Preview of 16.1- 16.6

16.1 Introduction

- the *Kinetics* of **Rigid Bodies**

  Relations between the forces acting on a rigid body, the shape and mass of the body, and the motion produced.

  Motion of the body as a whole + Motion about its mass center

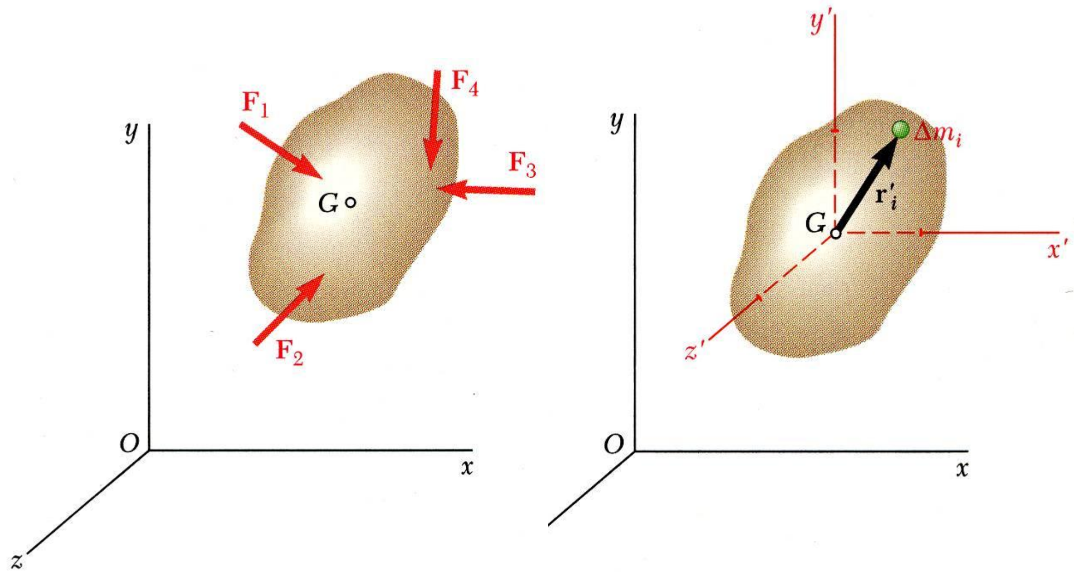
- Results of this chapter will be restricted to:
  - plane motion of rigid bodies
  - rigid bodies consisting of plane slabs or bodies which are symmetrical with respect to the reference plane.

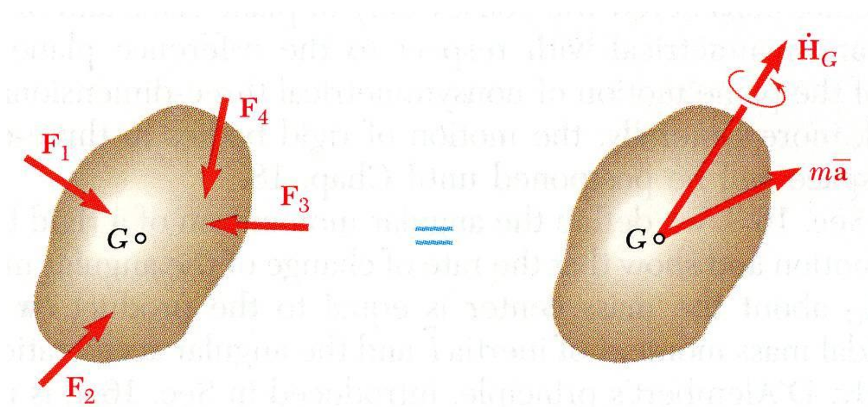
- Our approach will be to consider rigid bodies as made of large numbers of particles and to use the results of Chapter 14 for the motion of systems of particles. Specifically,

$$\sum \vec{F} = m\vec{\bar{a}} \qquad \text{and} \qquad \sum \vec{M}_G = \dot{\vec{H}}_G$$

- D'Alembert's principle is applied to prove that the external forces acting on a rigid body are equivalent a vector $m\vec{\bar{a}}$ attached to the mass center and a couple of moment $\bar{I}\alpha$.

