Topics in Magnetic Materials

I. Basic Magnetic Quantities & Units

(1) Magnetic Field, ***H*** and Magnetic Induction, ***B***

(2) Magnetic Moment, ***m*** & Magnetization, ***M***

(3) Magnetization Curve & Magnetic Hysteresis Curve

(4) Magnetostatics

(1) Magnetic Field & Magnetic Induction
(a) Magnetic Filed, *H*

- ***H*** represents a magnetic force generated in a volume of the space due to a change in magnetic energy.

Examples of the magnetic force

- A force on a current-carrying conductor
- A torque on a magnetic dipole
- A reorientation of spins on electrons within atoms

- ***H*** is fundamentally generated by an electrical charge in motion.

  Earth (~ 0.7Oe)
  Bulk magnets (~ 5,000Oe)
  Current-carrying conductors (~30,000Oe)
  Superconductors (>100,000Oe)
  (ref. Table 1.1 in David Jiles)

- Unit

  mks or SI(Systeme Interrnational): [A/m]
  cgs: [Oe]

$$1Oe = \frac{1000}{4\pi} A/m (\sim 79.6 A/m)$$

(a) Magnetic Field, *H*

▶ Magnetic Field Calculations

Simple cases : Biot-Savart law & Ampere's law
Complex cases : numerical methods

Biot-Savart law (empirical)

$$dH = \frac{1}{4\pi r^2} dl \times u$$

where, $\boldsymbol{dl}$ = an elemental line vector of the conductor along the current direction
$\boldsymbol{u}$ = a unit vector along the radial direction

Ampere's law : $Ni = \int H \cdot dl$

where, N = number of conductors, i = electric current

(a) Magnetic Field, *H*

For a linear current-carrying conductor, *H* at a distance of *a*?

By applying Biot-Savart law

$$H = \int_{-\pi/2}^{\pi/2} dH = \frac{i}{2\pi a} \; A/m \quad In\ cgs,\ \frac{2i}{10a}\,[Oe]$$

By applying Ampere's law, $i = \int H \cdot dl = 2\pi a H$

Thus, $H = \dfrac{i}{2\pi a}$

For a circular loop with a radius of *a*,

At the center, $H = \dfrac{i}{2a}\,[A/m] \;\; or \;\; \dfrac{2\pi i}{10a}\,[Oe]$

At a point *x* located on the axis of a circular loop,
where a is the angle between a point on the loop and *x*.

$$H = \sin^3 a \quad or \quad \frac{ia^3}{2(a^2 + x^2)^{3/2}}$$

At the off-axis of a circular loop?
No analytical solution -> need a numerical technique using computer

For a long thin solenoid (usually used for the production of a uniform magnetic field)

$$H = \frac{Ni}{L} = ni$$

For a toroid

$$H = \frac{Ni}{2\pi r}$$

For other cases (ref. David Jiles),

- Two coaxial coils (Helmholtz coils)
- A solenoid of finite length

(b) Magnetic Induction, *B*

- ***B*** is the response of a medium to an applied magnetic field ***H***
- **Definition** $B = \dfrac{\Phi}{A}$ ***B*** is the magnetic flux, Φ [Wb] passing through a unit cross-sectional area.

Magnetic flux, Φ? Generated by the presence of a magnetic field in a medium.

By Lentz law, the voltage *V* is induced as Φ changes

$$V = -N\frac{d\Phi}{dt} = -NA\frac{dB}{dt}$$: "electromagnetic induction"

Maxwell's equation (Gauss's law) $1volt = -(1)\dfrac{\Phi_i - \Phi_f}{1\sec}$

If $\Phi_f = 0$, $\Phi_i = 1$ Wb (= 1 volt•sec)

$$\nabla \cdot B = 0 \rightarrow \oint B \cdot dA$$ **Always form a closed path!**

Relation between ***B*** and ***H***

$\boldsymbol{B} = \mu \boldsymbol{H}$, where μ is permeability(투자율)

$\mu = \mu_o$ in free space

$= 4\pi \times 10^{-7}$ H/m (or Wb/A)

Relative permeability (상대투자율), μ_r

$\mu_r = \mu/\mu_o$

$\mu_r = 1$ in a perfect vacuum(free space)

