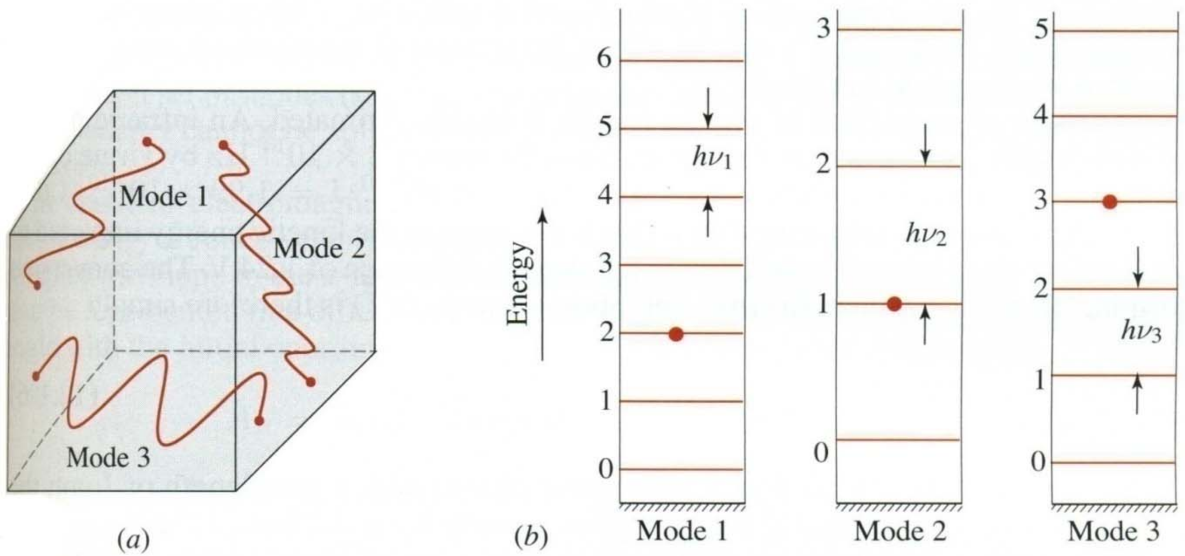
Ch. 12. Photon Optics

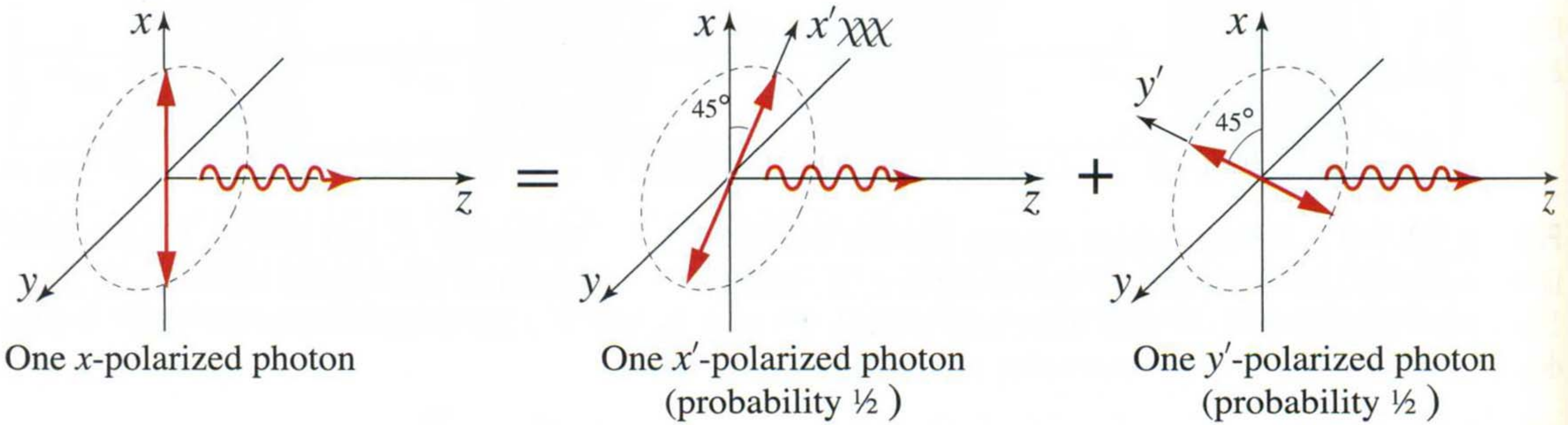
$$\mathbf{E}(\mathbf{r},t) = \sum_{\mathbf{q}} A_{\mathbf{q}}\, U_{\mathbf{q}}(\mathbf{r})\, \exp(j2\pi\nu_{\mathbf{q}} t)\, \widehat{\mathbf{e}}_{\mathbf{q}}$$

