Energy and Momentum Methods for Plane Motion of Rigid Bodies

Preview of 17.1- 17.7

Review of Ch.13

- *Newton Method*: Uses fundamental equation of motion directly relates force, mass, velocity and displacement.

- *Method of work and energy*: directly relates force, mass, velocity and displacement.

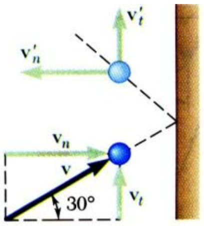
- *Method of impulse and momentum*: directly relates force, mass, velocity, and time.

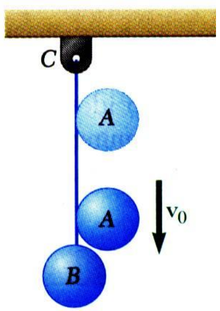
Review of Ch.13

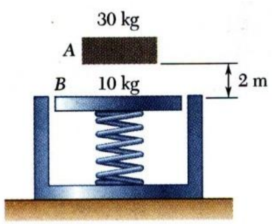
- Topics in Work and Energy Principle
  - Work of Gravity
  - Work of forces exerted by spring
  - Work of gravitational force
  - Conservative forces → Potential Energy Change
  - Work of forces equals change of kinetic energy

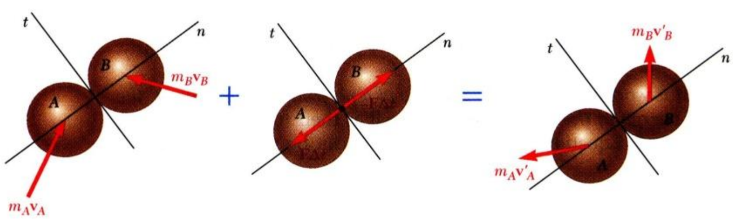
Review of Ch.13

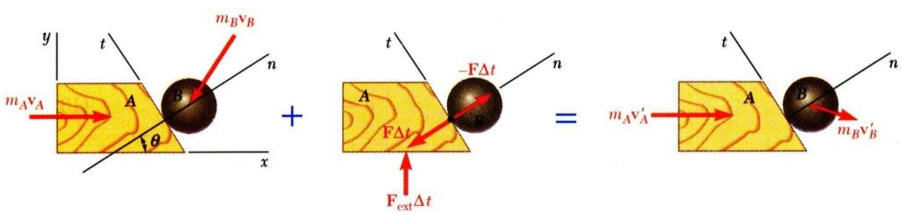
- Topics in Impulse and Momentum
  - Definition of Impulse
  - Change of Momentum
  - Line of Impact
  - Coefficient of restitution
  - Oblique Central Impact











17.1 Introduction

- Method of work and energy and the method of impulse and momentum for Rigid Bodies: What's the difference with the particles?

- Principle of work and energy is well suited to the solution of problems involving displacements and velocities.

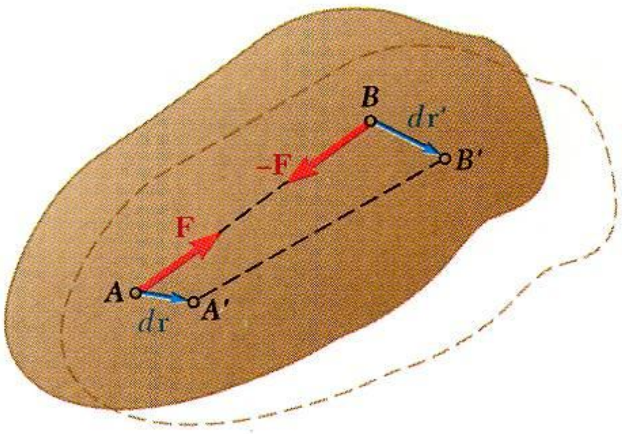
$$T_1 + U_{1\to 2} = T_2$$

- Principle of impulse and momentum is appropriate for problems involving velocities and time.

$$\vec{L}_1 + \sum \int_{t_1}^{t_2} \vec{F} dt = \vec{L}_2 \qquad \left(\vec{H}_O\right)_1 + \sum \int_{t_1}^{t_2} \vec{M}_O dt = \left(\vec{H}_O\right)_2$$

- Problems involving eccentric impact are solved by supplementing the principle of impulse and momentum with the application of the coefficient of restitution.

17.2 Principle of Work and Energy for a Rigid Body



- Method of work and energy is well adapted to problems involving velocities and displacements. Main advantage is that the work and kinetic energy are scalar quantities.
- Assume that the rigid body is made of a large number of particles.

