Introduction to Geophysics

1. In a broad sense: it designates the study of physical phenomena of the earth and its surroundings

   – It includes Meteorology, Hydrology, Oceanography, Seismology, Volcanology, Geomagnetism, Geodesy, Tectonophysics, Geothermometry, etc

2. In a narrow sense: it designates the study of the earth's interior from the surface to the interior (Solid earth geophysics)

   - Pure (global) geophysics: it is mainly related to revealing deep structures of the earth using seismology (earthquake)

   - Applied geophysics: it only deals with crust or surface
     It designates the study of the earth using physical measurements at or above the surface
     It has been applied in determining the depth of the crust or the building sites, and exploring mineral resources such as oil, gas, and methane hydrate

     ✓ Engineering geophysics: it is applied to investigate subsurface materials and structures that have an important meaning in engineering
     ✓ Environmental geophysics: it is used for environment management We apply geophysical methods to investigate physical and chemical phenomena near surface.

3. Applied geophysics: consists of Gravity method, Magnetic method, Seismic method, Electric method, Electromagnetic method, Well logging

   - In gravity prospecting: one measures minute variations in the pull of gravity from rocks within the first few miles of the earth's surface.
   - Different types of rocks have different densities, and the denser rocks have the greater gravitational attraction.

- Gravity instrument are designed to measure variations in the force of gravity from one place to another rather than the absolute force itself.
- Gravity method is useful wherever the formations of interest have densities that are appreciably different from those of surrounding formations.
- Gravity method is an effective means of mapping sedimentary basins where the basement rocks have a consistently higher density than the sediments.

- Magnetic prospecting maps variations in the magnetic field of the earth that are attributable to changes of structure, magnetic susceptibility, or remanence in certain near-surface rocks.
- Sedimentary rocks generally have a very small susceptibility compared with igneous or metamorphic rocks, which tend to have a much higher magnetite content
- The magnetic method was initially used for petroleum exploration in areas where the structure in oil-bearing sedimentary layers appeared to be controlled by topographic features, such as ridges or faults, on the basement surface.

- Electric methods: The resistivity method is designed to yield information on formations or bodies having anomalous electric conductivity.

- Electromagnetic method: Magnetotelluric methods:

- Radioactive methods

4. Geophysical methods

- Passive method: natural sources are used: gravity methods, magnetic methods, magnetotelluric method

- Active method: artificial sources are used: seismic method, electric method

표 1.1 물리탐사 방법과 적용분야

탐사방법	장	이용되는 물성	적용분야 1	2	3	4	5	6	7	8	9	10
중력탐사	2	밀도	**P**	**P**	s	s	s	s	!	!	s	!
자력탐사	3	대자율	**P**	**P**	**P**	s	!	m	!	**P**	**P**	!
굴절법 탄성파탐사	4, 5	탄성계수, 밀도	**P**	**P**	m	**P**	s	s	!	!	!	!
반사법 탄성파탐사	4, 6	탄성계수, 밀도	**P**	**P**	m	s	s	m	!	!	!	!
전기비저항탐사	7	전기비저항	m	m	**P**	**P**	**P**	**P**	**P**	s	**P**	m
자연전위(SP)탐사	8	전위차	!	!	**P**	m	**P**	m	m	m	!	!
유도분극(IP)탐사	9	전기비저항, 커패시턴스	m	m	**P**	m	s	m	m	m	m	m
전자(EM)탐사	10	컨덕턴스, 인덕턴스	s	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	m
VLF 전자탐사	11	컨덕턴스, 인덕턴스	m	m	**P**	m	s	s	s	m	m	!
GPR탐사	12	유전율, 전기전도도	!	!	m	**P**	**P**	**P**	s	**P**	**P**	**P**
MT탐사	11	전기비저항	s	**P**	**P**	m	m	!	!	!	!	!

P = 1차로 우선 적용, s = 2차 적용, m = 사용 가능하나 최적의 방법은 될 수 없음. 또는 현 상황에서는 이 분야에 적용되지 못하고 있음,
! = 부적합

적용분야

1 : 탄화수소 탐사(석탄, 석유, 가스)
2 : 광역 지질 연구(수백 km^2 정도의 넓은 지역)
3 : 광물 탐사 및 개발
4 : 토목 · 건설 공사부지 조사
5 : 수리지질학적 조사
6 : 지하공동 탐지
7 : 침출수 및 오염물질의 오염범위 파악
8 : 매몰된 금속물의 위치 및 종류 파악
9 : 고고학적 유물 조사
10 : 법과학 분야(예 : 범죄 수사시 매몰 시신 및 은닉 증거물 등 탐지)

Table 1.1 Geophysical methods and their main applications

Geophysical method	Chapter number	Dependent physical property	Applications (see key below) 1	2	3	4	5	6	7	8	9	10
Gravity	2	Density	**P**	**P**	s	s	s	s	!	!	s	!
Magnetic	3	Susceptibility	**P**	**P**	**P**	s	!	m	!	**P**	**P**	!
Seismic refraction	4,5	Elastic moduli; density	**P**	**P**	m	**P**	s	s	!	!	!	!
Seismic reflection	4,6	Elastic moduli; density	**P**	**P**	m	s	s	m	!	!	!	!
Resistivity	7	Resistivity	m	m	**P**	**P**	**P**	**P**	**P**	s	**P**	m
Spontaneous potential	8	Potential differences	!	!	**P**	m	**P**	m	m	m	!	!
Induced polarization	9	Resistivity; capacitance	m	m	**P**	m	s	m	m	m	m	m
Electromagnetic (EM)	10	Conductance; inductance	s	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	m
EM–VLF	11	Conductance; inductance	m	m	**P**	m	s	s	s	m	m	!
EM – ground penetrating radar	12	Permitivity; conductivity	!	!	m	**P**	**P**	**P**	s	**P**	**P**	**P**
Magneto-telluric	11	Resistivity	s	**P**	**P**	m	m	!	!	!	!	!

P = primary method; s = secondary method; m = may be used but not necessarily the best approach, or has not been developed for this application; (!) = unsuitable

Applications

1 Hydrocarbon exploration (coal, gas, oil)
2 Regional geological studies (over areas of 100s of km^2)
3 Exploration/development of mineral deposits
4 Engineering site investigations
5 Hydrogeological investigations
6 Detection of sub-surface cavities
7 Mapping of leachate and contaminant plumes
8 Location and definition of buried metallic objects
9 Archaeogeophysics
10 Forensic geophysics

Electric Resistivity Methods

1. Introduction

- Electric resistivity methods were developed in the early 1900s, but have become more widely used since the 1970s with the availability of computers to process and analyze the data

- Main application
  - ✓ In **groundwater** survey: they are extensively used in the search for suitable groundwater sources and to monitor types of groundwater pollution.
  - ✓ In **engineering** surveys: they are used to locate subsurface cavities, faults and fissures, permafrost, mineshafts, etc.
  - ✓ In **archaeology**: they are employed for mapping out the areal extent of remnants of buried foundations of ancient buildings.

- Electric resistivity is the physical property determined by the electric resistivity methods

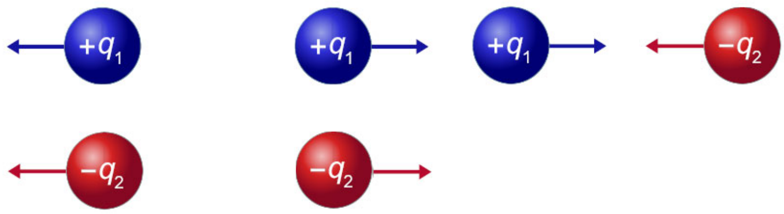
- Electrical resistivity, which is a fundamental and diagnostic physical property, can be determined by a wide variety of techniques, including electromagnetic induction. These methods will be discussed later.

2. Principles and elementary theory

(1) Coulomb Forces and Electric Field Intensity

A. Coulomb's law

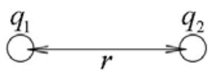
- If two charged particles are brought near each other, they each exert a force on the other. If the particles have the same sign of charge, they repel each other. That is, the force on each particle is directed away from the other particle. If the particles have opposite signs of charge, they attract each other



- This force of repulsion or attraction due to the charge properties of objects is called an electrostatic force. The equation giving the force for charged particles is called Coulomb's law
- In Coulomb's law, the electrostatic force between two charges is directly proportional to the charge magnitudes and inversely proportional to the square of the separation distance.

  The unit of charge is the Coulomb

$$\vec{\mathbf{F}} = k\frac{q_1 q_2}{r^2}\vec{r}$$



attraction or repulsion

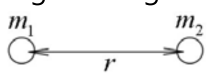
k : electrostatic constant $= {}^{1}\!/_{4\pi\varepsilon_0} = 9\times10^{9}\, m\,/\,farad$

ε_0 : permittivityconstant $= 8.85\times10^{-12}\, C^2 / N\cdot m^2$

$= 8.85\times10^{-12}\, farad\,/\,m$

This force is analogous to gravitational force

$$\vec{\mathbf{F}} = G\frac{m_1 m_2}{r^2}\vec{r}$$



always attraction

- Electric current is the rate *dq/dt* at which charge moves past a point or through a region.

  $$i = \frac{dq}{dt},\ dq = i \cdot dt,\ C = A \cdot s$$

B. Electric Field Intensity

- Suppose that one of the charges q_2 is sufficiently small so as not to disturb significantly the field of the fixed point charge q_1:

  $$\vec{E} = \frac{\vec{F}}{q_2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{q_2 r^2}\vec{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\vec{r}$$

  $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\vec{r}$$

C. Charge Distributions

- **Point Charges**
  - ✓ Electric field due to several point charges are vector sum of electric fields due to each charge.

    $$\vec{E} = \sum_{i=1}^{N}\frac{q_i}{4\pi\varepsilon_0 r_i^2}$$

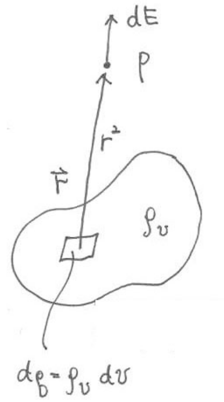
- **Volume Charge**
  - ✓ Assume that charge is distributed throughout a specified volume, and distributions are continuous rather than discrete. When charge density is defined by ρ_v

    $$\rho_v = \frac{dq}{dv},\quad dq = \rho_v dv$$

  - ✓ With reference to volume v, each differential charge dq produces a differential electric field at the observation point P as follows

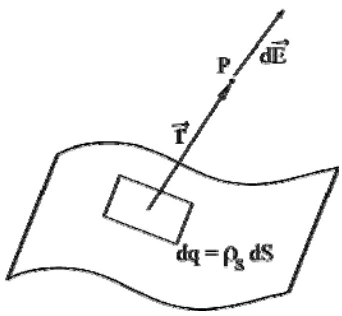
$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\vec{r}$$

- ✓ The total electric field at P is obtained by integration over the volume



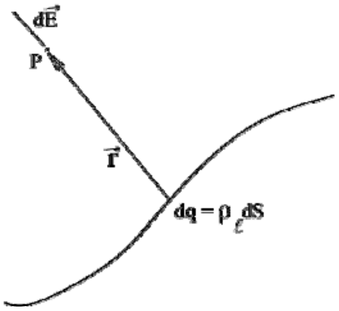
$$E = \int_v d\vec{E} = \int_v \frac{\rho_v dv}{4\pi\varepsilon_0 r^2}$$

- **Surface Charge**
  - ✓ Charge may also be distributed over a surface or a sheet. Differential charge dq on the sheet results in a differential electric field at point P as $dq = \rho_s ds$
  - ✓ The total electric field at P is



$$\vec{E} = \int d\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2}\vec{r} = \int_S \frac{\rho_v \vec{r}}{4\pi\varepsilon_0 r^2} dS$$

- **Line charge**
  - ✓ If charge is distributed over a line, each differential charge dQ along the line produces a differential electric field at P

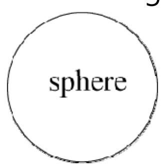


$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\vec{r}$$

- ✓ The total electric field at P is

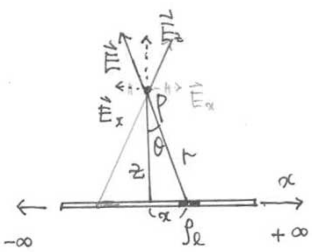
$$\vec{E} = \int_L \frac{\rho_v \vec{r}}{4\pi\varepsilon_0 r^2} dl$$

✧ Point Charge



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_v \vec{r}}{r^2} dV = \frac{1}{4\pi\varepsilon_0}\frac{\rho}{r^2}\frac{4\pi R^3}{3}\vec{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\vec{r}$$

