Absorption

$$\chi = \chi' + j\chi''$$

$$k = \omega\sqrt{\epsilon\mu_o} = k_o\sqrt{1+\chi} = k_o\sqrt{1+\chi'+j\chi''}$$

$$\beta - j\tfrac{1}{2}\alpha = k_o\sqrt{1+\chi'+j\chi''}$$

$$k = \beta - j\tfrac{1}{2}\alpha$$

$$U = A\exp(-\tfrac{1}{2}\alpha z)\exp(-j\beta z)$$

$$\beta = nk_o$$

$$n - j\frac{1}{2}\frac{\alpha}{k_o} = \sqrt{\epsilon/\epsilon_o} = \sqrt{1+\chi'+j\chi''}$$

$$\eta = \sqrt{\frac{\mu_o}{\epsilon}} = \frac{\eta_o}{\sqrt{1+\chi}}$$

Weakly absorbing media

$$n \approx \sqrt{1+\chi'}$$

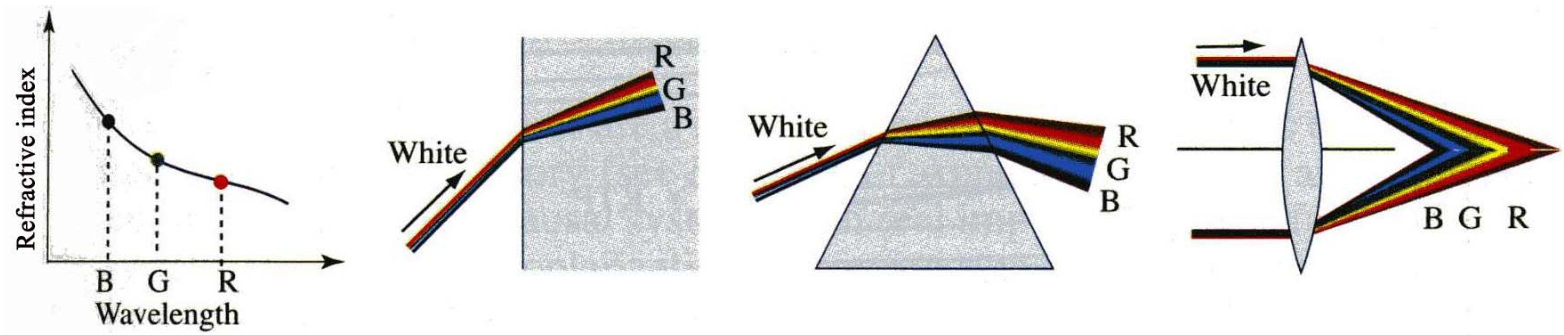
$$\alpha \approx -\frac{k_o}{n}\chi''$$

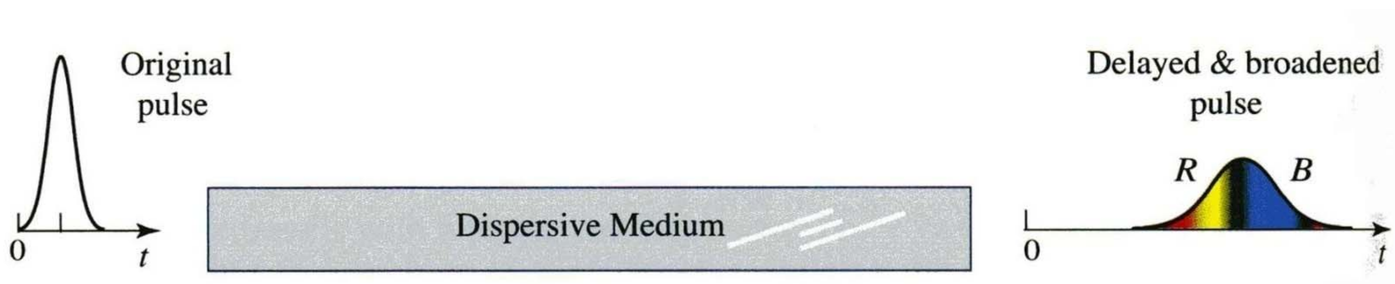
Strongly absorbing media

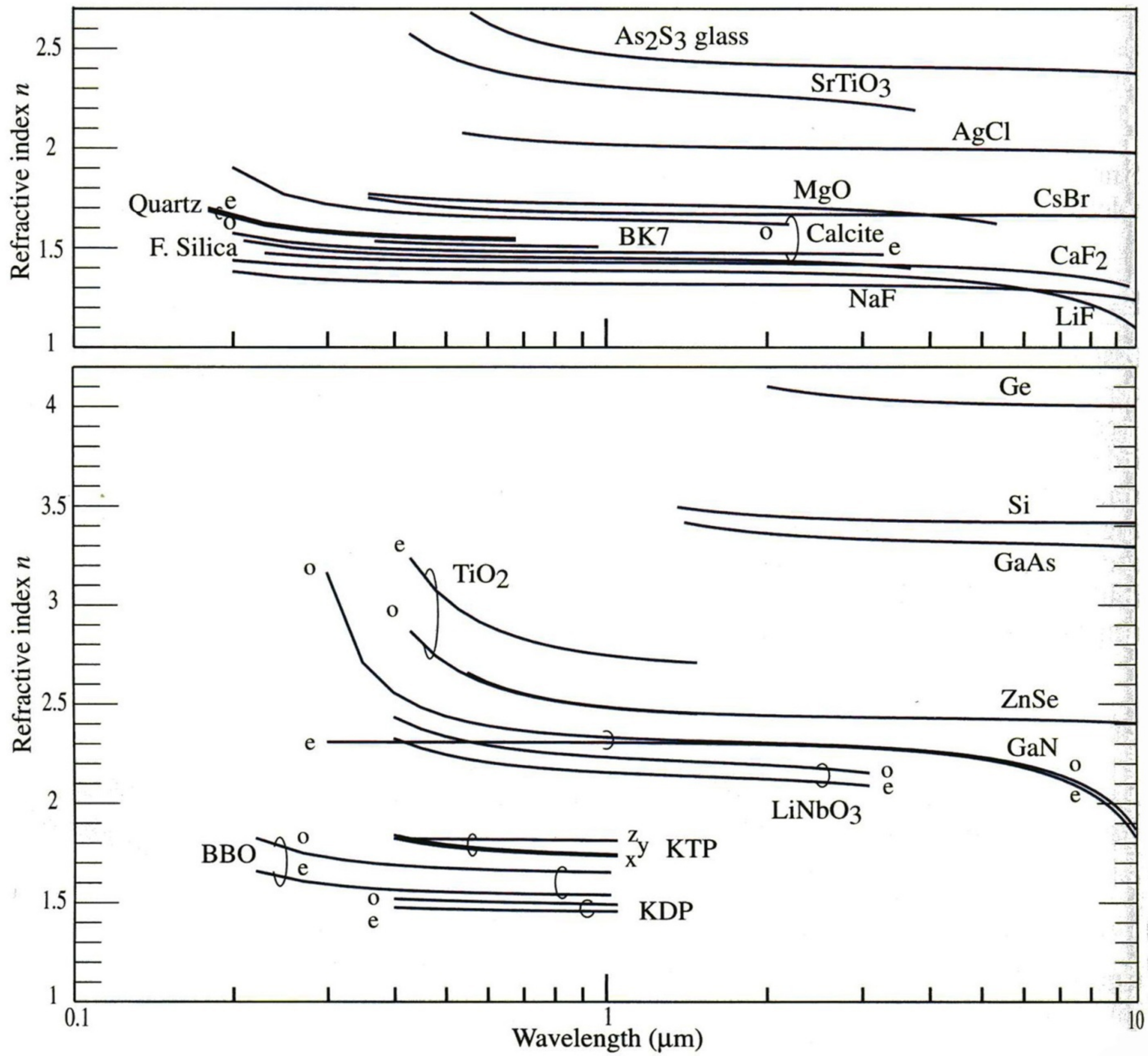
$$n \approx \sqrt{(-\chi'')/2}$$

$$\alpha \approx 2k_o\sqrt{(-\chi'')/2}$$

Dispersion


Refractive index
B
G
R
Wavelength
White
R
G
B
White
R
G
B
White
B G R


Original
pulse
0
t
Dispersive Medium
Delayed & broadened
pulse
R
B
0
t



As2S3 glass
SrTiO3
AgCl
MgO
CsBr
Quartz
e
o
F. Silica
BK7
o
Calcite
e
CaF2
NaF
LiF
Refractive index n
2.5
2
1.5
1
Ge
Si
GaAs
e
o
TiO2
o
ZnSe
GaN
e
o
e
LiNbO3
o
e
BBO
o
e
z
y
x
KTP
o
e
KDP
4
3.5
3
0.1
1
10
Wavelength (μm)

