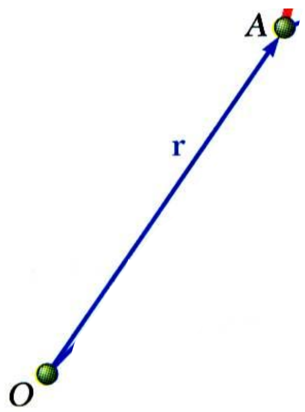
13.1 Introduction

- Previously, problems dealing with the motion of particles were solved through the fundamental equation of motion, $\vec{F} = m\vec{a}.$ Current chapter introduces two additional methods of analysis.

- *Method of work and energy*: directly relates force, mass, velocity and displacement.

- *Method of impulse and momentum*: directly relates force, mass, velocity, and time.

13.2 Work of a Force



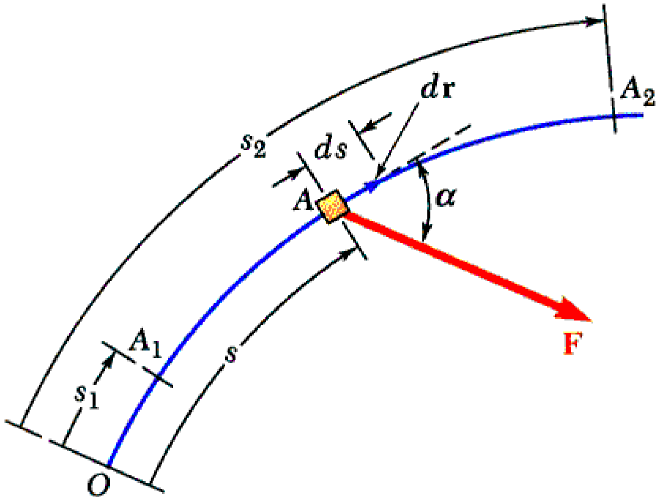
- Differential vector $d\vec{r}$ is the *particle displacement*.
- *Work of the force* is

$$dU =$$
$$=$$
$$=$$

- Work is a scalar quantity, i.e., it has magnitude and sign but not direction.
- Dimensions of work are length $\times$ force. Units are

$$1\,\mathrm{J}\,(joule) = (1\,\mathrm{N})(1\,\mathrm{m})$$

13.2 Work of a Force

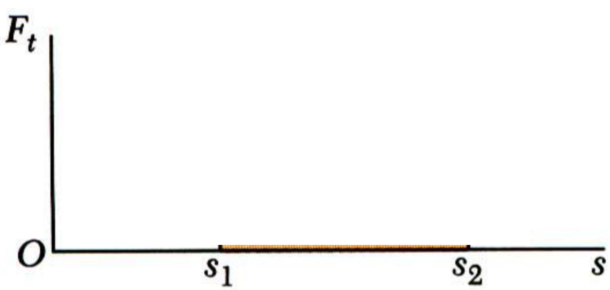


- Work of a force during a finite displacement,

$$U_{1\to 2} =$$

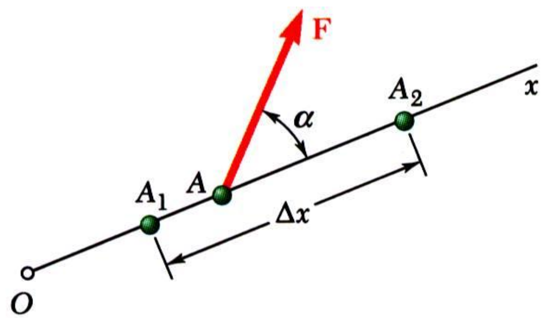
$$=$$ $$=$$

$$=$$



- Work is represented by curve of F_t plotted against s.