✧ Infinite Line Charge

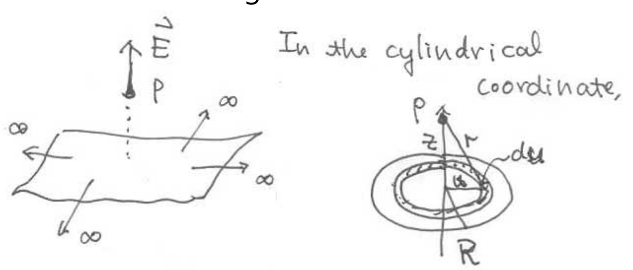


perpendicular components to z axis are cancelled

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{\infty} \frac{\rho_l \vec{z} dx}{r^2}\cos\theta = \frac{\rho_l}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{z}{r^2}dx = \frac{\rho_l}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{z}{\left(x^2+z^2\right)^{3/2}}dx$$

$$= \frac{\rho_l}{4\pi\varepsilon_0}\left[\frac{zx}{z^2(x^2+z^2)^{1/2}}\right]_{-\infty}^{\infty} = \frac{\rho_l}{2\pi\varepsilon_0 z}\vec{z}$$

✧ Infinite Plane Charge



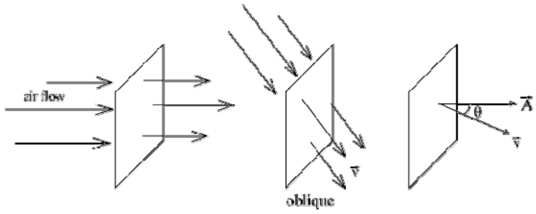
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{\rho_S \cdot 2\pi u \cdot du}{r^2}\cos\theta = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{\rho_S \cdot 2\pi u \cdot z}{(u^2+z^2)^{3/2}}du$$

$$= \frac{\rho_S \cdot z}{4\varepsilon_0}(-2)\left[\frac{1}{(u^2+z^2)^{1/2}}\right]_0^R = \frac{\rho_S z}{2\varepsilon_0}\left[\frac{1}{z} - \frac{1}{(R^2+z^2)^{1/2}}\right] \approx \frac{\rho_S}{2\varepsilon_0}\vec{z}\left(R\to\infty\right)$$

(2) Electric Flux and Gauss' Law

A. Electric Flux

- **Flux**
  - ✓ Consider a wide airstream of uniform velocity v at a small square loop of area A. Let Φ represent the volume flow rate (volume per unit time) at which air flows through the loop.



$$\phi = (v\cos\theta)A = \vec{v}\cdot\vec{A}$$

  - ✓ This rate depends on the angle between v and the plane of the loop. If v is perpendicular to the plane, the rate Φ is equal to (vA). If v is parallel to the plane of the loop, Φ is equal to (0). For an intermediate angle θ, the rate Φ depends of the component of v normal to the plane. Φ = ($vA\cos\theta = \vec{v}\cdot\vec{A}$).

  - ✓ This rate of flow through an area is an example of a flux. The word "flux" comes from the Latin word meaning "to flow."

- **Flux of an Electric field**

  - ✓ To define the flux of an electric field, consider an arbitrary Gaussian surface immersed in a nonuniform electric field.
  - ✓ Let us divide the surface into small squares of area ΔA, each square being small enough to permit us to neglect any curvature and to consider the individual square to be flat.
  - ✓ Because the squares have been taken to be arbitrarily small, the electric field E may be taken as constant over any given square. The vectors A and E make some angle θ with each other.
  - ✓ The flux of the electric field for the Gaussian surface (that is defined

as a imaginary closed surface enclosing the charge distribution) is

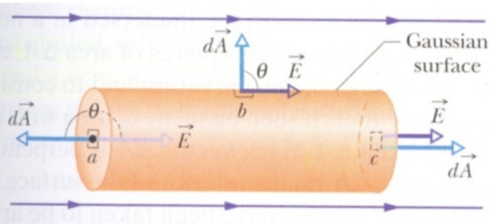
$$\phi = \sum \vec{E} \cdot \Delta\vec{A}$$

- ✓ By allowing the area of the squares to become smaller and smaller, the area vector of the squares can be written as dA, and the sum can be expressed by an integral.

  $\phi = \oint \vec{E} \cdot d\vec{A}$: electric flux through a Gaussian surface

- ✓ The flux of the electric field is a scalar, and its SI unit is ($N \cdot m^2 / C$). The flux is proportional to the number of electric field lines passing through area dA.

Problem: Consider a Gaussian surface in the form of a cylinder of radius R immersed in a uniform electric field E, with the cylinder axis parallel to the field. What is the flux Φ of the electric field through this closed surface?



$$\phi = \oint \vec{E} \cdot d\vec{A} = \oint_a \vec{E} \cdot d\vec{A} + \oint_b \vec{E} \cdot d\vec{A} + \oint_c \vec{E} \cdot d\vec{A}$$

$$= EA\cos 180 + EA\cos 90 + EA\cos 0 = -EA + 0 + EA = 0$$

it represents electric fields entering through the left end cap. leaving through the right end cap, and giving a net flux of zero

B. Gauss' Law

- ➢ Gauss' law relates the net flux Φ of an electric field through a closed surface (a Gaussian surface) to the new charge q_{enc} that is enclosed by that surface. It tells us that

  $\varepsilon_0 \phi = q_{enc}$

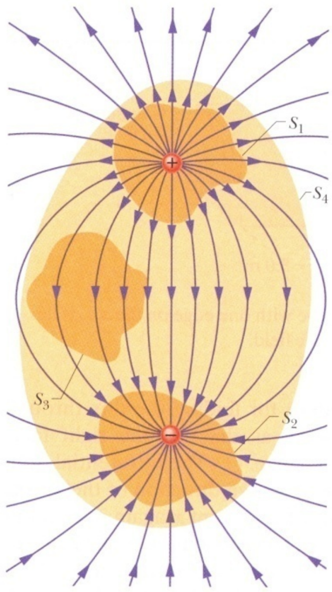
  $\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$

- ➢ These equations hold only when the net charge is located in a vacuum or in air.
- ➢ The net charge q_{enc} is the algebraic sum of all the enclosed positive

and negative charges, and it can be positive, negative, or zero.

- If qenc is positive, the net flux is outward; if q_{enc} is negative, the net flux is inward.
- Charge outside the surface, no matter how large or how close it may be, is not included in the term q_{enc} in Gauss' law. The exact form and location of the charges inside the Gaussian surface are also of no concern; the only things that matter are the magnitude and sign of the net enclosed charge.

Problem: Determine the direction of the electric field and the sign of the net charge for the surface S_1, S_2, S_3, and S_4.



Surface S_1: The electric field is outward for all points on the surface. the net charge is positive

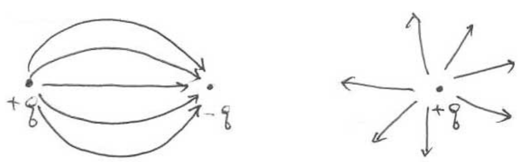
Surface S_2: inward negative

Surface S_3: 0

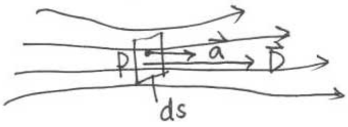
the net flux of the electric field through this surface is zero

Surface S_4: no net change. net flux is zero

- Electric flux Φ originates on positive charge and terminates on negative charge. One coulomb of electric charge gives rise to one coulomb of electric flux

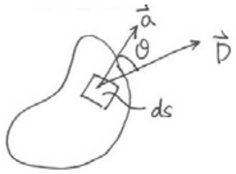


- If an amount of flux dΦ crosses the differential area dS whose normal vector is **a**, then the electric flux density at P is



$$\vec{D} = \frac{d\phi}{dS}\vec{a} \quad (\vec{a} \text{ is the normal vector})$$

- The electric flux density **D** may vary in magnitude and direction from point to point. If **D** makes an angle θ with the normal, then the differential flux crossing d**S** is given by



$$d\phi = D \cdot dS \cdot \cos\theta = \vec{D} \cdot dS \cdot \vec{a} = \vec{D} \cdot d\vec{S}$$

- Gauss' law state that the total flux of a closed surface is equal to the net charge within the surface
  $\oint \vec{D} \cdot d\vec{S} = q_{enc}$

- Consider a point charge q (assumed positive) at the origin. If this is enclosed by a spherical surface of radius r, **D** due to q is of constant magnitude over the surface and everywhere normal to the surface.

$$q = \oint \vec{D} \cdot d\vec{S} = D\int dS = D(4\pi r^2)$$

$$D = \frac{q}{4\pi r^2}$$

$$\vec{D} = \frac{q}{4\pi r^2}\vec{a}$$

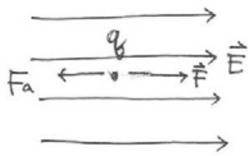
$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\vec{a}, \ \vec{D} = \varepsilon_0\vec{E}$$

$\vec{D} = \varepsilon\vec{E}$: more generally, for any electric field in an isotropic medium of permittivity ε

(3) The electrostatic field: work, energy, and potential

A. Work done in moving a point charge

- A charge q experiences a force **F** in an electric field E. In order to maintain the charge in equilibrium, a force $\mathbf{F_a}$ must be applied in opposition

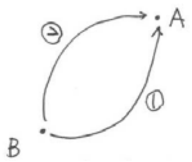


$$\vec{\mathbf{F}} = q\vec{E}$$
$$\vec{F}_a = -q\vec{E}$$

- Work is defined as the force acting over a distance. A differential amount of work dW is done when the applied force $\mathbf{F_a}$ produces a differential displacement d**l** of the charge; i.e., moves the charge through the distance dl = |d**l**|.
  $$dW = \vec{F}_a \cdot d\vec{l} = -q\vec{E} \cdot d\vec{l}$$

  Note that when q is positive and d**l** is in the direction of **E**, dW=$-qEdl$ < 0, indicating that work was done by the electric field. On the other hand, a positive dW indicates work done against the electric field

B. Conservative property of the electrostatic field
- The work, done in moving a point charge from one location B to another A in a static electric field, is independent of the path taken.



$$\int_1 \vec{\mathbf{E}} \cdot d\vec{l} = \int_2 \vec{\mathbf{E}} \cdot d\vec{l} \quad \Rightarrow \quad \int_{1-2} \vec{\mathbf{E}} \cdot d\vec{l} = 0$$

C. Electric potential between two points
- The potential of point A with respect to point B is defined as the work done in moving a unit positive charge, q_u, from B to A
  $$V_{AB} = \frac{W}{q_u} = -\int_B^A \vec{\mathbf{E}} \cdot d\vec{l} \quad (J/C \; or \; V)$$

  $*\vec{\mathbf{E}}$ is a conservative field

  $V_{AB} = V_{AC} - V_{BC}$: is considered as the potential difference between point A and B. when is positive, work must be done to move the unit positive charge from B to A, and point A is said to be at a higher potential than point B.

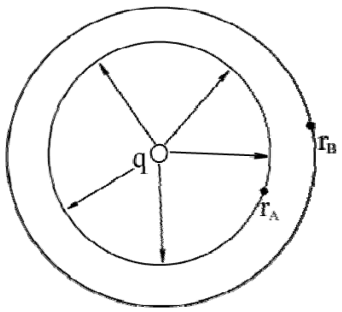
D. Potential of a point charge
- The electric field due to a point charge q is completely in the radial

direction:

$$V_{AB} = -\int_B^A \vec{\mathbf{E}} \cdot d\vec{l} = -\int_{r_B}^{r_A} \vec{\mathbf{E}} \cdot d\vec{r} = -\int_{r_B}^{r_A} \mathbf{E} \cdot dr$$

$$= -\frac{q}{4\pi\varepsilon_0}\int_{r_B}^{r_A} \frac{1}{r^2} \cdot dr = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

when $r_A < r_B$, for a positive charge q, point A is at a higher potential than B.

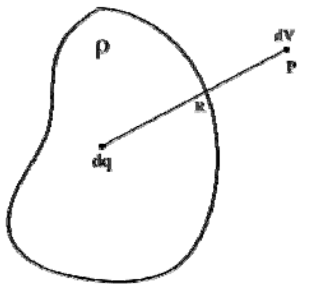


If the reference point B is allowed to move out to infinity,

$$V_{A\infty} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{\infty}\right) \Rightarrow V = \frac{q}{4\pi\varepsilon_0 r}$$

E. Potential of a charge distribution

- If charge is distributed throughout some finite volume with a known charge density ρ (C/m^3), then the potential at some external point can be determined.



$$dV = \frac{dq}{4\pi\varepsilon_0 R} \qquad dq = \rho dV$$

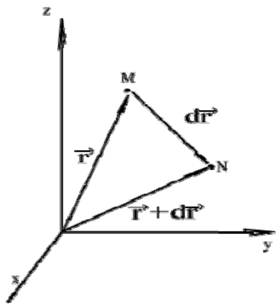
- Integration over the volume gives the total potential at P:

$$V = \int_V \frac{\rho \cdot dV}{4\pi\varepsilon_0 R}$$

- If charge is distributed over a surface or a line, the above expression for V holds, provided that the integration is over the surface or the line and that ρ_s or ρ_l is used in place of ρ. It must be emphasized that all these expressions for the potential at an external point are based upon a zero reference at infinity.