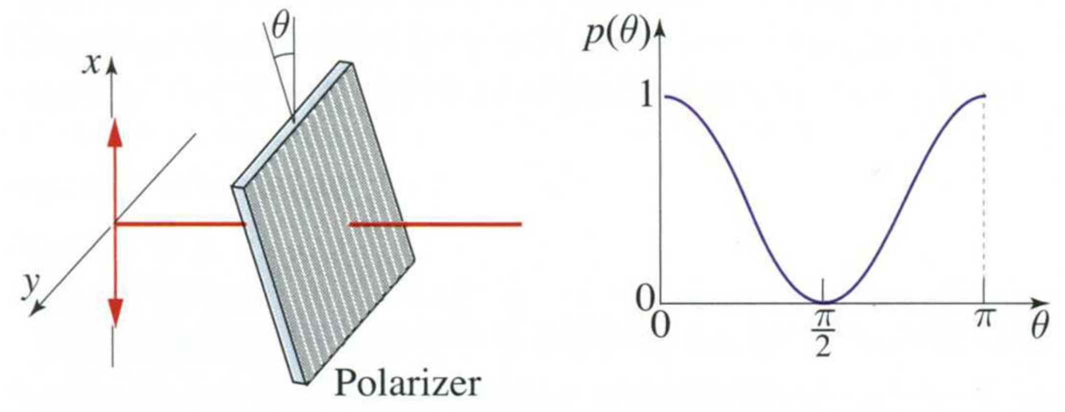
$$U_{\mathbf{q}}(\mathbf{r}) = \left(\frac{2}{d}\right)^{3/2} \sin\left(q_x \frac{\pi}{d} x\right)\, \sin\left(q_y \frac{\pi}{d} y\right)\, \sin\left(q_z \frac{\pi}{d} z\right) \qquad E_{\mathbf{q}} = \tfrac{1}{2}\epsilon \int_V |A_{\mathbf{q}}|^2\, |U_{\mathbf{q}}(\mathbf{r})|^2\, d\mathbf{r} = \tfrac{1}{2}\epsilon |A_{\mathbf{q}}|^2$$

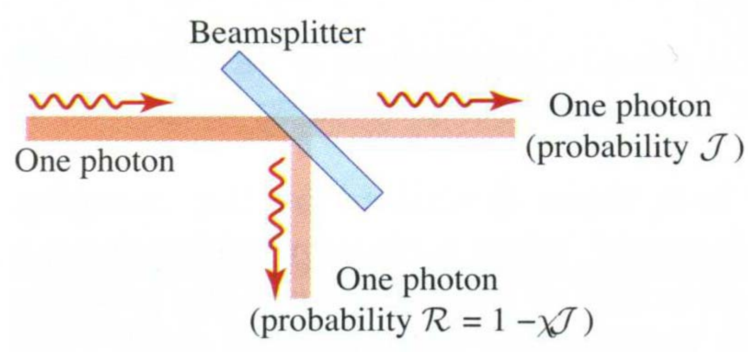


$$E = h\nu = \hbar\omega \qquad E_n = \left(n + \tfrac{1}{2}\right) h\nu, \qquad n = 0, 1, 2, \ldots.$$

$$E\,(\text{eV}) = \frac{1.24}{\lambda_o\,(\mu\text{m})}$$




θ
x
y
Polarizer
p(θ)
1
0
0
π/2
π
θ


Beamsplitter
One photon
One photon
(probability 𝒯)
One photon
(probability ℛ = 1 − 𝒯)