13.2 Work of a Force



- **Work of a constant force in rectilinear motion,**

$$U_{1\to2} =$$

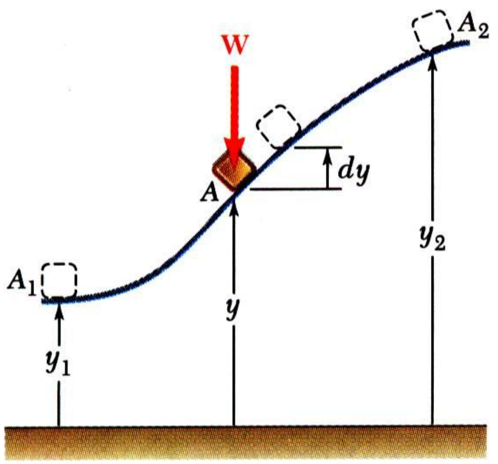
- **Work of the force of gravity,**

$$dU =$$

$$=$$

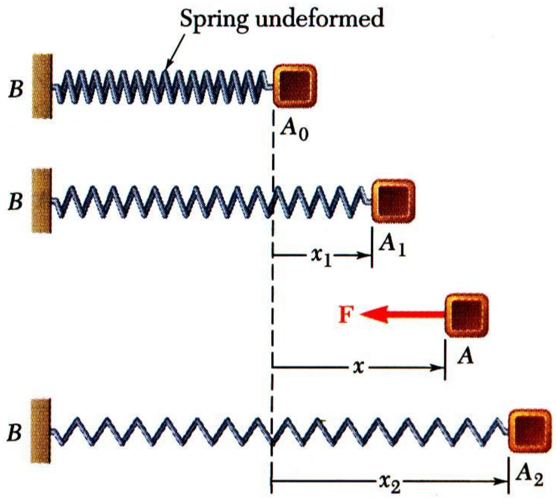
$$U_{1\to2} =$$

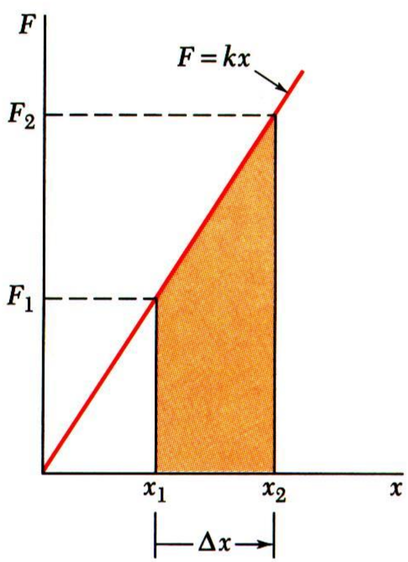
$$=$$



- Work *of the weight* is equal to product of weight W and vertical displacement Δy.
- Work *of the weight* is positive when i.e., when the weight moves down.

13.2 Work of a Force





- Magnitude of the force exerted by a spring is proportional to deflection,

  $F =$

  $k =$ spring constant (N/m or lb/in.)

- **Work *of the force exerted by spring,***

  $dU =$

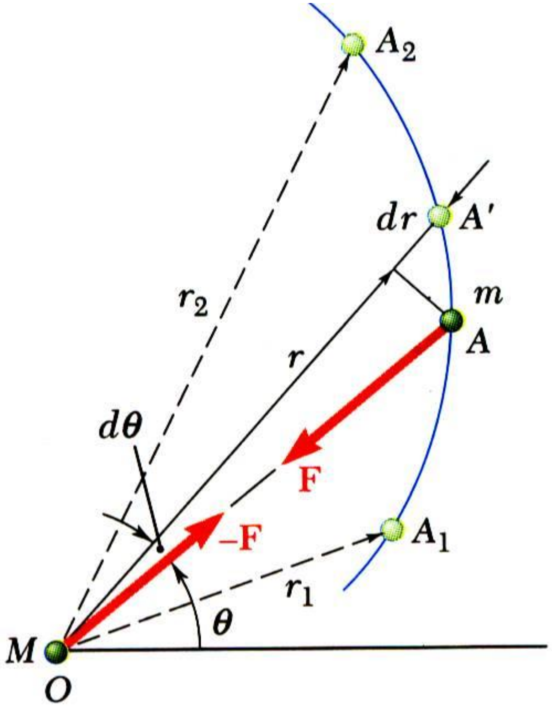
  $U_{1\to2} =$

- Work *of the force exerted by spring* is positive when i.e., when the spring is returning to its undeformed position.