16.2 Equations of Motion for a Rigid Body





- Consider a rigid body acted upon by several external forces. Why?
  →
- Assume that the body is made of a large number of particles.
- For the motion of the G of the body with respect to the Newtonian frame *Oxyz*,

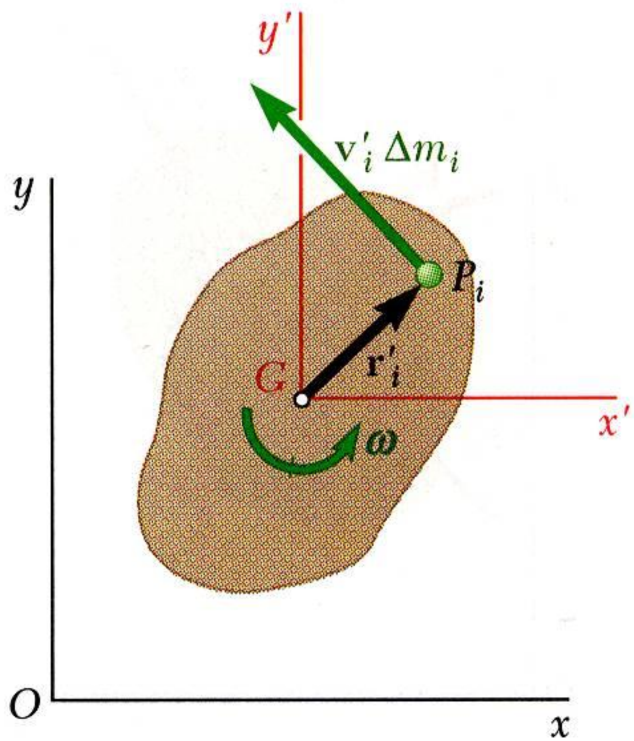
$$\sum \vec{F} = m\vec{\bar{a}}$$

- For the motion of the body with respect to the *Gx'y'z'*,

$$\sum \vec{M}_G = \dot{\vec{H}}_G$$

- System of external forces is equipollent to the system consisting of $m\vec{\bar{a}}$ and $\dot{\vec{H}}_G$.

16.3 Angular Momentum of a Rigid Body in Plane Motion



- Consider a rigid slab in plane motion.

- Angular momentum of a rotating particle w.r.t O:

- Angular momentum of the slab w.r.t G :

$$\begin{aligned}\vec{H}_G &= \sum_{i=1}^{n}\left(\vec{r}_i' \times \vec{v}_i' \Delta m_i\right) \\ &= \sum_{i=1}^{n}\left[\vec{r}_i' \times (\vec{\omega} \times \vec{r}_i') \Delta m_i\right] \\ &= \vec{\omega}\sum\left(r_i'^2 \Delta m_i\right) \\ &= \bar{I}\vec{\omega}\end{aligned}$$

- After differentiation,

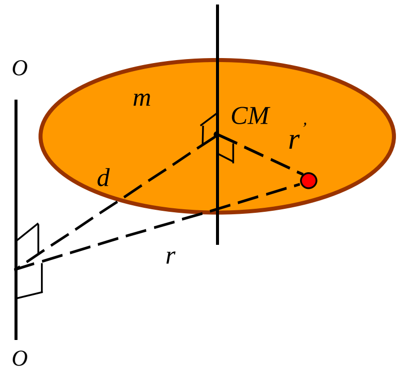
$$\dot{\vec{H}}_G = \bar{I}\dot{\vec{\omega}} = \bar{I}\vec{\alpha}$$

Mass Moments of Inertia

Mass Moment of Inertia:

$$I = \Sigma m_i r_i^2 = \int r^2 dm$$

Parallel-axis theorem: The moment of inertia around any axis can be calculated from the moment of inertia around parallel axis which passes through the center of mass. The equation to calculate this is called the parallel axis theorem and is given as



$$I = \bar{I} + md^2$$

where d is the distance between the original axis and the axis passing through the center of mass, m is the total mass of the body, and $\bar{I}$ is the moment of inertia around the axis passing through the center of mass.

