PHYSOR 2010
Advances in Reactor Physics
to Power the Nuclear renaissance

Adjoint-Weighted Kinetics Parameters in Monte Carlo Forward Calculations

Shim, Hyung Jin (SNU)

May 11, 2010
Pittsburgh, PA, USA

Contents

Definitions of β_{eff} and Λ

- Assuming that the angular flux and the cross-sections are time-independent, **the effective delayed neutron fraction, β_{eff}** and **the prompt neutron generation time, Λ** in the point kinetics equation are defined with **the adjoint flux** by

$$\beta_{eff} = \beta'_{eff} / F, \quad \cdots\cdots \quad (1)$$

$$\Lambda = \Lambda' / F; \quad \cdots\cdots \quad (2)$$

$$\beta'_{eff} = \int d\mathbf{r} \int dE \int d\mathbf{\Omega} \int dE' \int d\mathbf{\Omega}' \phi^*(\mathbf{r}, E, \mathbf{\Omega}) \frac{1}{4\pi} \chi_d(E, E') \nu_d(E') \Sigma_f(\mathbf{r}, E') \phi(\mathbf{r}, E', \mathbf{\Omega}') \quad \cdots\cdots \quad (3)$$

$$\Lambda' = \int d\mathbf{r} \int dE \int d\mathbf{\Omega} \phi^*(\mathbf{r}, E, \mathbf{\Omega}) \frac{1}{v(E)} \phi(\mathbf{r}, E, \mathbf{\Omega}) \quad \cdots\cdots \quad (4)$$

$$F = \int d\mathbf{r} \int dE \int d\mathbf{\Omega} \int dE' \int d\mathbf{\Omega}' \phi^*(\mathbf{r}, E, \mathbf{\Omega}) \frac{1}{4\pi} \chi(E, E') \nu(E') \Sigma_f(\mathbf{r}, E') \phi(\mathbf{r}, E', \mathbf{\Omega}') \quad \cdots\cdots \quad (5)$$

Two Approaches for Adjoint-Weighted β_{eff} and Λ

- The existing methods to calculate the adjoint weighted kinetics parameters in the forward Monte Carlo calculations can be grouped into two categories according to the adopted adjoint solution: **the constant source adjoint function, ϕ_S^*** and **the self-consistent adjoint function, ϕ_0^***.
- ϕ_S^* is the solution of

$$\mathbf{M}^*\phi_S^* = \Sigma_d \qquad \cdots\cdots\cdots \quad (6)$$

while ϕ_0^* is the fundamental-mode solution of the adjoint eigenvalue equation:

$$\mathbf{M}^*\phi_0^* = \frac{1}{k_0}\mathbf{F}^*\phi_0^* \qquad \cdots\cdots\cdots \quad (7)$$

where adjoint operators are defined as

$$\mathbf{M}^*\phi^* = \left[-\mathbf{\Omega}\cdot\nabla + \Sigma_t(\mathbf{r},E)\right]\phi^*(\mathbf{r},E,\mathbf{\Omega}) - \int dE' \int d\mathbf{\Omega}'\Sigma_s(\mathbf{r};E,\mathbf{\Omega}\rightarrow E',\mathbf{\Omega}')\phi^*(\mathbf{r},E',\mathbf{\Omega}'),$$

$$\mathbf{F}^*\phi^* = \int dE' \int d\mathbf{\Omega}' \frac{1}{4\pi}\chi(E',E)\nu(E)\Sigma_f(\mathbf{r},E)\phi^*(\mathbf{r},E',\mathbf{\Omega}').$$

Histories of Estimating Adjoint-Weighted β_{eff} and Λ in MC Forward Calculations

- Nauchi and Kameyama [1] proposed a kinetics parameter calculation method based on the adjoint solution of Eq. (6) with $\nu\Sigma_f$ as Σ_d, while Meulekamp and van der Marck [2] suggested the same method using the adjoint solution of Eq. (6) with Σ_f as Σ_d.

  [1] Y. Nauchi and T. Kameyama, "Proposal of Direct Calculation of Kinetic Parameters b_{eff} and L Based on Continuous Energy Monte Carlo Method," *J. Nucl. Sci. Technol.*, **42**[6], pp. 503-514 (2005).

