$\lambda = 2.0$						
0.0	-0.5958	-0.0481	0.4382	0.2615	0.1611	0.1101
0.1	-0.6094	-0.0642	0.4101	0.3259	0.1351	0.0875
0.2	-0.6534	-0.1851	0.3652	0.3854	0.0861	0.0734
0.3	-0.6871	-0.3288	0.3116	0.4129	0.0417	0.0654
0.4	-0.6846	-0.4401	0.2570	0.3953	0.0152	0.0606
0.5	-0.6304	-0.4796	0.2128	0.3332	0.0135	0.0575
0.6	-0.5493	-0.4254	0.1919	0.2404	0.0385	0.0559
0.7	-0.4476	-0.2804	0.2067	0.1430	-0.0867	0.0580
0.8	-0.3743	-0.0832	0.2644	0.0781	0.1470	-0.0672
0.9	-0.3956	-0.0672	0.3588	0.0965	0.1932	-0.0869

TABLE 2f

 $s/c = 1, \xi = 75 \text{ deg}$ 

$\beta/2\pi$	$C_{Fy}$	$C_{Fz}$	$C_{Fw}$	$C_{Mq}$	$C_{Mz}$	$C_{Mw}$
$\lambda = 0.2$						
0.0	1.1428	-0.2324	1.1382	-0.3631	0.3589	0.3616
0.1	0.5012	-0.7407	0.5900	-0.1346	0.1072	0.1585
0.2	0.6002	-0.5212	0.6703	-0.6111	0.4253	0.1672
0.3	0.7246	-0.6688	0.7710	-0.4741	0.1304	0.0943
0.4	0.8084	-0.3646	0.8257	-0.3064	0.2310	0.0454
0.5	0.8236	-0.1166	0.8096	-0.0599	0.1962	0.1994
0.6	0.7607	-0.0757	0.7180	0.1318	0.1994	0.0320
0.7	0.6267	0.2457	0.5625	0.3021	0.1917	0.1666
0.8	0.4410	0.3646	0.3669	0.4898	0.1667	0.1410
0.9	0.2240	0.3702	0.1598	0.4302	0.0759	0.0434
$\lambda = 0.5$						
0.0	1.1428	-0.5811	1.1141	-0.1953	0.3371	0.3539
0.1	0.6951	-0.9524	0.8762	-0.4047	0.1110	0.2347
0.2	0.6929	-0.5347	0.8248	-0.2446	0.1168	0.2022
0.3	0.7674	-0.6507	0.8328	-0.0526	0.1488	0.1982
0.4	0.8133	-0.4819	0.8046	0.1361	0.1761	0.1918
0.5	0.7949	-0.2890	0.7148	0.2984	0.1887	0.1736
0.6	0.7018	-0.1739	0.5651	0.4112	0.1820	0.1412
0.7	0.5419	0.2845	0.3756	0.4530	0.1555	0.0917
0.8	0.3392	0.3029	0.1845	0.3959	0.1104	0.0972
0.9	0.1626	0.1114	0.0945	0.1554	0.0501	0.0495
$\lambda = 1.0$						
0.0	1.1425	-1.1619	1.0317	-0.3711	0.2591	0.3278
0.1	0.8278	-1.2784	0.9774	-0.1165	0.0638	0.2611
0.2	0.7606	-1.1043	0.8474	0.1011	0.0649	0.2034
0.3	0.7773	-0.9467	0.7476	0.2444	0.1007	0.1726
0.4	0.7751	-0.7687	0.6259	0.3652	0.1329	0.1459
0.5	0.7170	-0.5639	0.4729	0.4311	0.1486	0.1148
0.6	0.5952	-0.3535	0.3043	0.4241	0.1415	0.0786
0.7	0.4254	0.1836	0.1956	0.3354	0.1093	0.0431
0.8	0.2552	0.1403	0.0943	0.1624	0.0531	0.0766
0.9	0.2469	-0.4340	0.2877	-0.0628	0.0052	0.0248
$\lambda = 2.0$						
0.0	1.1410	-2.3217	0.7539	-0.6032	0.0530	0.2395
0.1	0.8322	-1.9937	0.6592	-0.4369	0.1803	0.1754
0.2	0.7351	-1.7287	0.5017	-0.2694	0.1457	0.1159
0.3	0.7009	-1.5224	0.3565	-0.4227	0.0885	0.0767
0.4	0.6556	-1.3095	0.2313	-0.3860	0.0436	0.0507
0.5	0.5725	-1.0762	0.1390	-0.1786	0.0246	0.0338
0.6	0.4540	-0.8534	0.1130	-0.1596	0.0384	0.0264
0.7	0.3301	0.2205	0.0837	0.1786	0.0871	0.0433
0.8	0.2686	0.7822	0.0685	0.2264	0.1658	0.0335
0.9	0.4255	1.2769	0.1160	0.3156	0.2358	0.0661
						0.1421
						0.0213

