Theoretical Aspects for PMD Understanding

Byoungho Lee

School of Electrical Engineering

Seoul National University, Seoul 151-744, Korea

byoungho@plaza.snu.ac.kr

Outline

- Introduction
- Propagation of polarized light & SOP
- PMD vector

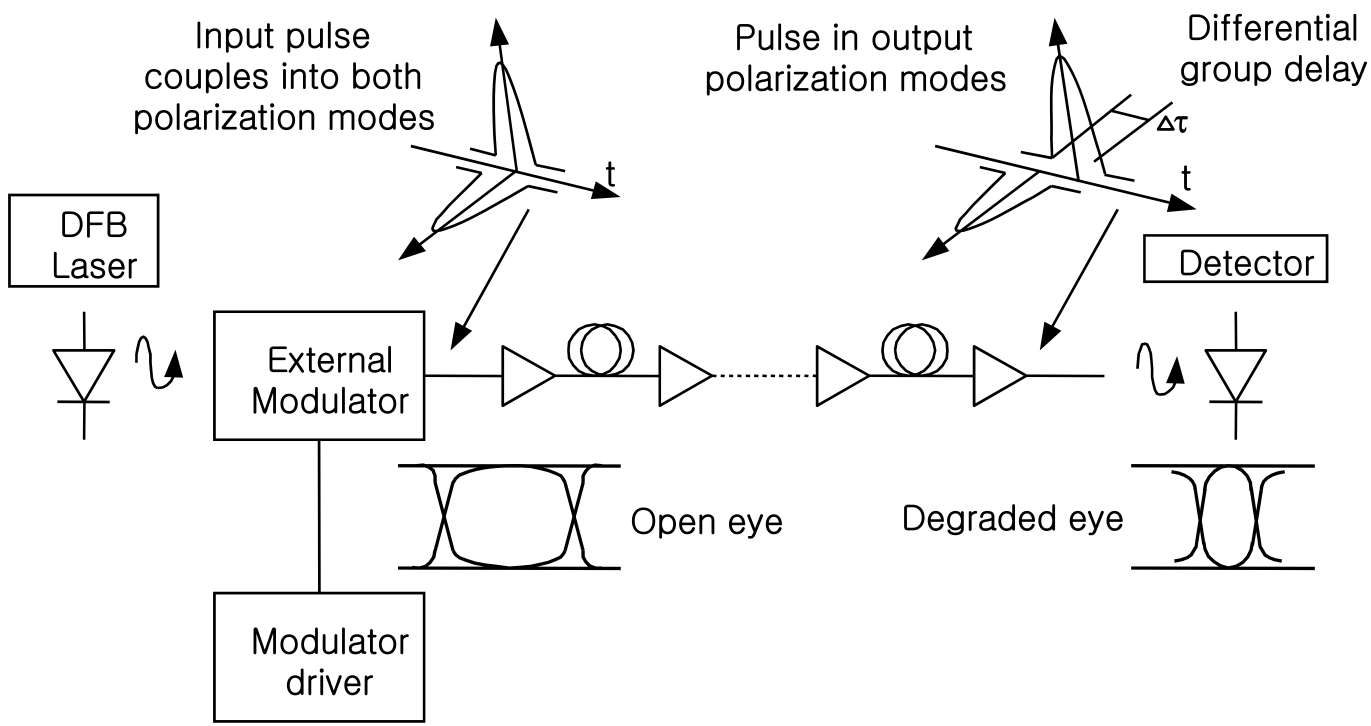
Introduction

- Bit rate of a single channel in WDM system tends to increase.
- Polarization mode dispersion (PMD) turns out to be the ultimate obstacle to the bit rate increase.
- Power penalty due to the PMD increases quadratically as the bit rate increases.
- Due to the higher-order components and the stochastic characteristics of the PMD, each channel in WDM system needs to be compensated separately in dynamic manner.
- To fully understand the PMD representation methods, some theoretical (mathematical) background is needed.

PMD

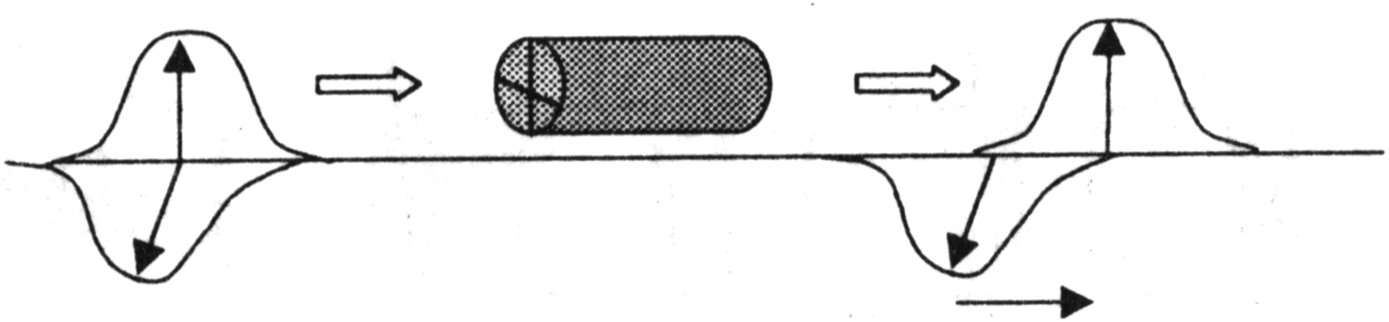
- **DGD $\Delta\tau$: Differential group delay between two polarization modes**
- **The effect of PMD in a digital communication system**

  → **Pulse Broadening**



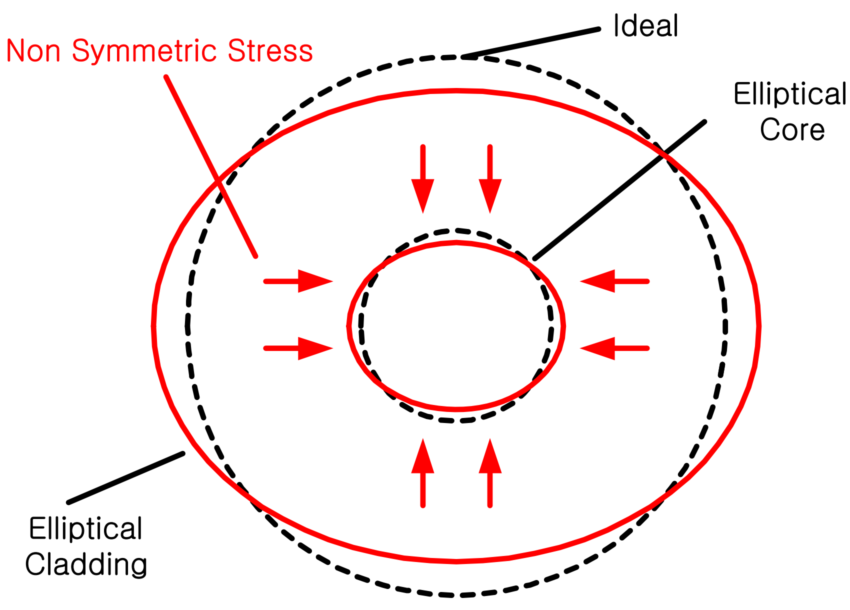
Origins of PMD

- PMD caused by local birefringence of the fiber
  - The intrinsic fiber core eccentricity
  - The stress induced by environmental factors

