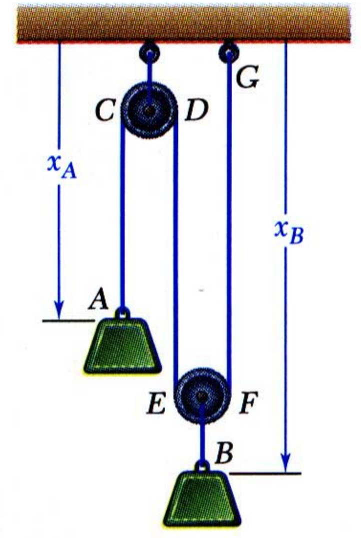
Kinematics of Particles

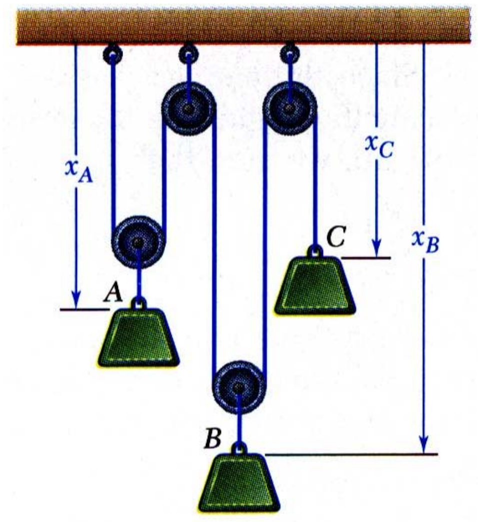
Today

- Motion of several particles: Dependent motion
- Definition of curvilinear motion
- Representation of curvilinear motion
  - Using rectangular components
  - Using tangential and normal components
  - Using radial and transverse components

Motion of Several Particles: Dependent Motion

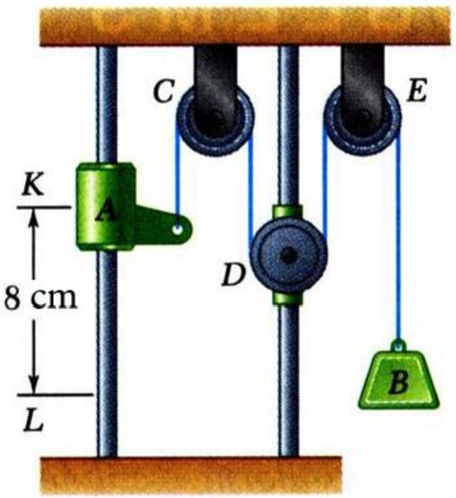


- Position of a particle may *depend* on position of one or more other particles.
- Position of block *B* depends on position of block *A.*
- *Why?* What's the relation?



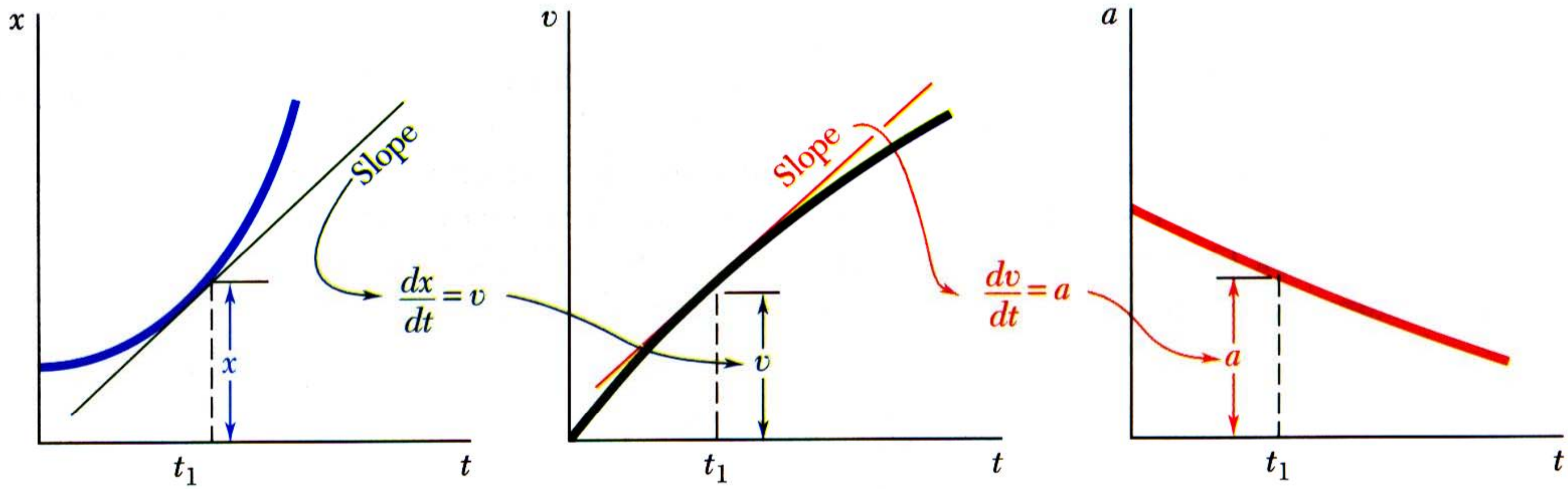
- Positions of three blocks are dependent.
- For linearly related positions, similar relations hold between velocities and accelerations.