TABLE 2g

 $s/c = (\xi = 0 \text{ deg})$ 

$\beta/2\pi$	$C_{Fy}$	$C_{Fz}$	$C_{Fw}$	$C_{Fy}$	$C_{Fz}$	$C_{Fw}$	$C_{Fz}$	$C_{Fw}$
$\lambda = 0.2$								
0.0	-0.5030	-0.0350	-0.5000	0.1128	0.0425	0.1142	0.0204	0.0097
0.1	-0.7063	0.1513	0.7318	0.0440	0.2559	0.1697	0.0239	0.0615
0.2	-0.8744	0.1522	0.8998	0.0210	0.2806	0.2182	0.0255	0.0700
0.3	-0.9658	0.1366	0.9886	0.0071	0.2777	0.2471	0.0224	0.0710
0.4	-1.0142	0.1271	1.0356	0.0231	0.2748	0.2631	0.0204	0.0713
0.5	-1.0297	0.1241	1.0506	0.0281	0.2738	0.2683	0.0198	0.0714
0.6	-1.0142	0.1271	1.0356	0.0231	0.2748	0.2631	0.0204	0.0713
0.7	-0.9658	0.1366	0.9886	0.0071	0.2777	0.2471	0.0224	0.0710
0.8	-0.8744	0.1522	0.8998	0.0210	0.2806	0.2182	0.0255	0.0700
0.9	-0.7063	0.1513	0.7318	0.0440	0.2806	0.1697	0.0239	0.0615
$\lambda = 0.5$								
0.0	0.4967	0.0895	0.4777	0.2814	0.0990	0.1128	0.0514	0.0225
0.1	0.5757	0.0221	0.5997	0.1966	0.2277	0.1383	0.0258	0.0547
0.2	0.6835	0.0913	0.7333	0.1652	0.3282	0.1706	0.0084	0.0819
0.3	0.7723	0.1230	0.8337	0.1644	0.3876	0.1976	0.0002	0.0992
0.4	0.8270	0.1356	0.8929	0.1706	0.4177	0.2145	0.0038	0.1084
0.5	0.8452	0.1389	0.9122	0.1735	0.4269	0.2202	0.0048	0.1112
0.6	0.8270	0.1356	0.8929	0.1706	0.4177	0.2145	0.0038	0.1084
0.7	0.7723	0.1230	0.8337	0.1644	0.3876	0.1976	0.0002	0.0992
0.8	0.6835	0.0913	0.7333	0.1652	0.3282	0.1706	0.0084	0.0819
0.9	0.5757	0.0221	0.5997	0.1966	0.2277	0.1383	0.0258	0.0547
$\lambda = 1.0$								
0.0	0.4803	0.1892	0.3962	0.5604	0.1600	0.1091	0.1051	0.0363
0.1	0.5302	0.1394	0.4866	0.5422	0.2219	0.1274	0.0957	0.0533
0.2	0.5847	0.0943	0.5763	0.5331	0.2851	0.1459	0.0860	0.0712
0.3	0.6327	0.0611	0.6498	0.5319	0.3370	0.1619	0.0782	0.0862
0.4	0.6651	0.0414	0.6972	0.5339	0.3703	0.1725	0.0735	0.0961
0.5	0.6765	0.0349	0.7134	0.5351	0.3817	0.1763	0.0719	0.0995
0.6	0.6651	0.0414	0.6972	0.5339	0.3703	0.1725	0.0735	0.0961
0.7	0.6327	0.0611	0.6498	0.5319	0.3370	0.1619	0.0782	0.0862
0.8	0.5847	0.0943	0.5763	0.5331	0.2851	0.1459	0.0860	0.0712
0.9	0.5302	0.1394	0.4866	0.5422	0.2219	0.1274	0.0957	0.0533
$\lambda = 2.0$								
0.0	0.4520	0.4135	0.0615	1.1121	0.1876	0.1026	0.2183	0.0426
0.1	0.4961	0.4075	0.1259	1.1614	0.2139	0.1192	0.2224	0.0731
0.2	0.5346	0.3997	0.1844	1.2020	0.2385	0.1334	0.2246	0.0595
0.3	0.5646	0.3923	0.2311	1.2325	0.2585	0.1444	0.2256	0.0778
0.4	0.5837	0.3871	0.2611	1.2516	0.2716	0.1514	0.2259	0.0705
0.5	0.5902	0.3853	0.2715	1.2581	0.2762	0.1538	0.2260	0.0720
0.6	0.5837	0.3871	0.2611	1.2516	0.2716	0.1514	0.2259	0.0705
0.7	0.5646	0.3923	0.2311	1.2325	0.2585	0.1444	0.2256	0.0778
0.8	0.5346	0.3997	0.1844	1.2020	0.2385	0.1334	0.2246	0.0760
0.9	0.4961	0.4075	0.1259	1.1614	0.2139	0.1192	0.2224	0.0514