  [2] R. K. Meulekamp and S. C. van der Marck, "Calculating the Effective Delayed Neutron Fraction with Monte Carlo," *Nucl. Sci. Eng.*, **152**, pp. 142-148 (2006).

- Feghhi et al. [3] proposed a method to calculate the importance-weighted neutron generation time based on a physical interpretation of the adjoint solution of Eq. (7). Kiedrowski and Brown [4] developed an adjoint-weighted kinetics parameter calculation method based on the iterated fission probability, which is proportional to the adjoint solution of Eq. (7).

  [3] S. A. H. Feghhi, M. Shahriari, H. Afarideh, "Calculation of the importance-weighted neutron generation time using MCNIC method," *Ann. Nucl. Energy*, **35**, pp. 1397-1402 (2008).

  [4] B. C. Kiedrowski and F. Brown, "Adjoint-Weighted Kinetics Parameters with Continuous Energy Monte Carlo," *Trans. Am. Nucl. Soc.*, 100, pp. 297-299 (2009).

ϕ_S^* vs. ϕ_0^* - Physical Meaning of Importance?

- **A robust MC algorithm for estimating a nuclear parameter can be reduced more easily from an approach using its physical meaning interpreted in neutron's microscopic behavior rather than from its mathematical equations.**
- Kobayashi [7] said that **ϕ_S^* has a physical meaning which can be calculated by the MC method, although ϕ_0^* has, in general, no clear physical meaning**.

  (K. Kobayashi, "Physical Meaning of Kinetics Parameter "Lifetime" Used in the New Multi-Point Reactor Kinetics Equations Derived Using Green's Function," *Ann. Nucl. Energy*, **23**[10], pp. 827-841 (1996).)

ϕ_S^* vs. ϕ_0^* - Physical Meaning of Importance? (Contd.)

- It is well known that ϕ_0^* is better than ϕ_S^* as a weighting function in order to improve the accuracy of the reactivity estimate in the point kinetics equation.
- Then **what is the physical meaning of ϕ_0^*?**
  - Hurwitz [5] said that the iterated fission probability, $F(P)$, is proportional to ϕ_0^* where $F(P)$ is defined by **the number of fissions produced in the *n*-th generation** from a neutron introduced at the location of P as n approaches infinity.

    ([5] H. Hurwitz, "Physical Interpretation of the Adjoint Flux: Iterated Fission Probability," *Naval Reactor Physics Handbook*, Vol. I, pp. 864-869, A. Radkowsky, Ed., U.S. Atomic Energy Commission (1964).)
  - The importance of a neutron "introduced isotropically" at location **r** and energy E is **the asymptotic increase in the total neutron population** of a critical reactor per neutron added at time zero from the neutron [6].

    ([6] A. F. Henry, "Nuclear-Reactor Analysis," MIT Press (1975).)

Objectives of This Paper

- **We derive the precise physical meaning of using ϕ_0^* the fission operator**.

- Recently, the kinetics parameter calculation capability has been implemented into McCARD with both the constant source adjoint function and the self-consistent adjoint function.
- β_{eff}'s weighted by ϕ_S^* and ϕ_0^* are calculated by the Nauchi and Kameyama's method and the Kiedrowski and Brown's method, respectively.
- However Λ weighted by ϕ_S^* is estimated by an algorithm slightly modified from the Nauchi and Kameyama's method and that weighted by ϕ_0^* is calculated by an algorithm consistent to the method using ϕ_S^*.
- The purpose of this paper is to **present a comparison of McCARD results of both methods with analytic solutions for infinite homogenous 2-group problems and with measurements for several critical facility systems**.

Physical Meaning of Green's Function, *G*

- For a nuclear system without external sources, the Green's function, G can be defined with the loss and migration operator $\mathbf{M}$ as

$$\mathbf{MG}=\mathbf{I}; \qquad \text{(A.1)}$$

$$\mathbf{G}\equiv G(\mathbf{r}_0,E_0,\mathbf{\Omega}_0\to\mathbf{r},E,\mathbf{\Omega}), \qquad \mathbf{I}\equiv\delta(\mathbf{r}-\mathbf{r}_0)\delta(E-E_0)\delta(\mathbf{\Omega}-\mathbf{\Omega}_0),$$

where $\mathbf{I}$ is the identity operator and δ is the Dirac's delta function.