Sample Problem 11.5



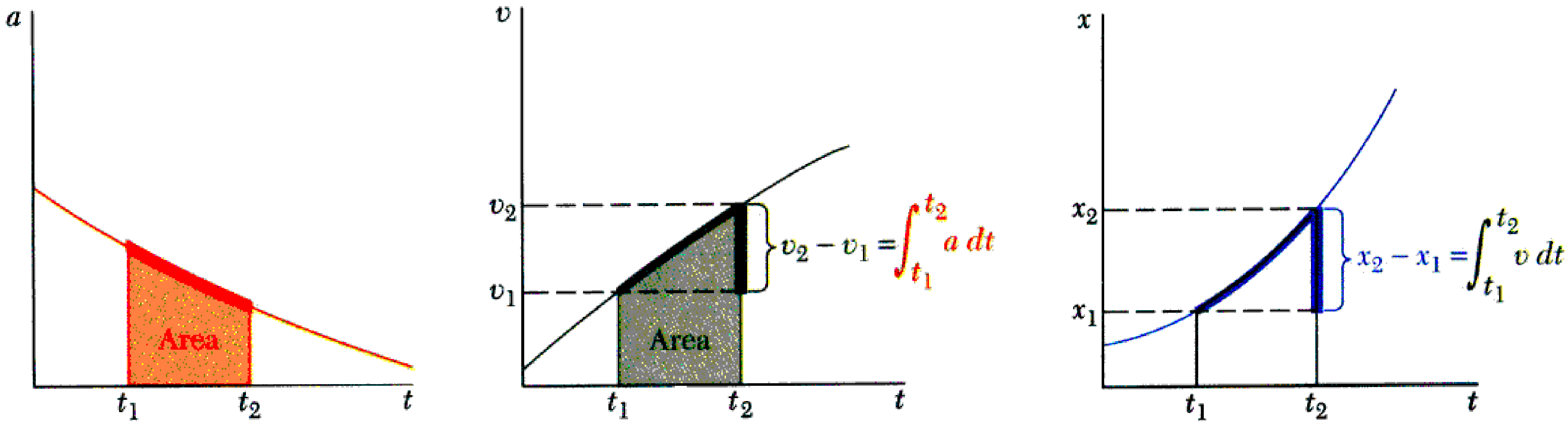
Pulley *D* is attached to a collar which is pulled down at 3 cm/s. At $t = 0$, collar *A* starts moving down from *K* with constant acceleration and zero initial velocity. Knowing that velocity of collar *A* is 12 cm/s as it passes *L*, determine the change in elevation, velocity, and acceleration of block *B* when block *A* is at *L*.

Graphical Solution of Rectilinear-Motion Problems



- Given the *x*-*t* curve, the *v*-*t* curve is equal to the *x*-*t* curve slope.
- Given the *v*-*t* curve, the *a*-*t* curve is equal to the *v*-*t* curve slope.

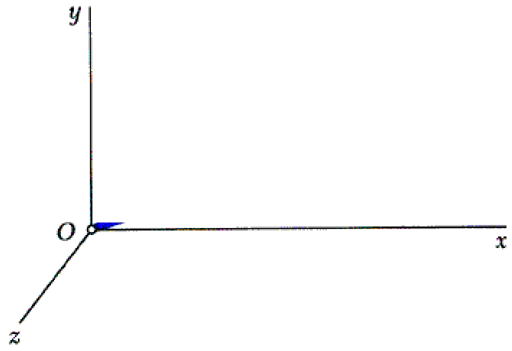
Graphical Solution of Rectilinear-Motion Problems



- Given the *a*-*t* curve, the change in velocity between t_1 and t_2 is equal to the area under the *a*-*t* curve between t_1 and t_2.