F. Gradient

- The vector separation of the two neighboring points M and N is



$$d\vec{r} = dx\vec{x} + dy\vec{y} + dz\vec{z}$$

$$\nabla V = \frac{\partial V}{\partial x}\vec{x} + \frac{\partial V}{\partial x}\vec{y} + \frac{\partial V}{\partial z}\vec{z} \quad \text{(cartesian)}$$

$$\nabla V = \frac{\partial V}{\partial r}\vec{r} + \frac{\partial V}{r\partial \phi}\vec{\phi} + \frac{\partial V}{\partial z}\vec{z} \quad \text{(cylindrical)}$$

$$\nabla V = \frac{\partial V}{\partial r}\vec{r} + \frac{\partial V}{r\partial \theta}\vec{\theta} + \frac{\partial V}{r\sin\theta\partial \phi}\vec{\phi} \quad \text{(spherical)}$$

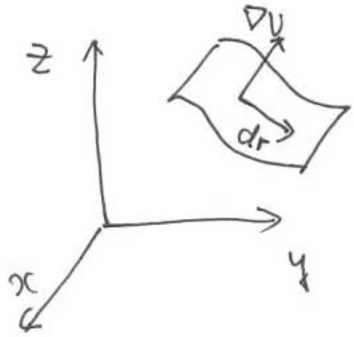
- From the calculus, the change in V from M to N is given by

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

- The del operator operating on V gives

$$\nabla V = \frac{\partial V}{\partial x}\vec{x} + \frac{\partial V}{\partial x}\vec{y} + \frac{\partial V}{\partial z}\vec{z}$$

- Gradient



$$dV = \nabla V \bullet d\vec{r}$$

∇V : the gradient of the scalar function V

The change in V in a given direction $d\vec{r}$ is proportional to the projection of ∇V in that direction.

∇V lies in the direction of maximum increase of the function V

* equipotential surface: V(x,y,z)=c, dV=0

which implies that ∇V is perpendicular to $d\vec{r}$.

The gradient of a potential function is a vector field that is every where normal to the equipotential surface.

G. Relationship between E and V

- From the integral expression for the potential of A with respect to B, the differential of V may be written

$$dV = -\vec{E} \bullet d\vec{l}$$

- The differential of V can also be expressed by

$$dV = \nabla V \bullet d\vec{r}$$

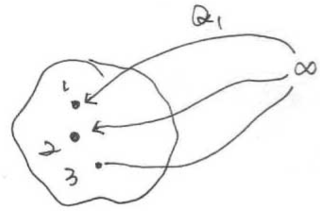
- Since dl=dr is an arbitrary small displacement, it follows that

$$\vec{E} = -\nabla V$$

- The electric field intensity E may be obtained when the potential function V is known by simply taking the negative of the gradient of V. The gradient was found to be a vector normal to the equipotential surfaces, directed to a positive change in V. With the negative sign here, the E field is found to be directed from higher to lower levels of potential V.

H. Energy in static electric fields

- Consider the work required to assemble, charge by charge, a distribution of n=3 point charges. The region is assumed initially to be charge-free and with E=0 throughout.
- The work required to place the first charge q_1 into position 1 is zero. Then, when q_2 is moved toward the region, work equal to the product of this charge and potential due to q_1 is required. The total work to position the three charges is



$$W_E = W_1 + W_2 + W_3$$
$$= 0 + (q_2 V_{2,1}) + (q_3 V_{3,1} + q_3 V_{3,2})$$

$V_{2,1}$: the portential at point 2 due to charge q_1 at position 1."

- If the three charges were brought into place in reverse order, the total work would be

$$W_E = W_3 + W_2 + W_1$$
$$= 0 + (q_2 V_{2,3}) + (q_1 V_{1,3} + q_1 V_{1,2})$$

- When the two expressions above are added, the result is twice the stored energy:

$$2W_E = q_1(V_{1,2} + V_{1,3}) + q_2(V_{2,1} + V_{2,3}) + q_3(V_{3,1} + V_{3,2})$$

* $q_1(V_{1,2} + V_{1,3})$ is the work done against the fields due to q_2 and q_3.

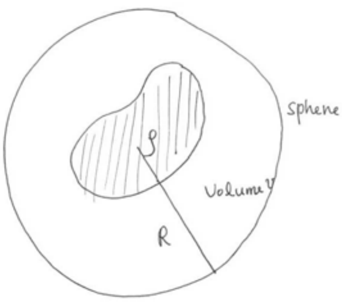
$$W_E = \frac{1}{2}[q_1V_1 + q_2V_2 + q_3V_3] = \frac{1}{2}\sum_{m=1}^{n} q_m V_m$$

- Integral form of the work:

$$W_E = \frac{1}{2}\int qVdV$$

- Other forms of the expression for stored energy are

$$W_E = \frac{1}{2}\int\left(\vec{D}\bullet\vec{E}\right)dv = \frac{1}{2}\int \varepsilon E^2 dv = \frac{1}{2}\int \frac{D^2}{\varepsilon}dv$$



$$W_E = \frac{1}{2}\int_v qVdv = \frac{1}{2}\int_v \left(\nabla\bullet\vec{D}\right)Vdv = \frac{1}{2}\int_v \left(\nabla\bullet V\vec{D} - \vec{D}\bullet\nabla V\right)dv$$

$$\int_V qdv = \oint \vec{D}\bullet d\vec{s} = q_{enc} = \int\left(\nabla\bullet D\right)dv$$

$$q = \nabla\bullet D$$

$$\begin{pmatrix} \nabla\bullet a\vec{A} = \vec{A}\bullet\nabla a + a\left(\nabla\bullet\vec{A}\right) \\ \therefore a\left(\nabla\bullet\vec{A}\right) = \nabla\bullet a\vec{A} - \vec{A}\bullet\nabla a \end{pmatrix}$$

$$\frac{1}{2}\int_v \nabla\bullet V\vec{D}dv + \frac{1}{2}\int_v \vec{D}\bullet\vec{E}dv$$

$\left(\frac{1}{2}\oint V\vec{D}\bullet d\vec{s}\right)$ as the enclosing sphere becomes very large, the enclosed volume charge looks like a point charge.

At the surface, D appears as k_1/R^2,

V appears as k_2/R

Integral is decreasing as $1/R^3$

$$W_E = \frac{1}{2}\int\left(\vec{D}\bullet\vec{E}\right)dv = \frac{1}{2}\int \varepsilon E^2 dv = \frac{1}{2}\int \frac{D^2}{\varepsilon}dv$$

(4) Ohm's Law

A. Electric current

- Although an electric current is a stream of moving charges, not all moving charges constitute an electric current.
- If there is to be an electric current through a given surface, there must be a net flow of charge through that surface.

$$i = \frac{dq}{dt} \text{(definition of current)}$$

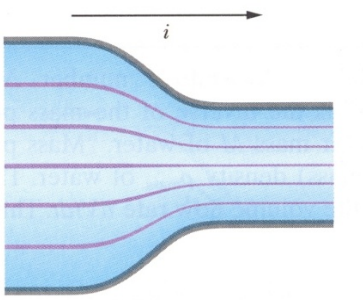
$$q = \int dq = \int_0^t i dt \quad \text{unit:} \; c/s = 1A$$

B. Current Density

- To describe the electric current, we can use the current density J.
- For each element of the cross section, the magnitude J is equal to the current per unit area through that element.
- We can write the amount of current though the element as ($\vec{J} \bullet d\vec{A}$).
- The total current through the surface is

$$i = \int \vec{J} \bullet d\vec{A}$$

- If the current is uniform across the surface and parallel to dA, then J is also uniform and parallel to dA. Then,



$$i = \int J dA = J \int dA = JA$$

$$J = \frac{i}{A}, \text{ unit: } A/m^2$$

- The current, which is toward the right in the figure, makes a transition from the wider conductor at the left to the narrower conductor at the right.
- Because charge is conserved during the transition, the amount of charge and the amount of current cannot change.
- However, the current density does change: it is greater in the narrower conductor.

C. Resistance and Resistivity

- The resistance is determined by a potential difference V between two points and the current i:

$$R = \frac{V}{i} \quad \text{(definition of R)}$$

$$1ohm = 1\Omega = 1V/A$$

- Resistivity is defined by the electric field and the current density

$$\rho = \frac{E}{J} \quad \text{(definition of } \rho\text{), unit: } \frac{V/m}{A/m^2} = \frac{V}{A}m = \Omega m$$

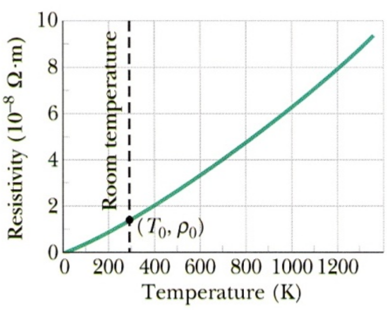
- Conductivity is

$$\sigma = \frac{1}{\rho} \quad \text{(definition of } \sigma\text{)}$$

- Calculating Resistance from Resistivity
  - Let A be the cross-sectional area of the wire, let L be its length, and let a potential difference V exist between the ends of the wire.
  - The electric field and current density are constant for all points within the wire.

$$E = V/L, \quad J = i/A, \quad \rho = \frac{E}{J} = \frac{V/L}{i/A} = \frac{V}{i}\frac{A}{L} = R\frac{A}{L}, \quad R = \rho\frac{L}{A}$$

- Variation with temperature
  - The values of most physical properties vary with temperature, and resistivity is no exception.
  - Figure shows the variation of this property for copper over a wide temperature range. The relation between temperature and resistivity for copper–and for metals in general–is fairly linear over a rather broad temperature range.



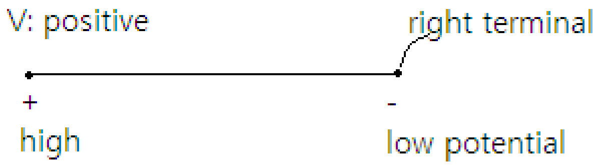
For such linear relations, we can write an empirical approximation

$$\rho - \rho_0 = \rho_0 \propto (T - T_0)$$

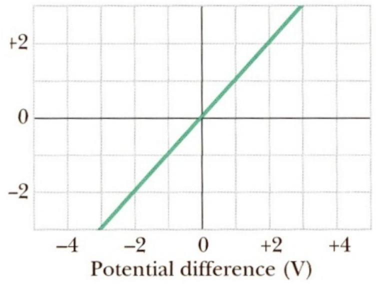
T_0 is a selected reference temperature.

ρ_0 is the resistivity at the reference temperature.

$T_0 = 293K, \quad \rho_0 = 1.69\times 10^{-8}\Omega m$ for copper



D. Ohm's Law



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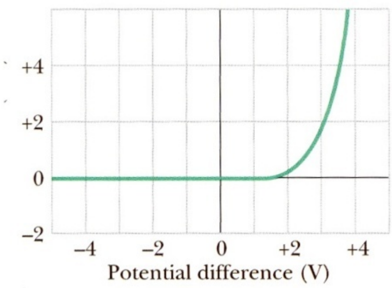


(*c*)

- Ohm's law is an assertion that the current through a device is always directly proportional to the potential difference applied to the device. $V \propto i$
- A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference

$$V = iR$$

3. Electric resistivity method

(1) Introduction

- The objective of most modern electric resistivity surveys is to obtain true resistivity models for the subsurface because they have geological meaning.

- Electrical prospecting involves the detection of surface effects produced by electric current flow in the ground

- In electric resistivity method, an artificial source of current is introduced into the ground through point electrodes. We measure potentials at other electrodes in the vicinity of the current flow.
  In practice, potential differences are measured rather than potentials, potential is not easy to measure.

- Because we measure the currents as well as the potentials, it is possible to determine an apparent resistivity of the subsurface.

- The resistivity that we obtain in field data is apparent resistivity rather than true resistivity, because geometrical factors are included. If the subsurface is homogeneous, then the apparent resistivity is the same as the true resistivity. In most cases, subsurface media are not homogeneous.

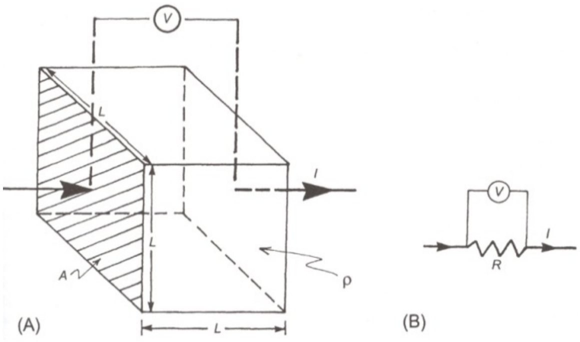
- In order to describe subsurface resistivity, one may process and interpret them.

(2) Electrical properties of rocks and minerals

- Several electrical properties of rocks and minerals are significant in electrical prospecting. They can be natural electrical potentials, electrical conductivity (or the inverse, electrical resistivity), and the dielectric constant. Magnetic permeability is also an indirect factor.

$$electric\ conductivity = \frac{1}{electric\ resistivity}$$

- Among them, electrical conductivity is the most important, whereas the others are of minor significant.
- Resistivity and conductivity



$$\rho = \frac{E}{J}, \quad E = \frac{V}{L}, \quad J = \frac{I}{A}$$
$$\therefore \rho = \frac{V/L}{I/A} = \frac{VA}{IL} = R\frac{A}{L}$$

- A conductor is usually defined as a material of resistivity less than $10^{-5}\Omega m$. The conductors contain a large number of free electrons whose mobility is very great.