- Inverting $\mathbf{M}$ in Eq. (A.1), $\mathbf{G}$ can be written as

$$\mathbf{G}=\mathbf{M}^{-1} \qquad \text{(A.2)}$$

- Then using the Green's function, the direct flux, ϕ for the forward transport equation expressed as

$$\mathbf{M}\phi=\frac{1}{k}\mathbf{F}\phi \qquad \text{(A.3)}$$

can be written as

$$\phi(\mathbf{r},E,\mathbf{\Omega})=\mathbf{M}^{-1}S=\mathbf{G}S=\int d\mathbf{r}_0\int dE_0\int d\mathbf{\Omega}_0\ G(\mathbf{r}_0,E_0,\mathbf{\Omega}_0\to\mathbf{r},E,\mathbf{\Omega})S(\mathbf{r}_0,E_0,\mathbf{\Omega}_0); \qquad \text{(A.4)}$$

$$S(\mathbf{r}_0,E_0,\mathbf{\Omega}_0)=\frac{1}{k}\mathbf{F}\phi=\frac{1}{k}\int dE'\int d\mathbf{\Omega}'\frac{1}{4\pi}\chi(E_0,E')\nu(E')\Sigma_f(\mathbf{r}_0,E')\phi(\mathbf{r}_0,E',\mathbf{\Omega}'). \qquad \text{(A.5)}$$

- From Eq. (A.5), G($\mathbf{r}_0$, E_0, $\Omega_0\to\mathbf{r}$, E, Ω) means the neutron angular flux at $\mathbf{r}$, E, Ω due to a unit fission source neutron located at $\mathbf{r}_0$, E_0, Ω_0.

Application of Neumann Series Solution of CDE

- $\phi(\mathbf{r}, E, \mathbf{\Omega})$ from the Neumann series solution to the integral transport equation can be expressed as

$$\phi(\mathbf{r},E,\mathbf{\Omega}) = \frac{1}{\Sigma_t(\mathbf{r},E)} \sum_{j=0}^{\infty} \int d\mathbf{r}' \int dE_0 \int d\mathbf{\Omega}_0 K_{s,j}(\mathbf{r}',E_0,\mathbf{\Omega}_0 \to \mathbf{r},E,\mathbf{\Omega}) \int d\mathbf{r}_0 T(E_0,\mathbf{\Omega}_0;\mathbf{r}_0 \to \mathbf{r}') S(\mathbf{r}_0,E_0,\mathbf{\Omega}_0)\,, \quad \cdots\cdots \text{(A.6)}$$

$$K_{s,j}(\mathbf{r}',E_0,\mathbf{\Omega}_0 \to \mathbf{r},E,\mathbf{\Omega}) = \int d\mathbf{r}_1 \int dE_1 \int d\mathbf{\Omega}_1 \cdots \int d\mathbf{r}_{j-1} \int dE_{j-1} \int d\mathbf{\Omega}_{j-1}$$
$$\times K_s(\mathbf{r}_{j-1},E_{j-1},\mathbf{\Omega}_{j-1} \to \mathbf{r},E,\mathbf{\Omega}) \cdots K_s(\mathbf{r}',E_0,\mathbf{\Omega}_0 \to \mathbf{r}_1,E_1,\mathbf{\Omega}_1)$$

$$K_s(\mathbf{r}',E',\mathbf{\Omega}' \to \mathbf{r},E,\mathbf{\Omega}) = T(E',\mathbf{\Omega}';\mathbf{r}' \to \mathbf{r}) C_s(\mathbf{r};E',\mathbf{\Omega}' \to E,\mathbf{\Omega})$$

$$T(E,\mathbf{\Omega};\mathbf{r}' \to \mathbf{r}) = \Sigma_t(\mathbf{r},E) \exp\left[-\int_0^{|\mathbf{r}-\mathbf{r}'|} \Sigma_t\left(\mathbf{r} - s\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}, E \right) ds \right] \frac{\delta\left(\mathbf{\Omega} \cdot \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} - 1 \right)}{|\mathbf{r}-\mathbf{r}'|^2}$$

$$C_s(\mathbf{r};E',\mathbf{\Omega}' \to E,\mathbf{\Omega}) = \frac{\Sigma_s(\mathbf{r};E',\mathbf{\Omega}' \to E,\mathbf{\Omega})}{\Sigma_t(\mathbf{r},E')}$$