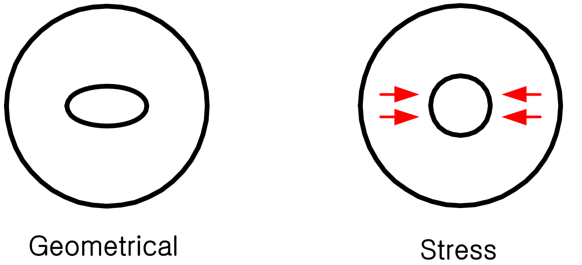


$$\Delta\tau = \Delta n l / c$$

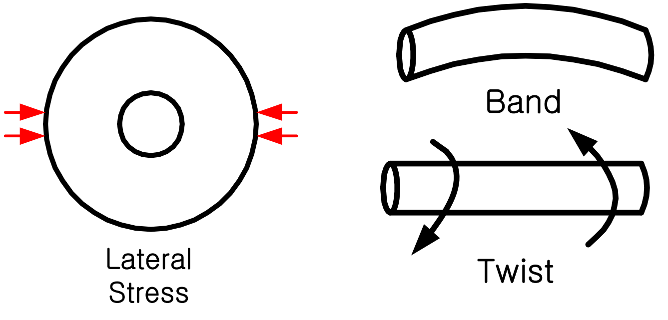
Causes of Birefringence



- Intrinsic : Oval waveguide

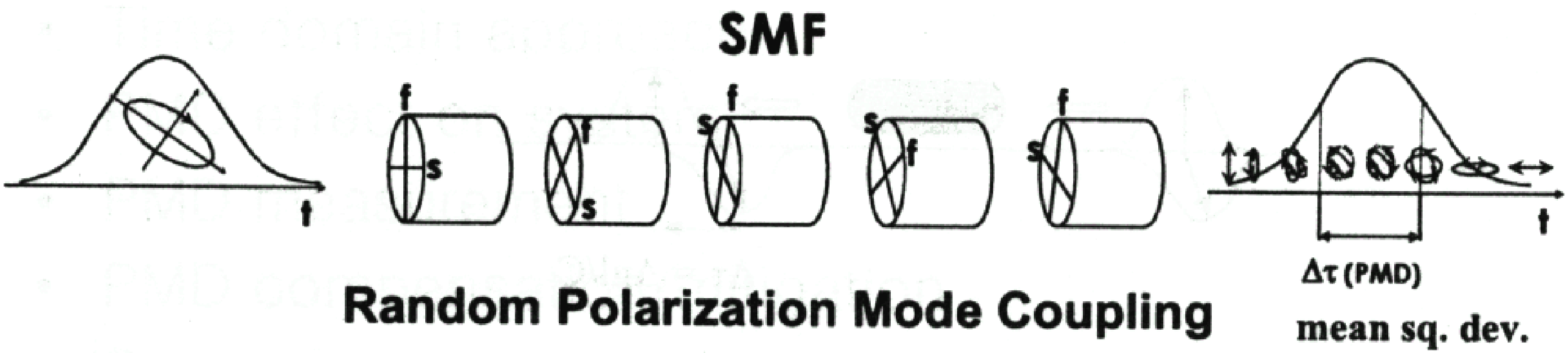


- Extrinsic : Mechanical stress



Realistic Model of Fiber

- Multiple concatenation of randomly oriented birefringent elements



Polarized Light – Jones Vector

- For TEM waves, E field is represented by

$$E = f(t)\left(a_x e^{i\phi_x}\hat{x} + a_y e^{i\phi_y}\hat{y}\right)$$

- Complex Jones vector

$$\begin{pmatrix} a_x e^{j\phi_x} \\ a_y e^{j\phi_y} \end{pmatrix} = \begin{pmatrix} s_x \\ s_y \end{pmatrix} \Rightarrow \quad E = f(t)\begin{pmatrix} s_x \\ s_y \end{pmatrix} \qquad s_x s_x^* + s_y s_y^* = 1$$

- Bra-ket notation of Jones vector

$$\boxed{\left|s\right\rangle \equiv \begin{pmatrix} s_x \\ s_y \end{pmatrix}, \qquad \left\langle s\right| \equiv \left(s_x^*, s_y^*\right)} \Rightarrow \quad \begin{aligned} E &= f\left|s\right\rangle \\ \left\langle s | s \right\rangle &= 1 \end{aligned}$$

Polarized Light – Stokes Vector

- Poincare sphere & Stokes vector

$$\hat{s} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

$$s_1 = \frac{a_x{}^2 - a_y{}^2}{a_x{}^2 + a_y{}^2} = s_x s_x^* - s_y s_y^*$$

$$s_2 = \frac{2a_x a_y \cos\delta}{a_x{}^2 + a_y{}^2} = s_x s_y^* + s_x^* s_y$$

$$s_3 = \frac{2a_x a_y \sin\delta}{a_x{}^2 + a_y{}^2} = j\left(s_x s_y^* - s_x^* s_y\right)$$

$$s_1{}^2 + s_2{}^2 + s_3{}^2 = 1$$

Relation between Jones Vector and Stokes Vector

- 2 X 2 Pauli spin matrix

$$\sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 0 & -j \\ j & 0 \end{pmatrix}$$

- Stokes vector vs. Jones vector

$$s_i = \langle s | \sigma_i | s \rangle, \qquad i = 1,2,3$$

- Pauli spin vector

$$\vec{\sigma} \equiv (\sigma_1, \sigma_2, \sigma_3) \Rightarrow \boxed{\hat{s} = \langle s | \vec{\sigma} | s \rangle}$$

Properties of Pauli Spin Matrices I

$$\sigma_i = \sigma_i^+, \quad \sigma_i^+ = \sigma_i^{-1}$$

$$\sigma_i^2 = I, \quad \sigma_p \sigma_q = -\sigma_q \sigma_p = j\sigma_r$$

$$\vec{\sigma}(\vec{a}\cdot\vec{\sigma}) = \vec{a}I + j\vec{a}\times\vec{\sigma}$$

$$(\vec{a}\cdot\vec{\sigma})\vec{\sigma} = \vec{a}I - j\vec{a}\times\vec{\sigma}$$

$$(\vec{a}\cdot\vec{\sigma})(\vec{a}\cdot\vec{\sigma}) = a^2 I$$

$$(\vec{a}\cdot\vec{\sigma})(\vec{b}\cdot\vec{\sigma}) = (\vec{a}\cdot\vec{b})I + j(\vec{a}\times\vec{b})\cdot\vec{\sigma}$$

Properties of Pauli Spin Matrices II

- Any 2 x 2 matrix may be expanded with Pauli spin matrices and identity matrix

$$M = a_0 I + a_1\sigma_1 + a_2\sigma_2 + a_3\sigma_3 = a_0 I + \vec{a}\cdot\vec{\sigma}$$

$$a_0 = \frac{1}{2}Tr(M), \qquad a_i = \frac{1}{2}Tr(\sigma_i M)$$

  - For Hermitian matrices, the coefficients are real.

- Trace of a matrix

$$Tr(M) = \sum_i m_{ii} = \sum_i \lambda_i$$

Properties of Pauli Spin Matrices III

- Some practical usage of Pauli spin vector

$$\langle s|\vec{a}\cdot\vec{\sigma}|s\rangle = \vec{a}\cdot\langle s|\vec{\sigma}|s\rangle = \vec{a}\cdot\hat{s}$$

$$\langle s|\vec{a}\times\vec{\sigma}|s\rangle = \vec{a}\times\langle s|\vec{\sigma}|s\rangle = \vec{a}\times\hat{s}$$

$$\langle s|R\vec{\sigma}|s\rangle = R\langle s|\vec{\sigma}|s\rangle = R\hat{s}$$

Transmission Matrices

- Assuming no polarization-dependent loss exists and fiber's loss is factored out, we can describe a fiber with a unitary transmission matrix.

- 2 x 2 Jones matrix

$$\left|t\right\rangle = T\left|s\right\rangle = e^{j\phi_0}U\left|s\right\rangle$$

- 3 x 3 Mueller matrix

$$\hat{t} = R\hat{s}$$

Fiber link

$$\left|s\right\rangle,\ \hat{s} \longrightarrow \boxed{T,U,R} \longrightarrow \left|t\right\rangle,\ \hat{t}$$

Unitary, Hermitian Matrices

- Definition of unitary matrix

$$MM^{+} = I, \qquad \det(M) = 1$$

where $^{+}$ denotes Hermitian conjugate.

- Definition of Hermitian matrix

$$M = M^{+}$$

  – Hermitian matrices have all real eigenvalues.