- Semiconductors lie between conductor and insulator. The semiconductors also carry current by mobile electrons but have fewer of them

- An insulator is one having a resistivity greater than $10^{7}\Omega m$. Insulators are characterized by ionic bonding so that the valence electrons are not free to move; the charge carriers are ions that must overcome larger barrier potentials than exist either in the semiconductors or conductors.

- A further difference between conductors and semiconductors is found in their respective variation with temperature. The former vary inversely with temperature and have their highest conductivities in the region of 0 K. The semiconductors, on the other hand, are practically insulators at low temperatures.

- In a loose classification, rocks and minerals are considered to be good, intermediate, and poor conductors within the following ranges.
  - Minerals of resistivity (10^{-8}) to about ($1\Omega m$)-> good conductor
  - Minerals and rocks of resistivity (1) to ($10^{7}\Omega m$)-> intermediate

- Minerals and rocks of resistivity above ($10^7 \Omega m$)-> poor conductor

➢ The resistivity of geological materials exhibits one of the largest ranges of all physical properties, from 1.6×10^{-8} Ωm for native silver to 10^{16} Ωm for pure sulphur.

- Igneous rocks tend to have the highest resistivities
- Sedimentary rocks tend to be most conductive, largely due to their high pore fluid content
- Metamorphic rocks have intermediate but overlapping resistivities.

- The age of a rock is also an important consideration: for example, the resistivity range of Precambrian volcanic rocks is 200 – 5,000 Ωm, whereas for Quaternary volcanic rocks it is 10-200 Ωm.
  The older a rocki is, the larger the resistivity is. -> due to compaction

- The effect of water content on the bulk resistivity of rocks is evident. In most cases a small change in the percentage of water affects the resistivity enormously. Saline groundwater may have a resistivity as low as 0.05 Ωm and some groundwater and glacial melt water can have resistivities in excess of 1000 Ωm.

✧ Refer to Tables

Table 5.1. *Resistivities of minerals*

Mineral	Formula	Resistivity (Ωm) Range	Average
Bismuthinite	Bi_2S_3	$18-570$	
Covellite	CuS	$3\times10^{-7}-8\times10^{-5}$	2×10^{-5}
Chalcocite	CU_2S	$3\times10^{-5}-0.6$	10^{-4}
Chalcopyrite	$CuFeS_2$	$1.2\times10^{-5}-0.3$	4×10^{-3}
Bornite	Cu_5FeS_4	$2.5\times10^{-5}-0.5$	3×10^{-3}
Pyrite	FeS_2	$2.9\times10^{-5}-1.5$	3×10^{-1}
Pyrrhotite	Fe_nS_m	$6.5\times10^{-6}-5\times10^{-2}$	10^{-4}
Cinnabar	HgS		2×10^{7}
Molybdenite	MoS_2	$10^{-3}-10^{6}$	10
Galena	PbS	$3\times10^{-5}-3\times10^{2}$	2×10^{-3}
Millerite	NiS		3×10^{-7}
Stannite	Cu_2FeSnS_2	$10^{-3}-6\times10^{3}$	
Stibnite	Sb_2S_3	$10^{5}-10^{12}$	5×10^{6}
Sphalerite	ZnS	$1.5-10^{7}$	10^{2}
Cobaltite	$CoAsS$	$3.5\times10^{-4}-10^{-1}$	
Arsenopyrite	$FeAsS$	$2\times10^{-5}-15$	10^{-3}
Niccolite	$NiAs$	$10^{-7}-2\times10^{-3}$	2×10^{-5}
Bauxite	$Al_2O_3\cdot nH_2O$	$2\times10^{2}-6\times10^{3}$	
Cuprite	Cu_2O	$10^{-3}-300$	30
Chromite	$FeCr_2O_4$	$1-10^{6}$	
Specularite	Fe_2O_3		6×10^{-3}
Hematite	Fe_2O_3	$3.5\times10^{-3}-10^{7}$	
Limonite	$2Fe_2O_3\cdot 3H_2O$	$10^{3}-10^{7}$	
Magnetite	Fe_3O_4	$5\times10^{-5}-5.7\times10^{3}$	
Ilmenite	$FeTiO_3$	$10^{-3}-50$	
Wolframite	Fe, Mn, WO_4	$10-10^{5}$	
Pyrolusite	MnO_2	$5\times10^{-3}-10$	
Quartz	SiO_2	$4\times10^{10}-2\times10^{14}$	
Cassiterite	SnO_2	$4\times10^{-4}-10^{4}$	0.2
Rutile	TiO_2	$30-1000$	500
Uraninite (pitchblende)	UO_2	$1-200$	
Anhydrite	$CaSO_4$		10^{9}
Calcite	$CaCO_3$		2×10^{12}
Fluorite	CaF_2		8×10^{13}
Siderite	$Fe_2(CO_3)_3$		70
Rock salt	$NaCl$	$30-10^{13}$	
Sylvite	KCl	$10^{11}-10^{12}$	
Diamond	C	$10-10^{14}$	
Serpentine		$2\times10^{2}-3\times10^{3}$	
Hornblende		$2\times10^{2}-10^{6}$	
Mica		$9\times10^{2}-10^{14}$	
Biotite		$2\times10^{2}-10^{6}$	
Bitum. coal		$0.6-10^{5}$	
Anthracite		$10^{-3}-2\times10^{5}$	
Lignite		$9-200$	
Fire clay			30
Meteoric waters		$30-10^{3}$	
Surface waters (ign. rocks)		$0.1-3\times10^{3}$	
Surface waters (sediments)		$10-100$	
Soil waters			100
Natural waters (ign. rocks)		$0.5-150$	9
Natural waters (sediments)		$1-100$	3
Sea water			0.2
Saline waters, 3%			0.15
Saline waters, 20%			0.05

Table 5.2. *Resistivities of various ores*

Ore	Other minerals	Gangue	ρ (Ωm)
Pyrite			
18%	2% (chalco)	80%	300
60%	5% (ZnS) + 15%	20%	0.9
95%	5% (ZnS)		1.0
Pyrrhotite			
41%		59%	2.2×10^{-4}
79%		21%	1.4×10^{-5}
95%		5%	1.4×10^{-5}
SbS_2 in quartz			$4 \times 10^3 - 3 \times 10^7$
FeAsS 60%	FeS 20%	20% SiO_2	0.39
FeAsS			$10^{-4} - 10^{-2}$
Cu_5FeS_4			3×10^{-3}
Cu_5FeS_4 40%		60% SiO_2	7×10^{-2}
Fe, Mn, WO_4	CoAsS		$10^3 - 10^7$
PbS, near massive			0.8
Fe_2O_3			0.1 – 300
Fe_2O_3, massive			2.5×10^3
Iron			
Fe_3O_4 60%			45
75% brown iron oxide		25%	$2 \times 10^4 - 8 \times 10^5$
Fe_3O_4			$5 \times 10^3 - 8 \times 10^3$
Zinc			
30%	5% PbS, 15% FeS	50%	0.75
80%	10% PbS, 10% FeS		1.7×10^3
90%	5% PbS	5%	130
Graphitic slate			0.13
Graphite, massive			$10^{-4} - 5 \times 10^{-3}$
MoS_2			$2 \times 10^2 - 4 \times 10^3$
MnO_2 colloidal ore			1.6
Cu_2S			3×10^{-2}
$CuFeS_2$			$10^{-4} - 1$
$CuFeS_2$ 90%	2% FeS	8% SiO_2	0.65
$FeCr_2O_4$			10^3

Table 5.3. *Resistivities of various rocks and sediments*

Rock type	Resistivity range (Ωm)
Granite porphyry	4.5×10^3 (wet) – 1.3×10^6 (dry)
Feldspar porphyry	4×10^3 (wet)
Syenite	$10^2 - 10^6$
Diorite porphyry	1.9×10^3 (wet) – 2.8×10^4 (dry)
Porphyrite	$10 - 5 \times 10^4$ (wet) – 3.3×10^3 (dry)
Carbonatized porphyry	2.5×10^3 (wet) – 6×10^4 (dry)
Quartz diorite	$2 \times 10^4 - 2 \times 10^6$ (wet) -1.8×10^5 (dry)
Porphyry (various)	$60 - 10^4$
Dacite	2×10^4 (wet)
Andesite	4.5×10^4 (wet) – 1.7×10^2 (dry)
Diabase (various)	$20 - 5 \times 10^7$
Lavas	$10^2 - 5 \times 10^4$
Gabbro	$10^3 - 10^6$
Basalt	$10 - 1.3 \times 10^7$ (dry)
Olivine norite	$10^3 - 6 \times 10^4$ (wet)
Peridotite	3×10^3 (wet) – 6.5×10^3 (dry)
Hornfels	8×10^3 (wet) – 6×10^7 (dry)
Schists (calcareous and mica)	$20 - 10^4$
Tuffs	2×10^3 (wet) – 10^5 (dry)
Graphite schist	$10 - 10^2$
Slates (various)	$6 \times 10^2 - 4 \times 10^7$
Gneiss (various)	6.8×10^4 (wet) – 3×10^6 (dry)
Marble	$10^2 - 2.5 \times 10^8$ (dry)
Skarn	2.5×10^2 (wet) – 2.5×10^8 (dry)
Quartzites (various)	$10 - 2 \times 10^8$
Consolidated shales	$20 - 2 \times 10^3$
Argillites	$10 - 8 \times 10^2$
Conglomerates	$2 \times 10^3 - 10^4$
Sandstones	$1 - 6.4 \times 10^8$
Limestones	$50 - 10^7$
Dolomite	$3.5 \times 10^2 - 5 \times 10^3$
Unconsolidated wet clay	20
Marls	3 – 70
Clays	1 – 100
Oil sands	4 – 800

Table 5.4. *Variation of rock resistivity with water content*

Rock	% H_2O	ρ (Ωm)
Siltstone	0.54	1.5×10^4
Siltstone	0.38	5.6×10^8
Coarse grain SS	0.39	9.6×10^5
Coarse grain SS	0.18	10^8
Medium grain SS	1.0	4.2×10^3
Medium grain SS	0.1	1.4×10^8
Graywacke SS	1.16	4.7×10^3
Graywacke SS	0.45	5.8×10^4
Arkosic SS	1.0	1.4×10^3
Organic limestone	11	0.6×10^3
Dolomite	1.3	6×10^3
Dolomite	0.96	8×10^3
Peridotite	0.1	3×10^3
Peridotite	0	1.8×10^7
Pyrophyllite	0.76	6×10^6
Pyrophyllite	0	10^{11}
Granite	0.31	4.4×10^3
Granite	0.19	1.8×10^6
Granite	0	10^{10}
Diorite	0.02	5.8×10^5
Diorite	0	6×10^6
Basalt	0.95	4×10^4
Basalt	0	1.3×10^8
Olivine-pyrox.	0.028	2×10^4
Olivine-pyrox.	0	5.6×10^7

(3) Current flow in a homogeneous earth

- Potentials in Homogeneous Media
  - ✓ Consider a continuous current flowing in an isotropic homogeneous medium. If d**A** is an element of surface and **J** is the current density, then the current passing through d**A** is ($\vec{J} \bullet d\vec{A}$),
  - ✓ The current density **J** and the electric field **E** are related through Ohm's law: ($\rho = \vec{E}/\vec{J}$) $\vec{J} = \sigma \vec{E}$
  - ✓ The electric field is the gradient of a scalar potential ($\vec{E} = -\nabla V$)
  - ✓ The electric current density can be expressed by ($\vec{J} = -\sigma \nabla V$)
  - ✓ If electric current density is constant, then ($\nabla \bullet \vec{J} = 0$)

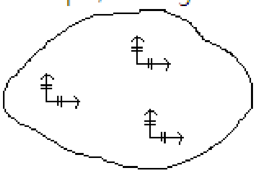
$$[\nabla \bullet (c\vec{B}) = \nabla c \bullet \vec{B} + c \nabla \bullet \vec{B}, \quad c: scalr, \vec{B}: vector]$$

$$\nabla \bullet (\sigma \nabla V) = 0$$
$$\nabla \sigma \bullet \nabla V + \sigma (\nabla \bullet \nabla V) = 0$$
$$\nabla \sigma \bullet \nabla V + \sigma \nabla^2 V = 0$$
$$\text{if } \sigma \text{ is constant, } \nabla \sigma = 0$$
$$\sigma \nabla^2 V = 0$$

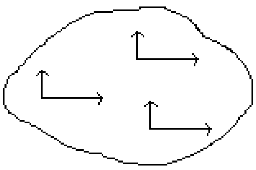
We denote material properties by ⮡

$\therefore \nabla^2 V = 0$ -> Laplace equation, potential is harmonic.