- **Unit: [G], [T]**

1 Tesla = 1 Wb/m^2 (= 1 volt•sec/m^2)
A force of 1 N/m on a conductor carrying 1 A perpendicular to the direction of ***B***

(2) Magnetic Moment & Magnetization
(a) Magnetic Moment, *m*

▶ **Definition of a magnetic dipole moment, *m***

$\boldsymbol{m} = pl$ in a bar magnet

$\boldsymbol{m} = iA$ in a conductor loop

cf) atomic magnetic moment

▶ **Unit**

SI	cgs
$[Am^2]$	[emu]
[Wbm]	[erg/Oe]

$$1 \text{ Wbm} = \frac{1}{4\pi} 10^{10} \text{Gcm}^3$$

▶ **Measurements of *m***

(i) Torque measurement : $\boldsymbol{\tau} = \mu_o \boldsymbol{m} \times \boldsymbol{H} = \boldsymbol{m} \times \boldsymbol{B}$

$\tau = \tau_{max}$ if m is perpendicular to H (or B), and then

$\boldsymbol{m} = \tau_{max}/\mu_o \boldsymbol{H}$

Since $\boldsymbol{m} = pl$ and $\boldsymbol{p} = \Phi/\mu_o$ in the Sommerfeld conversion

$\boldsymbol{m} = \Phi l/\mu_o$

(ii) Magnetization measurement

$\boldsymbol{m} = \boldsymbol{MV}$

(b) Magnetization, *M*

- ***M*** **is the magnetic moment *m* per unit volume (cf. *m* per unit mass = specific magnetization σ)**

  $M =$ (cf. $\sigma = M/\rho$ [emu/g]), where ρ is density

  Since $m = \Phi l/\mu_o$, $V = Al$

  $M = \Phi/\mu_o A = B/\mu_o$

  Therefore, $B = \mu_o M$ when $H = 0$

- **Saturation magnetization**

  M_o : complete saturation, where all atomic moments are aligned parallel to H_a

  M_s : technical saturation, where multiple-domains become single domain

- **Relation between *M* and *H***

  $M = \chi H$, where χ is susceptibility(자화율) ↔ $B = \mu H$ (μ is permeability(투자율))

  μ and χ are not useful for ferromagnets.

  Need differential values: $\mu' = dB/dH$, $\chi' = dM/dH$

- **Relationship between *H*, *B*, and *M***

  A universal relationship

  $B = \mu_o(H + M)$: SI(Sommerfeld)

  $= \mu_o H + I$: SI(Kennelly)

  $= H + 4\pi M$: cgs(Gaussian)

  $B = \mu_o(H + M) = \mu_o(H + \chi H) = \mu_o(1 + \chi)H$, Since $B = \mu H = \mu_o \mu_r H$, $\mu_r = 1 + \chi$

μ_r and χ are different ways of describing the response of a material to magnetic fields.

Magnetization Curves & Magnetic Hysteresis Curves (or Loops)

Diamagnets $\chi(T)$

Normal diamagnets (see Fig. 3.4-5 in O'Handley)

Superconductors (perfect diamagnetism)

Paramagnets $\chi(T)$

Typical paramagnets (Curie & Curie-Weiss law)
(see Fig. 3.4-5, 3.17 in O'Handley/ Fig. 9.2-3 in Jiles/ Fig. 3.5-6 in Cullity))

Pauli paramagnets (see Fig. 3.5 in O'Handley)

cf) superparamagnetism

Ferromagnets $\chi(T)$, $M_s(T)$, $M(H)$

$T > T_c$: Typical paramagnetic $\chi(T)$ (Curie-Weiss law)

$T < T_c$: $M_s(T)$ (see Fig 3.20 in O'Handley/ Fig. 11.7 in Jiles), $M(H)$

Antiferromagnets $\chi(T)$ (see Fig 9.6 in Jiles/ Fig 5.7, 5.8, 5.9 in Cullity)

$T > T_N$: Typical paramagnetic $\chi(T)$ (Curie-Weiss law)

$T < T_N$: $\chi(T)$

Ferrimagnets $\chi(T)$, $Ms(T)$, $M(H)$

$T > T_c$: Peculiar paramagnetic $\chi(T)$ (not following Curie-Weiss law)
(see Fig. 4.10 in O'Handley/ Fig 6.1, 6.7 in Cullity)