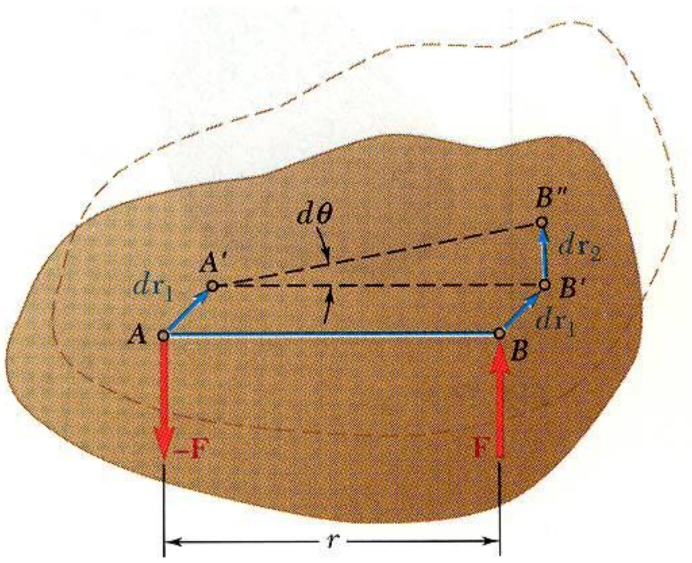
$$T_1 + U_{1\to 2} = T_2$$

$T_1, T_2 =$ initial and final total kinetic energy of particles forming body

$U_{1\to 2} =$ total work of internal and external forces acting on particles of body.

- Internal forces between particles *A* and *B* are equal and opposite.
- In general, small displacements of the particles *A* and *B* are not equal but the components of the displacements along *AB* are equal.
- Therefore, *the net work of internal forces is zero.*

17.3 Work of Forces Acting on a Rigid Body (Rotation)



- Work of a force during a displacement of its point of application,

$$U_{1\to 2} = \int_{A_1}^{A_2} \vec{F} \cdot d\vec{r} = \int_{s_1}^{s_2} (F\cos\alpha)ds$$

- Consider the net work of two forces $\vec{F}$ and $-\vec{F}$ forming a couple of moment $\vec{M}$ during a displacement of their points of application.

$$\begin{aligned} dU &= \vec{F} \cdot d\vec{r}_1 - \vec{F} \cdot d\vec{r}_1 + \vec{F} \cdot d\vec{r}_2 \\ &= F\, ds_2 = Fr\, d\theta \\ &= M\, d\theta \end{aligned}$$

- Work of a Moment M

$$U_{1\to 2} = \boxed{}$$

$$= \boxed{}$$ if M is constant.

17.3 Work of Forces Acting on a Rigid Body

Forces acting on rigid bodies which do no work:

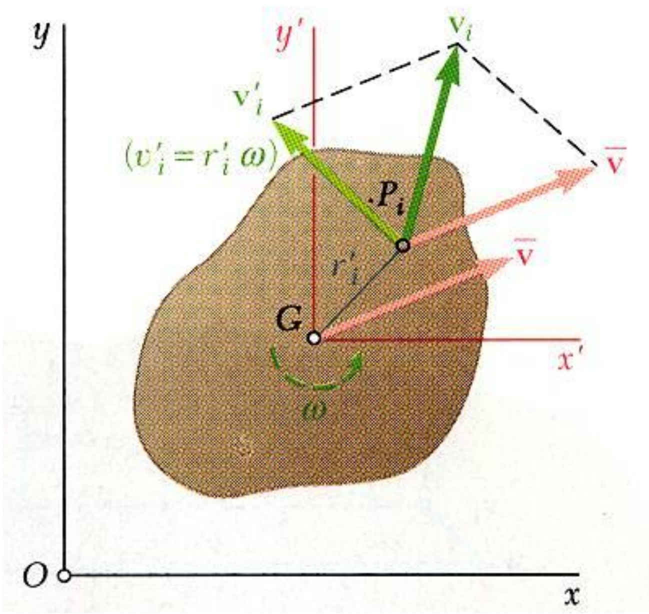
- Forces applied to fixed points:
  - reactions at a frictionless pin when the supported body rotates about the pin.