TABLE 2b

$s/c = 2, \xi = 15 \text{ deg}$

$\beta/2\pi$	$C_{Fy}$	$C_{Fz}$	$C_{Fw}$	$C_{My}$	$C_{Mz}$	$C_{Mw}$
$\lambda = 0.2$						
0.0	-0.5157	-0.5128	-0.5119	-0.1183	-0.1160	-0.1174
0.1	-0.7718	-0.7866	-0.7473	-0.1857	-0.1879	-0.1796
0.2	-0.9130	-0.9261	-0.8886	-0.2274	-0.2295	-0.2210
0.3	-0.9847	-0.9983	-0.9590	-0.2511	-0.2533	-0.2443
0.4	-1.0192	-1.0356	-0.9904	-0.2635	-0.2663	-0.2559
0.5	-1.0230	-1.0435	-0.9900	-0.2658	-0.2696	-0.2572
0.6	-0.9961	-1.0217	-0.9584	-0.2580	-0.2628	-0.2484
0.7	-0.9345	-0.9656	-0.8920	-0.2392	-0.2453	-0.2286
0.8	-0.8256	-0.8617	-0.7793	-0.2068	-0.2139	-0.1955
0.9	-0.6381	-0.6719	-0.5963	-0.1543	-0.1608	-0.1444
$\lambda = 0.5$						
0.0	-0.5092	-0.4903	-0.4872	-0.1168	-0.1027	-0.1117
0.1	-0.6262	-0.6444	-0.5599	-0.1507	-0.1457	-0.1343
0.2	-0.7348	-0.7730	-0.6416	-0.1829	-0.1833	-0.1591
0.3	-0.8079	-0.8580	-0.6982	-0.2058	-0.2094	-0.1773
0.4	-0.8427	-0.9022	-0.7218	-0.2177	-0.2236	-0.1861
0.5	-0.8406	-0.9079	-0.7124	-0.2184	-0.2261	-0.1851
0.6	-0.8024	-0.8748	-0.6719	-0.2079	-0.2166	-0.1745
0.7	-0.7293	-0.8009	-0.6048	-0.1868	-0.1948	-0.1555
0.8	-0.6292	-0.6878	-0.5245	-0.1576	-0.1620	-0.1319
0.9	-0.5343	-0.5598	-0.4682	-0.1290	-0.1251	-0.1134
$\lambda = 1.0$						
0.0	-0.4922	-0.4085	-0.4216	-0.1129	-0.0547	-0.0967
0.1	-0.5576	-0.5143	-0.4352	-0.1343	-0.0856	-0.0594
0.2	-0.6150	-0.6048	-0.4494	-0.1532	-0.1126	-0.1043
0.3	-0.6557	-0.6702	-0.4585	-0.1671	-0.1326	-0.1161
0.4	-0.6744	-0.7054	-0.4588	-0.1743	-0.1437	-0.1181
0.5	-0.6698	-0.7080	-0.4496	-0.1740	-0.1448	-0.1168
0.6	-0.6437	-0.6785	-0.4337	-0.1667	-0.1361	-0.1127
0.7	-0.6019	-0.6209	-0.4170	-0.1539	-0.1189	-0.1073
0.8	-0.5546	-0.5447	-0.4069	-0.1386	-0.0963	-0.1024
0.9	-0.5149	-0.4672	-0.4094	-0.1241	-0.0730	-0.0992
$\lambda = 2.0$						
0.0	-0.4620	-0.0710	-0.3041	-0.1060	-0.1395	-0.0697
0.1	-0.5081	-0.1400	-0.3040	-0.1227	-0.1208	-0.0727
0.2	-0.5451	-0.1980	-0.3027	-0.1362	-0.1049	-0.0550
0.3	-0.5706	-0.2402	-0.3013	-0.1459	-0.0935	-0.0762
0.4	-0.5836	-0.2633	-0.3006	-0.1510	-0.0876	-0.0773
0.5	-0.5838	-0.2658	-0.3009	-0.1517	-0.0876	-0.0782
0.6	-0.5725	-0.2487	-0.3026	-0.1480	-0.0931	-0.0787
0.7	-0.5518	-0.2148	-0.3052	-0.1307	-0.1030	-0.0786
0.8	-0.5247	-0.1695	-0.3076	-0.1307	-0.1156	-0.0775
0.9	-0.4940	-0.1192	-0.3081	-0.1188	-0.1285	-0.0748