Physical Meaning of *G*

- By comparing Eq. (A.4) with Eq. (A.6), the Green's function can be expressed as

$$\phi(\mathbf{r},E,\mathbf{\Omega}) = \mathbf{M}^{-1}S = \mathbf{G}S = \int d\mathbf{r}_0 \int dE_0 \int d\mathbf{\Omega}_0 \; G(\mathbf{r}_0,E_0,\mathbf{\Omega}_0 \to \mathbf{r},E,\mathbf{\Omega}) S(\mathbf{r}_0,E_0,\mathbf{\Omega}_0)\,; \quad \text{(A.4)}$$

$$\phi(\mathbf{r},E,\mathbf{\Omega}) = \frac{1}{\Sigma_t(\mathbf{r},E)} \sum_{j=0}^{\infty} \int d\mathbf{r}' \int dE_0 \int d\mathbf{\Omega}_0 K_{s,j}(\mathbf{r}',E_0,\mathbf{\Omega}_0 \to \mathbf{r},E,\mathbf{\Omega}) \int d\mathbf{r}_0 T(E_0,\mathbf{\Omega}_0;\mathbf{r}_0 \to \mathbf{r}') S(\mathbf{r}_0,E_0,\mathbf{\Omega}_0)\,, \quad \text{(A.6)}$$

$$\boxed{G(\mathbf{r}_0,E_0,\mathbf{\Omega}_0 \to \mathbf{r},E,\mathbf{\Omega}) = \frac{1}{\Sigma_t(\mathbf{r},E)} \sum_{j=0}^{\infty} \int d\mathbf{r}' K_{s,j}(\mathbf{r}',E_0,\mathbf{\Omega}_0 \to \mathbf{r},E,\mathbf{\Omega}) T(E_0,\mathbf{\Omega}_0;\mathbf{r}_0 \to \mathbf{r}')} \quad \text{(A.7)}$$

- Because of the physical meanings of the kernels in the right hand side (RHS) of Eq. (A.7), G($\mathbf{r}_0$, E_0, $\Omega_0 \to \mathbf{r}$, E, Ω) can be interpreted **as the probability that a neutron located at $\mathbf{r}_0$, E_0, Ω_0 migrates to r, E, Ω with no fission reactions**.

Physical Meaning of $\phi_S{}^*$

- From the relation of $\mathbf{G}=\mathbf{M}^{-1}$, $\phi_S{}^*$ of Eq. (6), $\mathrm{M}^* \phi_S{}^* = \Sigma_d$, can be written as

$$\begin{aligned}\phi_S^*(\mathbf{r},E,\mathbf{\Omega}) &= \left(\mathbf{M}^*\right)^{-1}\Sigma_d = \left(\mathbf{M}^{-1}\right)^*\Sigma_d \\ &= \int d\mathbf{r}'\int dE'\int d\mathbf{\Omega}'\,\Sigma_d(\mathbf{r}',E',\mathbf{\Omega}')\cdot G^*(\mathbf{r}',E',\mathbf{\Omega}'\to\mathbf{r},E,\mathbf{\Omega}) \\ &= \int d\mathbf{r}'\int dE'\int d\mathbf{\Omega}'\,\Sigma_d(\mathbf{r}',E',\mathbf{\Omega}')\cdot G(\mathbf{r},E,\mathbf{\Omega}\to\mathbf{r}',E',\mathbf{\Omega}').\end{aligned} \quad \text{(A.8)}$$