- Forces acting in a direction perpendicular to the displacement of their point of application:
  - reaction at a frictionless surface to a body moving along the surface
  - weight of a body when its center of gravity moves horizontally

- Friction force at the point of contact of a body rolling without sliding on a fixed surface.

$$dU = F\,ds_C = F\left(v_c dt\right) = 0$$

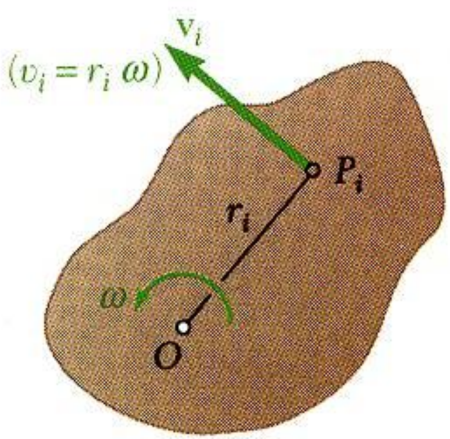
17.4 Kinetic Energy of a Rigid Body in Plane Motion



- Consider a rigid body of mass m in plane motion.

$$T = \tfrac{1}{2} m\bar{v}^2 + \tfrac{1}{2}\sum \Delta m_i v_i'^2$$
$$= \tfrac{1}{2} m\bar{v}^2 + \tfrac{1}{2}\left(\sum r_i'^2 \Delta m_i\right)\omega^2$$
$$= \tfrac{1}{2} m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2$$

- Kinetic energy of a rigid body can be separated into:
  - the kinetic energy associated with the motion of the mass center G and
  - the kinetic energy associated with the rotation of the body about G.



- Consider a rigid body rotating about a fixed axis through O.

$$T = \tfrac{1}{2}\sum \Delta m_i v_i^2 + \tfrac{1}{2}\sum \Delta m_i (r_i\omega)^2 + \tfrac{1}{2}\left(\sum r_i^2 \Delta m_i\right)\omega^2$$
$$= \boxed{}$$

17.5 Systems of Rigid Bodies

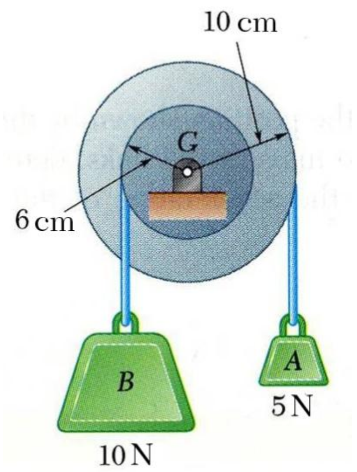
- For problems involving systems consisting of several rigid bodies, the principle of work and energy can be applied to **each body**.

- We may also apply the principle of work and energy **to the entire system**,

$$T_1 + U_{1\to2} = T_2$$

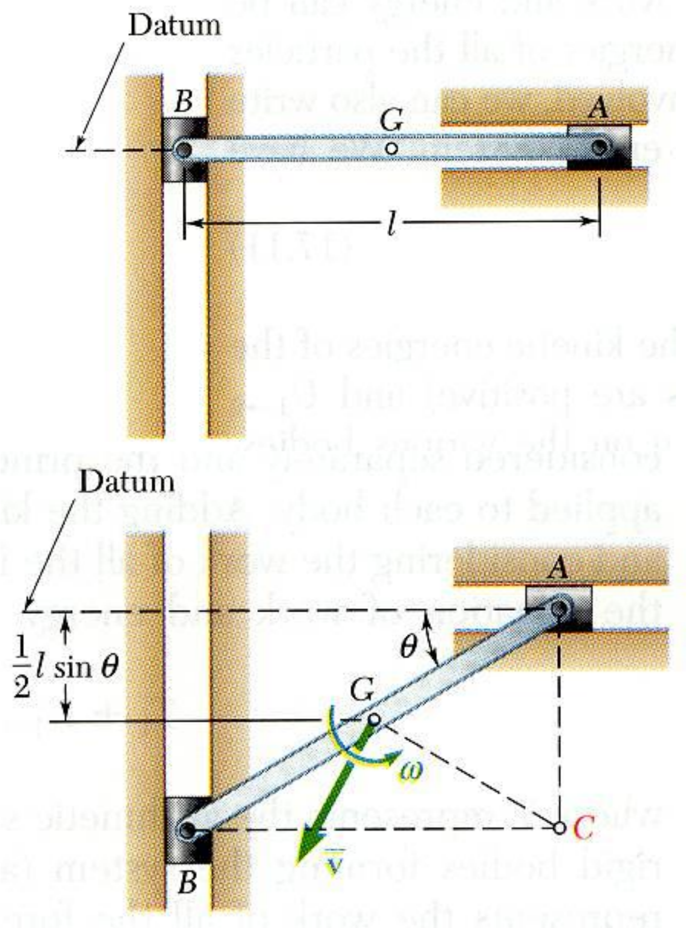
T_1, T_2 = arithmetic sum of the kinetic energies of all bodies forming the system

$U_{1\to2}$ = work of all forces acting on the various bodies, whether these forces are internal or external to the system as a whole.