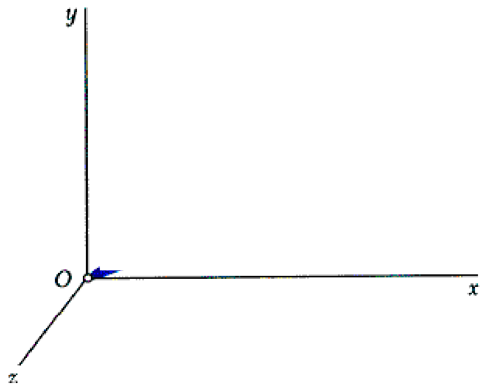
- Given the *v*-*t* curve, the change in position between t_1 and t_2 is equal to the area under the *v*-*t* curve between t_1 and t_2.

Curvilinear Motion: Position, Velocity & Acceleration



- Particle moving along a curve other than a straight line is in *curvilinear motion*.

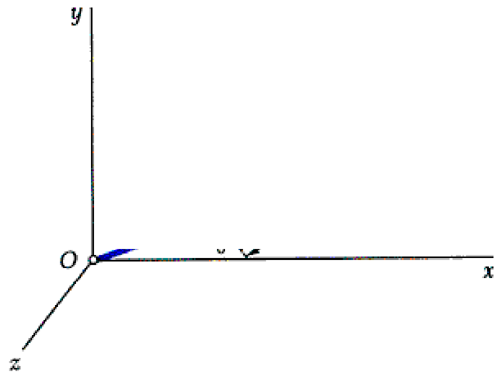
- *How is a Position vector* of a particle at time *t defined?*



- Consider particle which occupies position *P* defined by $\vec{r}$ at time *t* and *P'* defined by $\vec{r}\,'$ at $t + \Delta t$,

$$\vec{v} =$$

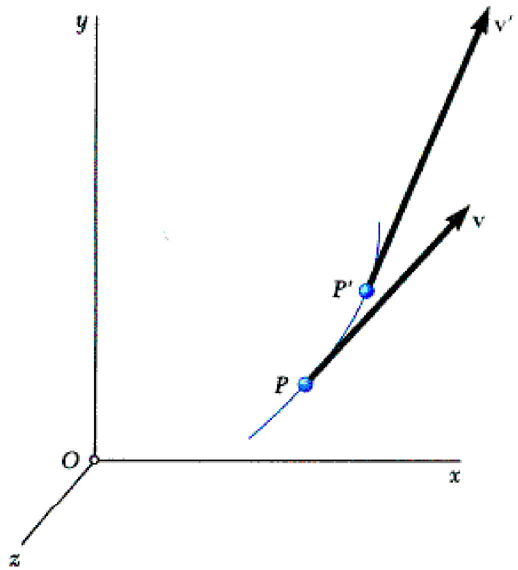
$$=$$



$$v =$$

$$=$$

Curvilinear Motion: Position, Velocity & Acceleration

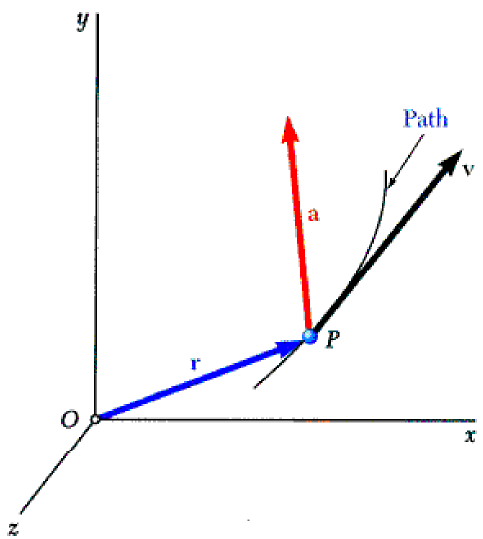


- Consider velocity $\vec{v}$ of particle at time t and velocity $\vec{v}'$ at $t + \Delta t$,