Property of Transmission Matrix I

- Projection operator

$$|s\rangle\langle s| = \begin{pmatrix} s_x \\ s_y \end{pmatrix}\left(s_x^*, s_y^*\right) = \begin{pmatrix} s_x s_x^* & s_x s_y^* \\ s_y s_x^* & s_y s_y^* \end{pmatrix} = \frac{1}{2}\left(I + \hat{s}\cdot\vec{\sigma}\right)$$

$$where \qquad \hat{s} = \langle s|\vec{\sigma}|s\rangle$$

$$|s\rangle = \hat{s}\cdot\vec{\sigma}|s\rangle$$

- Dot products

$$\langle p|q\rangle\langle q|p\rangle = \frac{1}{2}\left(I + \hat{p}\cdot\hat{q}\right)$$

$$If \quad \langle p|q\rangle = 0 \quad i.e., \quad |p\rangle \perp |q\rangle, \;\; then \quad \hat{p} = -\hat{q}$$

Property of Transmission Matrix II

- Conservation of dot products

$$\left|p_0\right\rangle = T\left|p_i\right\rangle, \quad \left|q_0\right\rangle = T\left|q_i\right\rangle$$

$$\left\langle p_0 \middle| q_0 \right\rangle = \left\langle p_i \middle| T^+ T \middle| q_i \right\rangle = \left\langle p_i \middle| q_i \right\rangle \qquad \left(\because T^+ T = I\right)$$

$$From \quad \left\langle q_i \middle| p_i \right\rangle\left\langle p_i \middle| q_i \right\rangle = \frac{1}{2}\left(I + \hat{p}_i \cdot \hat{q}_i\right), \quad \left\langle q_0 \middle| p_0 \right\rangle\left\langle p_0 \middle| q_0 \right\rangle = \frac{1}{2}\left(I + \hat{p}_0 \cdot \hat{q}_0\right)$$

$$\hat{p}_0 \cdot \hat{q}_0 = \hat{p}_i \cdot \hat{q}_i$$

- Dot products of vectors are conserved during transmission through fiber.

- Therefore, *the transmission through fiber is represented by a rotation of the Stokes vectors.*

Connection between *U* and *R*

$$\left| t \right\rangle = U \left| s \right\rangle$$

$$\hat{t} = R\hat{s} = R\left\langle s \middle| \vec{\sigma} \middle| s \right\rangle = \left\langle s \middle| R\vec{\sigma} \middle| s \right\rangle$$

$$\hat{t} = \left\langle t \middle| \vec{\sigma} \middle| t \right\rangle = \left\langle s \middle| U^{+}\vec{\sigma}U \middle| s \right\rangle$$

$$\boxed{\Rightarrow \quad R\vec{\sigma} = U^{+}\vec{\sigma}U}$$

Rotational Form of Jones Matrix I

- Recall projection operator

$$|s\rangle\langle s| = \frac{1}{2}\left(I + \hat{s}\cdot\vec{\sigma}\right), \quad where \quad \hat{s} = \langle s|\vec{\sigma}|s\rangle$$

- Consider a pair of orthogonal Jones vectors $|p\rangle, |p_-\rangle$, and corresponding Stokes vectors $\hat{p}, -\hat{p}.$

$$\langle p_-|p\rangle = 0$$

$$|p\rangle\langle p| = \frac{1}{2}\left(I + \hat{p}\cdot\vec{\sigma}\right), \quad |p_-\rangle\langle p_-| = \frac{1}{2}\left(I - \hat{p}\cdot\vec{\sigma}\right)$$

$$|p\rangle\langle p| + |p_-\rangle\langle p_-| = I$$

– Any pair of orthogonal vectors forms a complete orthogonal set in Jones space.

Rotational Form of Jones Matrix II

- Since $UU^{+}=I$ *and* $\det(U)=1$, the eigenvalues of U must be of unit magnitude and their product must be unity.
- Consequently

$$U = e^{-j\varphi/2}\left|r\right\rangle\left\langle r\right| + e^{j\varphi/2}\left|r\_\right\rangle\left\langle r\_\right|,$$

$$where \quad \left|r\right\rangle \; and \; \left|r\_\right\rangle \; are\ eigenvectors \;\; (and \;\; \left\langle r\_\middle|r\right\rangle = 0).$$

- Recalling the projection operator, we can rewrite

$$\boxed{U = I\cos(\varphi/2) - j\hat{r}\cdot\vec{\sigma}\sin(\varphi/2)}$$

- From the property of Pauli spin matrix

$$\boxed{U = e^{-j(\varphi/2)\hat{r}\cdot\vec{\sigma}}}$$

Rotational form in Stokes Space

$$U^{+}\vec{\sigma}U = (\cos\varphi)\vec{\sigma} + (1-\cos\varphi)\hat{r}(\hat{r}\cdot\vec{\sigma}) + (\sin\varphi)\hat{r}\times\vec{\sigma}$$

$$\textit{From} \quad R\vec{\sigma} = U^{+}\vec{\sigma}U$$

$$\boxed{R = (\cos\varphi)I + (1-\cos\varphi)\hat{r}\hat{r} + (\sin\varphi)\hat{r}\times = \hat{r}\hat{r} + (\sin\varphi)\hat{r}\times - (\cos\varphi)(\hat{r}\times)(\hat{r}\times)}$$

$$\textit{where} \quad \hat{r}\hat{r} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix}(r_1, r_2, r_3) = \begin{pmatrix} r_1 r_1 & r_1 r_2 & r_1 r_3 \\ r_2 r_1 & r_2 r_2 & r_2 r_3 \\ r_3 r_1 & r_3 r_2 & r_3 r_3 \end{pmatrix} \qquad \hat{r}\times = \begin{pmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ r_2 & r_1 & 0 \end{pmatrix}$$

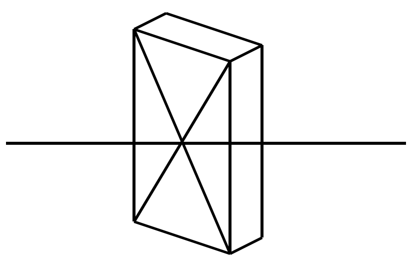
$$\text{Using} \quad (\hat{r}\times)(\hat{r}\times) = -I + \hat{r}\hat{r}, \quad (\hat{r}\times)(\hat{r}\times)(\hat{r}\times) = -\hat{r}\times$$

$$\boxed{R = e^{\varphi(\hat{r}\times)}}$$

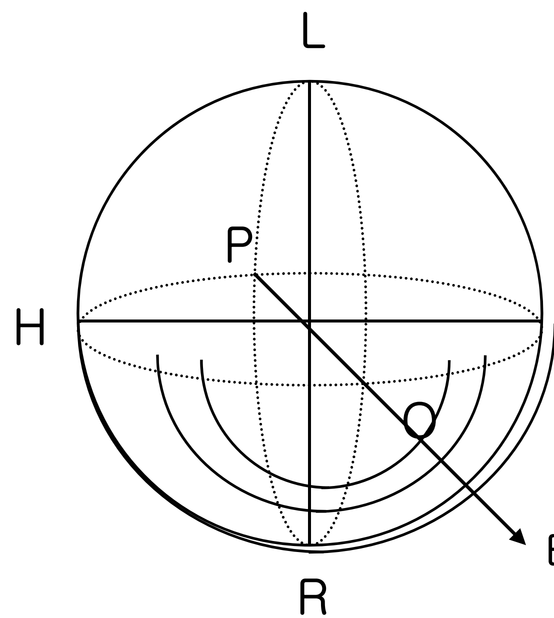
Poincare Sphere Representation of *R*

• The behavior of birefringence is described by a rotation on the Poincare sphere.

$$\hat{t} = R\hat{s}, \qquad R = e^{\varphi(\hat{r}\times)}$$



$$\delta = \frac{2\pi}{\lambda}\Delta n l, \qquad \varphi = \delta$$



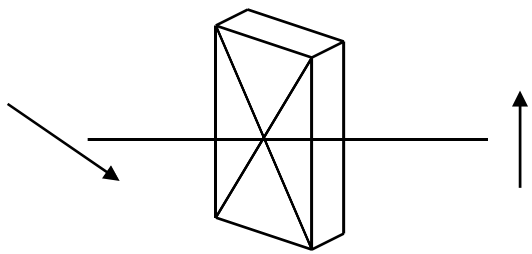
PMD Vector

- Contents
  - Frequency dependency of output SOP
  - Concept of PSP
  - Jones matrix eigenvector analysis
  - Pauli spin vector expansion
  - Mueller matrix expression
  - Input PMD vector
  - Evolution of PMD vector