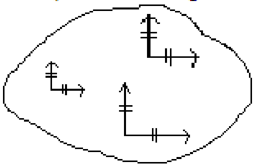
* Isotropic, homogeneous



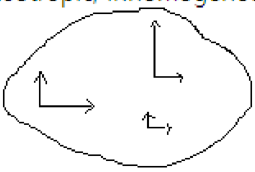
* Anisotropic, homogeneous



* Isotropic, inhomogeneous



* Anisotropic, inhomogeneous



* Boundary conditions

- must be satisfied at any interfaces between two regions of different conductivitivity

σ_1 1

σ_2 2

① potentials should be continuous, $V_1 = V_2$

② The normal components of $\vec{J}$ is continuous

$$J_{1n} = J_{2n}, \quad \sigma_1 E_{1n} = \sigma_1 E_{2n}$$

③ The tangential component of $\vec{E}$ is continuous

$$E_{1t} = E_{2t}$$

* Single current electrode at depth

- An electrode is buried in a homogeneous isotropic medium. The current circuit is completed through another electrode, usually at surface, but in any case, the electrode is located far away so that its influence is negligible.
- The current flows radially outward in all directions from the point electrode.

✓ In spherical coordinates

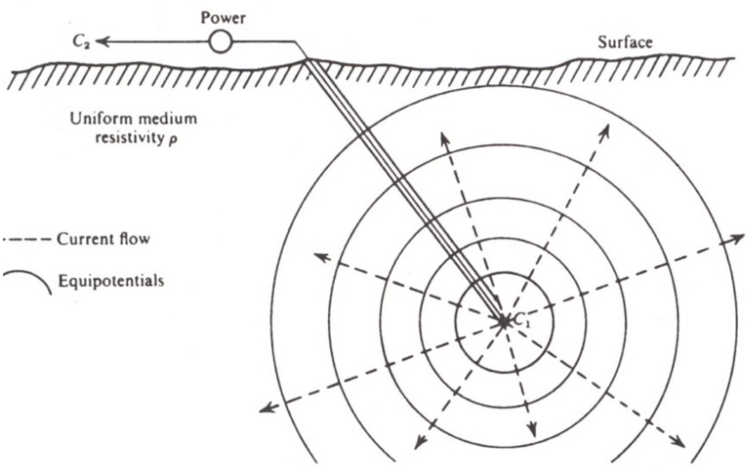


Figure 8.1. Buried point source of current in homogeneous ground.

We just consider the radial components

$$\nabla^2 V = \frac{d^2 V}{dr^2} + \frac{2}{r}\frac{dV}{dr} = 0$$

$$\times r^2$$

$$r^2 \frac{d^2 V}{dr^2} + 2r\frac{dV}{dr} = 0$$

$$\left(\frac{d}{dr}\left[r^2 \frac{d^2 V}{dr^2} \right] = r^2 \frac{d^2 V}{dr^2} + 2r\frac{dV}{dr} \right)$$

$$\frac{d}{dr}\left[r^2\frac{d^2V}{dr^2}\right]=0,\quad \text{intergrate}$$

$$\int\frac{d}{dr}\left[r^2\frac{d^2V}{dr^2}\right]dr=A$$

$$r^2\frac{d^2V}{dr^2}=A$$

$$\frac{d^2V}{dr^2}=\frac{A}{r^2}$$

$$V=-\frac{A}{r}+B$$

$$r\to\infty,\quad V\to 0$$

$$V=B=0$$

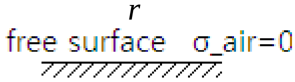
$$V=-\frac{A}{r}$$

Total current crossing a spherical surface

$$I=4\pi r^2J=-4\pi r^2\sigma\frac{dV}{dr}=-4\pi r^2\sigma\left(\frac{A}{r^2}\right)=-4\pi\sigma A$$

$$A=-\frac{I}{4\pi\sigma}=-\frac{I\rho}{4\pi}$$

$$V=\frac{I\rho}{4\pi}\frac{1}{r},\quad \rho=\frac{4\pi rV}{I}$$

- Single current electrode at surface
  (Figure 7.3 and Box 7.4 in the textbook)
  - ✓ Consider a single current electrode located at the surface of a homogeneous medium of resistivity ρ, current flows through a hemispherical surface in the lower medium. $V=-\frac{A}{r}$
  - ✓ Boundary conditions should be satisfied: 

    $$\sigma_1E_{1z}=\sigma_2E_{2z}$$

    $$0=\sigma_2\frac{\partial V}{\partial z}=\sigma_2\left[\frac{\partial V}{\partial r}\frac{\partial r}{\partial z}\right]=\sigma_2\frac{A}{r^2}\frac{z}{r}=\frac{Az}{r^3}$$

    at the interface z=0

    boundary conditioned is naturallysatisfied

- ✓ Remember that all the current flows through a hemispherical surface in the lower medium:

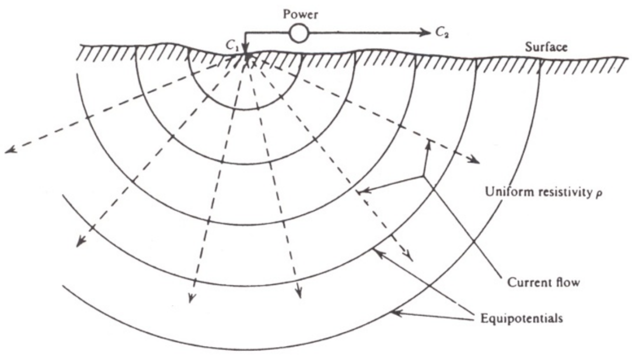
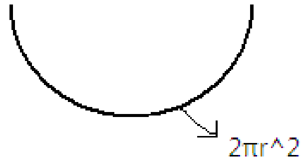


Figure 8.2. Point source of current at the surface of a homogeneous medium.



$$I = JS = J2\pi r^2$$

$$= 2\pi r^2\left(-\sigma\frac{dV}{dr}\right) = -2\pi r^2\sigma\left(\frac{A}{r^2}\right)$$

$$A = -\frac{I}{2\pi\sigma} = -\frac{I\rho}{2\pi}$$

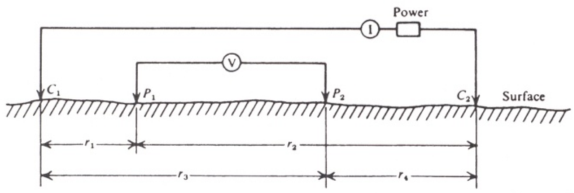


Figure 8.3. Two current and two potential electrodes on the surface of homogeneous isotropic ground of resistivity ρ.

$$V = \frac{I\rho}{2\pi}\frac{1}{r}$$

$$\rho = \frac{2\pi r}{I}V$$

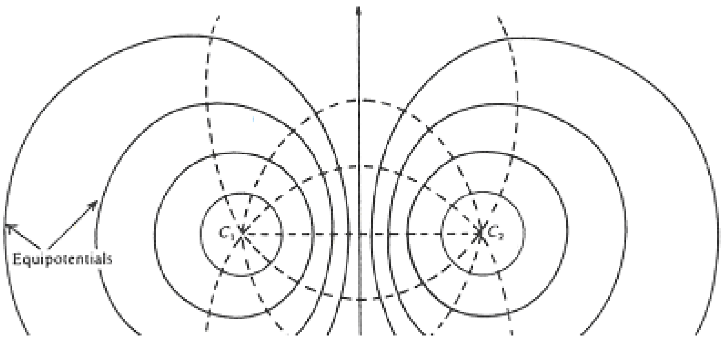
- ✓ Box 7.4 in the textbook

  The potential difference δV across ahemishphrical shell of incremental thickness δr is given by

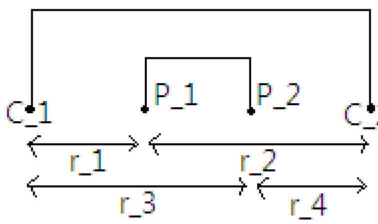
$$\frac{dV}{dr} = -\rho J = -\rho\frac{I}{2\pi r^2}$$

$$V_r = \int dV = -\int \rho\frac{I}{2\pi r^2}dr = \frac{\rho I}{2\pi}\frac{1}{r}$$

- ➢ Two current electrodes at surface (Figure 7.4 and Box 7.5 in the textbook)



$$V_1 = -\frac{A_1}{r_1},\quad V_2 = -\frac{A_2}{r_2}$$



$$A_1 = -\frac{I\rho}{2\pi}, \quad A_2 = \frac{I\rho}{2\pi} = -A_1$$

$$V_{P_1} = V_1 + V_2 = \frac{I\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$V_{P_2} = V_3 + V_4 = \frac{I\rho}{2\pi}\left(\frac{1}{r_3} - \frac{1}{r_4}\right)$$

poential difference

$$\begin{aligned}\Delta \mathrm{V} &= V_{P_1} - V_{P_2} \\ &= \frac{I\rho}{2\pi}\left\{\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - \left(\frac{1}{r_3} - \frac{1}{r_4}\right)\right\}\end{aligned}$$

$$\begin{aligned}\rho &= \frac{2\pi\Delta \mathrm{V}}{\mathrm{I}} 1 \Big/ \left\{\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - \left(\frac{1}{r_3} - \frac{1}{r_4}\right)\right\} \\ &= \frac{2\pi\Delta \mathrm{V}}{\mathrm{I}} p\end{aligned}$$

(4) Electrode configurations and geometric factors

- The above resistivity relationship has two parts, namely a resistance term R, and a term that describes the geometry of the electrode configuration being used, which is known as (geometrical factor).
- In reality, the subsurface ground is not homogeneous, and thus the resistivity obtained is no longer the ‘true’ resistivity. The resistivity that we obtain in fields is called apparent resistivity.
- The apparent resistivity is not a physical property of the subsurface media, unlike the true resistivity. Since all field resistivity data are apparent resistivity, we should try to obtain ‘true’ resistivities through interpretation techniques.
- (Figure 7.6) In order for at least 50 % of the current to flow through an interface at a depth of z meters into a second medium, the current electrode separation needs to be at least twice–and preferably more than three times – the depth.
- When the depths are of the order of several hundreds of meters, very long cable lengths are required, which can produce undesirable inductive

coupling effects.

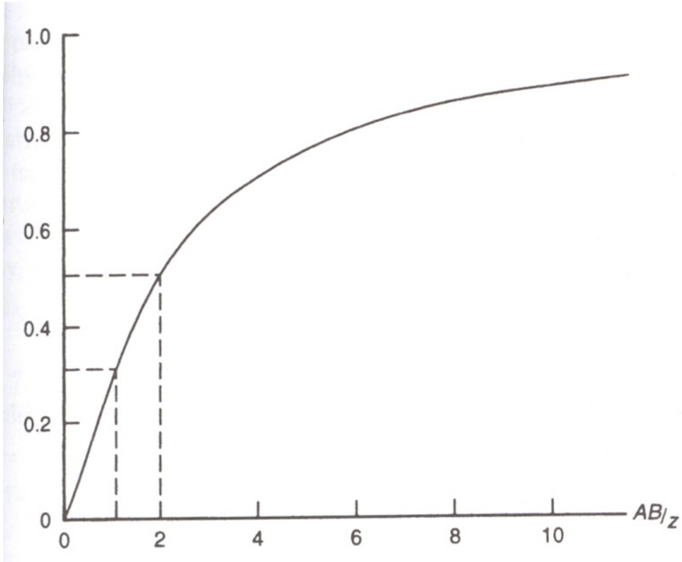


Figure 7.6 Proportion of current flowing below a depth z (m); AB is the current electrode half-separation

- An enormous number of electrode spreads have been used in resistivity at various times; not more than a half dozen have survived to any extent.
- In principle it is not necessary to use a collinear array. Practically, however, the electrodes are almost always in line; otherwise interpretation of results becomes difficult and the field work is complicated.

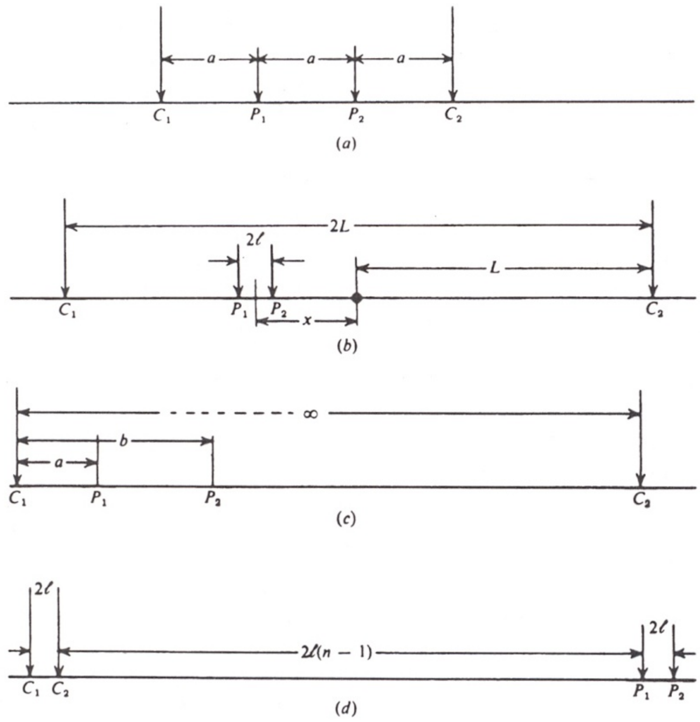


Figure 8.18. Electrode arrays in common use. (a) Wenner. (b) Schlumberger. (c) Pole-dipole. (d) Double-dipole.