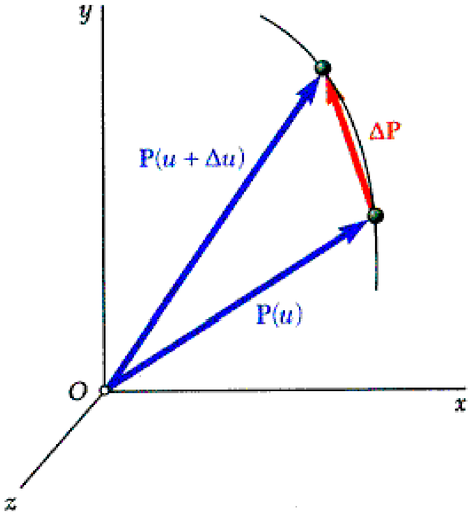
$$\vec{a} =$$

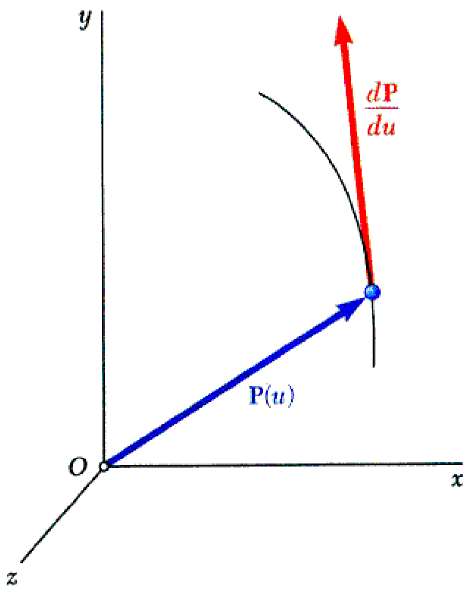
$$=$$

- Is velocity vector tangent to the path?
- Is acceleration vector tangent to the path?



Derivatives of Vector Functions





- Let $\vec{P}(u)$ be a vector function of scalar variable u,

$$\frac{d\vec{P}}{du} = \lim_{\Delta u \to 0} \frac{\Delta \vec{P}}{\Delta u} = \lim_{\Delta u \to 0} \frac{\vec{P}(u+\Delta u) - \vec{P}(u)}{\Delta u}$$

- Derivative of vector sum,

$$\frac{d\left(\vec{P}+\vec{Q}\right)}{du} = \frac{d\vec{P}}{du} + \frac{d\vec{Q}}{du}$$

- Derivative of product of scalar and vector functions,

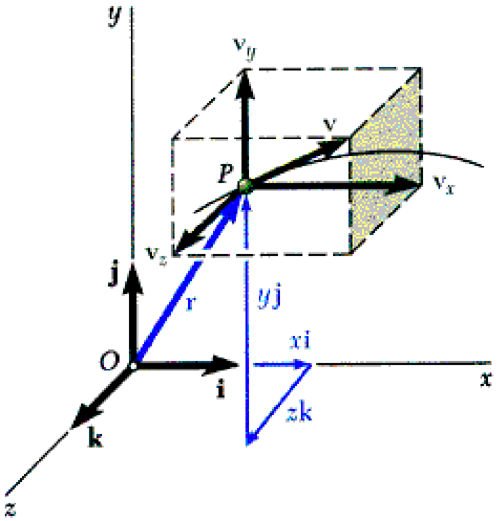
$$\frac{d\left(f\vec{P}\right)}{du} = \frac{df}{du}\vec{P} + f\frac{d\vec{P}}{du}$$

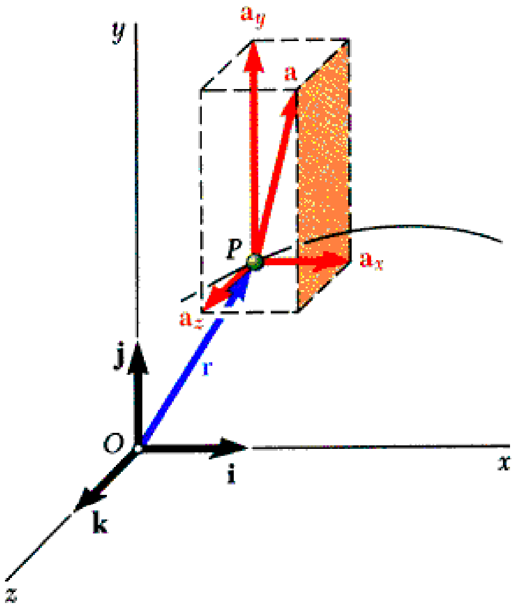
- Derivative of *scalar product* and *vector product*,

