Preview of 16.7-16.9

16.7 Constrained Plane Motion



- Most engineering applications involve rigid bodies which are moving under given constraints, e.g., cranks, connecting rods, and non-slipping wheels.
- *Constrained plane motion*: motions with definite relations between the components of acceleration of the mass center and the angular acceleration of the body.
- Solution of a problem involving constrained plane motion begins with a kinematic analysis.
- e.g., given θ , ω , and α , find P, N_A , and N_B .
 - kinematic analysis yields \overline{a}_x and \overline{a}_y .
 - application of d'Alembert's principle yields P, N_A , and N_B .

16.8 Constrained Motion: Noncentroidal Rotation





- *Noncentroidal rotation*: motion of a body is constrained to rotate about a fixed axis that does not pass through its mass center.
- Kinematic relation between the motion of the mass center *G* and the motion of the body about *G*,

$$\overline{a}_t = \overline{r}\alpha \qquad \overline{a}_n = \overline{r}\omega^2$$

- The kinematic relations are used to eliminate *ā_t* and *ā_n* from equations derived from d'Alembert's principle or from the method of dynamic equilibrium.
- Equate the moment about O:
- Motion with constant angular speed?

16.8 Constrained Plane Motion: Rolling Motion





- For a balanced disk constrained to roll without sliding, $\bar{x} = r\theta \rightarrow \bar{a} = r\alpha$
- Rolling, no sliding: $F \le \mu_s N$ $\overline{a} = r\alpha$ Rolling, sliding impending: $F = \mu_s N$ $\overline{a} = r\alpha$ Rotating and sliding: $F = \mu_k N$ $\overline{a}, r\alpha$ independent
- For the geometric center of an unbalanced disk,

 $a_0 = r\alpha$

The acceleration of the mass center,

$$\vec{\bar{a}}_G = \vec{a}_O +$$
$$= \vec{a}_O +$$

Sample Problem 16.6



The portion *AOB* of the mechanism is actuated by gear *D* and at the instant shown has a clockwise angular velocity of 8 rad/s and a counterclockwise angular acceleration of 40 rad/s².

Determine: a) tangential force exerted by gear *D*, and b) components of the reaction at shaft *O*.

Sample Problem 16.8



A sphere of weight *W* is released with no initial velocity and rolls without slipping on the incline.

Determine: *a*) the minimum value of the coefficient of friction, *b*) the velocity of *G* after the sphere has rolled 10 m and *c*) the velocity of *G* if the sphere were to move 10 m down a frictionless incline.

Sample Problem 16.9



A cord is wrapped around the inner hub of a wheel and pulled horizontally with a force of 200 N. The wheel has a mass of 50 kg and a radius of gyration of 70 mm. Knowing $\mu_s = 0.20$ and $\mu_k = 0.15$, determine the acceleration of *G* and the angular acceleration of the wheel.

Sample Problem 16.10



The extremities of a 4-m rod weighing 50 N can move freely and with no friction along two straight tracks. The rod is released with no velocity from the position shown.

Determine: *a*) the angular acceleration of the rod, and *b*) the reactions at *A* and *B*.