Kramers-Kronig relations

$$\chi'(\nu) = \frac{2}{\pi}\int_0^\infty \frac{s\chi''(s)}{s^2-\nu^2}\,ds$$

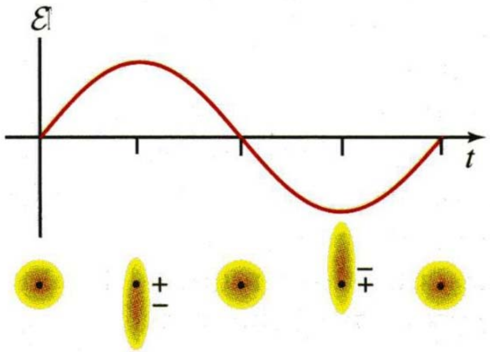
$$\chi''(\nu) = \frac{2}{\pi}\int_0^\infty \frac{\nu\chi'(s)}{\nu^2-s^2}\,ds$$

Lorentz oscillator model

$$\frac{d^2\mathcal{P}}{dt^2} + \sigma\frac{d\mathcal{P}}{dt} + \omega_0^2\,\mathcal{P} = \omega_0^2\epsilon_o\chi_0\,\mathcal{E}$$

$$\frac{d^2x}{dt^2} + \sigma\frac{dx}{dt} + \omega_0^2 x = \frac{\mathcal{F}}{m}$$

$$\chi_0 = \frac{Ne^2}{\epsilon_o m\omega_0^2}$$

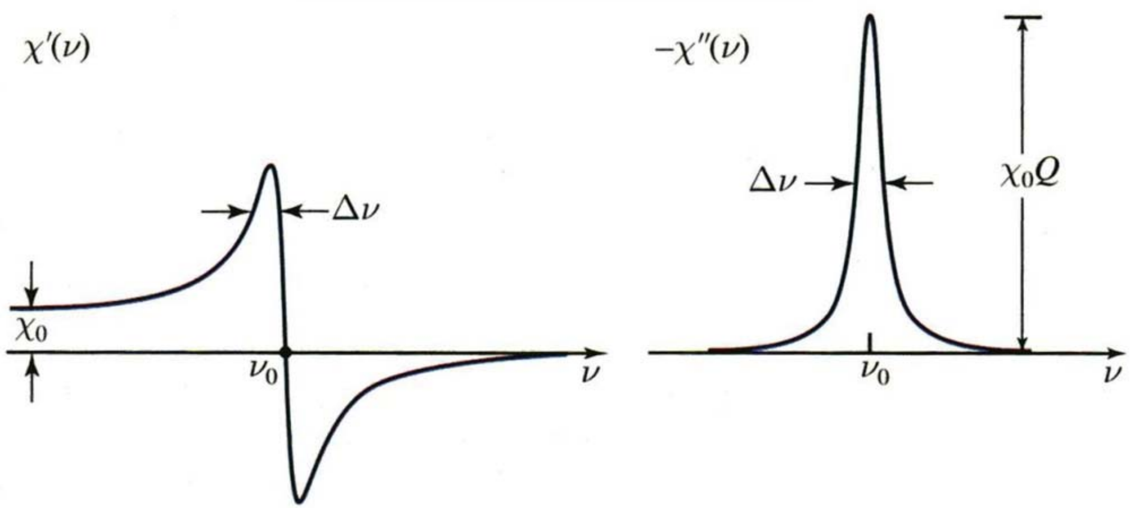


Lorentz oscillator model

$$\chi(\nu) = \chi_0 \frac{\nu_0^2}{\nu_0^2 - \nu^2 + j\nu\,\Delta\nu}$$

$$\chi'(\nu) = \chi_0 \frac{\nu_0^2\left(\nu_0^2 - \nu^2\right)}{\left(\nu_0^2 - \nu^2\right)^2 + (\nu\,\Delta\nu)^2}$$

$$\chi''(\nu) = -\chi_0 \frac{\nu_0^2\,\nu\,\Delta\nu}{\left(\nu_0^2 - \nu^2\right)^2 + (\nu\,\Delta\nu)^2}$$



Near resonance

$$\chi''(\nu) \approx -\chi_0 \frac{\nu_0 \Delta\nu}{4} \frac{1}{(\nu_0 - \nu)^2 + (\Delta\nu/2)^2}$$

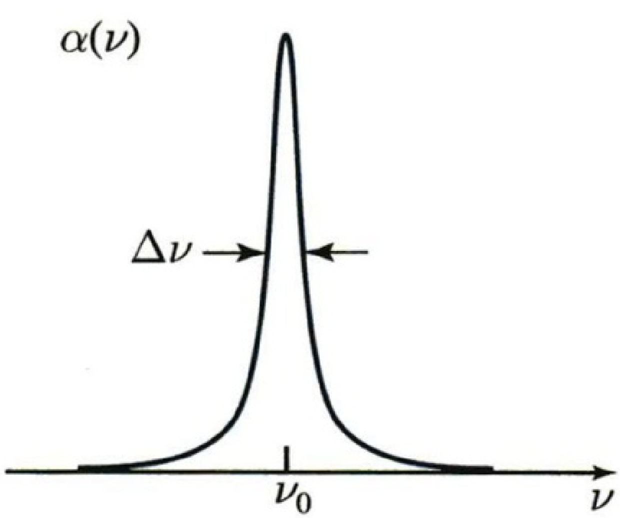
$$\chi'(\nu) \approx 2 \frac{\nu - \nu_0}{\Delta\nu} \chi''(\nu)$$

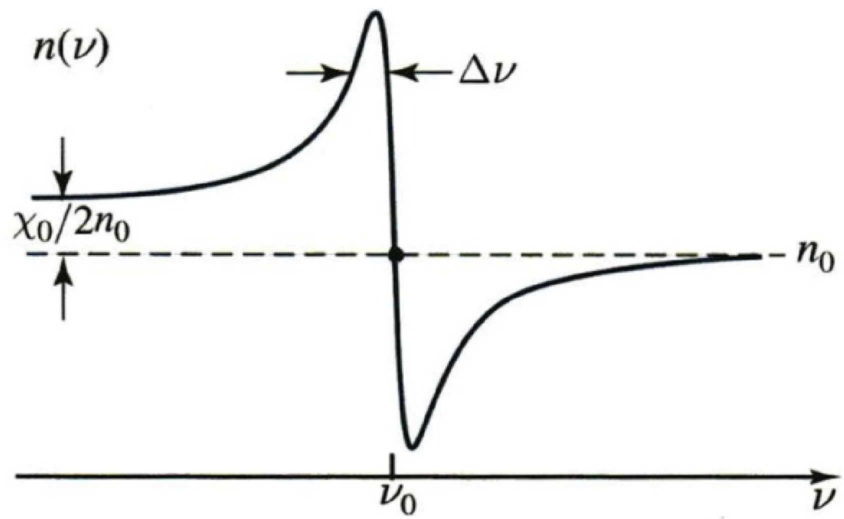
Far from resonance

$$\chi(\nu) \approx \chi_0 \frac{\nu_0^2}{\nu_0^2 - \nu^2}$$

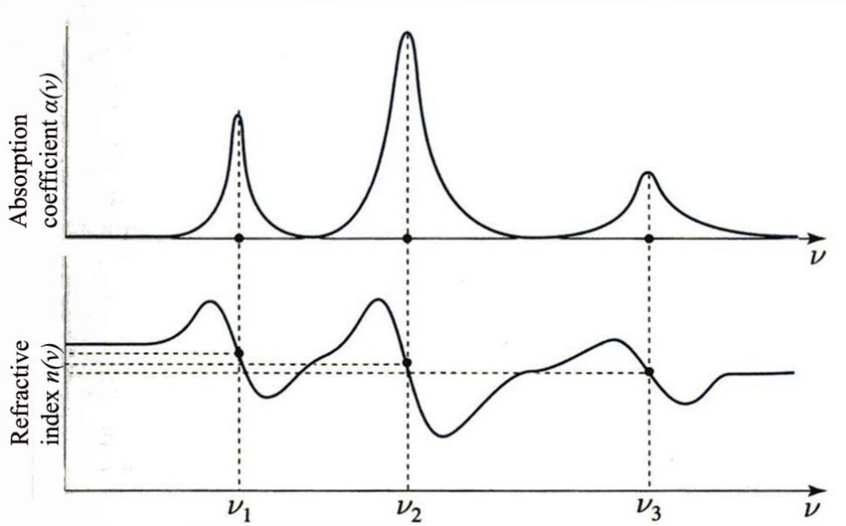
$$\alpha(\nu) \approx -\left(\frac{2\pi\nu}{n_0 c_o}\right)\chi''(\nu)$$

$$n(\nu) \approx n_0 + \frac{\chi'(\nu)}{2n_0}$$

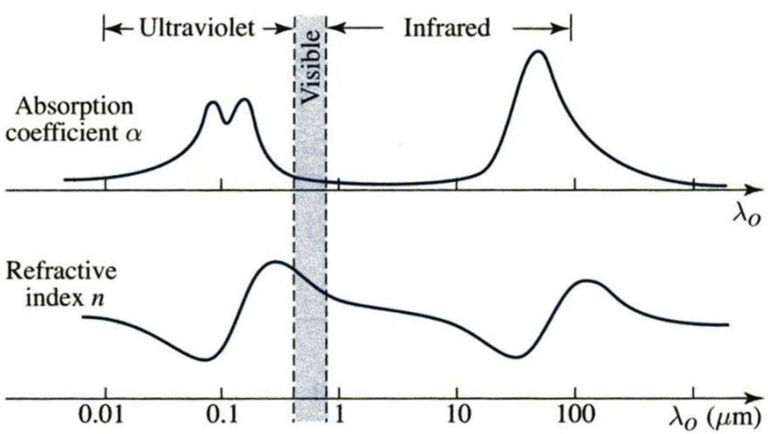




Sellmeier equation



$$n^2 \approx 1 + \sum_i \chi_{0i} \frac{\nu_i^2}{\nu_i^2 - \nu^2} = 1 + \sum_i \chi_{0i} \frac{\lambda^2}{\lambda^2 - \lambda_i^2}$$