$T < T_c$: $M_s(T)$ (see Fig. 1.11, 4.9, 4.11 in O'Handley/ Fig. 6.1, 6.8, 6.9, 6.11 in Cullity), $M(H)$

Diamagnets & Paramagnets

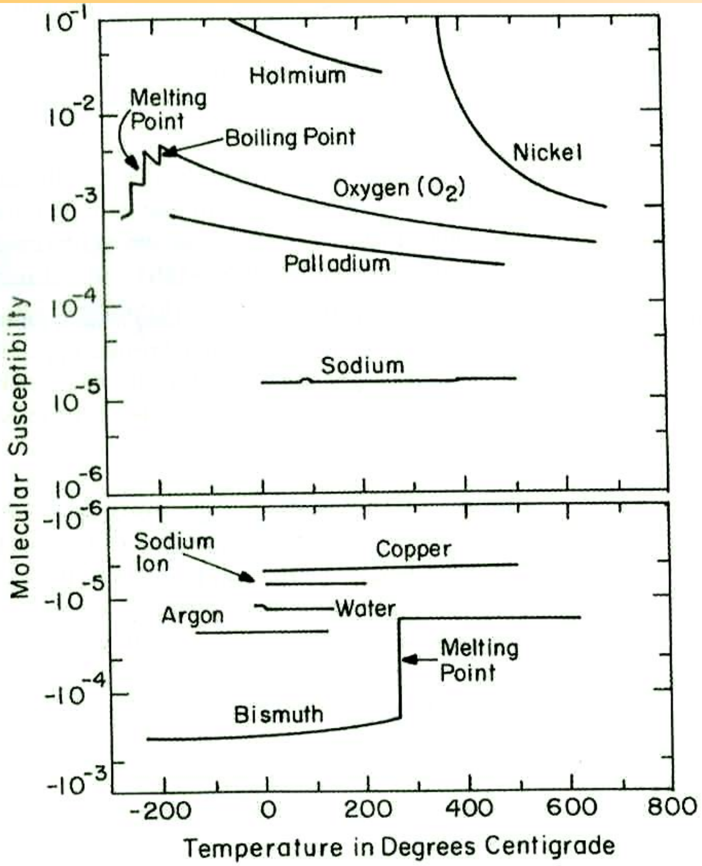

Figure 3.5 Temperature dependence of paramagnetic and diamagnetic susceptibility in some materials. [After Bozorth, copyright IEEE Press (1993)].

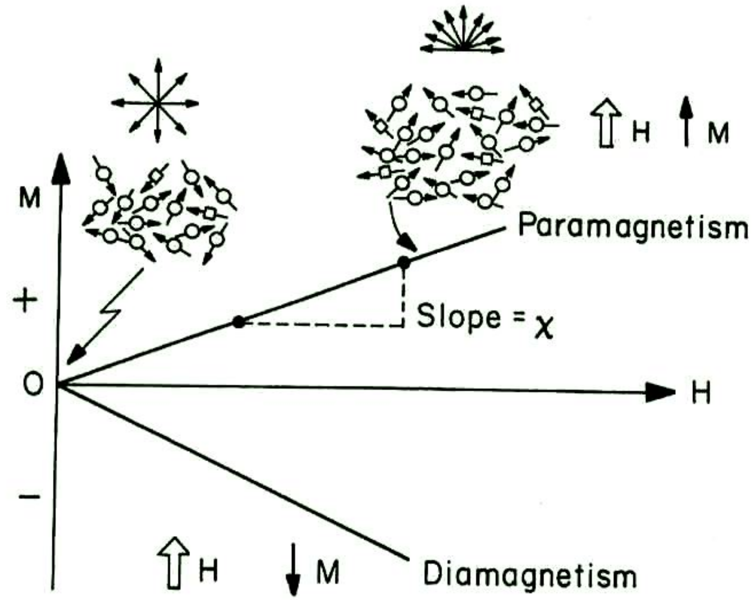

Figure 1.10 Field dependence of magnetization response in paramagnets and diamagnets. Inset shows a schematic of the distribution of local moments in the paramagnetic case.