Comments on Moment of Inertia

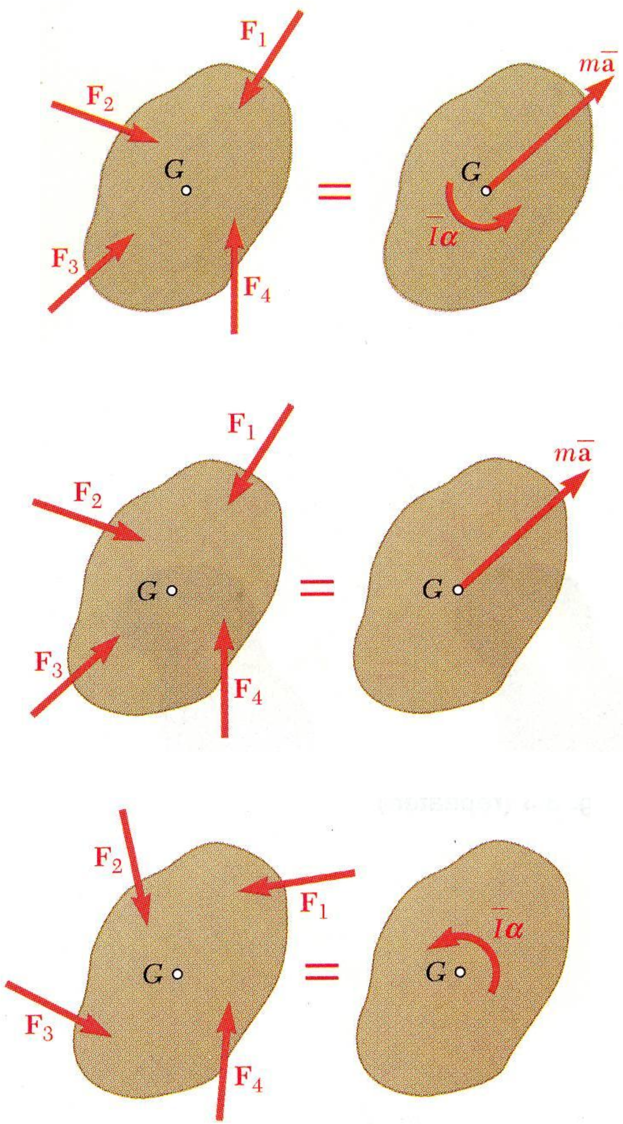
- Serves the same role for rotational motion as mass does for linear motion
- But since $I=\Sigma m_i r_i^2$ is a sum over many objects it depends on mass distribution
  - If of equal mass, a larger cylinder will have a greater moment of inertia than a smaller one. Something intuitively true.
  - When mass is far from the axis, it also hard to rotate something, again something familiar.
  - For rotational motion, the mass of a body cannot be considered as concentrated at the center of mass.
- Still it can be extended to the center of mass $(\Sigma M)_G = I_G \alpha_G$
- Experimentally done by measuring α for a known τ.