Material	Sellmeier Equation (Wavelength λ in μm)	Wavelength Range (μm)
Fused silica	$n^2 = 1 + \frac{0.6962\lambda^2}{\lambda^2 - (0.06840)^2} + \frac{0.4079\lambda^2}{\lambda^2 - (0.1162)^2} + \frac{0.8975\lambda^2}{\lambda^2 - (9.8962)^2}$	0.21–3.71
Si	$n^2 = 1 + \frac{10.6684\lambda^2}{\lambda^2 - (0.3015)^2} + \frac{0.0030\lambda^2}{\lambda^2 - (1.1347)^2} + \frac{1.5413\lambda^2}{\lambda^2 - (1104.0)^2}$	1.36–11
GaAs	$n^2 = 3.5 + \frac{7.4969\lambda^2}{\lambda^2 - (0.4082)^2} + \frac{1.9347\lambda^2}{\lambda^2 - (37.17)^2}$	1.4–11
BBO	$n_o^2 = 2.7359 + \frac{0.01878}{\lambda^2 - 0.01822} - 0.01354\lambda^2$	0.22–1.06
	$n_e^2 = 2.3753 + \frac{0.01224}{\lambda^2 - 0.01667} - 0.01516\lambda^2$	
KDP	$n_o^2 = 1 + \frac{1.2566\lambda^2}{\lambda^2 - (0.09191)^2} + \frac{33.8991\lambda^2}{\lambda^2 - (33.3752)^2}$	0.4–1.06
	$n_e^2 = 1 + \frac{1.1311\lambda^2}{\lambda^2 - (0.09026)^2} + \frac{5.7568\lambda^2}{\lambda^2 - (28.4913)^2}$	
$LiNbO_3$	$n_o^2 = 2.3920 + \frac{2.5112\lambda^2}{\lambda^2 - (0.217)^2} + \frac{7.1333\lambda^2}{\lambda^2 - (16.502)^2}$	0.4–3.1
	$n_e^2 = 2.3247 + \frac{2.2565\lambda^2}{\lambda^2 - (0.210)^2} + \frac{14.503\lambda^2}{\lambda^2 - (25.915)^2}$	

Optics of conductive media

$$\nabla \times \mathcal{H} = \frac{\partial \mathfrak{D}}{\partial t} + \mathfrak{J}$$

$$\nabla \times \mathbf{H} = j\omega \mathbf{D} + \mathbf{J}$$

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\nabla \times \mathbf{H} = j\omega \epsilon_e \mathbf{E}$$

$$\epsilon_e = \epsilon + \frac{\sigma}{j\omega}$$

$$n \approx \sqrt{\sigma / 2\omega\epsilon_o}$$

$$\alpha \approx \sqrt{2\omega\mu_o\sigma}$$

$$\eta \approx (1 + j)\sqrt{\omega\mu_o / 2\sigma}$$

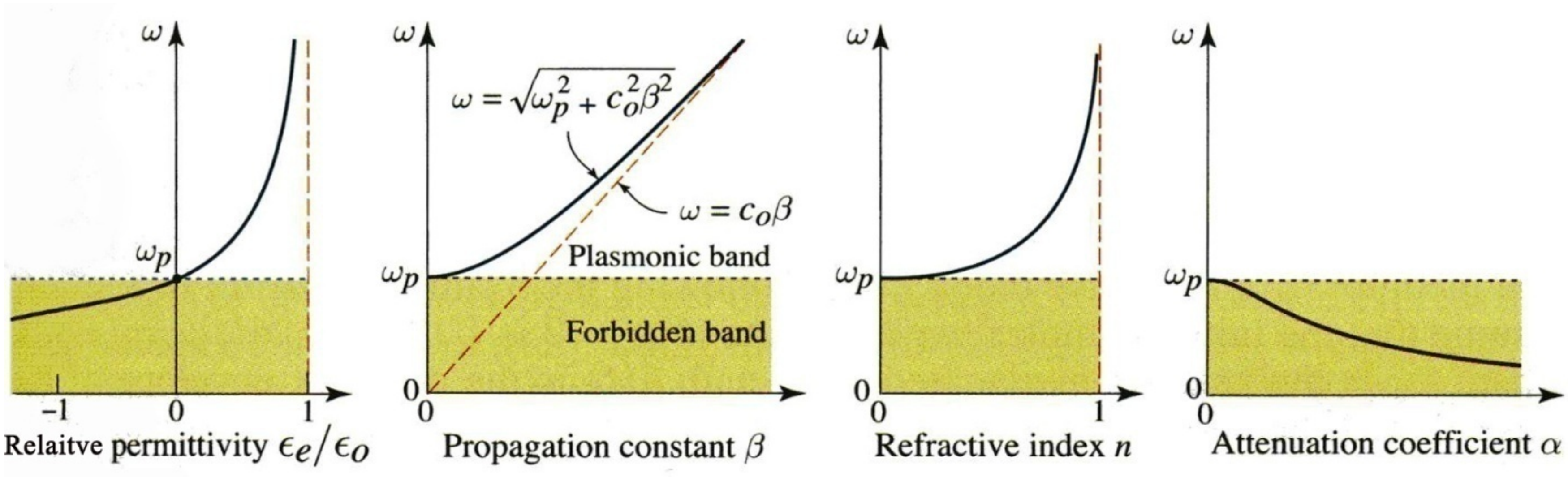
Optics of conductive media - Drude model

$$\sigma = \frac{\sigma_0}{1 + j\omega\tau}$$

$$\epsilon_e = \epsilon + \frac{\sigma_0}{j\omega(1 + j\omega\tau)}$$

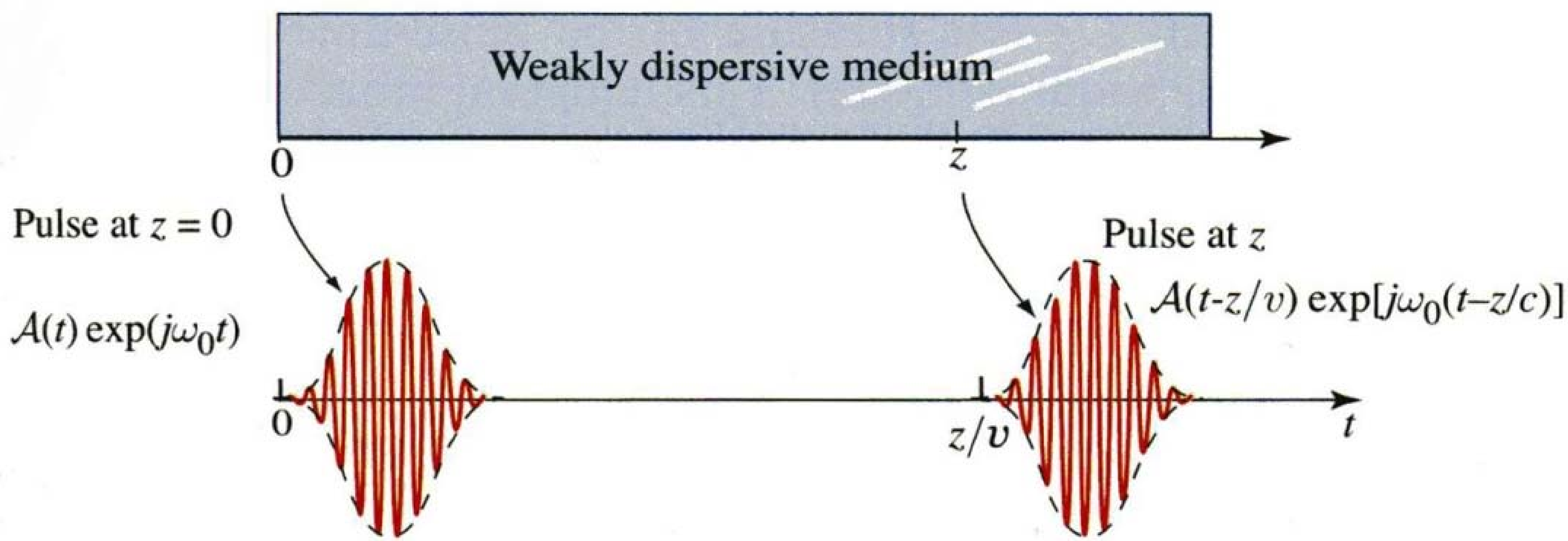
$$\epsilon_e = \epsilon_o\left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

$$\omega_p = \sqrt{\sigma_0/\epsilon_o\tau} \qquad \omega_p = \sqrt{\frac{Ne^2}{\epsilon_o m}}$$