- Work of the force exerted by the spring is equal to negative of area under curve of F plotted against x,

  $U_{1\to2} =$

13.2 Work of a Force



Work of a gravitational force (assume particle M occupies fixed position O while particle m follows path shown),

$$dU = -F\,dr = -G\frac{Mm}{r^2}dr$$

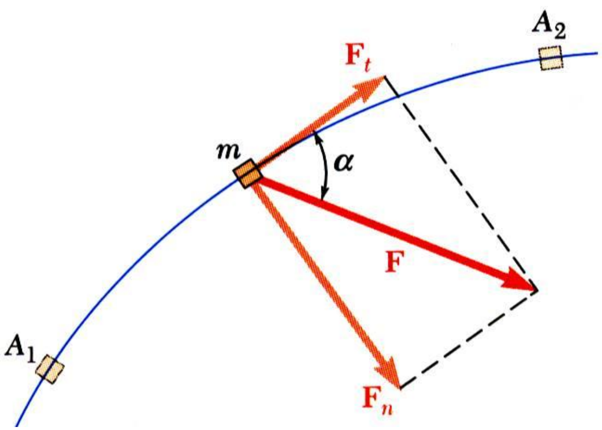
$$U_{1\to 2} = -\int_{r_1}^{r_2} G\frac{Mm}{r^2}dr = G\frac{Mm}{r_2} - G\frac{Mm}{r_1}$$

13.2 Work of a Force

Forces which *do not* do work ($ds = 0$ or $\cos \alpha = 0$):

- reaction at frictionless pin supporting rotating body,
- reaction at frictionless surface when body in contact moves along surface,
- reaction at a roller moving along its track, and
- weight of a body when its center of gravity moves horizontally.

13.3 Particle Kinetic Energy: Principle of Work & Energy



- Consider a particle of mass *m* acted upon by force $\vec{F}$

$$F_t =$$

$$=$$

$$F_t \, ds =$$

- Integrating from A_1 to A_2 ,

$$\int_{s_1}^{s_2} F_t ds =$$

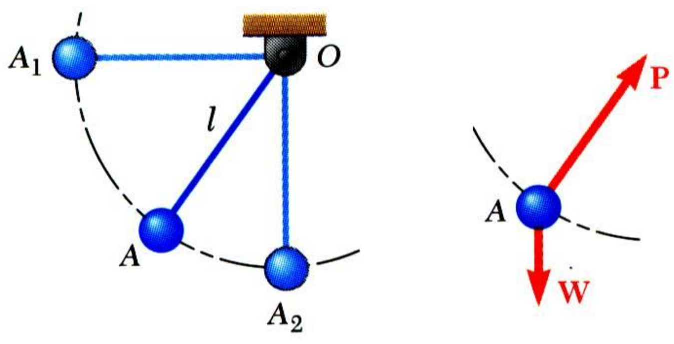
$$U_{1\to 2} = \qquad T = \tfrac{1}{2} m v^2 = kinetic\ energy$$

- *The work of the force* $\vec{F}$ *is equal to the change in kinetic energy of the particle.*

- Units of work and kinetic energy are the same:

$$T = \tfrac{1}{2} m v^2 = \mathrm{kg}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = \left(\mathrm{kg}\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)\mathrm{m} = \mathrm{N}\cdot\mathrm{m} = \mathrm{J}$$