Examples of Mass moment of Inertia

Slender Rod	$I = \frac{1}{12} \cdot M \cdot L^2$
Rectangular plane	$I = \frac{1}{12} \cdot M \cdot \left(a^2 + b^2\right)$
Solid Sphere	$I = \frac{2}{5} \cdot M \cdot R^2$
Disk	$I = \frac{1}{2} \cdot M \cdot R^2$
Thin walled hollow disk	$I = M \cdot R^2$
Hollow disk	$I = \frac{1}{2} \cdot M \cdot \left(R_i^2 + R_o^2\right)$

Radius of Gyration: $I = mk^2$

16.4 Plane Motion of a Rigid Body: D'Alembert's Principle



- Motion of a rigid body in plane motion is completely defined by the resultant and moment resultant about G of the external forces.

$$\sum F_x = m\bar{a}_x \quad \sum F_y = m\bar{a}_y \quad \sum M_G = \bar{I}\alpha$$

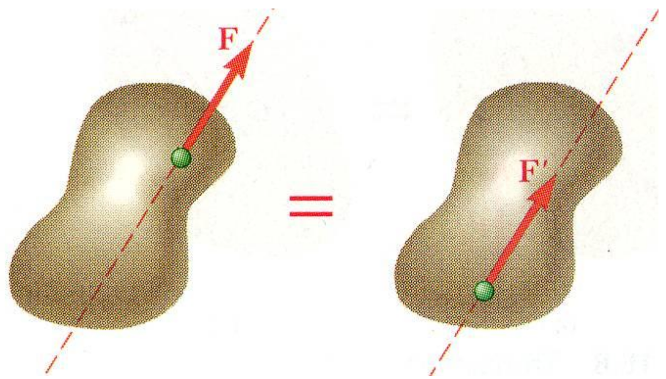
- The external forces and the collective effective forces of the slab particles are *equipollent* (reduce to the same resultant and moment resultant) and *equivalent* (have the same effect on the body).

- *d'Alembert's Principle*: The external forces acting on a rigid body are equivalent to the effective forces of the various particles forming the body.

- One can transform an accelerating rigid body into an equivalent **static system** subjected to "effective force" and "effective moment".

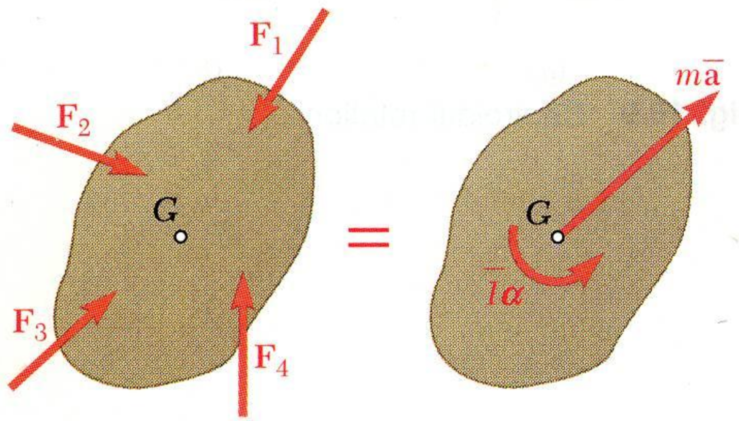
- One can apply the moment equation w.r.t any appropriate **point**!

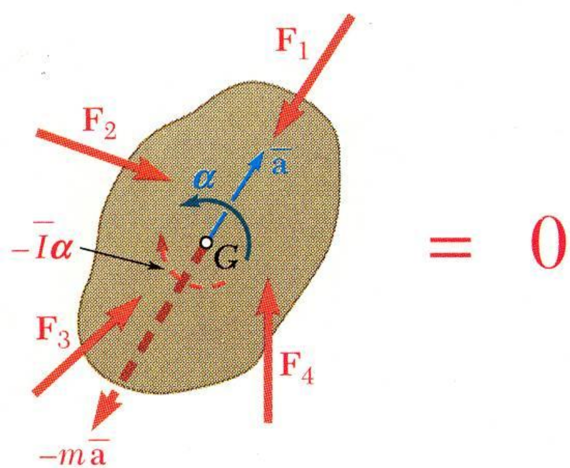
16.5 Axioms of the Mechanics of Rigid Bodies



- The forces $\vec{F}$ and $\vec{F}'$ act at different points on a rigid body but but have the same magnitude, direction, and line of action.
- The forces produce the same moment about any point and are therefore, equipollent external forces.
- This proves the principle of transmissibility whereas it was previously stated as an axiom.