$$\frac{d\left(\vec{P}\bullet\vec{Q}\right)}{du} = \frac{d\vec{P}}{du}\bullet\vec{Q} + \vec{P}\bullet\frac{d\vec{Q}}{du}$$

$$\frac{d\left(\vec{P}\times\vec{Q}\right)}{du} = \frac{d\vec{P}}{du}\times\vec{Q} + \vec{P}\times\frac{d\vec{Q}}{du}$$

Rectangular Components of Velocity & Acceleration





- When position vector of particle P is given by its rectangular components,

- Velocity vector,

$\vec{v} =$

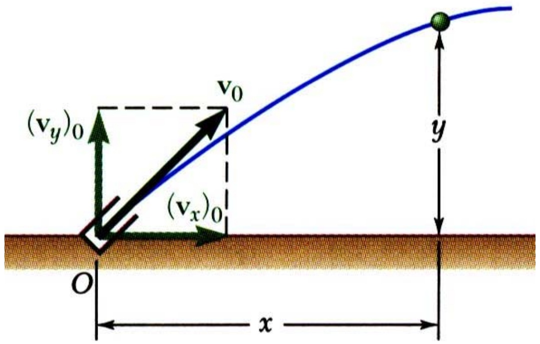
$=$

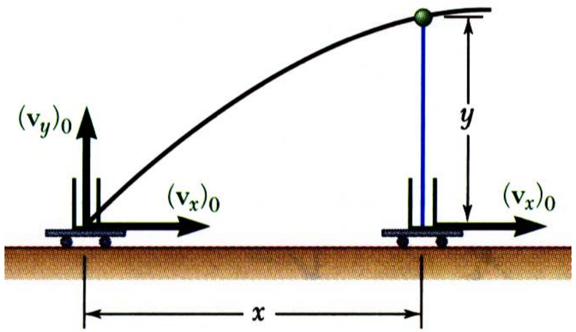
- Acceleration vector,

$\vec{a} =$

$=$

Rectangular Components of Velocity & Acceleration





- Rectangular components particularly effective when component accelerations can be integrated independently, e.g., motion of a projectile,

$$a_x = \ddot{x} = 0 \qquad a_y = \ddot{y} = -g \qquad a_z = \ddot{z} = 0$$

with initial conditions,

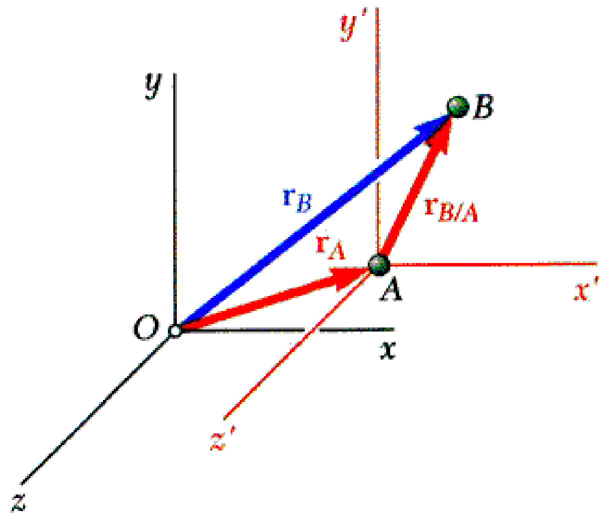
$$x_0 = y_0 = z_0 = 0 \qquad (v_x)_0, (v_y)_0, (v_z)_0 = 0$$

Integrating twice yields

$$v_x = (v_x)_0 \quad v_y = (v_y)_0 - gt \qquad v_z = 0$$
$$x = (v_x)_0 t \quad y = (v_y)_0 y - \tfrac{1}{2} g t^2 \quad z = 0$$

- Motion in horizontal direction is uniform.
- Motion in vertical direction is uniformly accelerated.
- Motion of projectile could be replaced by two independent rectilinear motions.