- Wenner and Schlumberger arrays were formerly most popular; Since the development of the pseudodepth section in IP work, the dipole-dipole (double dipole) configuration has become equally so. Nowadays, dipole-dipole method has popularly used for electric resistivity survey.

A. Wenner array

- ✓ The electrodes are uniformly spaced in a line.
- ✓ Compute the apparent resistivity using the basic relationship:

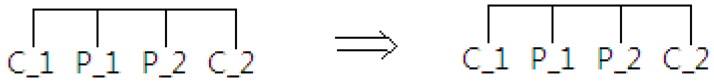
$$\rho = \frac{2\pi\Delta V}{I} 1 \Big/ \left\{ \left(\frac{1}{r_1} - \frac{1}{r_2} \right) - \left(\frac{1}{r_3} - \frac{1}{r_4} \right) \right\} = \left(\frac{2\pi\Delta V}{I} \right) G$$

$$r_1 = r_4 = a, \quad r_2 = r_3 = a$$

$$\rho = \frac{2\pi\Delta V}{I} 1 \Big/ \left\{ \left(\frac{1}{a} - \frac{1}{2a} \right) - \left(\frac{1}{2a} - \frac{1}{a} \right) \right\} = \frac{2\pi a\Delta V}{I}$$

- ✓ In spite of the simple geometry, this arrangement is often quite inconvenient for field work.
- ✓ For depth exploration using the Wenner spread, the electrodes are expanded about a fixed center, increasing the spacing *a* (e.g., Figure 8.18 A) in steps.
- ✓ For lateral exploration or mapping, the spacing remains constant and

all four electrodes are moved along the line, then along another line, and so on. In mapping, the apparent resistivity for each array position is plotted against the center of the spread.



moving

B. Schlumberger (gradient) array

- ✓ The current electrodes are spaced much further apart than the potential electrodes.
- ✓ Compute the apparent resistivity using the basic relationship

$$r_1 = (L-x)-l$$
$$r_2 = (L+x)+l$$
$$r_3 = (L-x)+l$$
$$r_4 = (L+x)-l$$

$$\rho_a = \frac{2\pi\Delta V}{I}\left\{\left[\frac{1}{(L-x)-l}-\frac{1}{(L+x)+l}\right]-\left[\frac{1}{(L-x)+l}-\frac{1}{(L+x)-l}\right]\right\}^{-1}$$

If the smallest current-potential electrode distance is always considerably greater than the distance between the two potential electrodes (by a factor of 10 or more), then we have.

$$(L-x)-l \gg 2l, \quad L-x \gg 3l$$
$$l^2 \approx 0$$

$$\begin{aligned}
\rho_a &= \frac{2\pi\Delta V}{I}\left\{\frac{1}{(L-x)-l}-\frac{1}{(L-x)+l}+\frac{1}{(L+x)-l}-\frac{1}{(L+x)+l}\right\}^{-1} \\
&= \frac{2\pi\Delta V}{I}\left[\frac{2l}{(L-x)^2-l^2}+\frac{2l}{(L+x)^2-l^2}\right]^{-1} \\
&= \frac{2\pi\Delta V}{I}\left[\frac{2l\left[(L+x)^2-l^2+(L-x)^2-l^2\right]}{\left[(L-x)^2-l^2\right]\left[(L+x)^2-l^2\right]}\right]^{-1} \\
&= \frac{2\pi\Delta V}{I}\left[\frac{2l\left[2(L^2+x^2-l^2)\right]}{\left(L^2-x^2\right)^2-l^2\left(L+x\right)^2-l^2\left(L-x\right)^2+l^4}\right]^{-1} \\
&= \frac{\pi\Delta V}{2Il}\left[\frac{\left(L^2-x^2\right)^2}{L^2+x^2}\right]
\end{aligned}$$

- ✓ In **vertical sounding**, the potential electrodes remain fixed while the current-electrode spacing is expanded symmetrically about the center of the spread.

- ✓ For large values of L, it may be necessary to increase ℓ in order to maintain a measurable potential.
- ✓ This procedure is more convenient than the Wenner expanding spread because we need to move only two electrodes.

- ✓ Lateral profiling may be done in two ways.
- ✓ With a very large fixed separation of the current electrodes (300 m or more), the potential pair is moved between them with fixed spacing, subject to the limitation $L - x \gg 3l$. Apparent resistivity is plotted against the midpoint of the potential electrodes.
- ✓ The other configuration is similar to the Wenner in that the electrode spacing remains fixed ($L \gg l$) and the whole array is moved along the line in suitable steps. This arrangement is less convenient than the first because it requires that all four electrodes be moved for each station.

C. Pole-dipole (three-point) array

- ✓ One of the current electrodes is fixed at a great distance from the other three, all of which can have various spacings.
- ✓ Compute the apparent resistivity using r_1=a, r_3=b, r_2=r_4=∞

$$\rho_a = \frac{2\pi\Delta V}{I}\frac{1}{\frac{1}{a}-\frac{1}{b}} = \frac{2\pi\Delta V}{I}\frac{ab}{b-a}$$

- ✓ When b=2a, we obtain

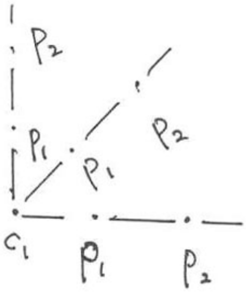
$$\rho_a = 4\pi a(\frac{\Delta V}{I})$$

- ✓ When the potential spacing is very small compared to the distance of either potential electrode from C_1 (C_2 still at ∞), we write $r_1 = a - \Delta a/2$, $r_3 = a + \Delta a/2$, and the apparent resistivity becomes

$$\rho_a = \frac{2\pi\Delta V}{I}\left[\frac{1}{a-\frac{\Delta a}{2}} - \frac{1}{a+\frac{\Delta a}{2}}\right]^{-1} = \frac{2\pi\Delta V}{I}\left[\frac{\Delta a}{a^2 - \frac{(\Delta a)^2}{4}}\right]^{-1}$$

$$= \frac{2\pi a^2}{I}\left[\frac{\Delta V}{\Delta a}\right] = \frac{2\pi a^2}{I}\frac{\partial V}{\partial a}$$

- ✓ This arrangement is equivalent to a half-Schlumberger array. In this case, the electrode configurations measure potential gradient.



- ✓ This permits lateral exploration on radial lines from a fixed position of C_1, by moving one or both potential electrodes, a particularly convenient method for resistivity mapping in the vicinity of a conductor of limited extent.

- ✓ A further variation on the pole-dipole array is obtained by moving one of the potential electrodes, say P_2, to a distant point, which is also remote from C_2. In this case, r_3=b=∞ as well, the apparent resistivity relationship is the same as that for the Wenner spread, hence this array is known as the half-Wenner array.

$$\rho_a = \frac{2\pi a \Delta V}{I}$$

- ✓ In field work the location of an electrode at infinity requires that it have very little influence on the rest of the array.
- ✓ For instance, when using a Wenner spread, the remote electrode or electrodes must be at least 10 times the spacing to reduce the effect to 10% or less.
- ✓ With the Schlumberger system, because the potential electrodes are close together, the far current electrode need only be about three times as far away as the one nearby to get the same result.

D. Dipole-dipole (double-dipole) system

- ✓ The potential electrodes are closely spaced and remote from the current electrodes, which are also close together.
- ✓ Compute the apparent resistivity when $r_1=r_4=2nl$, $r_2=2l(n-1)$, $r_3=2l(n+1)$, $n \gg l$

$$\rho_a = \frac{2\pi\Delta V}{I}\left[\frac{1}{2nl} - \frac{1}{2l(n-1)} - \frac{1}{2l(n+1)} + \frac{1}{2nl}\right]^{-1}$$

$$= \frac{2\pi\Delta V}{I}\left[\frac{2(n-1)(n+1) - n(n+1) - n(n-1)}{2nl(n-1)(n+1)}\right]^{-1}$$

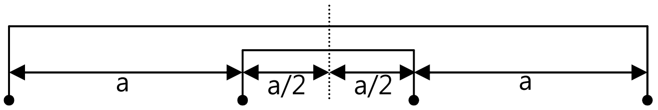
$$= \frac{2\pi\Delta V}{I} nl(n-1)(n+1)$$ (dropping the minus sign)

n=5 or less, this is the spread commonly used in IP work.

- ✓ When the dipoles are widely separated, n≫1, the apparent resistivity is expressed by $\rho_a = \dfrac{2\pi n^3 l \Delta V}{I}$

E. Lee-partitioning array

- ✓ Five electrodes are used. The configuration is very similar to Wenner array. The difference between Wenner and Lee configurations is that additional potential electrode is placed between two potential electrodes.

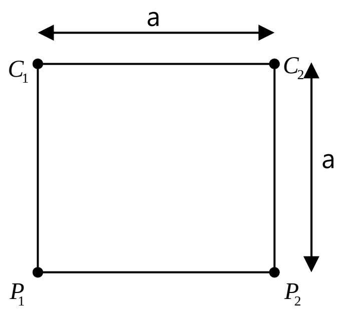


- ✓ Potential differences between the central potential electrode and the other two electrodes are measured

$$\rho_1 = 4\pi a \frac{\Delta V_1}{I}, \qquad \rho_2 = 4\pi a \frac{\Delta V_2}{I}$$

- ✓ This configurations is designed to reduce the undesirable effects of near-surface lateral inhomogeneities.

F. Square array



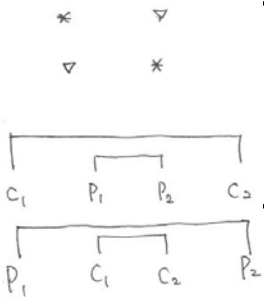
- ✓ Compute the apparent resistivity

$$\rho_a = \frac{2\pi\Delta V}{I}\left[\frac{1}{a} - \frac{1}{\sqrt{2}a} - \frac{1}{\sqrt{2}a} + \frac{1}{a}\right]^{-1}$$

$$= \frac{2\pi\Delta V}{I}\left[\frac{2}{a} - \frac{\sqrt{2}}{a}\right]^{-1} = \frac{2\pi\Delta V}{I}\left[\frac{a}{2-\sqrt{2}}\right] = \pi a \frac{\Delta V}{I}(2+\sqrt{2})$$

- ✓ The square array is particularly good for determining lateral azimuthal variations in resistivity.
- ✓ By switching P_1 and C_2, the square is effectively rotated through 90° and thus the apparent resistivity can be determined for two

orthogonal directions.



- ✓ For ground that is largely uniform, the two resistivities should be the same, but where there is a difference in resistivity due to a form of anisotropy, the two resistivities will differ from each other.

- ✓ In all the above electrode layouts the potential and current electrodes may be interchanged. By the principle of reciprocity, the apparent resistivity should be the same in both cases.
- ✓ The switching of current and potential electrodes could be desirable in using high voltages with large spreads in Schlumberger and, possible, Wenner layouts.

➢ Signal contribution section

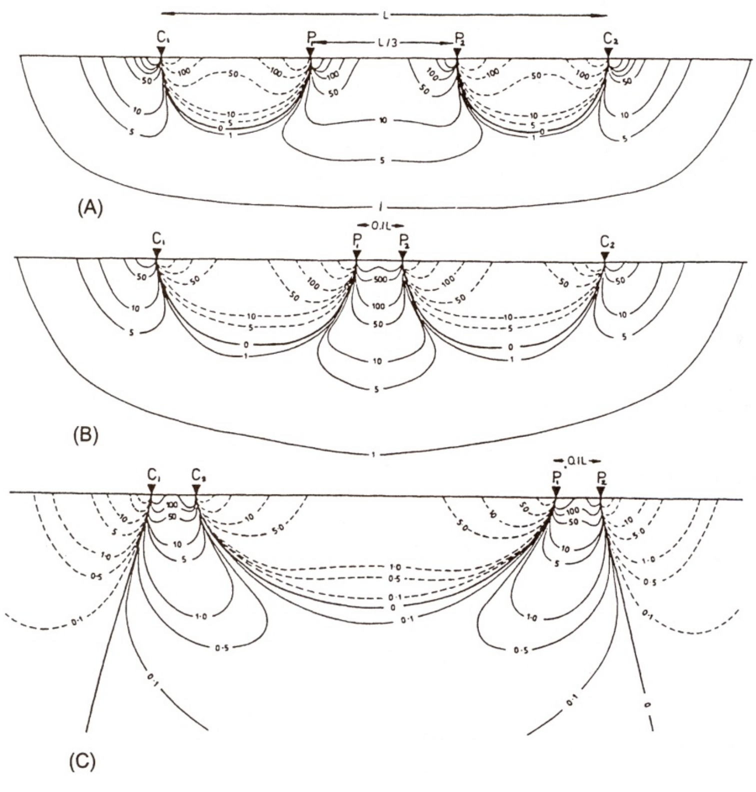


Figure 7.8 Signal contributi
tions for: (A) Wenner, (B) Schlur
and (C) dipole–dipole configu
Contours indicate the relative
butions made by discrete
elements of the sub-surface to t
potential difference measured b
the two potential electrodes P_1
From Barker (1979), by permis

- ✓ Figure 7.8 shows contoured plots of the contribution made by each unit volume of the sub-surface to the voltage measured at the surface.
- ✓ Figure 7.8A shows the signal contribution for a Wenner array. The main response, which originates from depth, is largely flat, which

indicates that for horizontally layered media, the Wenner array has a high vertical resolution.