ω
ω_p
−1
0
1
Relaitve permittivity ϵe/ϵo
ω
ω = √(ω_p² + c_o²β²)
ω = c_oβ
Plasmonic band
ω_p
Forbidden band
0
0
Propagation constant β
ω
ω_p
0
0
1
Refractive index n
ω
ω_p
0
0
Attenuation coefficient α

Pulse propagation in dispersive media

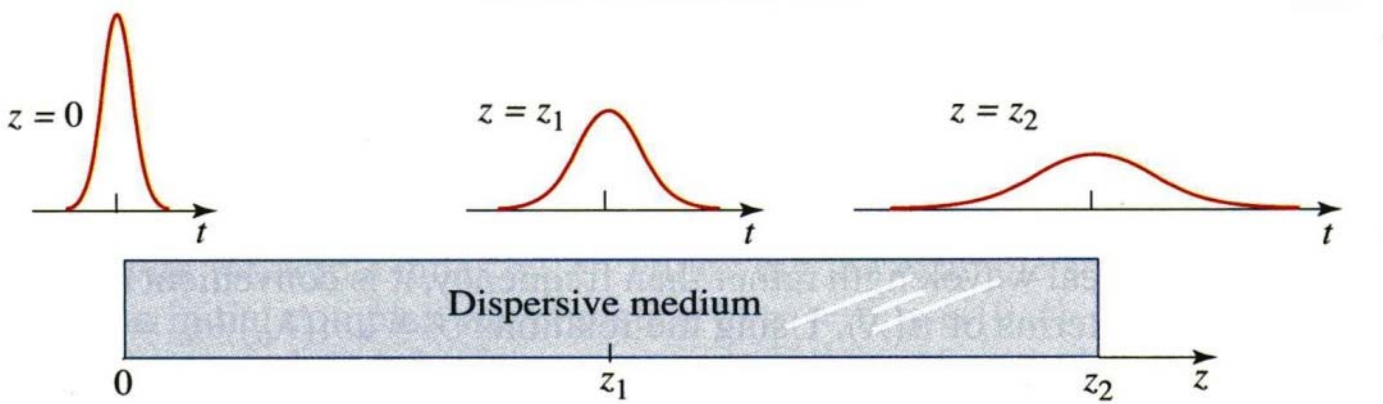


$$\frac{1}{v} = \beta' = \frac{d\beta}{d\omega}$$

$$\boxed{v = \frac{c_o}{N}}$$

$$\boxed{N = n - \lambda_o \frac{dn}{d\lambda_o}.}$$

Pulse propagation in dispersive media



$$D_\nu = \frac{d}{d\nu}\left(\frac{1}{v}\right) = 2\pi\beta''$$

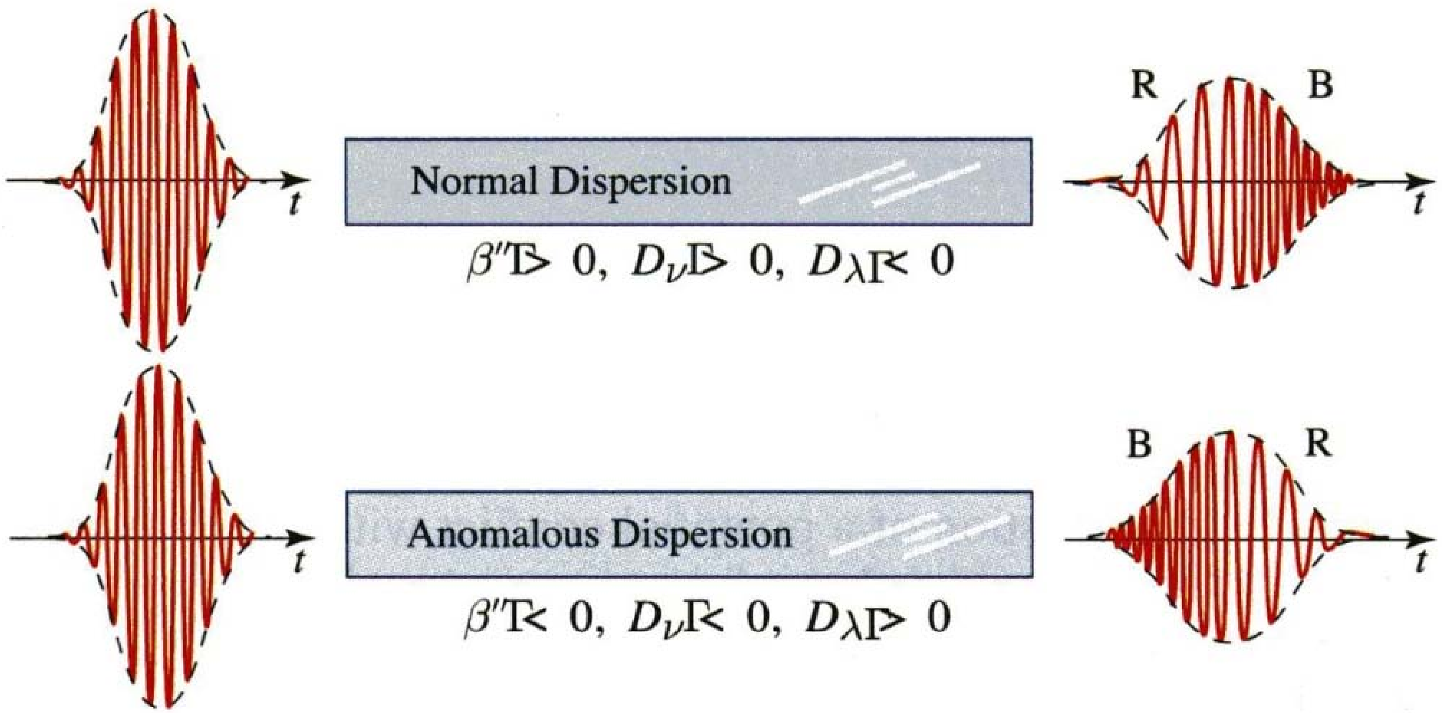
$$D_\nu = \frac{\lambda_o^3}{c_o^2}\frac{d^2n}{d\lambda_o^2}$$

$$\sigma_\tau = |D_\nu|\sigma_\nu z$$

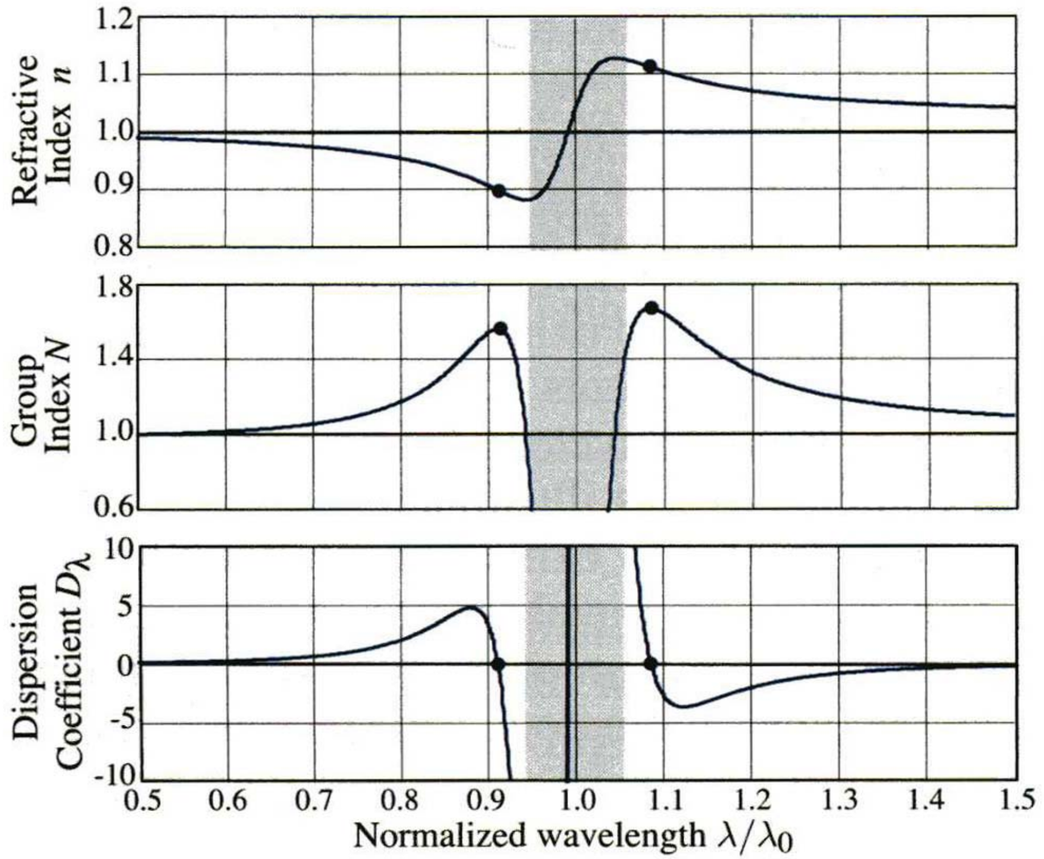
$$D_\lambda = -\frac{\lambda_o}{c_o}\frac{d^2n}{d\lambda_o^2}$$