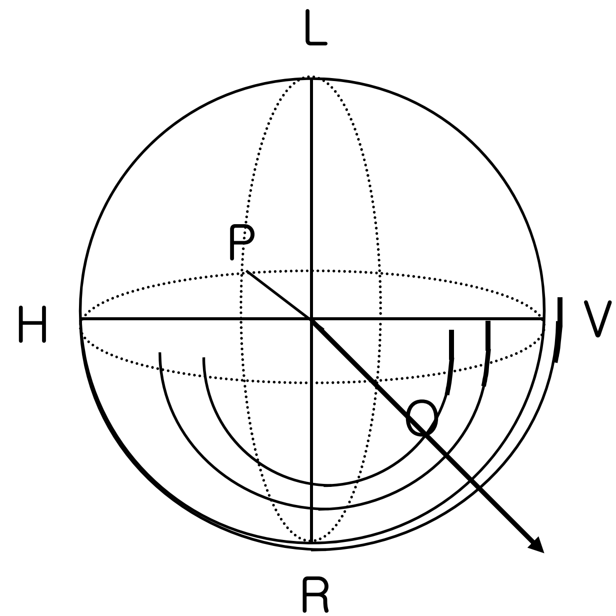
Frequency Dependency of Output SOP I

- Single birefrengent element

Half-wave plate



$$\delta = \frac{2\pi}{\lambda}\Delta nl = \frac{2\pi c}{\lambda}\frac{\Delta nl}{c} = \omega\Delta\tau$$



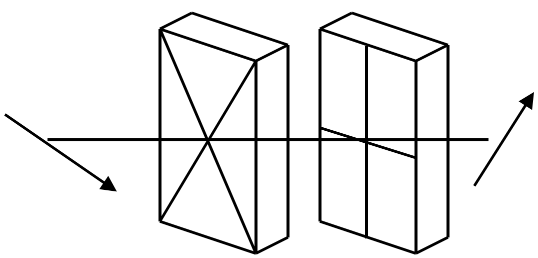
$$\Delta\delta = \Delta\omega\Delta\tau$$

– *If an input SOP is aligned to the eigenmode, the output SOP does not vary with optical frequency.*

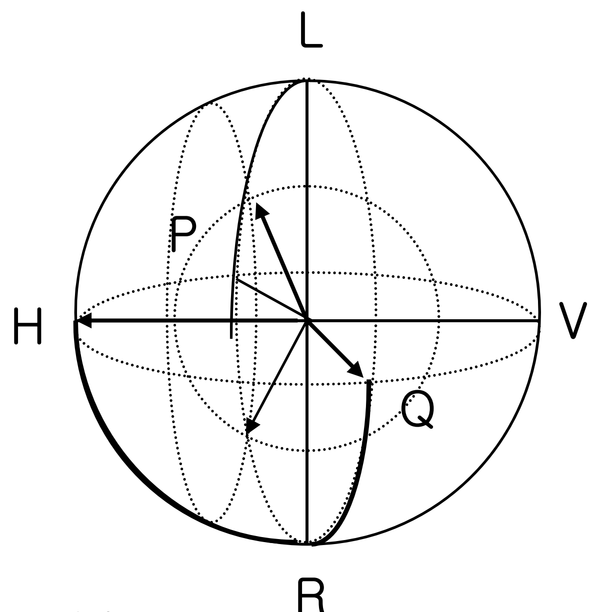
Frequency Dependency of Output SOP II

- Two birefrengent elements

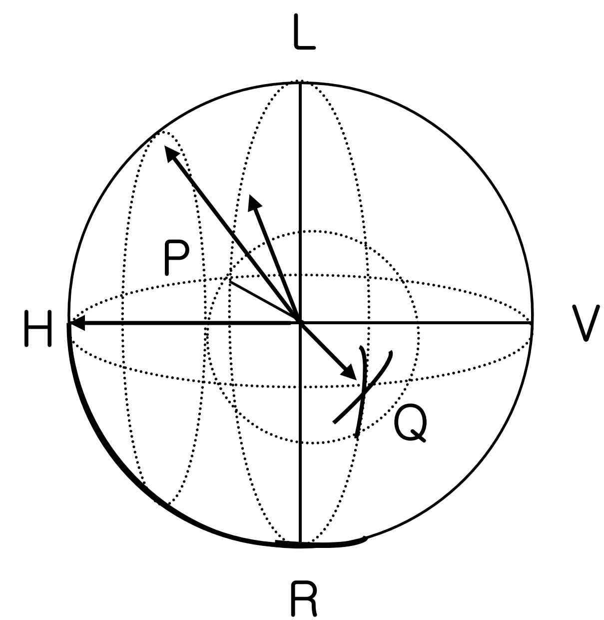
Two quarter-wave plates



Eigenmode



Principle state of polarization

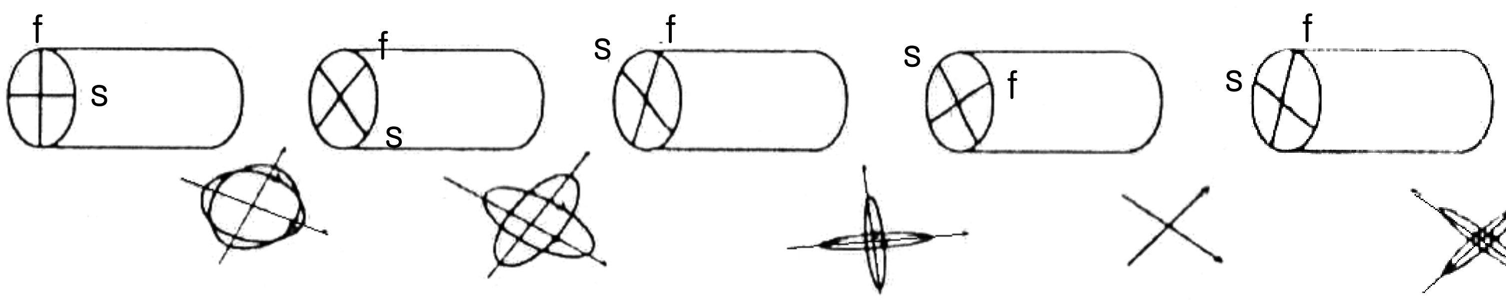


- *Eigenmode varies with optical frequency.*

- *There exist input/output SOPs that are invariant with optical frequency to first-order.*

Frequency Dependency of Output SOP III

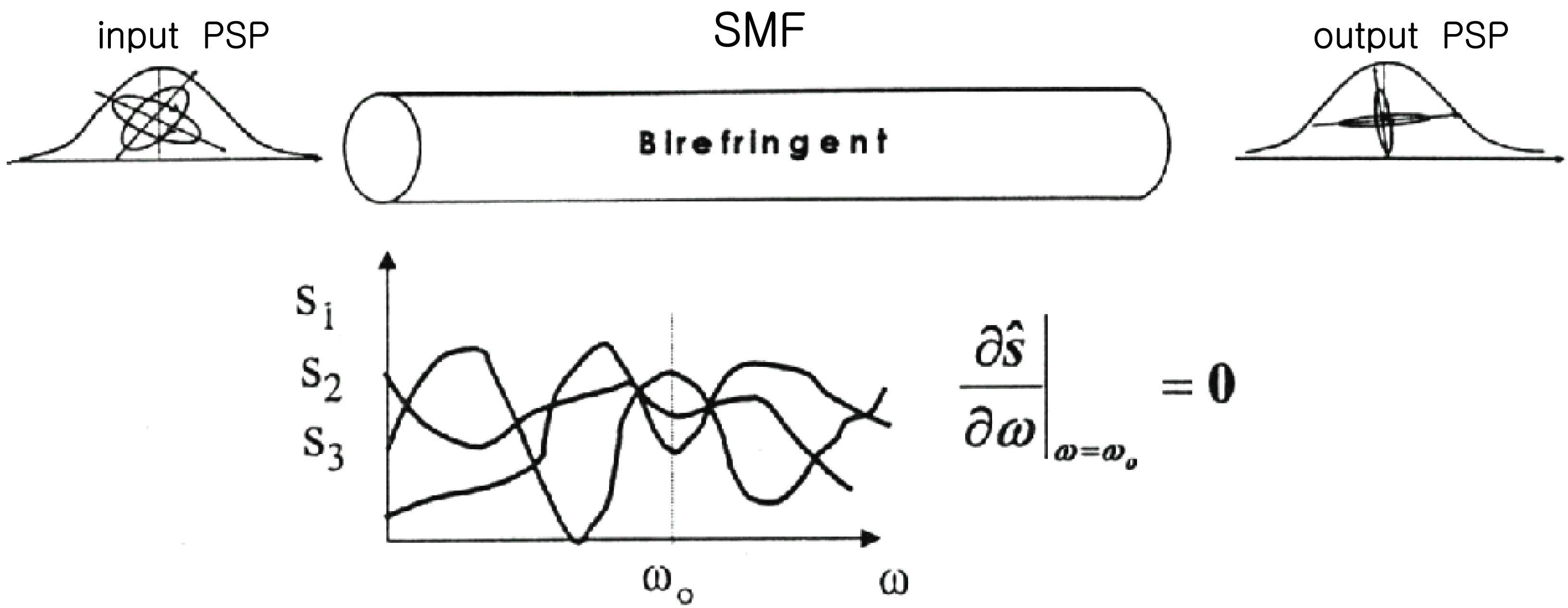
- Multiple birefrengent elements: fiber link