- By using the physical meaning of G derived from Eq. (A.7), $\phi_S{}^*$ indicates **the sum of the detector responses generated from a neutron of r, *E*, Ω with no fission reactions**.
- For a consistency with the self-consistent adjoint function, we choose $\nu\Sigma_f$ as Σ_d in Eq. (A.8).
- Then **$\phi_S{}^*$ means the number of fission neutrons produced for the next generation due to the source neutron located at r, *E*, Ω.**

Algorithm for Calculating β_{eff} weighted by ϕ_S^*

- Using the physical meaning of ϕ_S^*, the numerator and denominator of β_{eff} at cycle i can be reduced as

$$\beta'^{i}_{eff} = k_d^{i,-1}\beta_0^{i-1}k^{i-1} \qquad \text{(A.9)}$$

$$F^i = k^i k^{i-1} \qquad \text{(A.10)}$$

  where $k_d^{i,-1}$ is the number of next-generation fission neutrons from a delayed fission source neutron generated at the previous cycle and β_0^{i-1} is the ratio of the number of delayed neutrons to the total number of fission neutrons generated at cycle i-1.

- From Eqs. (A.9) and (A.10), β_{eff} at cycle i can be calculated by the collision estimator as

$$\overline{\beta^i_{eff}} = \frac{\overline{k_d^{i,-1}\beta_0^{i-1}}}{\overline{k^i}} = \frac{\dfrac{1}{M^i}\displaystyle\sum_{j\in D^i}\sum_{k=1}^{K^{ij}} w^{ijk}\frac{\nu\Sigma_f^{ijk}}{\Sigma_t^{ijk}}}{\dfrac{1}{M^i}\displaystyle\sum_{j=1}^{M^i}\sum_{k=1}^{K^{ij}} w^{ijk}\frac{\nu\Sigma_f^{ijk}}{\Sigma_t^{ijk}}} \qquad \text{(A.11)}$$

  where i, j, and k are cycle, history, and collision indices, respectively. D^i is the domain of delayed fission neutron sources among all the sources for cycle i.

Algorithm for Calculating Λ weighted by ϕ_S^*

- In the MC steady-state particle transport method, the angular flux, $\phi(\mathbf{r}, E, \Omega)$ can be estimated by summing track lengths. This means that a track length sampled for a neutron at $\mathbf{r}$, E, Ω can represent $\phi(\mathbf{r}, E, \Omega)$.
- Then the numerator of Λ can be expressed as

$$\Lambda' = E\left[\phi_S^*(\Delta l)\cdot\left\{\Delta l/v(\Delta l)\right\}\right] \qquad \text{(A.12)}$$

Δl is the random variable of the track length.

- From Eq. (A.12) and the physical meaning of ϕ_S^*, Λ' estimated at cycle i can be calculated by

$$\overline{\Lambda'^i} = \frac{1}{M^i}\sum_{j=1}^{M^i}\sum_{k=1}^{K^{ij}} w^{ijk}\frac{\Delta l^{ijk}}{v^{ijk}}\left(\frac{1}{w^{ijk}}\sum_{k'=k}^{K^{ij}} w^{ijk'}\frac{\nu\Sigma_f^{ijk'}}{\Sigma_t^{ijk'}}\right) \qquad \text{(A.13)}$$

- Introducing the flight time, Δt, Λ at cycle i can be estimated by

$$\overline{\Lambda^i} = \frac{1}{M^i}\sum_{j=1}^{M^i}\sum_{k=1}^{K^{ij}} w^{ijk}\frac{\nu\Sigma_f^{ijk}}{\Sigma_t^{ijk}}\sum_{k'=1}^{k}\Delta t^{ijk} \Bigg/ \frac{1}{M^i}\sum_{j=1}^{M^i}\sum_{k=1}^{K^{ij}} w^{ijk}\frac{\nu\Sigma_f^{ijk}}{\Sigma_t^{ijk}}\sum_{k'=1}^{k}\nu\Sigma_f^{ijk'}\Delta l^{ijk'} \qquad \text{(A.14)}$$