$$\sigma_\tau = |D_\lambda|\sigma_\lambda z$$

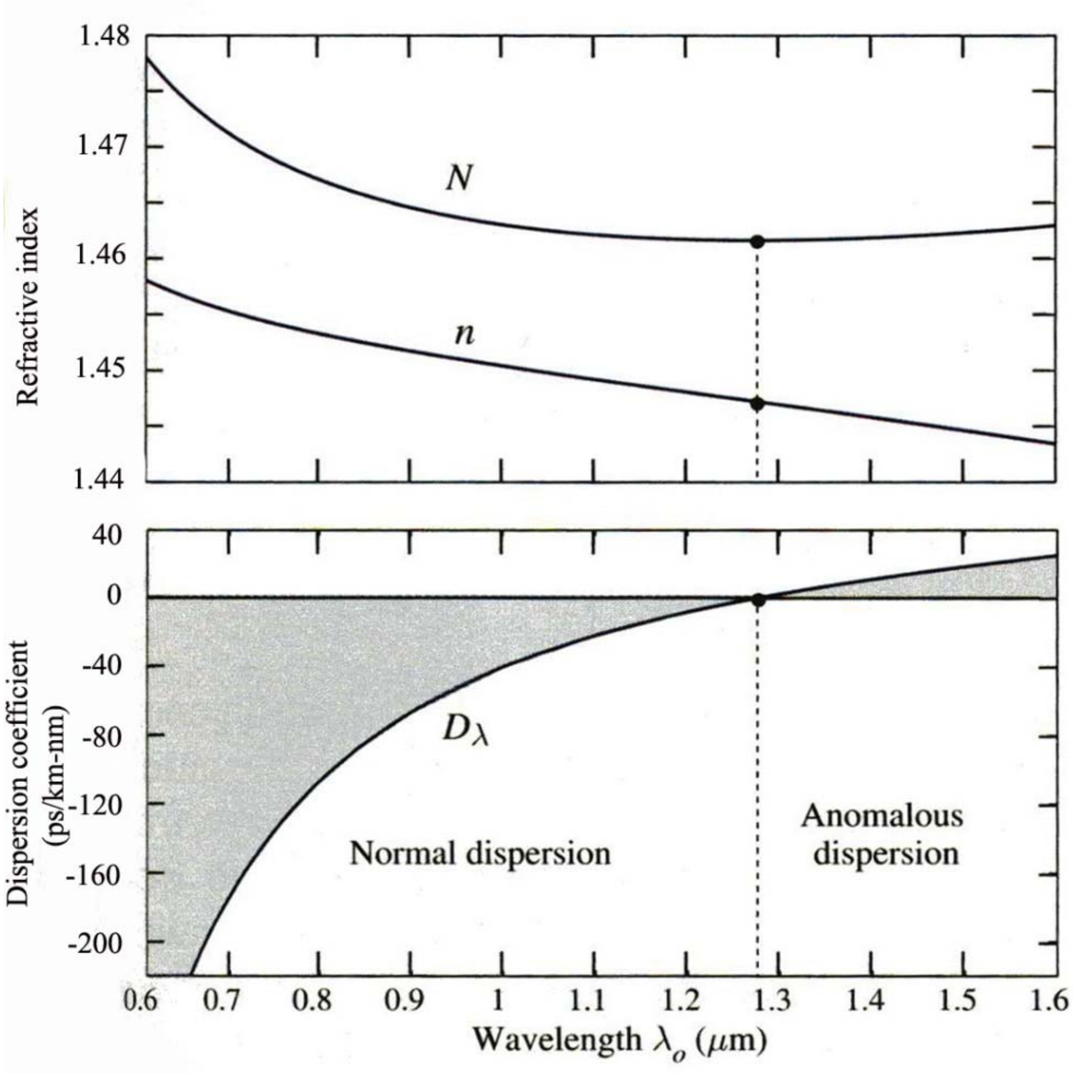
Normal and anomalous dispersion



Single resonance medium



Fused silica



$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 \exp(-j\mathbf{k}\cdot\mathbf{r})$$

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}_0 \exp(-j\mathbf{k}\cdot\mathbf{r})$$

$$\mathbf{k} \times \mathbf{H}_0 = -\omega\,\epsilon\,\mathbf{E}_0$$

$$\mathbf{k} \times \mathbf{E}_0 = \omega\,\mu\,\mathbf{H}_0$$

$$k = \omega\sqrt{\epsilon\mu}$$

$$\eta = \frac{E_0}{H_0} = \frac{\omega\mu}{k}$$

$$n = \sqrt{\frac{\epsilon}{\epsilon_o}\frac{\mu}{\mu_o}} = c_o\sqrt{\epsilon\mu}$$

$$nk_o - j\tfrac{1}{2}\alpha = \omega\sqrt{\epsilon\mu}$$

Doubly negative metamaterials

$$\epsilon = -|\epsilon| \qquad \mu = -|\mu|$$

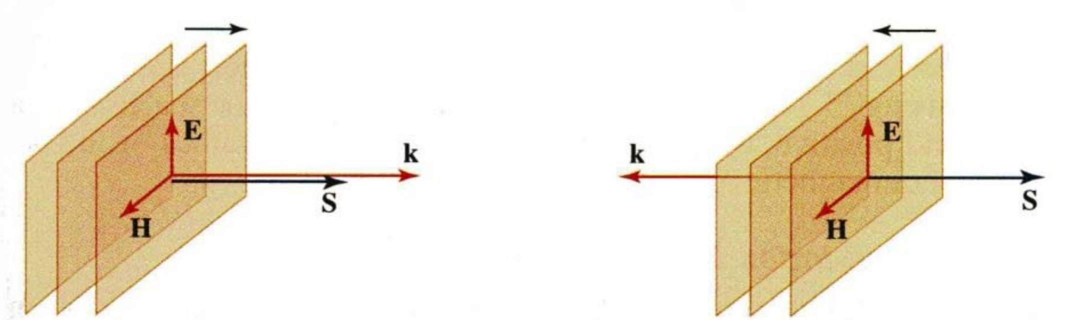
$$\mathbf{k} \times \mathbf{H}_0 = \omega\,|\epsilon|\,\mathbf{E}_0$$

$$\mathbf{k} \times \mathbf{E}_0 = -\omega\,|\mu|\,\mathbf{H}_0$$

$$\mathbf{E} = E_0 \exp(-jkz)\,\widehat{\mathbf{x}}$$

$$\mathbf{H} = H_0 \exp(-jkz)\,\widehat{\mathbf{y}}$$

$$n = -c_o\sqrt{|\epsilon||\mu|}$$

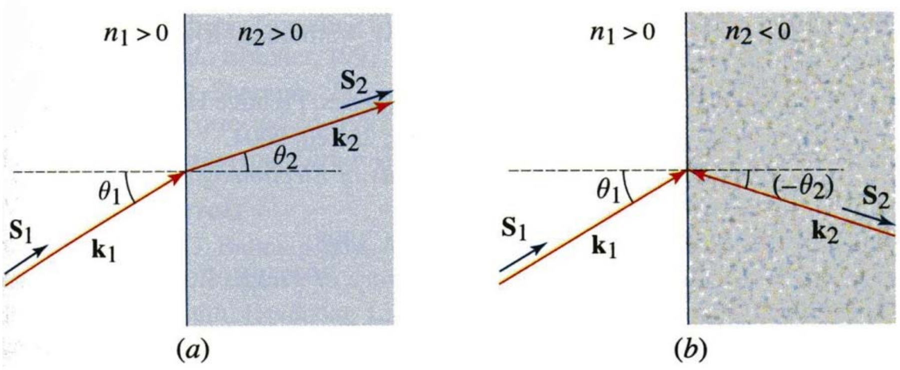


(*a*) Ordinary material (*b*) Doubly negative material

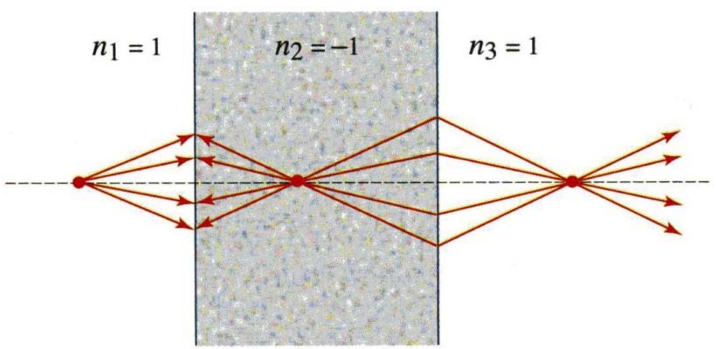
$$\left(|\epsilon| - \mathrm{Re}\{\epsilon\}\right)\left(|\mu| - \mathrm{Re}\{\mu\}\right) > \mathrm{Im}\{\epsilon\}\,\mathrm{Im}\{\mu\}$$

Left-handedness

Negative index materials



$$n_1 \sin\theta_1 = -|n_2| \sin\theta_2$$



Left-handed material

- V.G. Veselago, Soviet Physics Uspekhi 10, 509 (1968)

$$k \sin\varphi = k_1 \sin\varphi_1.$$

Однако последнее равенство удовлетворяется как при угле φ_1, так и при угле $\pi - \varphi_1$.