TABLE 2i

 $s/c = 2, \xi = 30 \text{ deg}$ 

$\beta/2\pi$	$C_{FV}$	$C_{Fz}$	$C_{Fw}$	$C_{Mv}$	$C_{Mz}$	$C_{Mw}$
$\lambda = 0.2$						
0.0	-0.5564	-0.1199	0.0498	-0.1313	-0.1291	-0.1303
0.1	-0.8338	-0.1292	0.1224	-0.2028	-0.2019	-0.1995
0.2	-0.9390	-0.1345	0.1268	-0.2340	-0.2329	-0.2307
0.3	-0.9895	-0.1346	0.1598	-0.2509	-0.2507	-0.2464
0.4	-1.0100	-0.0909	0.1598	-0.2588	-0.2603	-0.2525
0.5	-1.0027	-0.0336	0.2619	-0.2580	-0.2615	-0.2498
0.6	-0.9654	-0.0323	0.3160	-0.2480	-0.2537	-0.2378
0.7	-0.8922	0.1024	0.3643	-0.2276	-0.2353	0.2155
0.8	-0.7682	0.1665	0.3921	-0.1932	-0.2025	0.1799
0.9	-0.5663	0.1588	0.3259	-0.1389	-0.1468	0.1277
$\lambda = 0.5$						
0.0	-0.5495	-0.0892	0.1165	0.1297	-0.1156	-0.1238
0.1	-0.6889	-0.0104	0.2334	0.1674	-0.1603	-0.1518
0.2	-0.7858	-0.0250	0.3052	0.1954	-0.1925	-0.1738
0.3	-0.8376	-0.0628	0.3573	0.2119	-0.2126	-0.1857
0.4	-0.8506	-0.1026	0.3974	0.2176	-0.2220	-0.1875
0.5	-0.8274	-0.1408	0.4234	0.2129	-0.2206	-0.1799
0.6	-0.7689	-0.1167	0.4280	0.1978	-0.2079	-0.1636
0.7	-0.6775	-0.0802	0.3984	0.1730	-0.1831	-0.1407
0.8	-0.5690	-0.1258	0.3176	0.1429	-0.1485	-0.1172
0.9	-0.5003	-0.1757	0.1906	-0.1221	-0.1172	-0.1080
$\lambda = 1.0$						
0.0	-0.5304	-0.1891	0.1892	0.1252	-0.0668	-0.1059
0.1	-0.5993	-0.1387	0.2682	0.1457	-0.0969	-0.1125
0.2	-0.6485	-0.0965	0.3281	0.1614	-0.1206	-0.1169
0.3	-0.6735	-0.0622	0.3663	0.1704	-0.1362	-0.1181
0.4	-0.6731	-0.0376	0.3812	0.1723	-0.1425	-0.1157
0.5	-0.6494	-0.0265	0.3719	0.1671	-0.1391	-0.1107
0.6	-0.6082	-0.0337	0.3391	0.1563	-0.1265	-0.1045
0.7	-0.5607	-0.0628	0.2879	0.1427	-0.1072	-0.0997
0.8	-0.5234	-0.1107	0.2313	0.1308	-0.0861	-0.0989
0.9	-0.5126	-0.1621	0.1914	0.1247	-0.0706	-0.1024
$\lambda = 2.0$						
0.0	-0.4938	-0.1740	0.2148	0.1166	0.1321	-0.0736
0.1	-0.5303	-1.2054	0.2435	0.1294	0.1138	-0.0733
0.2	-0.5557	-1.2237	0.2619	0.1389	0.0999	-0.0668
0.3	-0.5691	-1.2301	0.2694	0.1446	0.0914	-0.0736
0.4	-0.5711	-1.2269	0.2672	0.1466	0.0890	-0.0746
0.5	-0.5636	-1.2182	0.2574	0.1450	0.0924	-0.0760
0.6	-0.5498	-1.2078	0.2436	0.1408	0.1004	-0.0617
0.7	-0.5333	-1.1990	0.2295	0.1350	0.1110	-0.0791
0.8	-0.5175	-1.1924	0.2189	0.1286	0.1216	-0.0569
0.9	-0.5043	-1.1859	0.2139	0.1223	0.1293	-0.0780