- Question : *Are there such input/output SOPs that are invariant with optical frequency to first-order even in fiber link? If so, how can we find them?*

Principal State of Polarization

• It has been found empirically that there always exist such SOPs that are invariant with optical frequency to first-order when traveling through linear birefringent medium.



$$\left.\frac{\partial \hat{s}}{\partial \omega}\right|_{\omega=\omega_o} = 0$$

Jones Matrix Eigenvector Analysis I

- Eigenvalue equation from the definition of PSP

$$\left|t\right\rangle = e^{-j\phi_0} U \left|s\right\rangle \qquad \left|t\right\rangle_\omega = -j\left(\frac{d\phi_0}{d\omega} + jU_\omega U^+\right)\left|t\right\rangle$$

$$Let \quad \left|t\right\rangle = e^{-j\phi}\left|\left(\left|t\right|\right)\right\rangle$$

$$For \;\; PSP,$$

$$\left|t\right\rangle_\omega = -j\frac{d\phi}{d\omega}e^{-j\phi}\left|\left(\left|t\right|\right)\right\rangle = -j\frac{d\phi}{d\omega}\left|t\right\rangle \quad \left(\because \;\; \left|\left(\left|t\right|\right)\right\rangle_\omega = 0\right)$$

$$Then \;\; \left(\tau_g - \tau_0\right)\left|t\right\rangle = jU_\omega U^+\left|t\right\rangle, \quad where \;\; \tau_g = \frac{d\phi}{d\omega}, \tau_0 = \frac{d\phi_0}{d\omega}$$

- τ_0 is a common delay and τ_g is a group delay.

Jones Matrix Eigenvector Analysis II

- jU_wU^{+} is a Hermitian matrix.

$$UU^{+} = I$$

$$\left(UU^{+}\right)_{\omega} = 0 \quad \Rightarrow \quad U_{\omega}U^{+} = -UU_{\omega}^{+}$$

$$\therefore \quad jU_{\omega}U^{+} = -jUU_{\omega}^{+} = \left(jU_{\omega}U^{+}\right)^{+}$$

- A Hermitian matrix has real eigenvalues and their eigenvectors are orthogonal.

- Frequency expansion of U

$$U(\omega + d\omega) = U + d\omega\, U_{\omega} = \left(I + d\omega U_{\omega}U^{+}\right)U$$

  - Since det(U(w))=det(U(w+dw))=1, Tr(U_wU^{+}) should be zero.

Jones Matrix Eigenvector Analysis III

- Therefore, eigenvalues of $jU_\omega U^+$ are given by

$$\boxed{\left(\tau_{\mathrm{g}} - \tau_0\right) = \pm \tau/2}$$

- Now, we have two orthogonal PSP's |p>, |p_> whose relative delays are $\tau/2, \ -\tau/2.$

- The directions of PSPs do not vary with frequency to first-order.

- PMD vector is defined as

$$\boxed{\vec{\tau} = \tau\hat{p}} \qquad \boxed{\vec{\Omega} = \tau\hat{p}}$$

- The magnitude of PMD vector is differential group delay (DGD) between the slow and the fast PSPs, and its direction represents the slow PSP.

Pauli Spin Matrix Expansion

- If we expand $jU_\omega U^+$ with Pauli spin vector, we obtain $jU_\omega U^+ = \frac{1}{2}\vec{a}\cdot\vec{\sigma}$ since $Tr\left(jU_\omega U^+\right) = 0.$

- Previously, we know that

$$jU_\omega U^+ \left| p \right\rangle = \frac{1}{2}\tau \left| p \right\rangle$$

- Substituting the Pauli spin vector expansion into the eigenvalue equation, we obtain

$$\vec{a}\cdot\vec{\sigma}\left| p \right\rangle = \tau \left| p \right\rangle$$

- This implies $\vec{a} = \tau\hat{p} = \vec{\tau}$

- Consequently, we obtain $\boxed{jU_\omega U^+ = \frac{1}{2}\vec{\tau}\cdot\vec{\sigma}}$

Mueller Matrix Expression I

- From $|t\rangle = e^{-j\phi_0} U |s\rangle$, we obtain

$$|t\rangle_\omega = -j\left(\tau_0 + jU_\omega U^+\right)|t\rangle = -j\left(\tau_0 + \frac{1}{2}\vec{\tau}\cdot\vec{\sigma}\right)|t\rangle$$

- From $\hat{t} = \langle t|\vec{\sigma}|t\rangle$, we obtain

$$\hat{t}_\omega = \langle t|_\omega \vec{\sigma}|t\rangle + \langle t|\vec{\sigma}\left(|t\rangle\right)_\omega$$

- Substituting the first equation in the second one and using the properties of Pauli spin vector, we obtain

$$\boxed{\hat{t}_\omega = \vec{\tau}\times\hat{t}}$$

Mueller Matrix Expression II

- Mueller Matrix vs. PMD vector

$$\hat{t} = R\hat{s}$$

$$\hat{t}_\omega = R_\omega \hat{s} = R_\omega R^+ \hat{t}$$

$$\hat{t}_\omega = \vec{\tau} \times \hat{t}$$

$$\Rightarrow \boxed{\vec{\tau}\times = R_\omega R^+}$$

Input PMD vector

- From the output PSP and output PMD vector, input PSP and input PMD vector are defined as

$$\boxed{\left| p_i \right\rangle = T^{+} \left| p_0 \right\rangle, \qquad \vec{\tau}_i = R^{+} \vec{\tau}_0}$$

- Matrix operator transform through transmission matrix.

$$M_0 = T M_i T^{+}$$

- Transmission matrix vs. input PMD vector

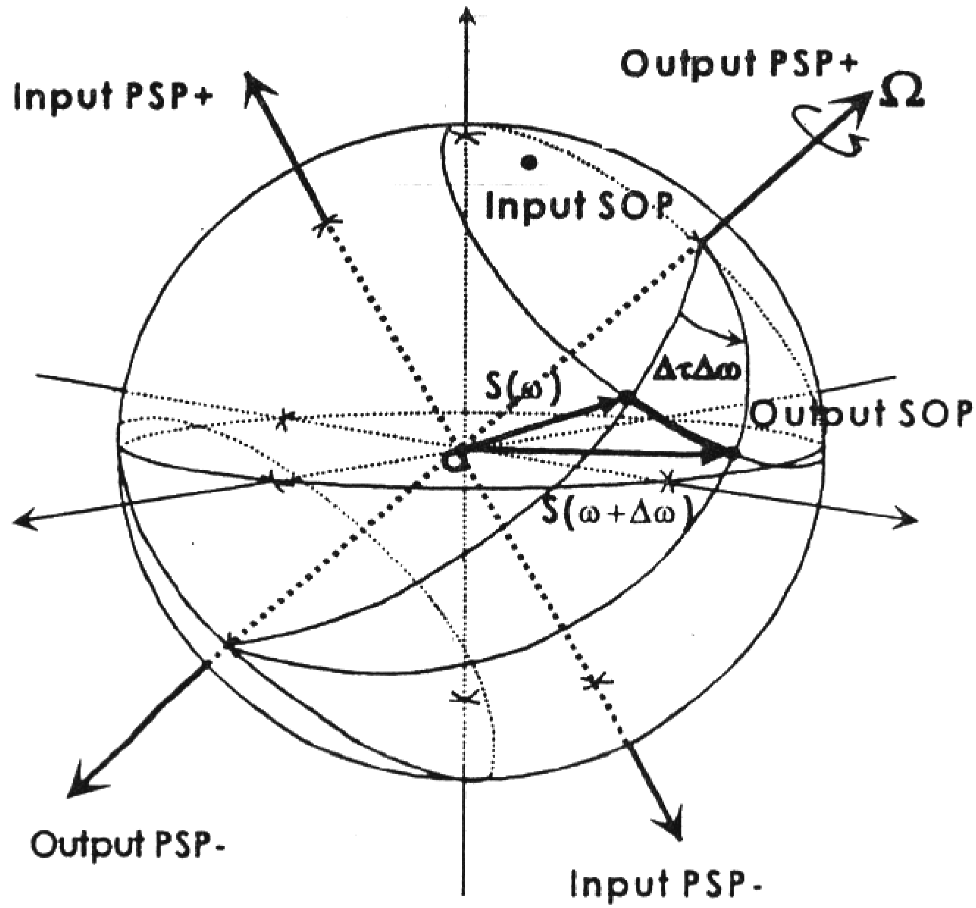
$$\boxed{\vec{\tau}_i \cdot \vec{\sigma} = U^{+} \vec{\tau}_0 \cdot \vec{\sigma} U = 2 j U^{+} U_{\omega}}$$