Paramagnets

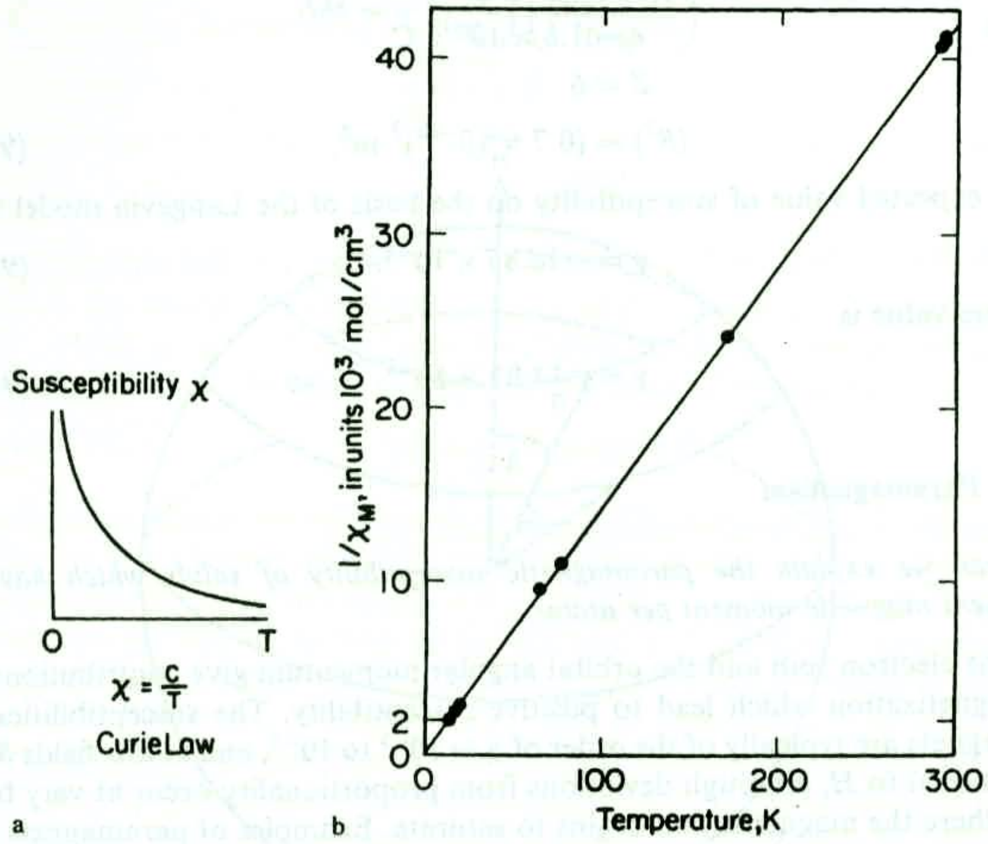

Fig. 9.2 Temperature dependence of paramagnetic susceptibility. Left-hand diagram shows a schematic of the variation of χ with T. The right-hand diagram shows the variation of $1/\chi$ with temperature for the paramagnetic salt Gd $(C_2H_5{\cdot}SO_4){\cdot}9H_2O$. The circles are experimental points; the straight line is the Curie law prediction.