- ✓ The Schlumberger array also has a high vertical resolution (Figure 7.8B).
- ✓ For the dipole-dipole array, the signal contribution indicates that there is a poor vertical resolution and that the array is particularly sensitive to deep lateral resistivity variations. This sensitivity can be used in resistivity profiling.

(5) Modes of Deployment

- ➢ There are two main modes of deployment of electrode arrays.
- ➢ One is to determine the vertical variation of resistivity. This is known as vertical electrical sounding (VES).
- ➢ The other is for horizontal traversing (horizontal variation of resistivity) and is called constant separation traversing (CST) (also called 'electrical resistivity traversing', ERT).

- ➢ Vertical electrical sounding (VES)
  - ✓ As the distance between the current electrodes is increased, so the depth to which the current penetrates is increased.
  - ✓ In the case of the dipole-dipole array, increased depth penetration is obtained by increasing the inter-dipole separation, not by lengthening the current electrode dipole.
  - ✓ The position of measurement is taken as the midpoint of the electrode array.
  - ✓ For a depth sounding, measurements of the resistance ($\delta V/I$) are made at the shortest electrode separation and then at progressively larger spacings.

  - ✓ In the normal Wenner array, all four electrodes have to be moved to new positions (Figure 7.17A).

- ✓ In the case of the Schlumberger array (Figure 7.17C), the potential electrodes (P_1P_2) are placed at a fixed spacing which is no more than one-fifth of the current-electrode half-spacing. The current electrodes are placed at progressively larger distances. When the measured voltage between P_1 and P_2 falls to very low values, the potential electrodes are spaced more widely apart.
- ✓ The dipole-dipole array is seldom used for vertical sounding as large and powerful electrical generators are normally required. In the dipole-dipole array, the distance between two dipoles is then increased progressively.

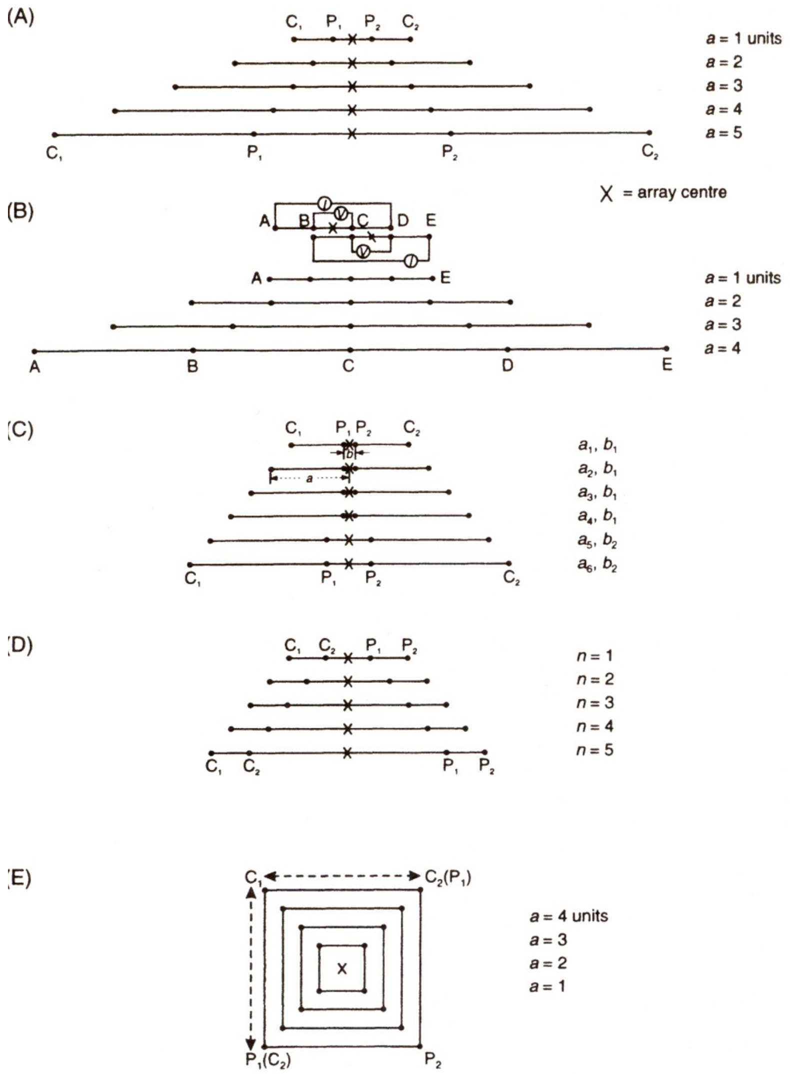


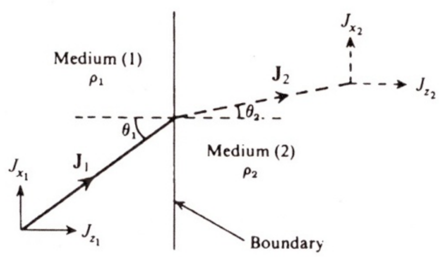
Figure 7.17 Expanded arrays (with successive positions displaced for clarity) for: (A) Wenner, (B) offset Wenner, (C) Schlumberger, (D) dipole–dipole and (E) square arrays

- ✓ The presence of horizontal or gently dipping beds of different resistivities is best detected by the expanding spread.
- ✓ The VES is useful in determining depth of overburden, depth, structure, and resistivity of flat-lying sedimentary beds
- ✓ It is frequently necessary to carry out this expansion procedure at several locations in an area, even when the main interest may be in lateral exploration, to establish proper electrode spacing for the lateral search

- Lateral profiling
  - This method is particularly useful in mineral exploration, where the detection is isolated bodies of anomalous resistivity is required.
  - In Wenner, Schlumberger, and pole-dipole surveys, the apparent resistivity is plotted at the midpoint of the potential electrodes. For dipole-dipole array, the station is at the array midpoint.
  - When the potential electrodes are closely spaced with respect to the current spread, the measurement is effectively of potential gradient at the midpoint.

(6) Effect of inhomogeneous ground

- Distortion of Current flow at a plane interface
  - Consider two homogeneous media.
  - Suppose that a current of density **J₁** is flowing in medium (1) in such a direction as to meet the boundary at an angle θ_1 to the normal. To determine the direction of this current in medium (2), we use the boundary conditions



$$E_{x1} = E_{x2}, \qquad \sigma_1 E_{z1} = \sigma_2 E_{z2}$$

$$\rho_1 J_{x1} = \rho_2 J_{x2}, \qquad \sigma_1 \rho_1 J_{z1} = \sigma_2 \rho_2 J_{z2}$$

$$\rho_1 \frac{J_{x1}}{J_{z1}} = \rho_2 \frac{J_{x2}}{J_{z2}}$$

$$\rho_1 \tan\theta_1 = \rho_2 \tan\theta_2$$

$$\frac{\rho_1}{\rho_2} = \frac{\tan\theta_2}{\tan\theta_1}$$

$$\rho_1 < \rho_2, \quad \theta_1 > \theta_2$$

- Distortion of Potential at a Plane Interface
  (Figure 8.10)

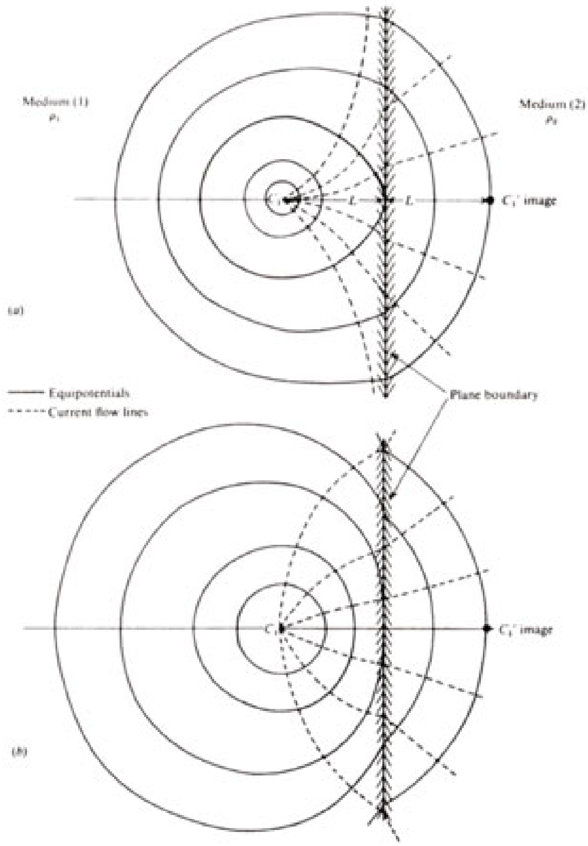


Fig. 8.10

- ✓ If the current flow is distorted in passing from a medium of one resistivity into another, the equipotentials also will be distorted.
- ✓ If the current flows from the high-resistivity medium towards the low-resistivity medium, the current flow lines converge towards the boundary, increasing the current density at the boundary but decreasing the potential gradient.
- ✓ When the current flows from the low-resistivity medium towards the high-resistivity medium, the current flow lines diverge towards the boundary, decreasing the current density at the boundary but increasing the potential gradient.

➢ Electric reflection coefficient, k

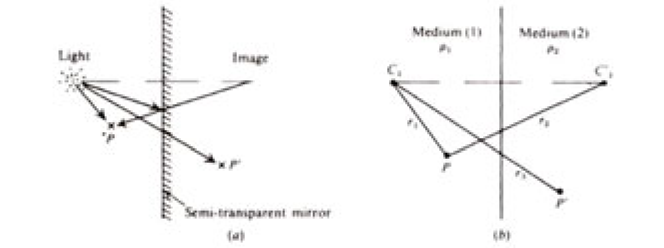


Figure 8.9 Analogy between optical and electrical images. (a) Optical image (b) Electrical image

(Figure 8.9)

- ✓ The analogy between the electrical situation and optics is based on the fact that current density, like light ray intensity, decreases with the inverse square of distance from a point source in a medium.
- ✓ If we assume that the electric reflection coefficient is k, then we can compute the potential at P in the first medium using the image of current source.

  $$V = \frac{I\rho_1}{4\pi}\left(\frac{1}{r_1} + \frac{k}{r_2}\right)$$

- ✓ In the second medium at P′, the potential is

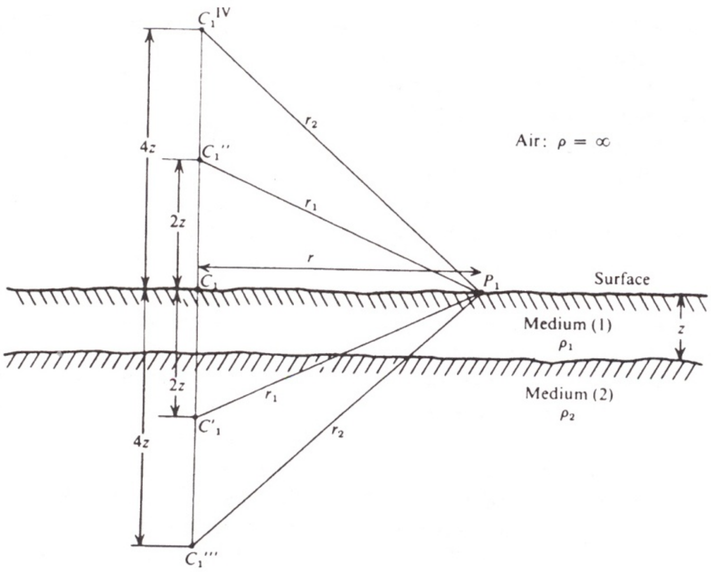
  $$V = \frac{I\rho_2}{4\pi}\frac{1-k}{r_3},$$ (transimission coefficient = 1-k)

- ✓ These potentials must be equal at the interface, when $r_1=r_2=r_3$

  $$\frac{I\rho_1}{4\pi}\left(\frac{1}{r_1} + \frac{k}{r_2}\right) = \frac{I\rho_2}{4\pi}\frac{1-k}{r_3}$$

  $$\frac{\rho_1}{\rho_2} = \frac{1-k}{1+k}, \qquad k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