Implication of PMD vector

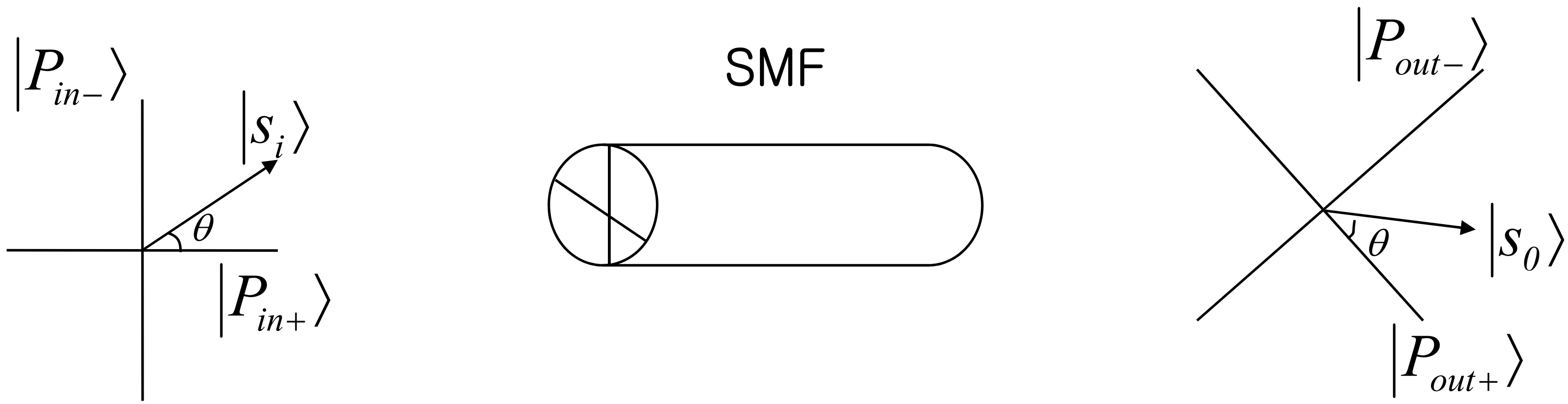
- PMD vector

$$\frac{\partial \hat{s}_{out}}{\partial \omega} = \vec{\tau} \times \hat{s}_{out}$$

- For small signal bandwidth, spectral resolved output SOP will form a part of circle about the PMD vector.



Meaning of PSP



$$\vec{E}_{out}(t) = c_+ |p_{out+}\rangle E_{in}\left(t+\frac{\tau}{2}\right) + c_- |p_{out-}\rangle E_{in}\left(t-\frac{\tau}{2}\right)$$

where

$$\vec{E}_{in} = E_{in}|e_{in}\rangle, \quad c_{\pm} = \langle p_{in\pm}|e_{in}\rangle$$

If the PSPs are known, the SMF can be treated as a simple birefringent medium to first-order.

PMD Vector Concatenation

- Concatenation of two PMD elements

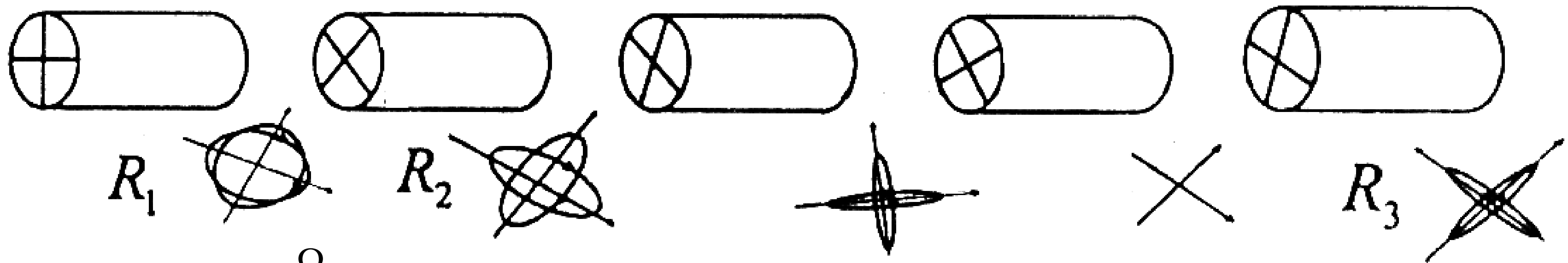
$$R_1,\ \vec{\tau}_1 \qquad\qquad R_2,\ \vec{\tau}_2$$

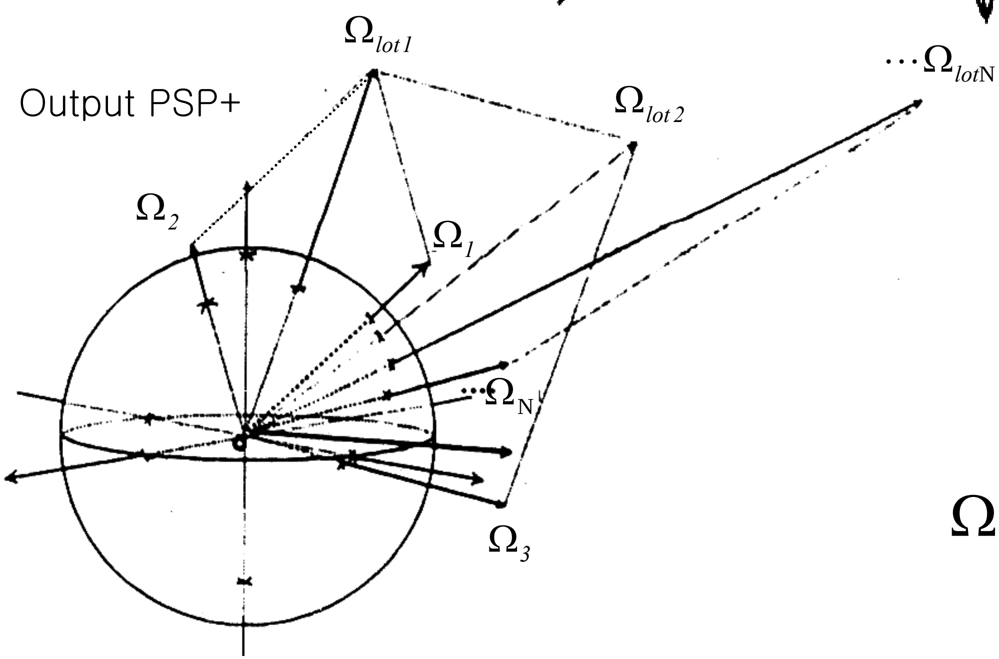
$$R_{tot} = R_2 R_1$$

$$\vec{\tau}_{tot}\times = (R_2R_1)'(R_2R_1)^{-1} = R_2'R_2^{-1} + R_2R_1'R_1^{-1}R_2^{-1} = (\vec{\tau}_2 + R_2\vec{\tau}_1)\times$$

$$\Rightarrow \boxed{\vec{\tau}_{tot} = \vec{\tau}_2 + R_2\vec{\tau}_1}$$

Evolution of PMD vector





$$\Omega_{tot} = \tilde{\Omega}_1 + \tilde{\Omega}_2 + \ldots \tilde{\Omega}_N, \tilde{\Omega}_l = R_{\rangle i}\Omega_l$$