13.4 Applications of the Principle of Work and Energy

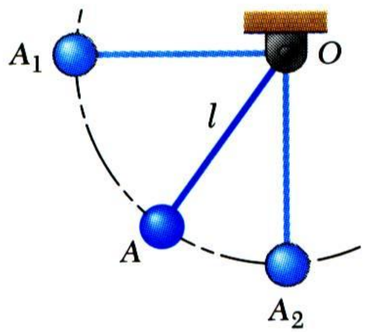


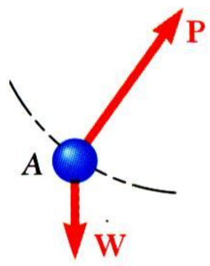
- Wish to determine velocity of pendulum bob at A_2. Consider work & kinetic energy.
- Force $\vec{P}$ acts normal to path and does no work.

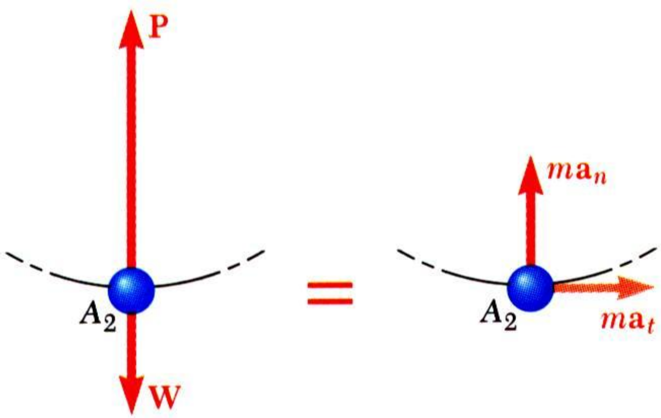
$$T_1 + U_{1\to 2} = T_2$$

- Velocity found without determining expression for acceleration and integrating.
- All quantities are scalars and can be added directly.
- Forces which do no work are eliminated from the problem.

13.4 Applications of the Principle of Work and Energy







$$v_2 = \sqrt{2gl}$$

- Principle of work and energy cannot be applied to directly determine the acceleration of the pendulum bob.
- Calculating the tension in the cord requires supplementing the method of work and energy with an application of Newton's second law.
- As the bob passes through A_2 ,

$$\sum F_n = m\,a_n$$

$$P - W =$$

$$P = W +$$

13.5 Power and Efficiency

- $Power =$ rate at which work is done.

  $=$

  $=$

- Dimensions of power are work/time or force*velocity. Units for power are

$$1\,\mathrm{W}\ (\mathrm{watt}) = 1\,\frac{\mathrm{J}}{\mathrm{s}} = 1\,\mathrm{N}\cdot\frac{\mathrm{m}}{\mathrm{s}} \quad \text{or} \quad 1\,\mathrm{hp} = 550\,\frac{\mathrm{ft}\cdot\mathrm{lb}}{\mathrm{s}} = 746\,\mathrm{W}$$

- $\eta =$ efficiency

  $=$

  $=$

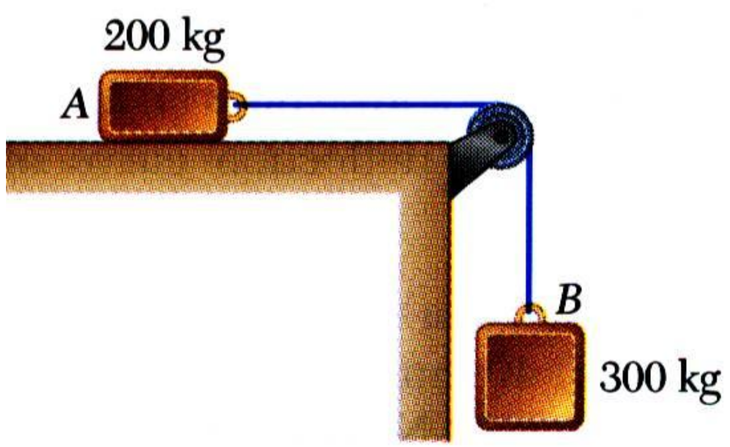
Sample Problem 13.1

An automobile weighing 4000 N is driven down a 5° incline at a speed of 88 m /s when the brakes are applied causing a constant total breaking force of 1500 N.