Motion Relative to a Frame in Translation



- Designate one frame as the *fixed frame of reference*. All other frames not rigidly attached to the fixed reference frame are *moving frames of reference*.
- Position vectors for particles *A* and *B* with respect to the fixed frame of reference *Oxyz* are $\vec{r}_A$ and $\vec{r}_B$.
- Vector $\vec{r}_{B/A}$ joining *A* and *B* defines the position of *B* with respect to the moving frame *Ax'y'z'* and

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

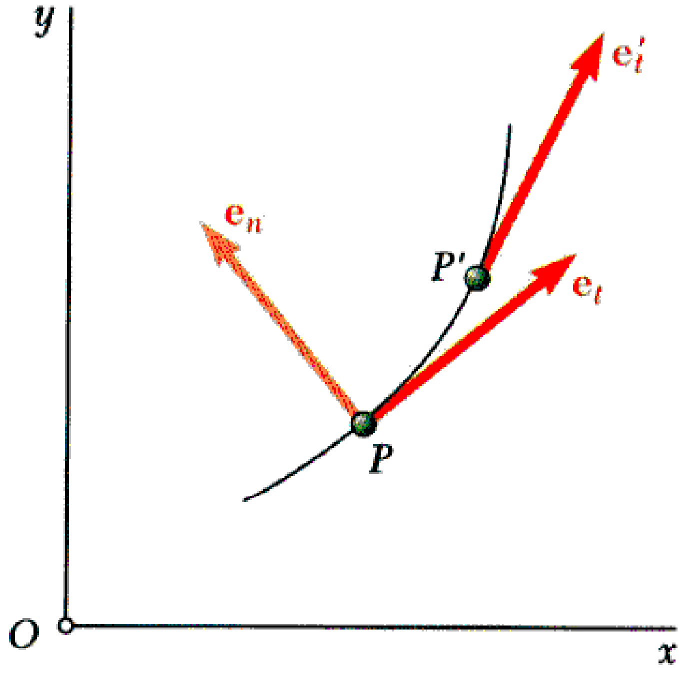
- Differentiating twice,

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad \vec{v}_{B/A} = \text{velocity of } B \text{ relative to } A.$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \qquad \vec{a}_{B/A} = \text{acceleration of } B \text{ relative to } A.$$

- Absolute motion of *B* can be obtained by combining motion of *A* with relative motion of *B* with respect to moving reference frame attached to *A*.

Tangential and Normal Components



- Velocity vector of particle is tangent to path of particle. In general, acceleration vector is not. Wish to express acceleration vector in terms of tangential and normal components. Why?

- $\vec{e}_t$ and $\vec{e}_t'$ are tangential **unit vectors** for the particle path at *P* and *P'*. When drawn with respect to the same origin, $\Delta\vec{e}_t = \vec{e}_t' - \vec{e}_t$ and $\Delta\theta$ is the angle between them. Why $\Delta\theta$?