TABLE 2j  
 $s/c = 2, \quad \xi = 45 \text{ deg}$

$\beta/2\pi$	$C_{Fy}$	$C_{Fz}$	$C_{Fw}$	$C_{My}$	$C_{Mz}$	$C_{Mw}$
$\lambda = 0.2$						
0.0	-0.6338	-0.0341	-0.6312	-0.1295	0.0610	-0.1565
0.1	-0.8891	-0.1100	-0.8744	-0.2437	0.0242	-0.2205
0.2	-0.9496	-0.1111	-0.9345	-0.2536	0.0318	-0.2372
0.3	-0.9786	-0.0572	-0.9717	-0.2038	0.0891	-0.2458
0.4	-0.9862	-0.0189	-0.9910	-0.1287	0.1652	-0.2485
0.5	-0.9692	0.1046	-0.9874	-0.0402	0.2472	-0.2479
0.6	-0.9232	0.1928	-0.9550	0.0548	0.3274	-0.2334
0.7	-0.8408	0.2759	-0.8854	0.1502	0.3972	-0.2124
0.8	-0.7046	0.3358	-0.7582	0.2304	0.4362	-0.1775
0.9	-0.4887	0.2736	-0.5327	0.2003	0.3428	-0.1221
$\lambda = 0.5$						
0.0	-0.6261	-0.0870	-0.6091	-0.3228	0.1436	-0.1546
0.1	-0.7684	-0.0569	-0.7613	-0.3459	0.2235	-0.1902
0.2	-0.8384	-0.0261	-0.8423	-0.3409	0.2770	-0.2088
0.3	-0.8627	-0.0229	-0.8850	-0.3004	0.3304	-0.2161
0.4	-0.8519	0.0828	-0.8974	-0.2359	0.3811	-0.2145
0.5	-0.8073	0.1428	-0.8762	-0.1589	0.4197	-0.2040
0.6	-0.7279	0.1888	-0.8149	-0.0830	0.4330	-0.1843
0.7	-0.6164	0.1982	-0.7075	0.0320	0.4012	-0.1559
0.8	-0.4985	0.1337	-0.5655	0.0529	0.2993	-0.1253
0.9	-0.4728	-0.0180	-0.4826	-0.1957	0.1508	-0.1174
$\lambda = 1.0$						
0.0	-0.6032	-0.1848	-0.5269	-0.6391	0.2348	-0.1490
0.1	-0.6619	-0.1417	-0.6157	-0.6397	0.3051	-0.1639
0.2	-0.6903	-0.0991	-0.6752	-0.6176	0.3539	-0.1718
0.3	-0.6899	-0.0588	-0.7054	-0.5759	0.3811	-0.1727
0.4	-0.6631	-0.0269	-0.7036	-0.5231	0.3846	-0.1669
0.5	-0.6149	-0.0125	-0.6677	-0.4721	0.3616	-0.1554
0.6	-0.5556	-0.0250	-0.6005	-0.4402	0.3128	-0.1405
0.7	-0.5046	-0.0698	-0.5169	-0.4473	0.2474	-0.1271
0.8	-0.4895	-0.1371	-0.4515	-0.5044	0.1905	-0.1222
0.9	-0.5291	-0.1910	-0.4500	-0.5891	0.1798	-0.1310
$\lambda = 2.0$						
0.0	-0.5528	-0.4193	-0.1728	-1.2519	0.2552	-0.1365
0.1	-0.5650	-0.3969	-0.2149	-1.2467	0.2694	-0.1402
0.2	-0.5663	-0.3790	-0.2422	-1.2288	0.2713	-0.1414
0.3	-0.5579	-0.3687	-0.2504	-1.2042	0.2628	-0.1402
0.4	-0.5430	-0.3683	-0.2385	-1.1802	0.2474	-0.1370
0.5	-0.5265	-0.3782	-0.2099	-1.1651	0.2301	-0.1330
0.6	-0.5141	-0.3961	-0.1730	-1.1652	0.2256	-0.1295
0.7	-0.5106	-0.4165	-0.1404	-1.1820	0.2122	-0.1278
0.8	-0.5180	-0.4317	-0.1251	-1.2103	0.2188	-0.1287
0.9	-0.5340	-0.4341	-0.1359	-1.2383	0.2348	-0.1319