Determine the distance traveled by the automobile as it comes to a stop.

Sample Problem 13.1

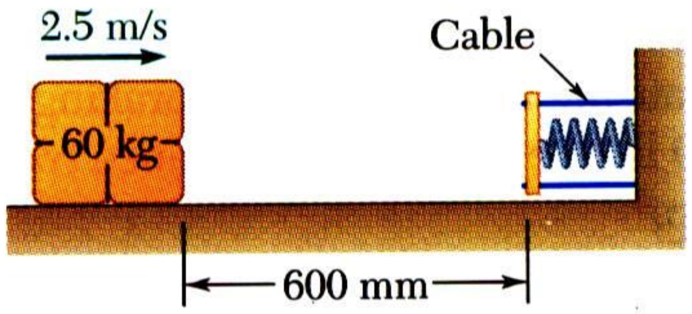
Sample Problem 13.2



Two blocks are joined by an inextensible cable as shown. If the system is released from rest, determine the velocity of block *A* after it has moved 2 m. Assume that the coefficient of friction between block *A* and the plane is $\mu_k = 0.25$ and that the pulley is weightless and frictionless.

Sample Problem 13.2

Sample Problem 13.3

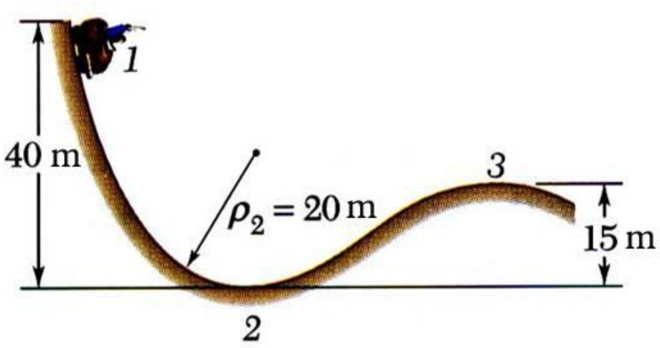


A spring is used to stop a 60 kg package which is sliding on a horizontal surface. The spring has a constant k = 20 kN/m and is held by cables so that it is initially compressed 120 mm. The package has a velocity of 2.5 m/s in the position shown and the maximum deflection of the spring is 40 mm.

Determine *(a)* the coefficient of kinetic friction between the package and surface and *(b)* the velocity of the package as it passes again through the position shown.

Sample Problem 13.3

Sample Problem 13.4



A 2000 N car starts from rest at point 1 and moves without friction down the track shown.

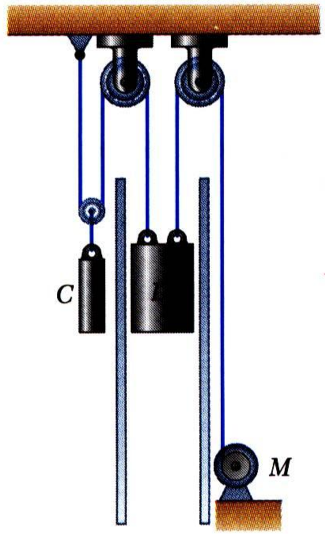
Determine:

a) the force exerted by the track on the car at point 2, and

b) the minimum safe value of the radius of curvature at point 3.

Sample Problem 13.4

Sample Problem 13.5



The dumbwaiter *D* and its load have a combined weight of 600 N, while the counterweight *C* weighs 800 N.

Determine the power delivered by the electric motor *M* when the dumbwaiter *(a)* is moving up at a constant speed of 8 m/s and *(b)* has an instantaneous velocity of 8 m/s and an acceleration of 2.5 m/s^2, both directed upwards.

Sample Problem 13.5