The probability $p(\mathbf{r})dA$ of observing a photon at a point $\mathbf{r}$ within an incremental area dA, at any time, is proportional to the local optical intensity $I(\mathbf{r}) \propto |U(\mathbf{r})|^2$, so that

$$p(\mathbf{r})\, dA \propto I(\mathbf{r})\, dA. \tag{12.1-8}$$

Photon Position

The linear momentum associated with a photon in a plane-wave mode of wavevector $\mathbf{k}$ is:

$$\mathbf{p} = \hbar \mathbf{k}. \qquad (12.1\text{-}9)$$

Its magnitude is $p = \hbar k = \hbar\omega/c = \hbar 2\pi/\lambda$, so that

$$p = E/c = h/\lambda. \qquad (12.1\text{-}10)$$

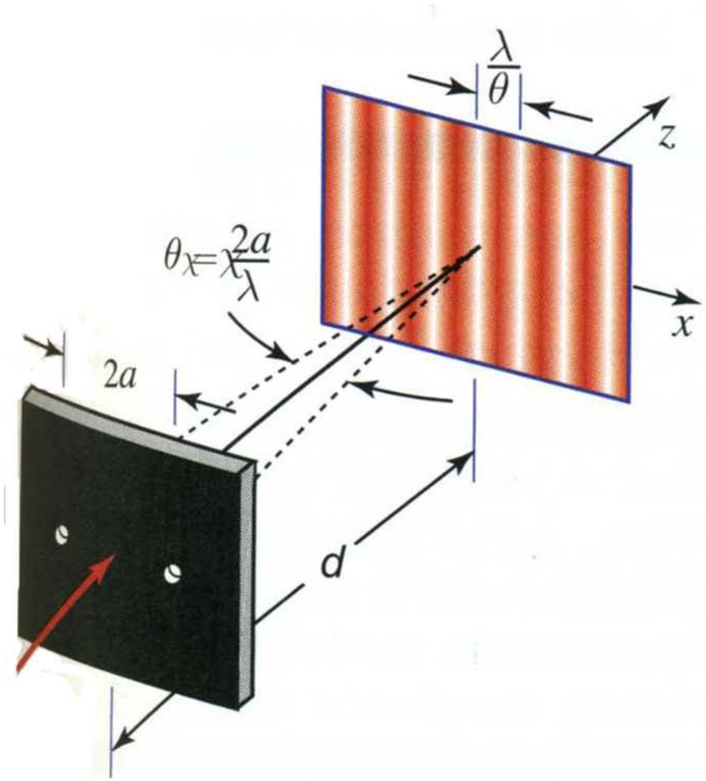
The momentum of a photon described by an arbitrary complex wavefunction $U(\mathbf{r}) \exp(j2\pi\nu t)$ is uncertain. It has the value