Требуя по-прежнему, чтобы энергия во второй среде *оттекала* от границы раздела, мы приходим тогда к тому, что фаза должна *набегать* на эту границу и, следовательно, направление распространения преломленной волны будет составлять с нормалью угол

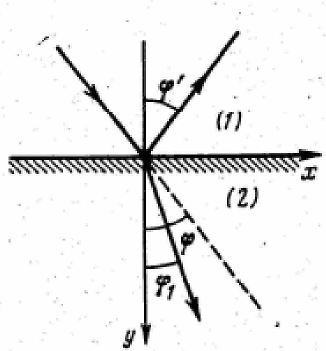


Рис. 12

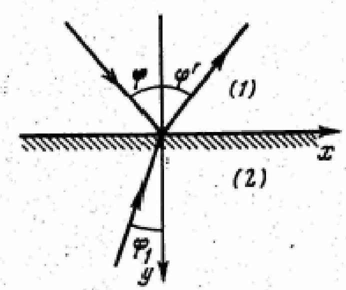


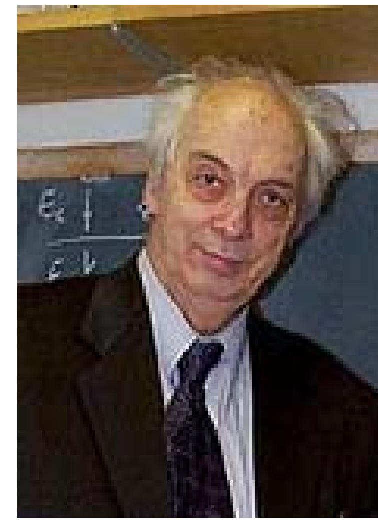
Рис. 13

$\pi - \varphi_1$. Как ни непривычно такое построение, но, конечно, ничего удивительного в нем нет, ибо фазовая скорость еще ничего не говорит о направлении потока энергии.

ЧЕТВЕРТАЯ ЛЕКЦИЯ

(5.V 1944 г.)

Victor Georgievich Veselago graduated from Moscow University in 1952, and was with P.N. Lebedev Physical Institute, Moscow, from 1952 to 1983. From 1983 up to now, he is the head of laboratory of magnetic materials in A.M. Prokhorov Institute of General Physics. He received his PhD degree in 1959, for radiospectropy investigation of molecular spectra, and degree Doctor of Science (solid state physics) in 1974 for investigation of solid state in high magnetic field., both in P.N. Lebedev Institute. From 1980 until now, is a professor of applied physics in Moscow Institute of Physics and Technology. The area of scientific interest of V.G. Veselago is magnetism, solid-state physics, electro-dynamics. In his papers, published in 1966-1972 was at the first time considered electrodynamics of material with negative value of refraction index (so called Left-Handed Materials, LHM). V.G. Veselago is a winner of State Prize for science of USSR (1976), and a winner of academician V.A. Fock prize (2004). He is also an Honored Scientist of Russian Federation (2002). He is an active expert in Russian Foundation for Fundamental Research, Russian Foundation for Humanitarian Research, and is vice-chairman of physics section of Supreme Attestation Committee of Russia (VAK). He is a founder and vice-editor of the electronic, scientific journal " **Исследовано в России** Investigated in Russia." Married, has 3 daughters and 1 son. His favorite animal is lady-cat, Fifa. His hobby is of railways (real, not models).



Metamaterials

news feature

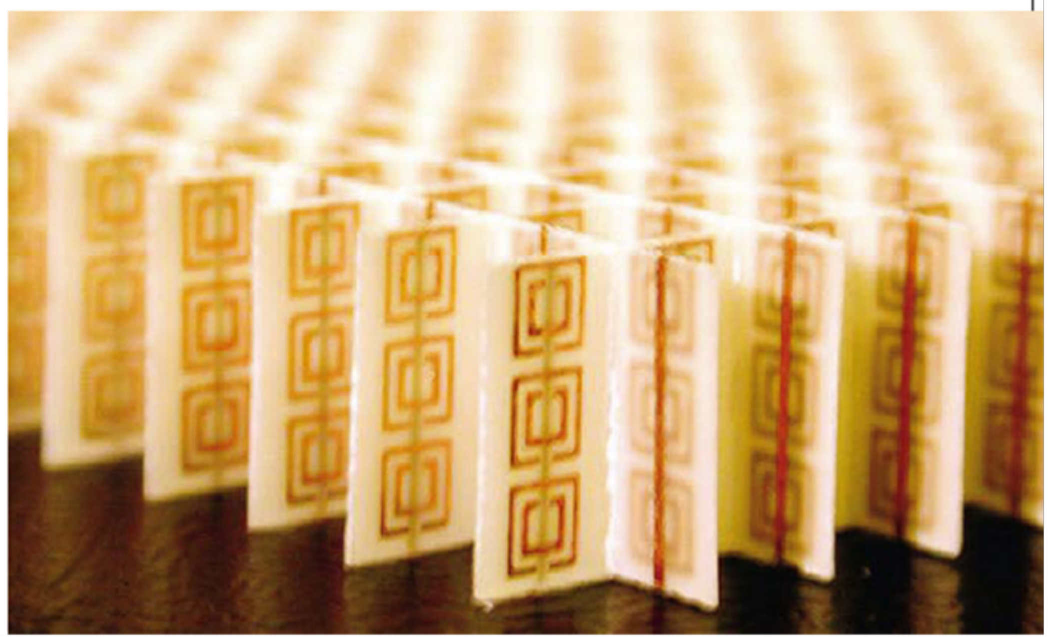
A lens less ordinary

In the 1960s, a Russian physicist considered the properties of a material that didn't yet exist. Now researchers appear to have fulfilled his predictions — but is everything as it seems? Liesbeth Venema investigates.

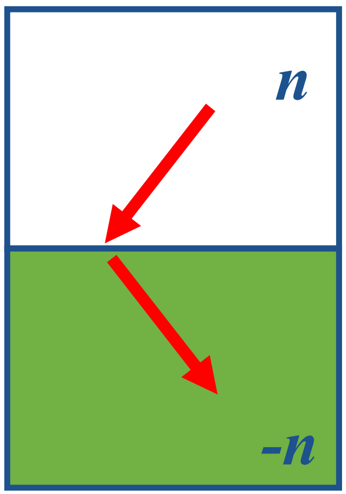
There are some truths in physics on which we have come to depend. Light rays, for example, bend when they cross the boundary between two materials. That's why an oar dipped into water appears to bend towards the surface, and why the pool itself looks shallower than it really is.

But this familiar phenomenon, called refraction, is beginning to look less straightforward. In the lab of David Smith, a physicist at the University of California, San Diego, a strange array of metal wires and loops has been pieced together. In April 2001, Smith and his team showed that this construction, which they refer to as a 'metamaterial', has a peculiar property: it bends electromagnetic waves in the opposite direction to normal materials[1].

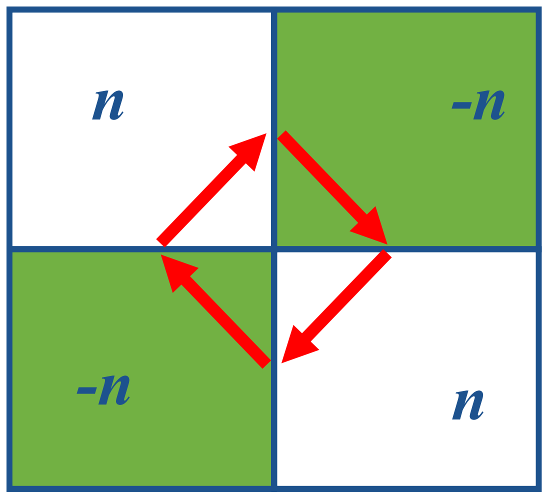
If a pool of water had this property, known as negative refraction, oars would bend away from the surface, and the pool



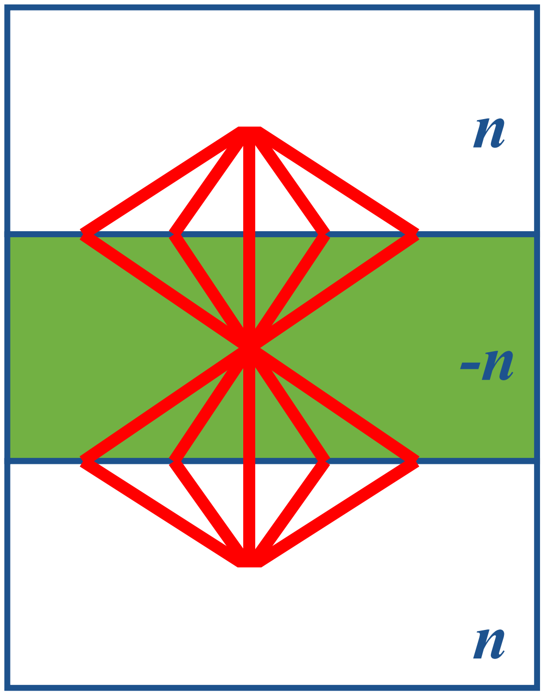
Light propagation in negative index material