- The denominator of RHS of Eq. (A.14) can be calculated as

$$\overline{F^i} = \overline{k^i}\,\overline{k^{i-1}} \qquad \text{(A.15)}$$

Physical Meaning of ϕ_0^*

- By inverting $\mathbf{M}^*$ in the adjoint eigenvalue equation, one can obtain

$$\phi_0^* = \frac{1}{k_0}\left(\mathbf{M}^*\right)^{-1}\mathbf{F}^*\phi_0^* = \frac{1}{k_0}\left(\mathbf{F}\mathbf{M}^{-1}\right)^*\phi_0^* \quad \text{(B.1)}$$

- By introducing the fission operator, $\mathbf{H}$ which is defined by

$$\mathbf{H} = \mathbf{F}\mathbf{M}^{-1};$$
$$\mathbf{H}\cdot = \int d\mathbf{r}'\int dE'\int d\mathbf{\Omega}'\, H(\mathbf{r}', E', \mathbf{\Omega}' \to \mathbf{r}, E, \mathbf{\Omega})\,\cdot\,, \quad \text{(B.2)}$$

where $H(\mathbf{r}', E', \mathbf{\Omega}' \to \mathbf{r}, E, \mathbf{\Omega})$ denotes the number of first-generation fission neutrons born per unit phase space volume about $\mathbf{r}$, E, $\mathbf{\Omega}$ due to a parent neutron located at $\mathbf{r}'$, E', $\mathbf{\Omega}'$, Eq. (B.1) can be expressed as

$$\phi_0^* = \frac{1}{k_0}\mathbf{H}^*\phi_0^* \quad \text{(B.3)}$$

- By applying the power method for the eigenvalue equation of Eq. (B.3), an unnormalized fundamental-mode eigenfunction can be calculated as

$$\phi_0^* = \lim_{n\to\infty}\phi_{0,n}^*;\ \phi_{0,n}^* = \left(\frac{1}{k_0}\mathbf{H}^*\right)^n\phi_{0,init.}^* = \frac{1}{k_0^{\,n}}\left(\mathbf{H}^n\right)^*\phi_{0,init.}^* \quad \text{(B.4)}$$

n is the iteration or generation index. $\phi_{0,n}^*$ denotes the n-th iterative solution and $\phi_{0,init.}^*$ can be an arbitrary non-zero function assumed as a starting distribution.

Physical Meaning of ϕ_0^* (Contd.)

- Because of the definition for **H** in Eq. (B.2), the integral notation for Eq. (B.4) can be written as

$$\phi_{0,n}^* = \frac{1}{k_0^{\,n}} \int d\mathbf{r}' \int dE' \int d\mathbf{\Omega}' H^n(\mathbf{r}, E, \mathbf{\Omega} \to \mathbf{r}', E', \mathbf{\Omega}') \phi_{0,init.}^*(\mathbf{r}', E', \mathbf{\Omega}') \quad \text{(B.5)}$$

where $H^n(\mathbf{r}, E, \Omega \to \mathbf{r}', E', \Omega')$ denotes the number of first-generation fission neutrons born per unit phase space volume about $\mathbf{r}'$, E', Ω' due to a parent neutron located at $\mathbf{r}$, E, Ω.

- Then when $\phi_{0,init.}^*(\mathbf{r}, E, \Omega) = 1$, **$\phi_0^*(\mathbf{r}, E, \Omega)$ of Eq. (B.4) can be interpreted as the number of fission neutrons produced in the *n*-th generation due to a unit source neutron located at r, *E*, Ω as *n* approaches infinity.**
- When n is large enough for the iterative solution to converge, ϕ_0^* can be approximated by $\phi_{0,n}^*$.

$$\phi_0^* \cong \phi_{0,n}^*; \; n >> 1 \quad \text{(B.6)}$$

Algorithm for Calculating β_{eff}, Λ weighted by ϕ_0^*

- Assuming that ϕ_0^* becomes converged after n power iterations, in the same way to derive an algorithm for β_{eff} weighted by ϕ_S^*, β_{eff} weighted by $\phi_{0,n}^*$ can be calculated as

$$\overline{\beta_{eff}^i} = \frac{\overline{k_d^{i,-n} \beta_0^{i-n}}}{\overline{k^i}} = \frac{\frac{1}{M^i} \sum_{j \in D^{i-n+1}} \sum_{k=1}^{K^{ij}} w^{ijk} \frac{\nu \Sigma_f^{ijk}}{\Sigma_t^{ijk}}}{\frac{1}{M^i} \sum_{j=1}^{M^i} \sum_{k=1}^{K^{ij}} w^{ijk} \frac{\nu \Sigma_f^{ijk}}{\Sigma_t^{ijk}}} \quad \text{..................} \quad \text{(B.7)}$$