$s/c = \xi = 60 \text{ deg}$

$\beta/2\pi$	$C_{FV}$	$C_{Fz}$	$C_{Fw}$	$C_{Mv}$	$C_{Mz}$	$C_{Mw}$
$\lambda = 0.2$						
0.0	-0.7648	-0.7628	-0.7575	0.1994	-0.1973	-0.1975
0.1	-0.9324	-0.8999	-0.9532	0.2372	-0.2274	-0.2424
0.2	-0.9436	-0.9141	-0.9612	0.2361	-0.2273	-0.2404
0.3	-0.9531	-0.9361	-0.9881	0.2359	-0.2302	-0.2370
0.4	-0.9500	-0.9490	-0.9991	0.2337	-0.2320	-0.2309
0.5	-0.9258	-0.9420	-0.9880	0.2274	-0.2298	-0.2205
0.6	-0.8738	-0.9074	-0.9295	0.2151	-0.2218	-0.2043
0.7	-0.7854	-0.8347	-0.8266	0.1947	-0.2052	-0.1802
0.8	-0.6401	-0.7003	-0.5722	0.1605	-0.1739	-0.1436
0.9	-0.4018	-0.4484	-0.3504	0.1024	-0.1127	-0.0894
$\lambda = 0.5$						
0.0	-0.7558	-0.7424	-0.7124	0.1970	-0.1837	-0.1857
0.1	-0.8686	-0.8405	-0.8301	0.2208	-0.2041	-0.2109
0.2	-0.8932	-0.8739	-0.8437	0.2233	-0.2090	-0.2106
0.3	-0.8847	-0.8904	-0.8111	0.2188	-0.2107	-0.2003
0.4	-0.8498	-0.8876	-0.7476	0.2089	-0.2087	-0.1836
0.5	-0.7848	-0.8552	-0.6560	0.1927	-0.2003	-0.1611
0.6	-0.6846	-0.7807	-0.5388	0.1686	-0.1824	-0.1329
0.7	-0.5484	-0.6511	-0.4069	0.1361	-0.1516	-0.1011
0.8	-0.4105	-0.4778	-0.3141	0.1029	-0.1099	-0.0788
0.9	-0.4500	-0.4331	-0.4325	0.1144	-0.1003	-0.1100
$\lambda = 1.0$						
0.0	-0.7269	-0.6657	-0.5788	0.1895	-0.1343	-0.1509
0.1	-0.7574	-0.7133	-0.5825	0.1925	-0.1424	-0.1478
0.2	-0.7499	-0.7377	-0.5469	0.1873	-0.1456	-0.1362
0.3	-0.7132	-0.7395	-0.4858	0.1763	-0.1442	-0.1197
0.4	-0.6508	-0.7106	-0.4117	0.1600	-0.1360	-0.1010
0.5	-0.5684	-0.6433	-0.3402	0.1396	-0.1191	-0.0836
0.6	-0.4813	-0.5385	-0.2946	0.1185	-0.0935	-0.0727
0.7	-0.4241	-0.4226	-0.2065	0.1080	-0.0655	-0.0455
0.8	-0.4502	-0.3678	-0.3971	0.1125	-0.0527	-0.0760
0.9	-0.5791	-0.4586	-0.5235	0.1472	-0.0772	-0.1332
$\lambda = 2.0$						
0.0	-0.6488	-0.3144	-0.3160	0.1691	0.0750	-0.0824
0.1	-0.6170	-0.3160	-0.2856	0.1569	0.0760	-0.0724
0.2	-0.5764	-0.3026	-0.2620	0.1440	0.0802	-0.0651
0.3	-0.5316	-0.2668	-0.2515	0.1315	0.0896	-0.0619
0.4	-0.4903	-0.2084	-0.2585	0.1206	0.1042	-0.0634
0.5	-0.4632	-0.1380	-0.2826	0.1138	0.1217	-0.0694
0.6	-0.4611	-0.0779	-0.3170	0.1134	0.1368	-0.0781
0.7	-0.4890	-0.0563	-0.3488	0.1209	0.1426	-0.0865
0.8	-0.5412	-0.0938	-0.3629	0.1352	0.1336	-0.0910
0.9	-0.6009	-0.1885	-0.3508	0.1527	0.1093	-0.0894
						0.0852
						0.0785
						0.0698
						0.0600
						0.0508
						0.0444
						0.0431
						0.0485
						0.0570
						0.0730