Dynamic Equation for PMD vector

$$\frac{\partial \hat{s}}{\partial l} = W(l,\omega) \times \hat{s}$$

$$\frac{\partial \hat{s}}{\partial \omega} = \vec{\tau}(l,\omega) \times \hat{s}$$

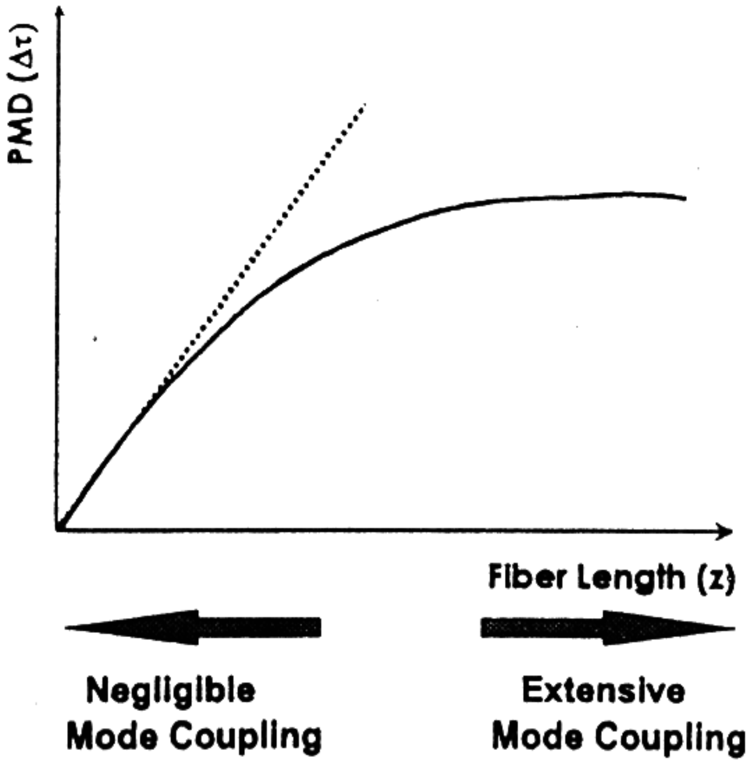
$$\Rightarrow \frac{\partial \vec{\tau}}{\partial l} = \frac{\partial W}{\partial \omega} + W \times \vec{\tau}$$

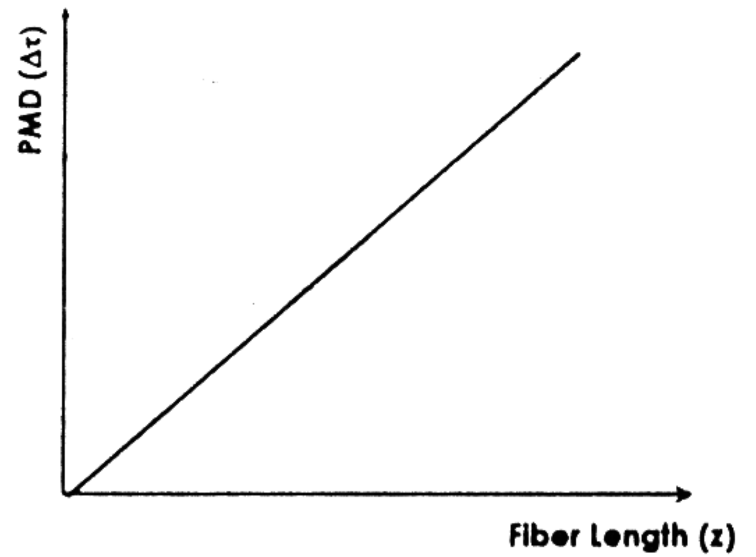
- *W* is a randomly varying birefringence vector.

- The statistical behavior of the PMD can be described by solving the dynamic equation with the martingale differential equation method.

PMD vs. fiber length

$$(\Delta\tau)^2 = \langle \tau^2 \rangle - \langle \tau \rangle^2 = \frac{\Delta\beta^2}{2h^2}\left(e^{-2hz} - 1 + 2hz\right)$$



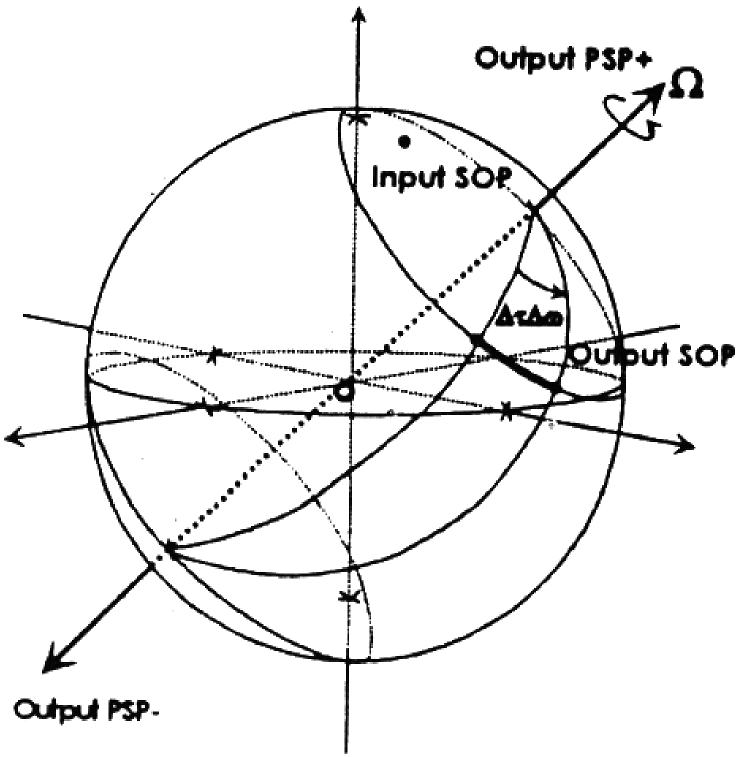


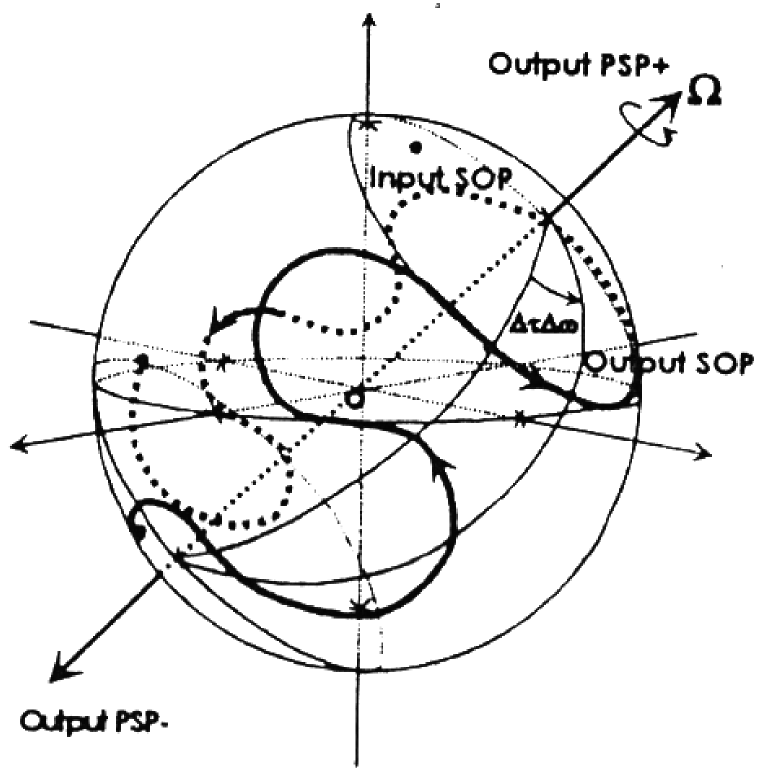
PM Fiber (No Mode Coupling)

High-Order PMD

$$\vec{\tau}(\Delta\omega) = \vec{\tau}^{(0)} + \vec{\tau}^{(1)}\Delta\omega + \vec{\tau}^{(2)}\frac{\Delta\omega^2}{2} + \cdots$$

where $\vec{\tau}^{(n)}$ denotes $(n+1)$th order PMD





Autocorrelation of PMD

- Frequency autocorrelation of PMD vector

$$\langle \Omega(\omega') \cdot \Omega(\omega) \rangle = \frac{3}{\Delta\omega^2}\left[1 - \exp\left(-\frac{\pi\Delta\omega^2}{8}\tau_{DGD}^2\right)\right]$$

- 3dB bandwidth of the autocorrelation function

$$B_{PMD} \cong \frac{0.64}{\tau_{DGD}}$$

– First-order approximation of PMD is valid when the signal bandwidth is much smaller than B_{PMD}.