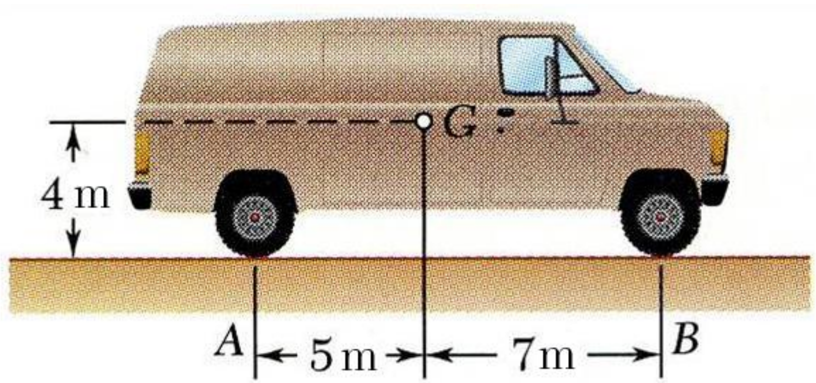
16.6 Problems Involving the Motion of a Rigid Body





- The fundamental relation between the forces acting on a rigid body in plane motion and the acceleration of its mass center and the angular acceleration of the body is illustrated in a free-body-diagram equation.
- The techniques for solving problems of static equilibrium may be applied to solve problems of plane motion by utilizing
  - d'Alembert's principle, or
  - principle of dynamic equilibrium
- These techniques may also be applied to problems involving plane motion of connected rigid bodies by drawing a free-body-diagram equation for each body and solving the corresponding equations of motion simultaneously.

Sample Problem 16.1

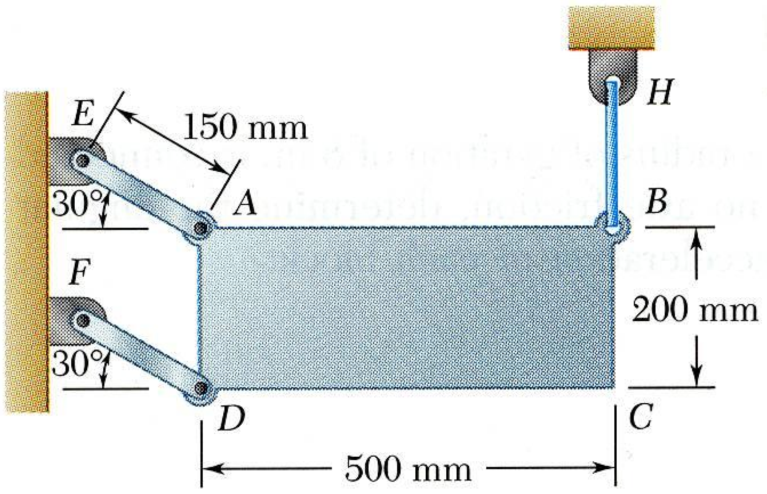


At a forward speed of 30 m/s, the truck brakes were applied, causing the wheels to stop rotating. It was observed that the truck to skidded to a stop in 200 m.

Determine the magnitude of the normal reaction and the friction force at each wheel as the truck skidded to a stop.