TABLE 21

$s/c = 2, \xi = 75 \text{ deg}$

$\beta/2\pi$	$C_{Pv}$	$C_{Pa}$	$C_{Pw}$	$C_{Mv}$	$C_{Ma}$	$C_{Mw}$
$\lambda = 0.2$						
0.0	-0.9734	-0.9728	0.9625	-0.2664	-0.2647	-0.2634
0.1	-0.9609	-0.9119	-0.9979	-0.2502	-0.2355	-0.2602
0.2	-0.9284	-0.8871	-0.9578	-0.2321	-0.2198	-0.2400
0.3	-0.9234	-0.8985	-0.9365	-0.2245	-0.2165	-0.2281
0.4	-0.9132	-0.9075	-0.9072	-0.2183	-0.2151	-0.2171
0.5	-0.8846	-0.8992	-0.8588	-0.2101	-0.2120	-0.2040
0.6	-0.8296	-0.8641	-0.7847	-0.1978	-0.2047	-0.1869
0.7	-0.7385	-0.7911	-0.6768	-0.1786	-0.1902	-0.1633
0.8	-0.5869	-0.6523	-0.5145	-0.1455	-0.1606	-0.1271
0.9	-0.3033	-0.3532	-0.2500	-0.0782	-0.0897	-0.0642
$\lambda = 0.5$						
0.0	-0.9626	-0.9581	-0.8967	-0.2634	-0.2525	-0.2454
0.1	-0.9876	-0.9341	-0.9658	-0.2578	-0.2332	-0.2530
0.2	-0.9494	-0.9063	-0.9189	-0.2382	-0.2165	-0.2317
0.3	-0.9074	-0.8975	-0.8468	-0.2212	-0.2080	-0.2074
0.4	-0.8518	-0.8832	-0.7548	-0.2039	-0.2011	-0.1813
0.5	-0.7705	-0.8437	-0.6392	-0.1830	-0.1906	-0.1518
0.6	-0.6519	-0.7594	-0.4968	-0.1552	-0.1714	-0.1178
0.7	-0.4847	-0.6042	-0.3312	-0.1168	-0.1365	-0.0791
0.8	-0.2923	-0.3630	-0.2016	-0.0725	-0.0806	-0.0496
0.9	-0.4152	-0.3401	-0.4573	-0.1090	-0.0798	-0.1198
$\lambda = 1.0$						
0.0	-0.9252	-0.9013	-0.6941	-0.2532	-0.2077	-0.1900
0.1	-0.9015	-0.8688	-0.6785	-0.2357	-0.1867	-0.1788
0.2	-0.8413	-0.8424	-0.6011	-0.2118	-0.1710	-0.1530
0.3	-0.7590	-0.8113	-0.4943	-0.1856	-0.1577	-0.1222
0.4	-0.6526	-0.7528	-0.3752	-0.1505	-0.1406	-0.0907
0.5	-0.5214	-0.6442	-0.2625	-0.1239	-0.1139	-0.0624
0.6	-0.3806	-0.4723	-0.1914	-0.0906	-0.0736	-0.0452
0.7	-0.2916	-0.2700	-0.2299	-0.0709	-0.0267	-0.0557
0.8	-0.3894	-0.2071	-0.4428	-0.0985	-0.0142	-0.1113
0.9	-0.6994	-0.4927	-0.6881	-0.1834	-0.0915	-0.1793
$\lambda = 2.0$						
0.0	-0.7975	-0.6078	-0.2591	-0.2183	-0.0105	-0.0709
0.1	-0.6967	-0.5409	-0.2149	-0.1823	-0.0165	-0.0573
0.2	-0.5874	-0.4575	-0.1814	-0.1479	-0.0432	-0.0468
0.3	-0.4782	-0.3356	-0.1761	-0.1168	-0.0758	-0.0437
0.4	-0.3863	-0.1701	-0.2115	-0.0924	-0.1164	-0.0508
0.5	-0.3402	-0.0117	-0.2878	-0.0808	-0.1594	-0.0684
0.6	-0.3706	-0.1392	-0.3821	-0.0890	-0.1594	-0.0884
0.7	-0.4826	-0.1244	-0.4473	-0.1185	-0.1842	-0.0911
0.8	-0.6323	-0.0618	-0.4409	-0.1598	-0.1371	-0.1094
0.9	-0.7510	-0.3452	-0.3643	-0.1966	-0.0630	-0.0939