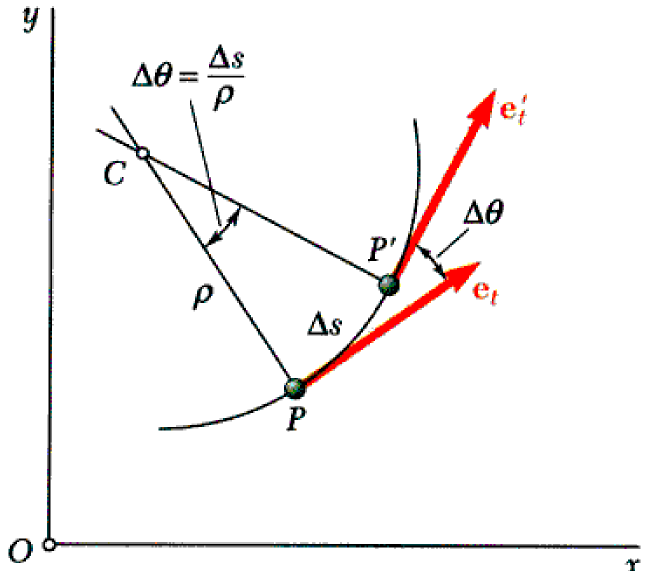
$$\Delta e_t =$$

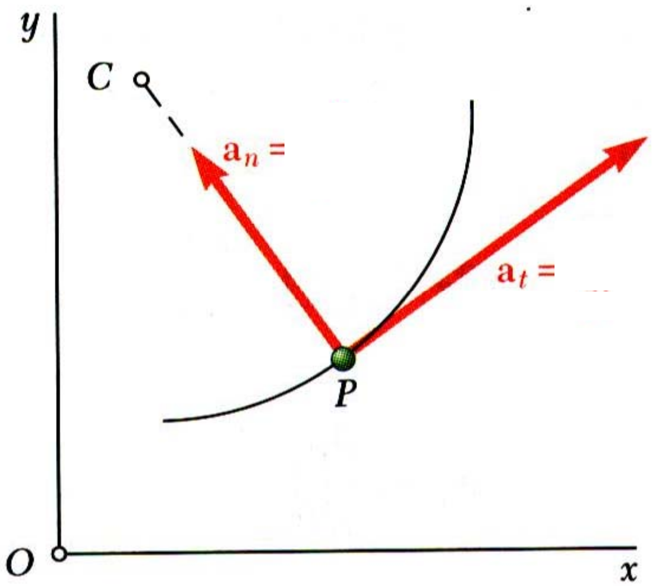
$$\lim_{\Delta\theta \to 0} \frac{\Delta\vec{e}_t}{\Delta\theta} =$$

$$\vec{e}_n =$$

Direction of change of tangential term of the acceleration

Tangential and Normal Components





- With the velocity vector expressed as $\vec{v} =$ the particle acceleration may be written as

$$\vec{a} =$$

but

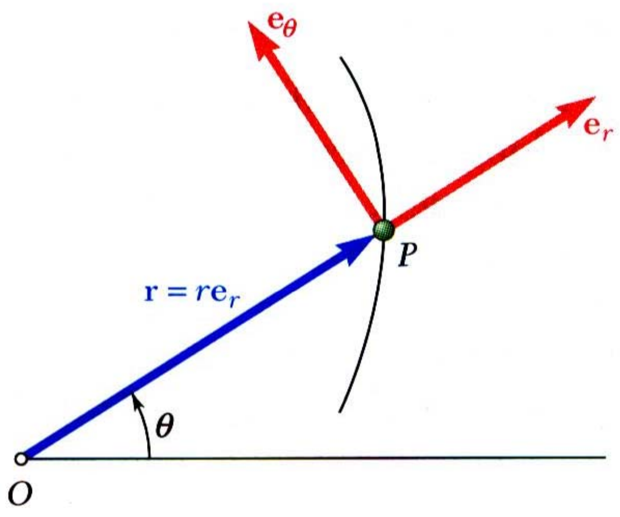
$$\frac{d\vec{e}_t}{d\theta} = \qquad \frac{d\theta}{ds} = \qquad \frac{ds}{dt} =$$

After substituting,

$$\vec{a} = \qquad a_t = \qquad a_n =$$

- Tangential component of acceleration reflects and normal component reflects
- Tangential component may be positive or negative. Normal component always points toward

Radial and Transverse Components



$$\vec{r} = r\vec{e}_r$$

$$\frac{d\vec{e}_r}{d\theta} = \qquad \frac{d\vec{e}_\theta}{d\theta} =$$

$$\frac{d\vec{e}_r}{dt} =$$

$$\frac{d\vec{e}_\theta}{dt} =$$

- When particle position is given in polar coordinates, it is convenient to express velocity and acceleration with components parallel and perpendicular to *OP*.

- The particle velocity vector is

$$\vec{v} =$$

$$=$$

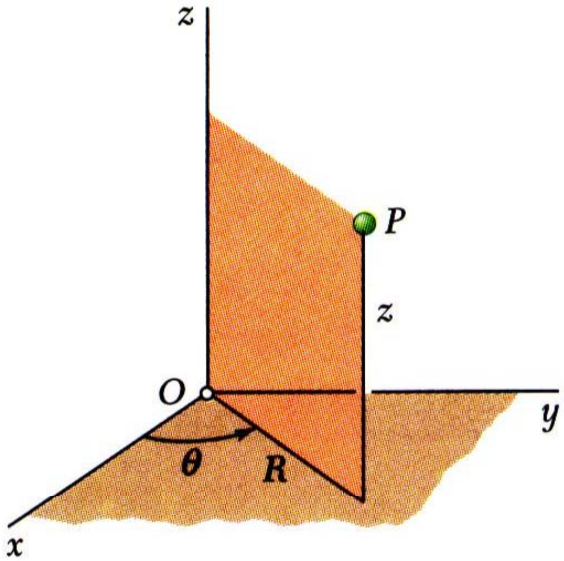
- Similarly, the particle acceleration vector is