Sample Problem 16.1

Sample Problem 16.2

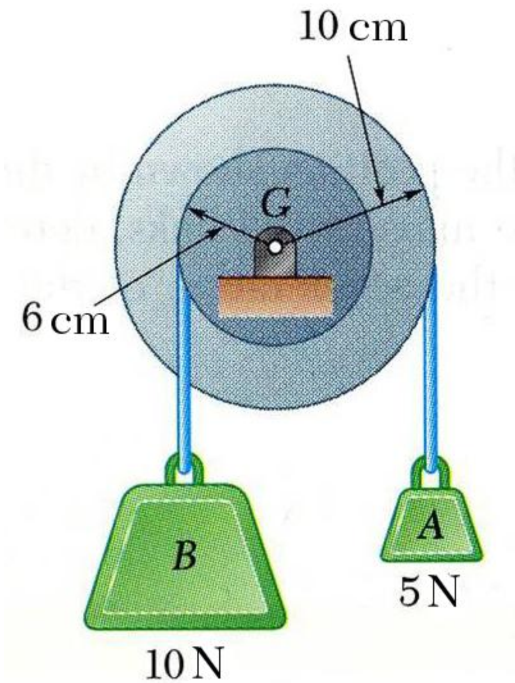


The thin plate of mass 8 kg is held in place as shown.

Neglecting the mass of the links, determine immediately after the wire has been cut (*a*) the acceleration of the plate, and (*b*) the force in each link.

Sample Problem 16.2

Sample Problem 16.3

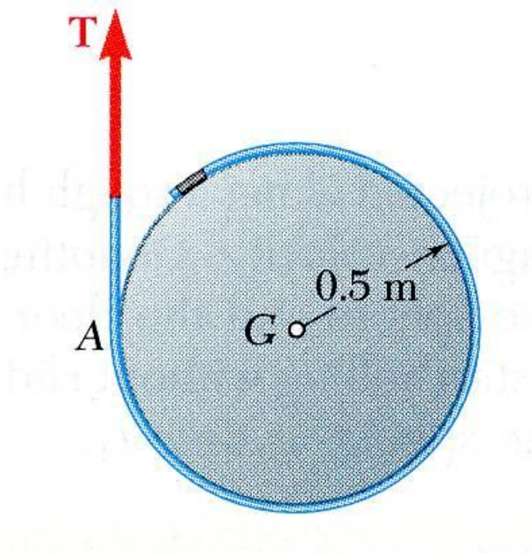


A pulley weighing 12 N and having a radius of gyration of 8 cm is connected to two blocks as shown.

Assuming no axle friction, determine the angular acceleration of the pulley and the acceleration of each block.

Sample Problem 16.3

Sample Problem 16.4

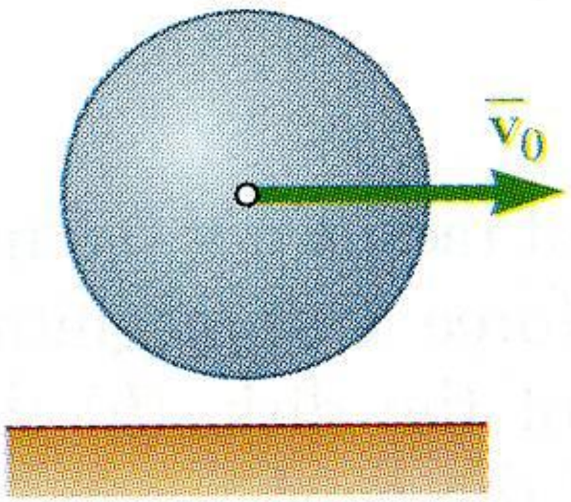


A cord is wrapped around a homogeneous disk of mass 15 kg. The cord is pulled upwards with a force $T = 180$ N.

Determine: (*a*) the acceleration of the center of the disk, (*b*) the angular acceleration of the disk, and (*c*) the acceleration of the cord.

Sample Problem 16.4

Sample Problem 16.5



A uniform sphere of mass *m* and radius *r* is projected along a rough horizontal surface with a linear velocity v_0. The coefficient of kinetic friction between the sphere and the surface is μ_k.

Determine: (*a*) the time t_1 at which the sphere will start rolling without sliding, and (*b*) the linear and angular velocities of the sphere at time t_1.

Sample Problem 16.5