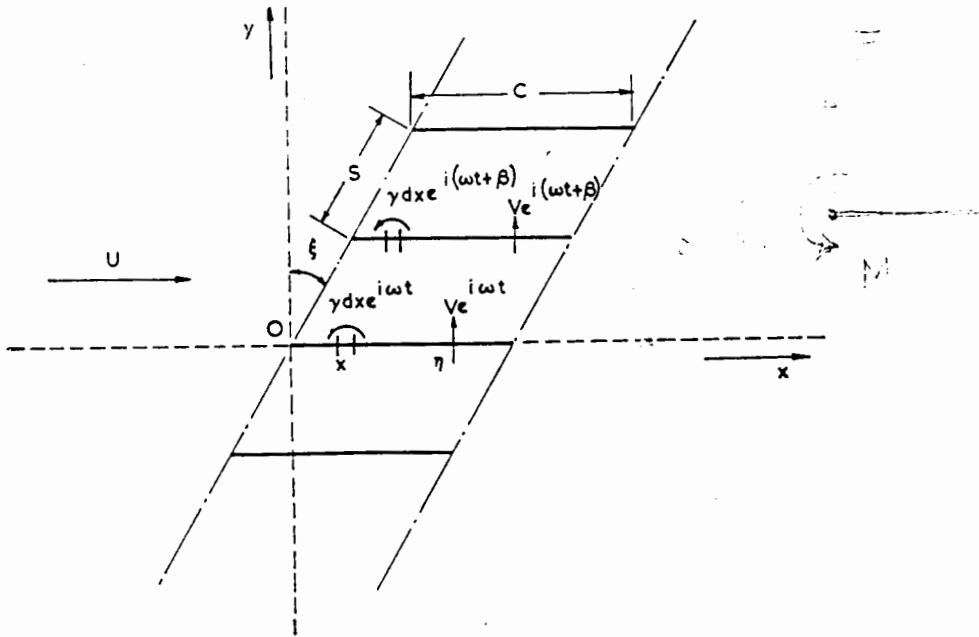


FIG. 1

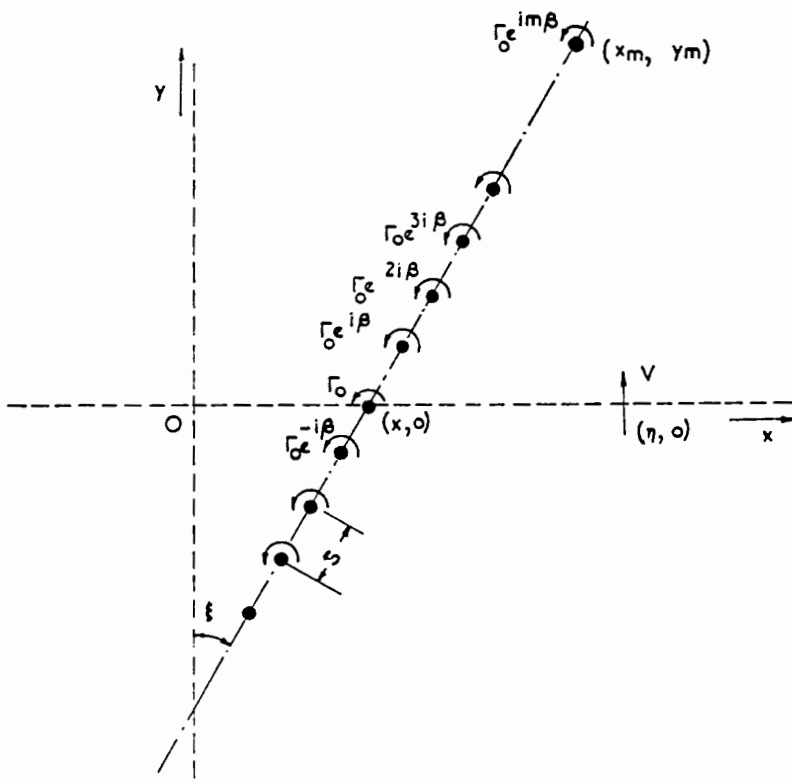


FIG. 2