- Λ^i weighted by $\phi_{0,n}^*$ can be estimated by

$$\overline{\Lambda^i} = \frac{\frac{1}{M^i} \sum_{j=1}^{M^i} \left(\sum_{k=1}^{K^{ij}} w^{ijk} \frac{\nu \Sigma_f^{ijk}}{\Sigma_t^{ijk}} \right) \left(w^{(i-n+1)j'1} \frac{\Delta l^{(i-n+1)j'k'}}{v^{(i-n+1)j'k'}} \right)}{\frac{1}{M^i} \sum_{j=1}^{M^i} \left(\sum_{k=1}^{K^{ij}} w^{ijk} \frac{\nu \Sigma_f^{ijk}}{\Sigma_t^{ijk}} \right) \left(w^{(i-n+1)j'1} \nu \Sigma_f^{(i-n+1)j'k'} \Delta l^{(i-n+1)j'k'} \right)} \quad \text{..................} \quad \text{(B.8)}$$

- The denominator of RHS of Eq. (B.8) can be calculated by

$$\overline{F^i} = \overline{k^i}\,\overline{k^{i-n}} \quad \text{..................} \quad \text{(B.9)}$$

Infinite Homogeneous 2-Group Problems

- The MC forward eigenvalue calculations with multi-group cross-section libraries were conducted for homogeneous infinite mediums characterized by 2-group cross-sections of Table I, varying the infinite multiplication factor, k_{inf}. In the table, the differential scattering cross-section of the first group, Σ_{s21} is set at 0.181905, 0.247143, or 0.312987 corresponding to k_{inf} of 0.9, 1.0, or 1.1.

< 2-group cross-sections for the infinite homogeneous problems >

Cross-section	First Gr. (g=1)	Second Gr. (g=2)
Σ_t	0.50	0.50
Σ_f	0.025	0.175
ν	2.0	2.0
Σ_{sgg}	0.10	0.20
$\Sigma_{sg'g}$ ($g \neq g'$)	variable	0.00
χ_1	0.80	0.5
χ_2	0.20	0.5
$\chi_{d,1}$	0.80	0.80
$\chi_{d,2}$	0.20	0.20
β_0 ($=\nu_d/\nu$)	0.006	0.006
$1/v$ [sec/cm]	2.28626×10^{-10}	1.29329×10^{-6}

Comparisons of Adjoint-Weighted β_{eff} and Λ for the Infinite Homogeneous 2-Group Problems