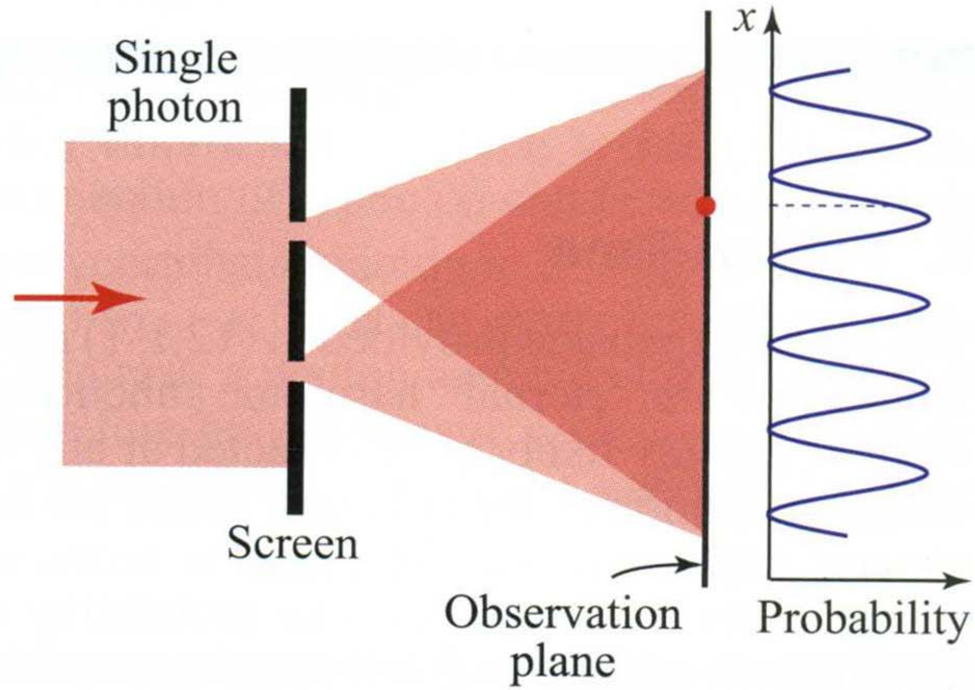
$$\mathbf{p} = \hbar\mathbf{k}, \tag{12.1-11}$$

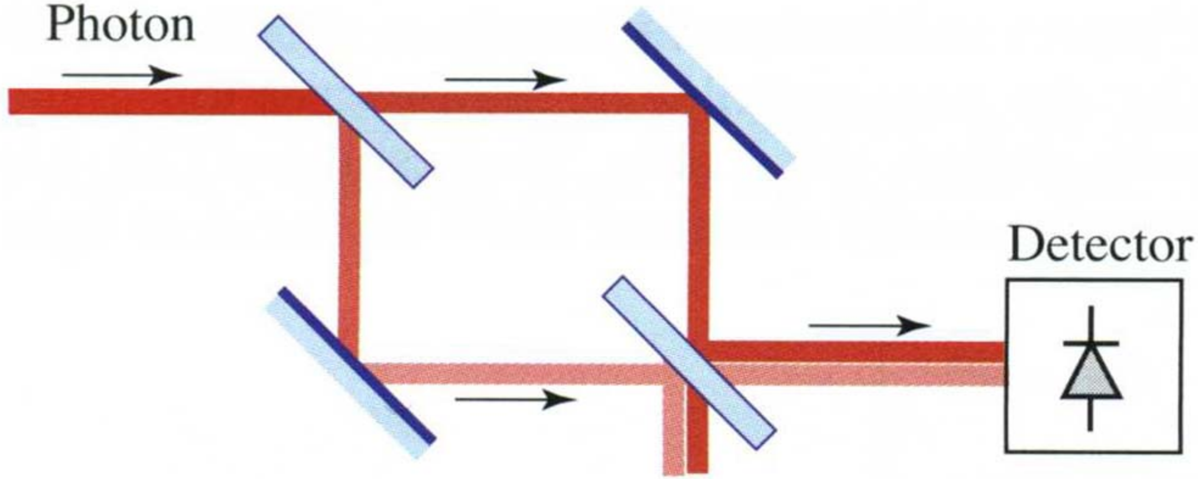
with probability proportional to $|A(\mathbf{k})|^2$, where $A(\mathbf{k})$ is the amplitude of the plane-wave Fourier component of $U(\mathbf{r})$ with wavevector $\mathbf{k}$.

$$\mathbb{S} = \pm\hbar$$

$$I(x) \approx 2I_0\left(1 + \cos\frac{2\pi x\theta}{\lambda}\right)$$






Photon
Detector

The probability of observing a photon at a point **r** within the incremental area dA, and during the incremental time interval dt following time t, is proportional to the intensity of the mode at **r** and t, so that

$$p(\mathbf{r}, t)\, dA\, dt \propto I(\mathbf{r}, t)\, dA\, dt \propto |U(\mathbf{r}, t)|^2\, dA\, dt. \quad (12.1\text{-}14)$$

Photon Position and Time

Time-Energy Uncertainty Relation

$$\sigma_E \sigma_t \geq \frac{\hbar}{2}$$