Negative refraction



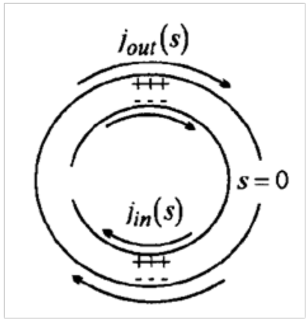
Formation of
an open cavity

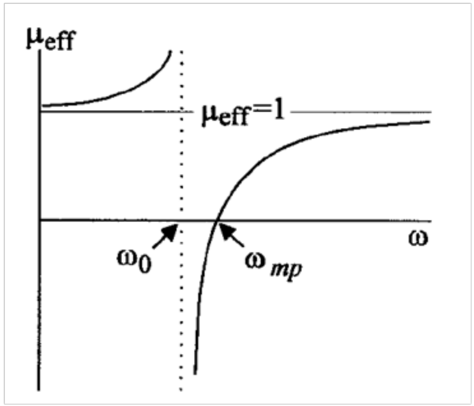


Imaging by a slab

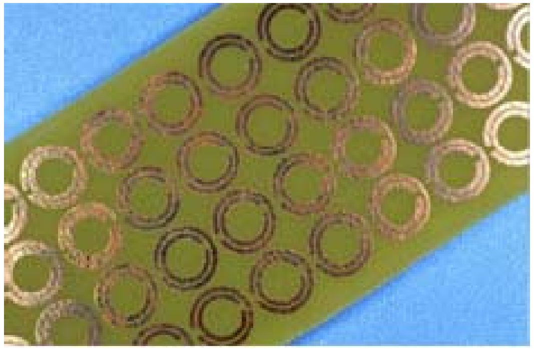
Split ring resonators

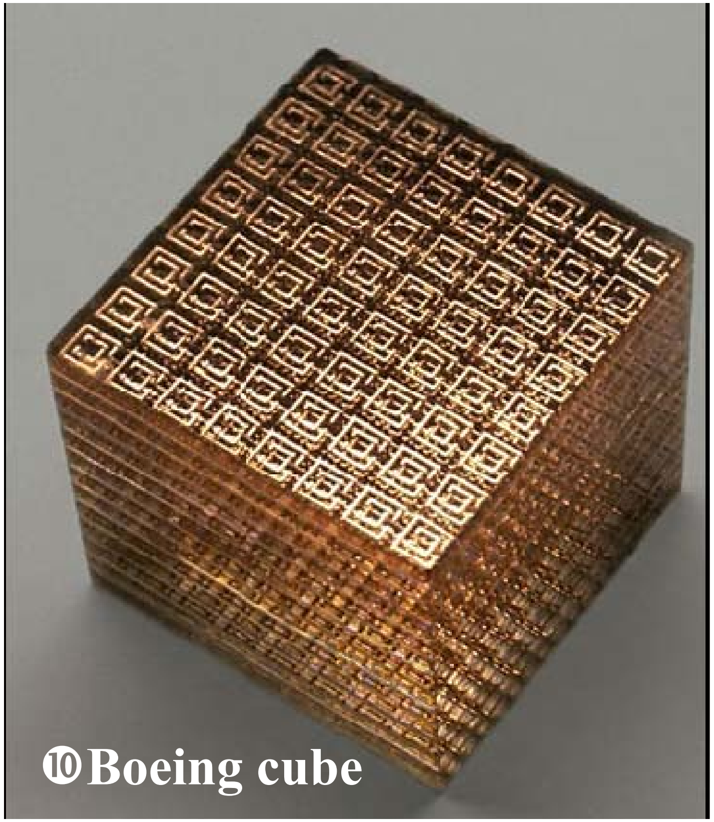
- J. B. Pendry *et al.*, IEEE Transactions on Microwave Theory and Techniques **47**, 2075 (1999).





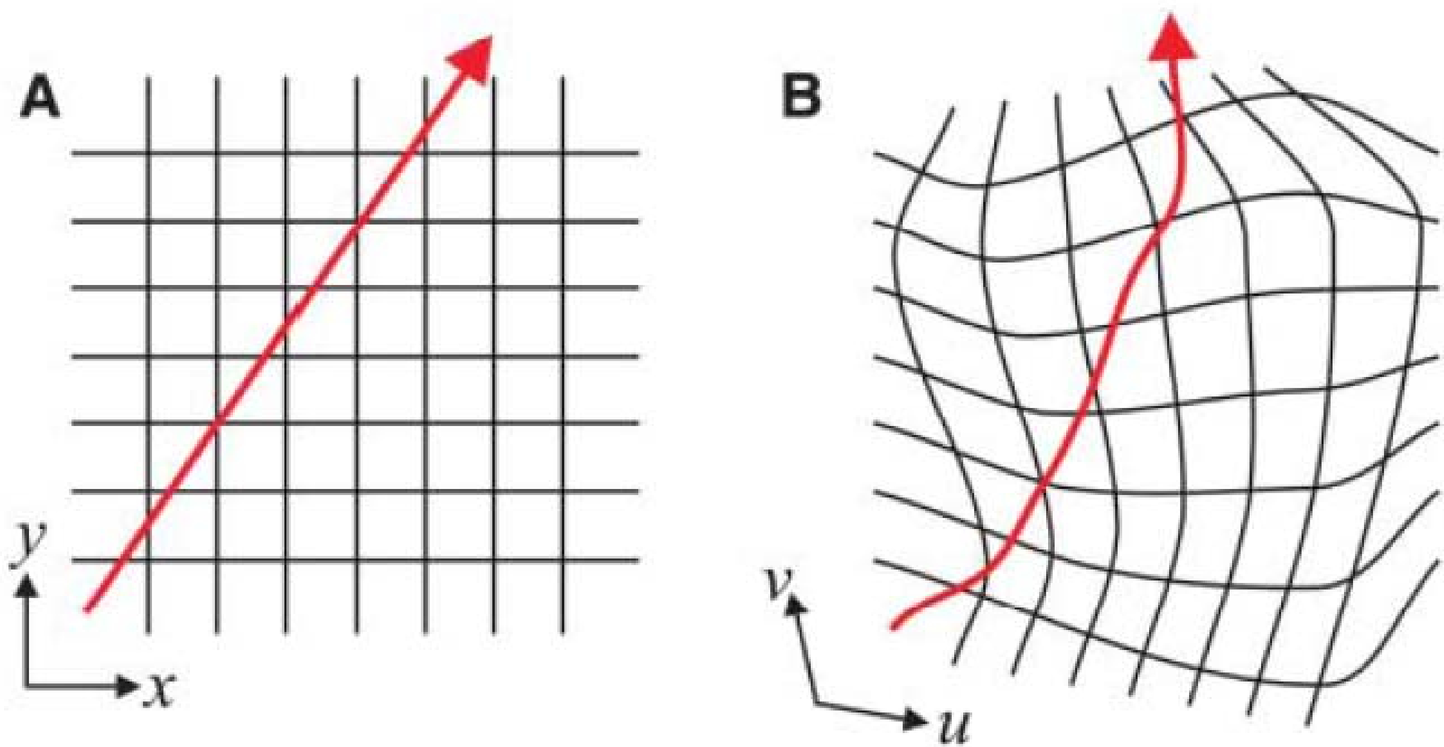
Effective magnetic permeability





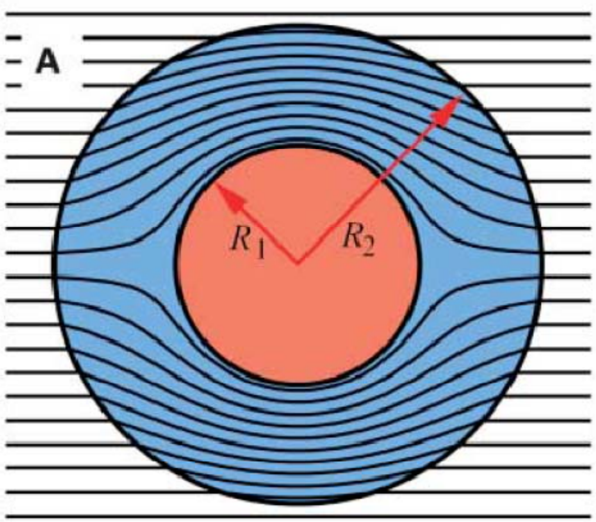
Coordinate transform

❖ G. R. Newkome *et al*., Science **312**, 1782 (2006).



When the coordinate is transformed, the Maxwell equations have exactly the same form but the ε and μ are scaled by a common factor.