Ferromagnets

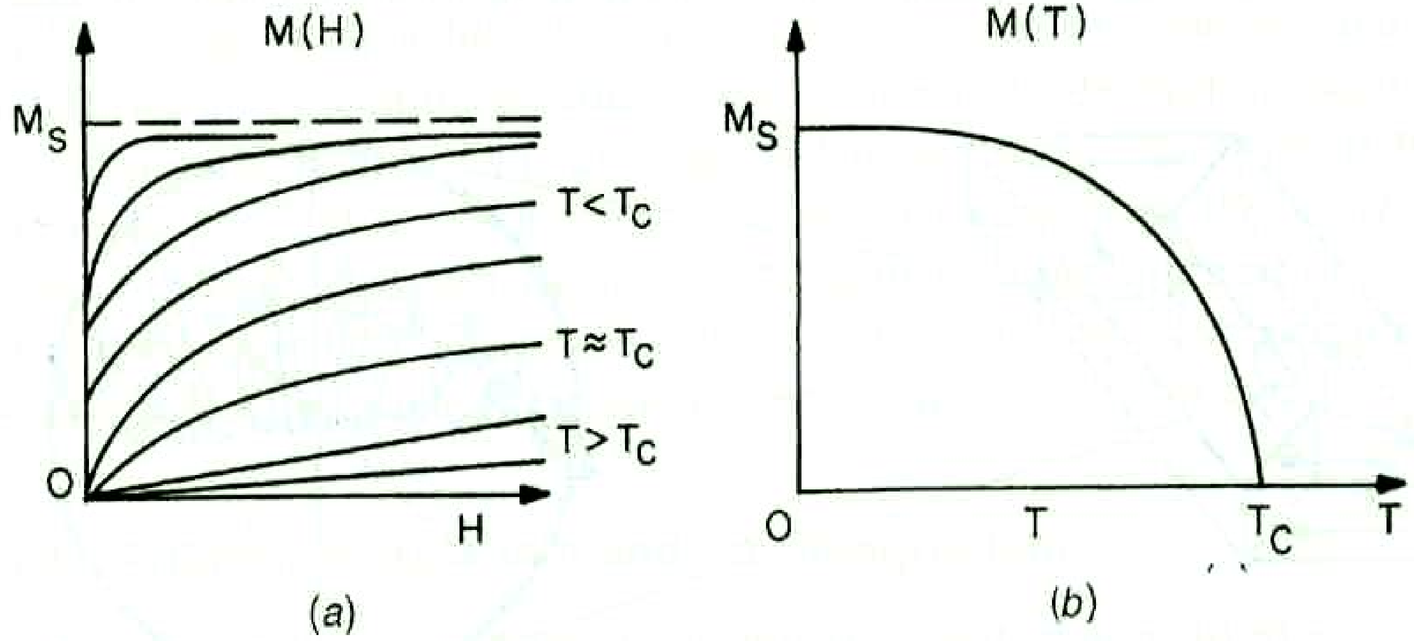

Figure 1.11 (*a*) Magnetization of a strongly magnetic material (e.g., a ferromagnet) versus field; (*b*) temperature dependence of the saturation magnetization. T_C is the Curie temperature.