Correlation between All Orders of PMD

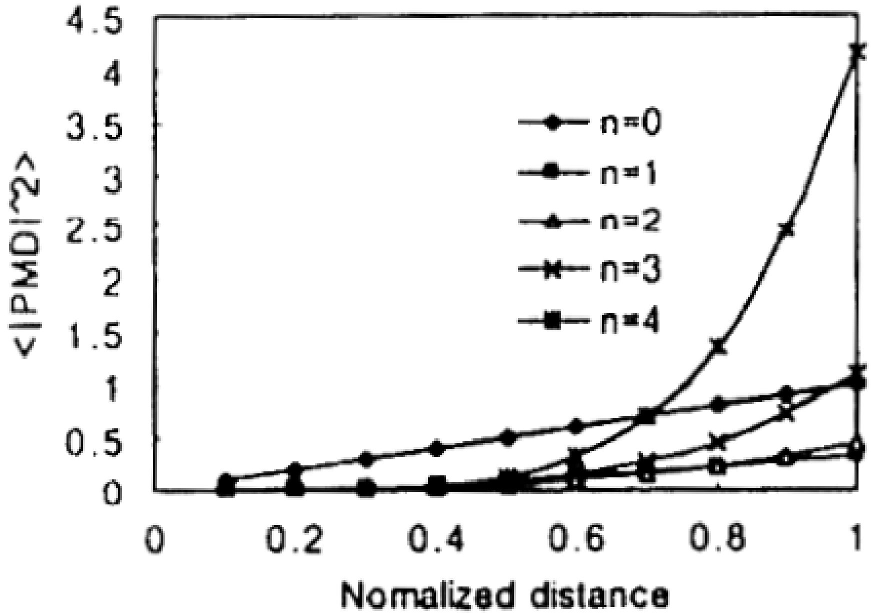
$$\left\langle \Omega^{(n)}(\omega') \cdot \Omega^{(n+1)}(\omega) \right\rangle = 0$$

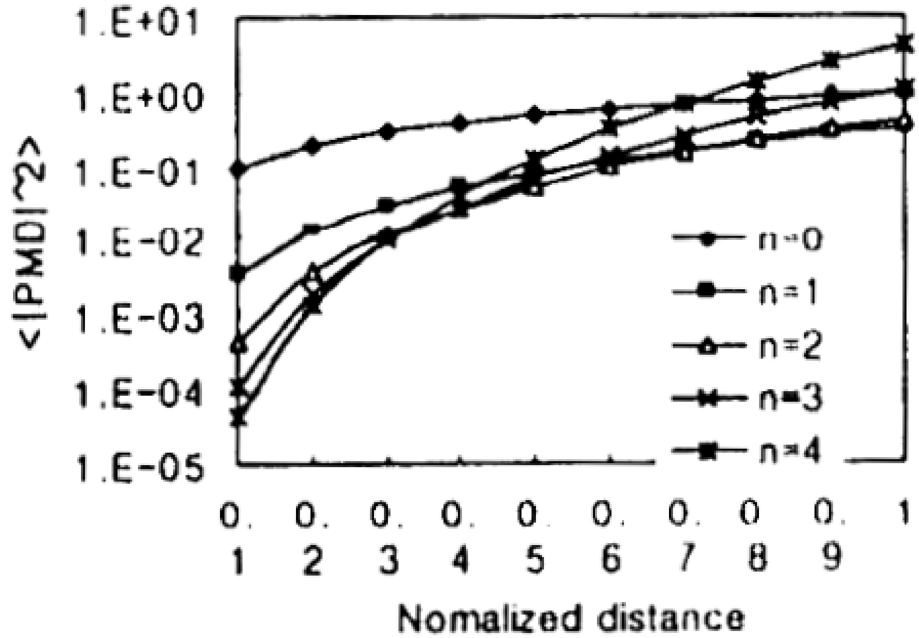
$$\left\langle \Omega^{(n)}(\omega') \cdot \Omega^{(n)}(\omega) \right\rangle = \frac{(2n)!}{3^n (n+1)!} \left\langle \left| \Omega^{(0)} \right|^2 \right\rangle^{n+1}$$

• The mean square value of the (n+1)th order of PMD is proportional to the (n+1)th power of $\left\langle \left| \Omega^{(0)} \right|^2 \right\rangle$

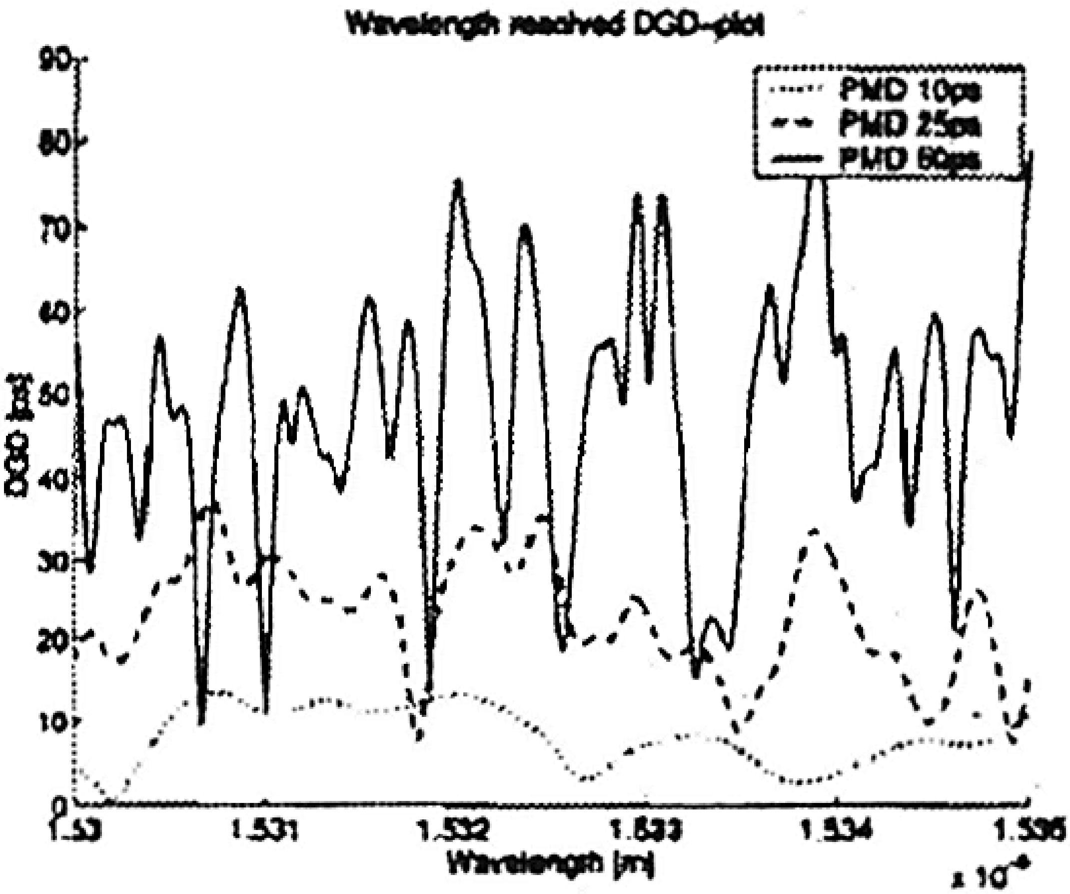
– The high order PMD increases very rapidly with fiber length.

High-Order PMD vs. Distance





Frequency Dependency of PMD



DGD vs. wavelength

Summary

• Mathematical description of polarized light propagation and PMD have been discussed.

• This mathematical description forms basics for understanding PMD representations for optical communication systems.