- For problems involving pin connected members, blocks and pulleys connected by inextensible cords, and meshed gears,
  - internal forces occur in pairs of equal and opposite forces
  - points of application of each pair move through equal distances
  - net work of the internal forces is zero
  - work on the system reduces to the work of the external forces

17.6 Conservation of Energy



- mass m
- released with zero velocity
- determine ω at θ

- Expressing the work of conservative forces as a change in potential energy, the principle of work and energy becomes

$$T_1 + V_1 = T_2 + V_2$$

- Consider the slender rod of mass m.

$$T_1 = 0, \quad V_1 = 0$$

$$T_2 = \boxed{}$$

$$= \boxed{} = \frac{1}{2}\frac{ml^2}{3}\omega^2$$

$$V_2 = \boxed{} = \boxed{}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 = \frac{1}{2}\frac{ml^2}{3}\omega^2 - \frac{1}{2} mgl\sin\theta$$

$$\omega = \left(\frac{3g}{l}\sin\theta\right)$$

17.7 Power

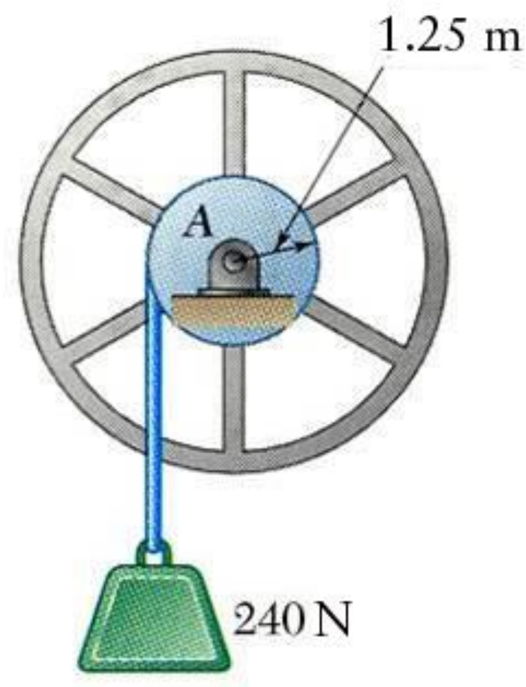
- Power = rate at which work is done
- For a body acted upon by force $\vec{F}$ and moving with velocity $\vec{v}$,

$$\text{Power} = \frac{dU}{dt} = \boxed{}$$

- For a rigid body rotating with an angular velocity $\vec{\omega}$ and acted upon by a couple of moment $\vec{M}$ parallel to the axis of rotation,

$$\text{Power} = \frac{dU}{dt} = \frac{M\,d\theta}{dt} = \boxed{}$$

Sample Problem 17.1

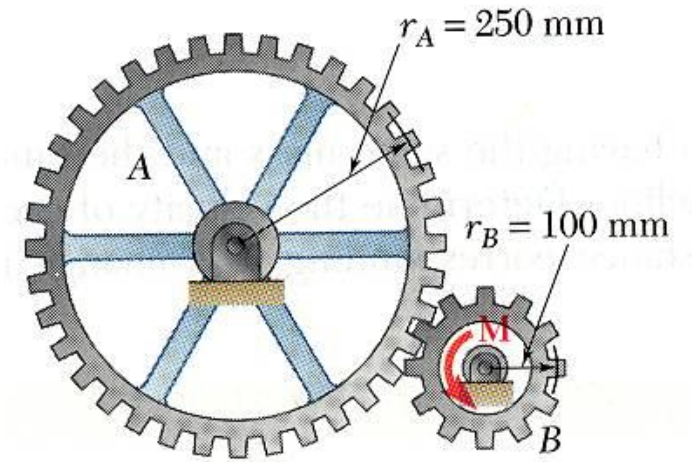


For the drum and flywheel, $\bar{I} = 10.5\,\text{kgm}^2$
The bearing friction is equivalent to a couple of $60\,\text{N}\cdot\text{m}$ At the instant shown, the block is moving downward at 6 m /s

Determine the velocity of the block after it has moved 4 m downward.