Basic concept of cloaking



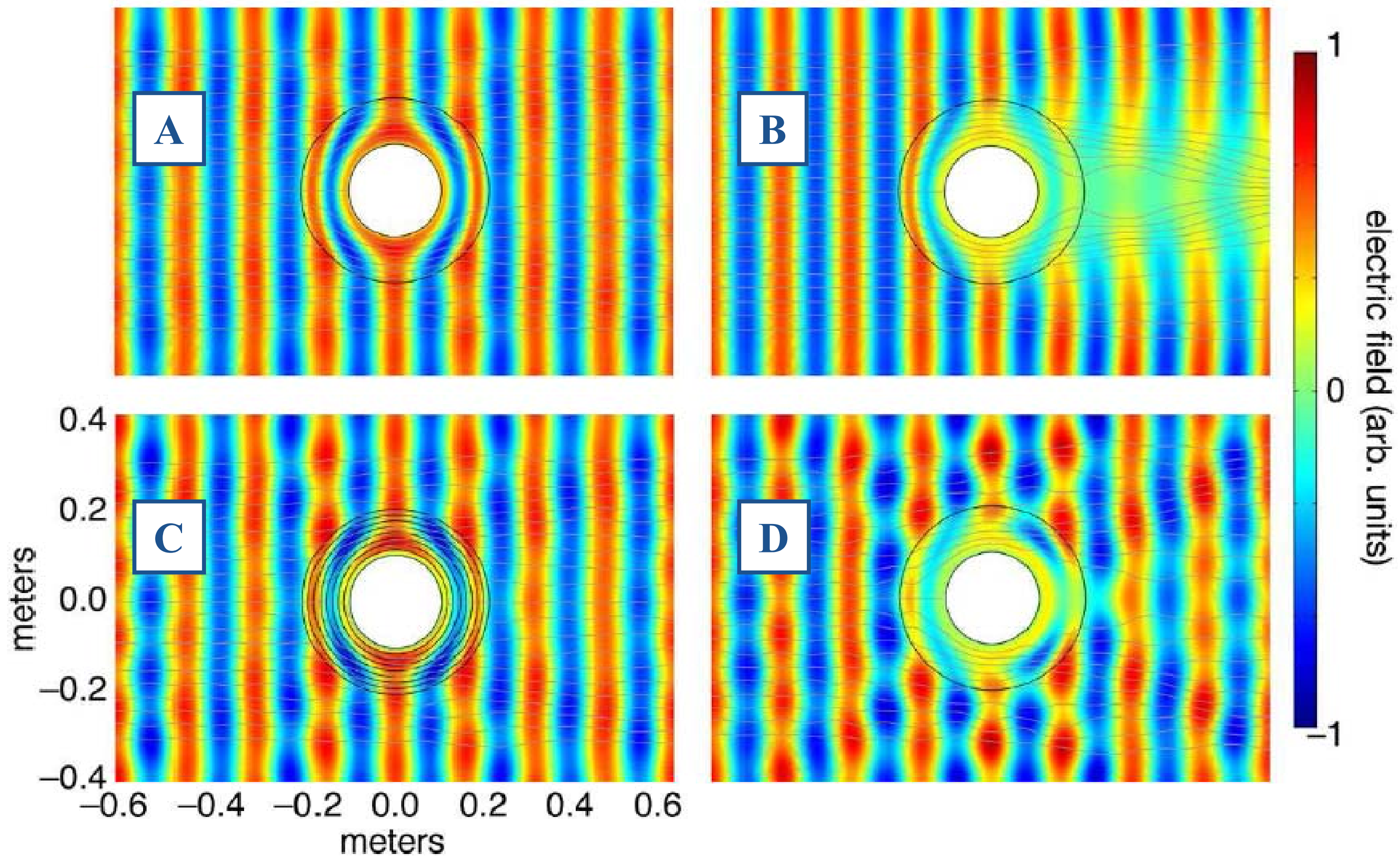
region $r<R_2$ → **region $R_1<r<R_2$**

$$r' = R_1 + r(R_2 - R_1)/R_2$$
$$\theta' = \theta$$
$$\phi' = \phi$$

$$\varepsilon'_{r'} = \mu'_{r'} = \frac{R_2}{R_2 - R_1}\frac{(r'-R_1)^2}{r'}$$
$$\varepsilon'_{\theta'} = \mu'_{\theta'} = \frac{R_2}{R_2 - R_1}$$
$$\varepsilon'_{\phi'} = \mu'_{\phi'} = \frac{R_2}{R_2 - R_1}$$

Any radiation attempting to penetrate the secure volume is smoothly guided around by the cloak to emerge traveling in the same direction as if it had passed through the empty volume of space.

Simulation of cloaking structures



Pendry

A : Ideal parameter & lossless

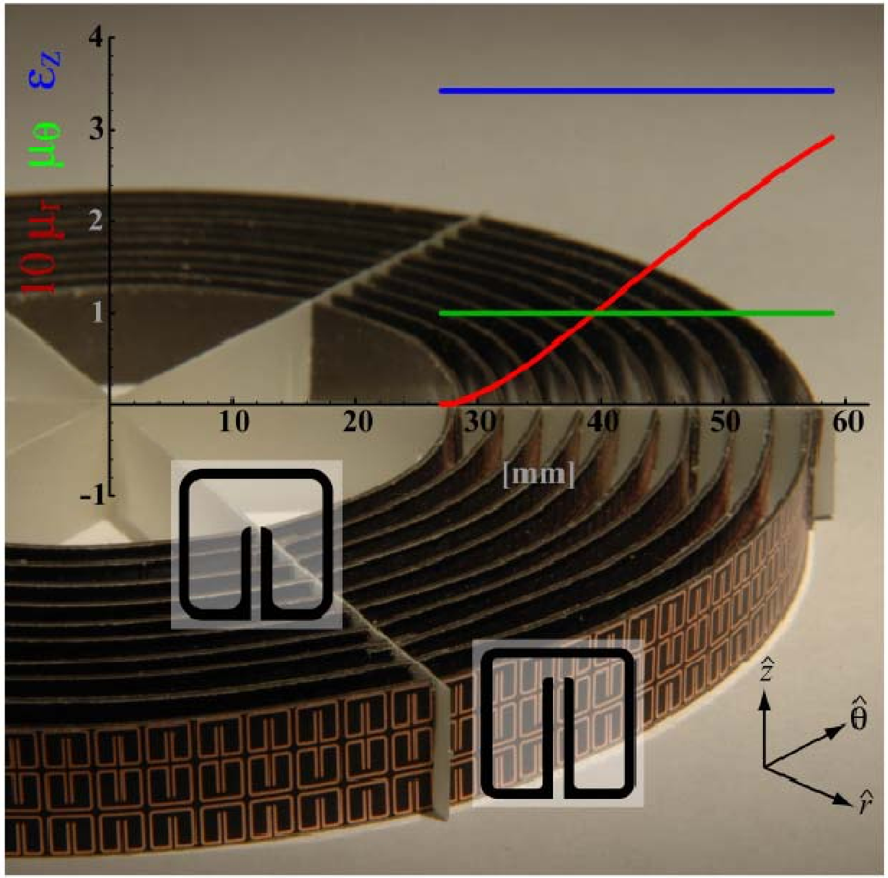
B : Ideal parameter & lossy

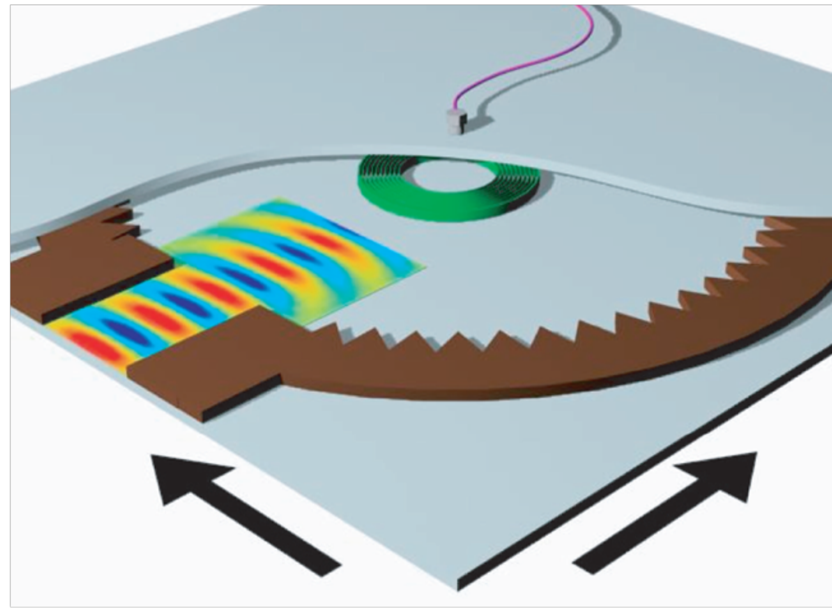
C : 8-layers approximation

D : Reduced approximation

Microwave cloaking structure

- D. Schurig *et al.*, Science (2006).





Split ring resonators