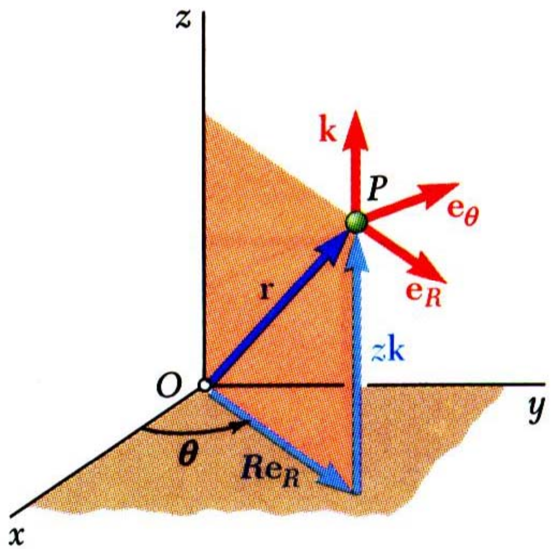
$$\vec{a} = \frac{d}{dt}\Big(\qquad \Big)$$

$$=$$

$$=$$

Radial and Transverse Components





- When particle position is given in cylindrical coordinates, it is convenient to express the velocity and acceleration vectors using the unit vectors $\vec{e}_R$, $\vec{e}_\theta$, and $\vec{k}$.

- Position vector,

$$\vec{r} = R\vec{e}_R + z\vec{k}$$

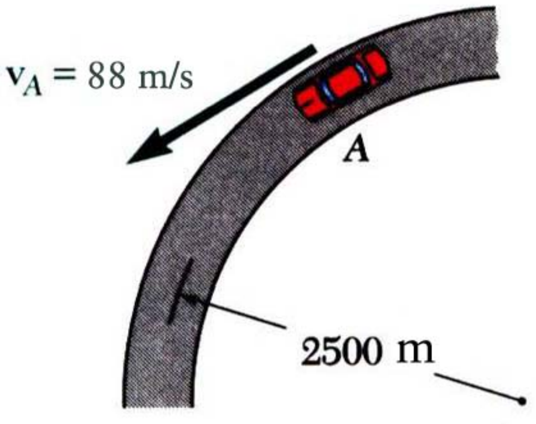
- Velocity vector,

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{R}\vec{e}_R + R\dot{\theta}\vec{e}_\theta + \dot{z}\vec{k}$$

- Acceleration vector,

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(\ddot{R} - R\dot{\theta}^2\right)\vec{e}_R + \left(R\ddot{\theta} + 2\dot{R}\dot{\theta}\right)\vec{e}_\theta + \ddot{z}\vec{k}$$

Sample Problem 11.10



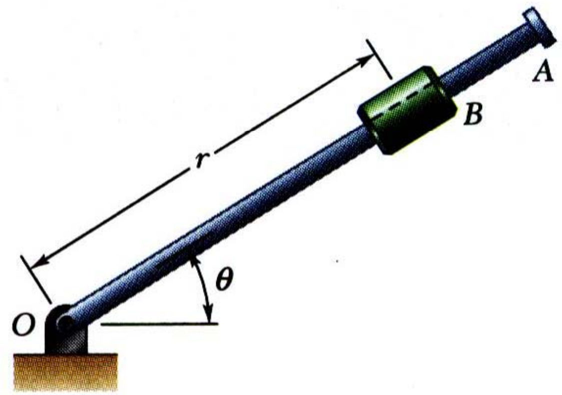
A motorist is traveling on curved section of highway at 60 km/h. The motorist applies brakes causing a constant deceleration rate.

Knowing that after 8 s the speed has been reduced to 66 m/s, determine the acceleration of the automobile immediately after the brakes are applied.

- Calculate tangential and normal components of acceleration.

- Determine acceleration magnitude and direction with respect to tangent to curve.

Sample Problem 11.12



Rotation of the arm about O is defined by $\theta = 0.15t^2$ where θ is in radians and t in seconds. Collar B slides along the arm such that $r = 0.9 - 0.12t^2$ where r is in meters.

After the arm has rotated through 30°, determine *(a)* the total velocity of the collar, *(b)* the total acceleration of the collar, and *(c)* the relative acceleration of the collar with respect to the arm.

Approach:

What kind of motion is this?

What are the knowns?

What do we want to know?