Sample Problem 17.1

Sample Problem 17.2



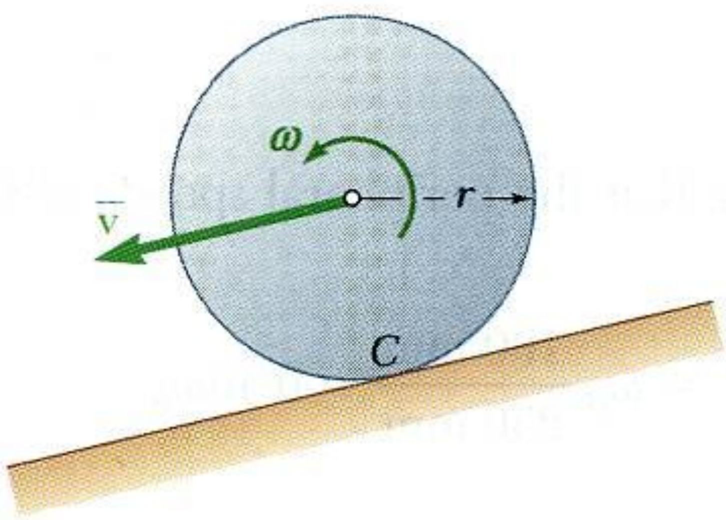
$$m_A = 10\,\text{kg} \qquad \bar{k}_A = 200\,\text{mm}$$
$$m_B = 3\,\text{kg} \qquad \bar{k}_B = 80\,\text{mm}$$

The system is at rest when a moment of $M = 6\,\text{N}\cdot\text{m}$ is applied to gear *B*.

Neglecting friction, *a*) determine the number of revolutions of gear *B* before its angular velocity reaches 600 rpm, and *b*) tangential force exerted by gear *B* on gear *A*.

Sample Problem 17.2

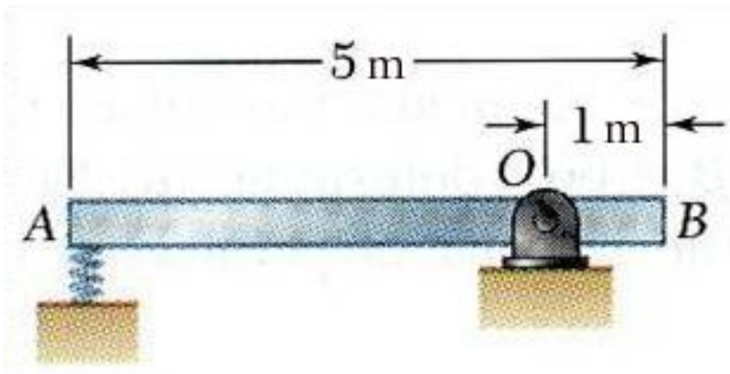
Sample Problem 17.3



A sphere, cylinder, and hoop, each having the same mass and radius, are released from rest on an incline. Determine the velocity of each body after it has rolled through a distance corresponding to a change of elevation *h*.

Sample Problem 17.3

Sample Problem 17.4

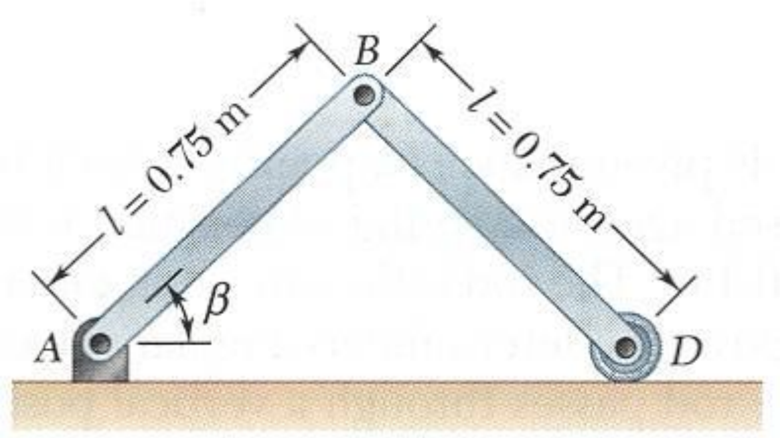


A 30-N slender rod pivots about the point *O*. The other end is pressed against a spring (k = 1800 N/m) until the spring is compressed 30 cm and the rod is in a horizontal position.

If the rod is released from this position, determine its angular velocity and the reaction at the pivot as the rod passes through a vertical position.

Sample Problem 17.4

Sample Problem 17.5



Each of the two slender rods has a mass of 6 kg. The system is released from rest with $\beta = 60^o$.

Determine *a*) the angular velocity of rod *AB* when $\beta = 20^o$, and *b*) the velocity of the point *D* at the same instant.

Sample Problem 17.5