Antifferomagnets

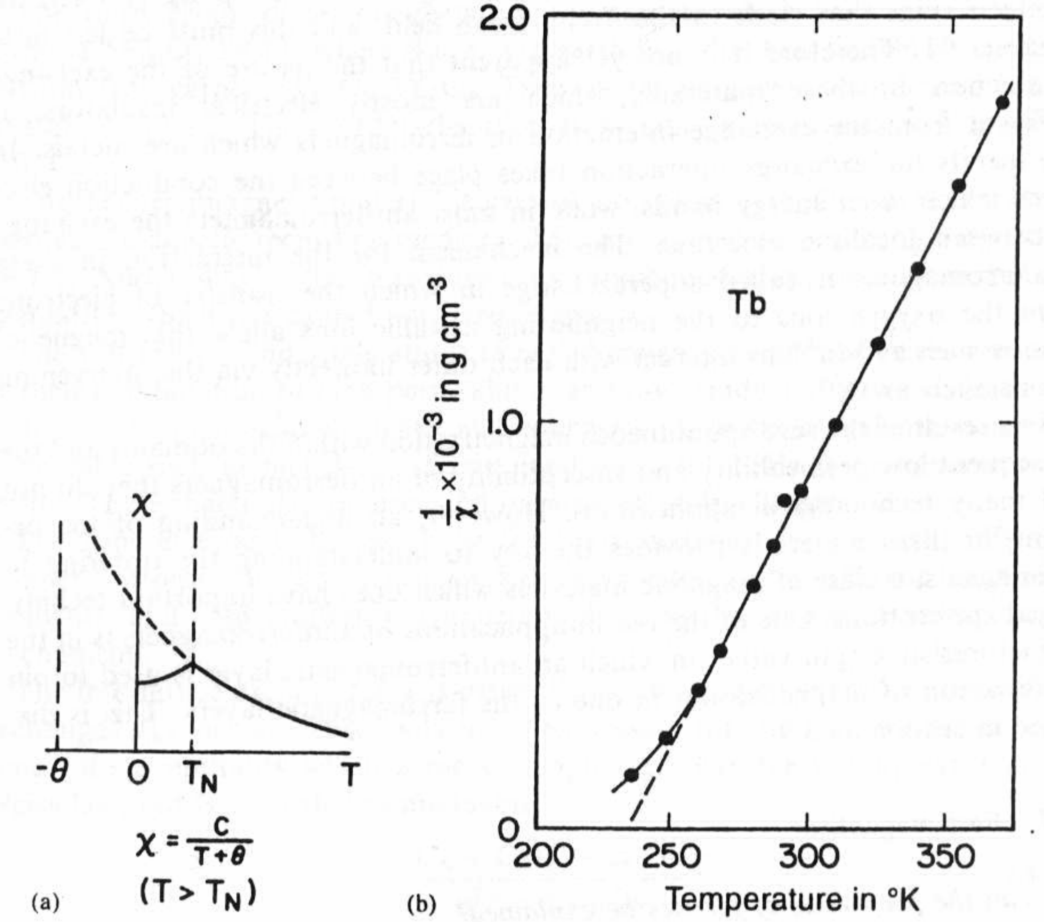

Fig. 9.6 The left-hand diagram (a) is the schematic variation of χ with temperature in the paramagnetic regime of materials which undergo a transformation to antiferromagnetism. The right-hand diagram (b) is $1/\chi$ versus data for terbium.

Ferrimagnets

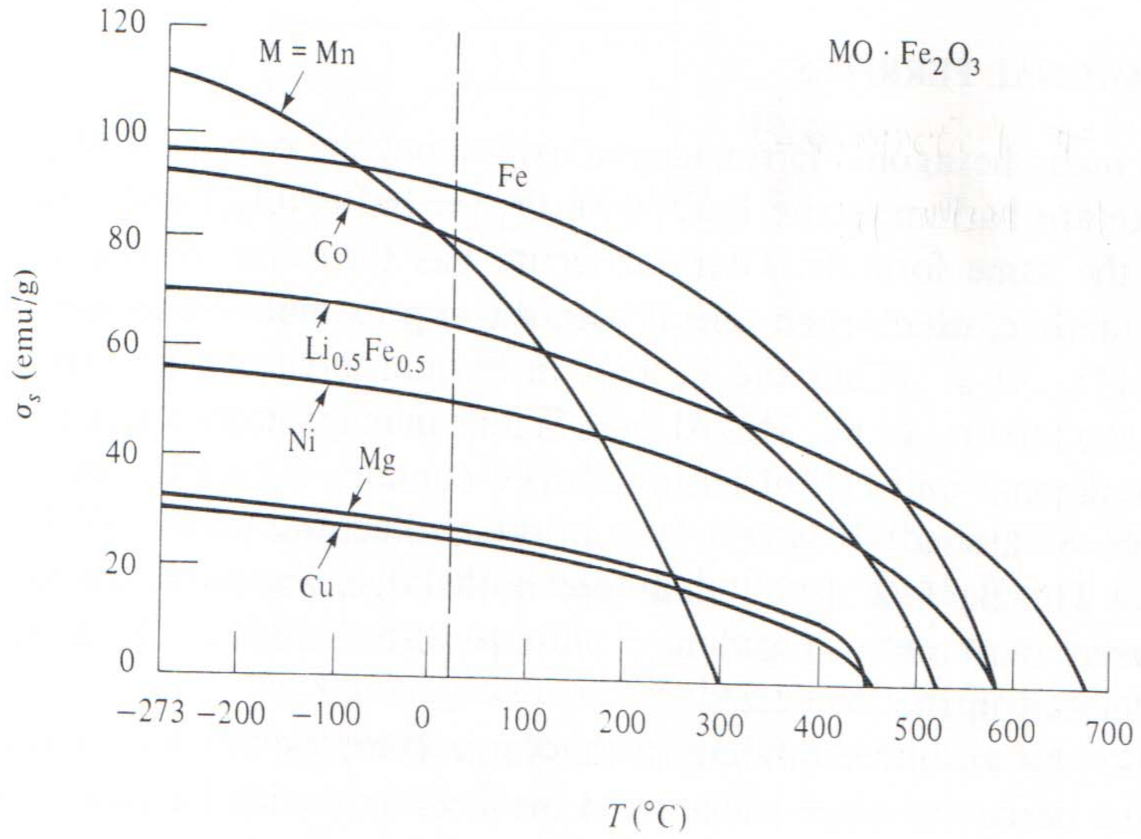

Fig. 6.11 Saturation magnetization of some cubic ferrites as a function of temperature, after Smit and Wijn [G.10].