Parameter	Ref.	McCARD Mean ($\bar{x}$)	McCARD $\sigma / \bar{x}$ [%]	McCARD $\bar{x}-2\sigma$	McCARD $\bar{x}+2\sigma$
Σ_{s21} = 0.181905					
k_{inf}	0.90000	0.89998	0.001	0.89996	0.90000
$\beta_{eff}(\phi_S^*)$	5.0519×10^{-3}	5.0528×10^{-3}	0.045	5.0482×10^{-3}	5.0574×10^{-3}
$\beta_{eff}(\phi_0^*)$	4.9977×10^{-3}	4.9897×10^{-3}	0.119	4.9778×10^{-3}	5.0015×10^{-3}
$\Lambda(\phi_S^*)$	4.4431×10^{-6}	4.4432×10^{-6}	0.003	4.4429×10^{-6}	4.4434×10^{-6}
$\Lambda(\phi_0^*)$	4.5011×10^{-6}	4.5016×10^{-6}	0.010	4.5007×10^{-6}	4.5025×10^{-6}
Σ_{s21} = 0.247143					
k_{inf}	1.00000	1.00002	0.001	1.00000	1.00004
$\beta_{eff}(\phi_S^*)$	5.4600×10^{-3}	5.4595×10^{-3}	0.044	5.4547×10^{-3}	5.4643×10^{-3}
$\beta_{eff}(\phi_0^*)$	5.4353×10^{-3}	5.4348×10^{-3}	0.106	5.4233×10^{-3}	5.4463×10^{-3}
$\Lambda(\phi_S^*)$	4.0313×10^{-6}	4.0313×10^{-6}	0.003	4.0310×10^{-6}	4.0315×10^{-6}
$\Lambda(\phi_0^*)$	4.0576×10^{-6}	4.0577×10^{-6}	0.009	4.0570×10^{-6}	4.0584×10^{-6}
Σ_{s21} = 0.312987					
k_{inf}	1.10000	1.10000	0.001	1.09998	1.10002
$\beta_{eff}(\phi_S^*)$	5.8017×10^{-3}	5.8034×10^{-3}	0.043	5.7984×10^{-3}	5.8083×10^{-3}
$\beta_{eff}(\phi_0^*)$	5.7942×10^{-3}	5.7933×10^{-3}	0.106	5.7811×10^{-3}	5.8055×10^{-3}
$\Lambda(\phi_S^*)$	3.6888×10^{-6}	3.6889×10^{-6}	0.002	3.6887×10^{-6}	3.6891×10^{-6}
$\Lambda(\phi_0^*)$	3.6967×10^{-6}	3.6968×10^{-6}	0.009	3.6961×10^{-6}	3.6975×10^{-6}

Comparisons of β_{eff} for Critical Facilities

Facility	Core Name	$(\beta_{eff})_{exp}$	$(\beta_{eff})_{MC}$ weighted by ϕ_S^*			$(\beta_{eff})_{MC}$ weighted by ϕ_0^*		
			Mean	RSD (%)	Ratio to Exp.	Mean	RSD (%)	Ratio to Exp.
Godiva	-	0.00640	0.00646	0.17	1.01	0.00649	0.42	1.01
TCA	1.50U	0.00771	0.00767	0.49	0.99	0.00774	1.32	1.00
	1.83U	0.00760	0.00758	0.53	1.00	0.00762	1.38	1.00
	2.48U	0.00765	0.00750	0.47	0.98	0.00748	1.31	0.98
	3.00U	0.00749	0.00754	0.44	1.01	0.00764	1.29	1.02

Comparisons of Λ for Critical Facilities

Facility	Core Name	$(\beta_{eff}/\Lambda)_{exp}$	$(\beta_{eff}/\Lambda)_{MC}$ weighted by ϕ_S^*			$(\beta_{eff}/\Lambda)_{MC}$ weighted by ϕ_0^*		
			Mean	RSD (%)	Ratio to Exp.	Mean	RSD (%)	Ratio to Exp.
Godiva	-	1.11×10^6	1.13×10^6	0.17	1.02	1.14×10^6	0.42	1.03
TCA	1.50U	219	191	0.49	0.87	220	1.34	1.01
	1.83U	201	175	0.53	0.87	197	1.39	0.98
	2.48U	175	154	0.47	0.88	170	1.32	0.97
	3.00U	161	143	0.44	0.89	158	1.29	0.98
STACY	#30	126.8	125.5	0.50	0.99	126.6	1.50	1.00
	#33	116.7	111.9	0.56	0.96	116.7	1.40	1.00

Conclusions

- We have derived two types of the MC algorithms for computing the kinetics parameters weighted by the constant source adjoint function, ϕ_S^*, and the self-consistent adjoint function, ϕ_0^*, for the MC forward eigenvalue calculations and implemented them in McCARD.
- From the comparisons with the analytic solutions for the infinite homogeneous 2-group problems, we demonstrated that the MC estimates of the effective delayed neutron fraction, β_{eff}, and the prompt neutron generation time, Λ, agree well within 95% confidence intervals.
- For some critical facility problems, it is demonstrated that β_{eff} and β_{eff}/Λ weighted by ϕ_0^* agree well with the measurements within 3% errors, while the maximum error of β_{eff}/Λ from the use of ϕ_S^* is 13%.