- ➢ Surface Potential due to horizontal beds



- The current source and potential point are located on the surface, above a horizontal boundary separating two media whose resistivities are ρ_1 and ρ_2.
- Because of the ground surface, there are now three media, separated by two interfaces. As a result, there is an infinite set of images above and below the current electrode.
- The effect of each successive image on the potential at P is reduced by the reflection coefficient between the boundaries.
- For the current source and its first image below ground, the potential is $V' = \frac{I\rho_1}{2\pi}\left(\frac{1}{r} + \frac{k}{r_1}\right)$
- The effect of the second image at C_1'' is

$$V'' = \frac{I\rho_1}{2\pi}\left(\frac{kk_a}{r_1}\right) = \frac{I\rho_1}{2\pi}\frac{k}{r_1}$$

k_a : the reflection coefficient at the surface boundary

$$k_a = \frac{\rho_a - \rho_1}{\rho_a + \rho_1} = \frac{\rho_a\left(1 - \frac{\rho_1}{\rho_a}\right)}{\rho_a\left(1 + \frac{\rho_1}{\rho_a}\right)} = 1, \qquad (\rho_a \to \infty)$$

- The potential due to the third image C_1''' and C_1^{IV} will be

$$V''' = \frac{I\rho_1}{2\pi}\left(\frac{kk}{r_2}\right)$$

$$V^{\mathrm{IV}} = \frac{I\rho_1}{2\pi}\left(\frac{kkk_a}{r_2}\right)$$

- ✓ The resultant total potential at P can be expressed as an infinite series of the form

$$V = V' + V'' + V''' + V^{\mathrm{IV}}$$
$$= \frac{I\rho_1}{2\pi}\left(\frac{1}{r_1} + \frac{2k}{r_1} + \frac{2k^2}{r_2} + \cdots + \frac{2k^m}{r_m} + \cdots\right)$$
$$r_1 = \sqrt{r^2 + (2z)^2}$$
$$r_2 = \sqrt{r^2 + (4z)^2}$$
$$\vdots$$
$$r_m = \sqrt{r^2 + (2mz)^2}$$
$$V = \frac{I\rho_1}{2\pi}\left(\frac{1}{r} + 2\sum_{m=1}^{\infty}\frac{k^m}{[1+(2mz)^2]^{1/2}}\right) = \frac{I\rho_1}{2\pi r}\left(1 + 2\sum_{m=1}^{\infty}\frac{k^m}{[1+(2mz/r)^2]^{1/2}}\right)$$

(7) Interpretation

- ➢ Vertical sounding for two horizontal layers

- ✓ The above equation relates the potential of a single electrode to the resistivity of the upper layer in terms of the electrode spacing, the depth to the interface, and the resistivity contrast between the two beds.
- ✓ We need an expression in the form of an apparent resistivity, which would be measured by a four electrode system.

- ✓ For a basic electrode configuration, the measured potential difference between P_1 and P_2:

$$\Delta V = V_1 - V_2$$
$$= \frac{I\rho_1}{2\pi}[(\frac{1}{r_1} - \frac{1}{r_2}) - (\frac{1}{r_3} - \frac{1}{r_4})$$
$$+2\sum_{m=1}^{\infty} k^m (\frac{1}{[r_1^2 + 4m^2z^2]^{1/2}} - \frac{1}{[r_2^2 + 4m^2z^2]^{1/2}}$$
$$- \frac{1}{[r_3^2 + 4m^2z^2]^{1/2}} + \frac{1}{[r_4^2 + 4m^2z^2]^{1/2}})]$$

- For Wenner array, because $r_1=r_4=a$, $r_2=r_3=2a$, the apparent resistivity is expressed as

$$\Delta V = \frac{I\rho_1}{2\pi a}\left[1 + \sum_{m=1}^{\infty} \frac{4k^m}{\{1+(2mz/a)^2\}^{1/2}} - \sum_{m=1}^{\infty} \frac{4k^m}{\{4+(2mz/a)^2\}^{1/2}}\right] = \frac{I\rho_1}{2\pi a}(1+4D_w)$$

where

$$D_w = \sum_{m=1}^{\infty} k^m \left[\frac{1}{\{1+(2mz/a)^2\}^{1/2}} - \frac{1}{\{4+(2mz/a)^2\}^{1/2}}\right]$$

$$\rho_a = \rho_1\left[1 + \sum_{m=1}^{\infty} \frac{4k^m}{\{1+(2mz/a)^2\}^{1/2}} - \sum_{m=1}^{\infty} \frac{4k^m}{\{4+(2mz/a)^2\}^{1/2}}\right] = \rho_1(1+4D_w) \quad (8.38)$$

- For Schlumberger spread, when $x=0$, $r_1=r_4=L-\ell$, $r_2=r_3=L+\ell$

$$\Delta V = \frac{I\rho_1}{2\pi}\left[\left(\frac{2}{L-\ell} - \frac{2}{L+\ell}\right) + 4\sum_{m=1}^{\infty} k^m \times \left\langle \frac{1}{(L-\ell)\{1+(2mz)^2/(L-\ell)^2\}^{1/2}} - \frac{1}{(L+\ell)\{1+(2mz)^2/(L+\ell)^2\}^{1/2}}\right\rangle\right]$$

$$= \frac{I\rho_1 2\ell}{\pi(L^2-\ell^2)}\left[1 + \left(\frac{L+\ell}{\ell}\right) \times \sum_{m=1}^{\infty} \frac{k^m}{\{1+(2mz)^2/(L-\ell)^2\}^{1/2}} - \left(\frac{L-\ell}{\ell}\right)\sum_{m=1}^{\infty} \frac{k^m}{\{1+(2mz)^2/(L+\ell)^2\}^{1/2}}\right]$$

$$\rho_a = \rho_1\left[1 + \left(\frac{L+\ell}{\ell}\right) \times \sum_{m=1}^{\infty} \frac{k^m}{\{1+(2mz)^2/(L-\ell)^2\}^{1/2}} - \left(\frac{L-\ell}{\ell}\right) \times \sum_{m=1}^{\infty} \frac{k^m}{\{1+(2mz)^2/(L+\ell)^2\}^{1/2}}\right] = \rho_1(1+D_s) \quad (8.39a)$$

where

$$D_s = \left(\frac{L+\ell}{\ell}\right)\sum_{m=1}^{\infty} \frac{k^m}{\{1+(2mz)^2/(L-\ell)^2\}^{1/2}} - \left(\frac{L-\ell}{\ell}\right)\sum_{m=1}^{\infty} \frac{k^m}{\{1+(2mz)^2/(L+\ell)^2\}^{1/2}}$$

When $L \gg \ell$

$$\Delta V \approx \frac{I\rho_1 2\ell}{\pi L^2}\left[1 + 2\sum_{m=1}^{\infty}\frac{k^m}{\left\{1+(2mz/L)^2\right\}^{1/2}}\right]$$

$$\approx \frac{I\rho_1 2\ell}{\pi L^2}(1 + 2D_s')$$

where

$$D_s' = \sum_{m=1}^{\infty}\frac{k^m}{\left\{1+(2mz/L)^2\right\}^{3/2}}$$

Approximately, we have

$$\rho_a \approx \rho_1\left[1 + 2\sum_{m=1}^{\infty}\frac{k^m}{\left\{1+(2mz/L)^2\right\}^{3/2}}\right]$$

$$= \rho_1(1 + 2D_s'). \qquad (8.39b)$$

- ✓ For dipole-dipole array, because $r_1=r_4=2n\ell$, $r_2=2(n-1)\ell$, $r_3=2(n+1)\ell$

$$\Delta V = -\frac{I\rho_1}{2\pi(n-1)n(n+1)\ell}\times\left[1 + n(n+1)\times\sum_{m=1}^{\infty}\frac{k^m}{\left[1+(2mz)^2/\{2(n-1)\ell\}^2\right]^{1/2}} + n(n-1)\times\sum_{m=1}^{\infty}\frac{k^m}{\left[1+(2mz)^2/\{2(n+1)\ell\}^2\right]^{1/2}} - 2(n-1)(n+1)\times\sum_{m=1}^{\infty}\frac{k^m}{\left\{1+(2mz/2n\ell)^2\right\}^{1/2}}\right]$$

$$\rho_a = \rho_1(1 + D_d) \qquad (8.40a)$$

$$\rho_a = \rho_1\left[1 - \sum_{m=1}^{\infty}\frac{k^m}{\left\{1+(2mz/r)^2\right\}^{3/2}} + 3\sum_{m=1}^{\infty}\frac{k^m}{\left\{1+(2mz/r)^2\right\}^{5/2}}\right] = \rho_1(1 + D_d') \qquad (8.40b)$$

- ✓ For r≪z
  the series terms in all cases tend to zero
  $\rho_a \simeq \rho_1$

- ✓ For r≫z
  the series expansions in all of the equations become the same
  (because the denominators ≃ 1 or 2)

$$\rho_a = \rho_1\left(1+2\sum_{m=1}^{\infty}k^m\right) \qquad k^m < 1 \qquad \sum_{m=1}^{\infty}k^m = \frac{1}{1-k} - 1$$

$$k = \frac{\rho_2-\rho_1}{\rho_2+\rho_1} \qquad \rho_a \simeq \rho_2$$

- ✓ **Assignment: Please compute the apparent resistivity for a Schlumberger spread as shown in Figure 8.20**

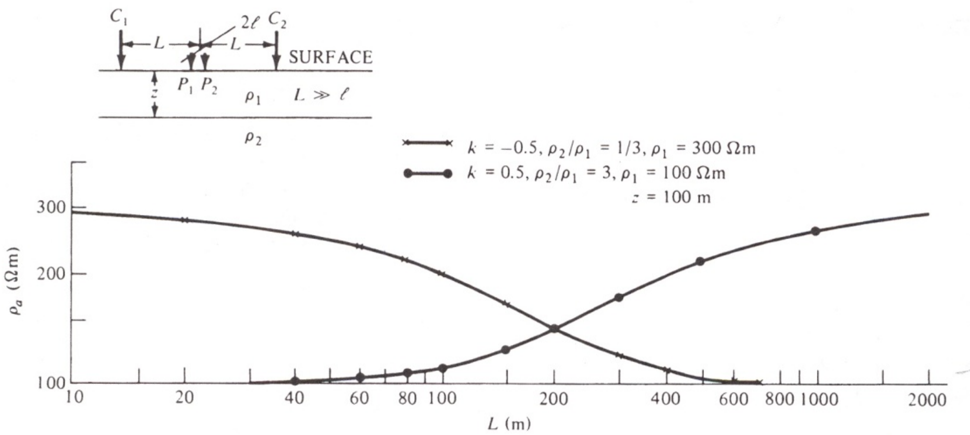


Figure 8.20. Plots of ρ_a versus L over two horizontal beds for a Schlumberger expanding spread. Curves were calculated from Equation (8.39b).

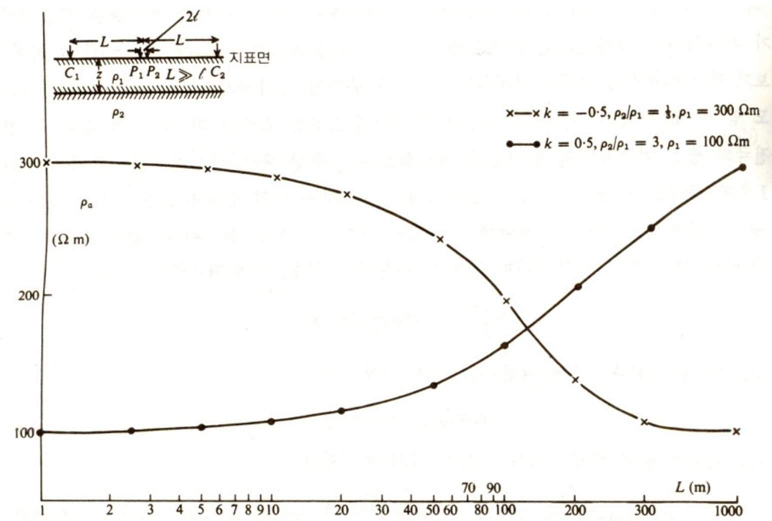


그림 4·36 슐럼버저 배열에 의한 수직탐사－수평 2층구조에 대한 ρ_a과 L의 그래프 (전기 비저항곡선은 z =100m로 하고, 식 (4.65b)으로부터 계산하였음)

- ✓ Crude interpretation

- Figures 8.20 and 4.36 show a pair of resistivity curves for two layers with contrasts of 3 and 1/3. The upper bed resistivity is 100 and 300 Ωm for the two cases, and the thickness is 100 m.
- The curve for ρ_2/ρ_1 = 1/3 is clearly asymptotic to ρ_1 and ρ_2 at the limits of small and large spacing, and its point of maximum slope is approximately at 100 m.
- Thus we can estimate the depth and the resistivities of the two beds for this simple example.

- The other curve gives the upper layer resistivity at small spacing, but it is not so clear what the value of ρ_2 may be. If the spacing were increased to several kilometers, it would be asymptotic to 300 Ωm.
- The point of inflexion is not at 100 m but at a larger spacing.

- We can get some idea of the unknown parameters ρ_1, ρ_2, and z from the field curve, provided the resistivity contrast is not two great and particularly if the lower bed is the more conductive

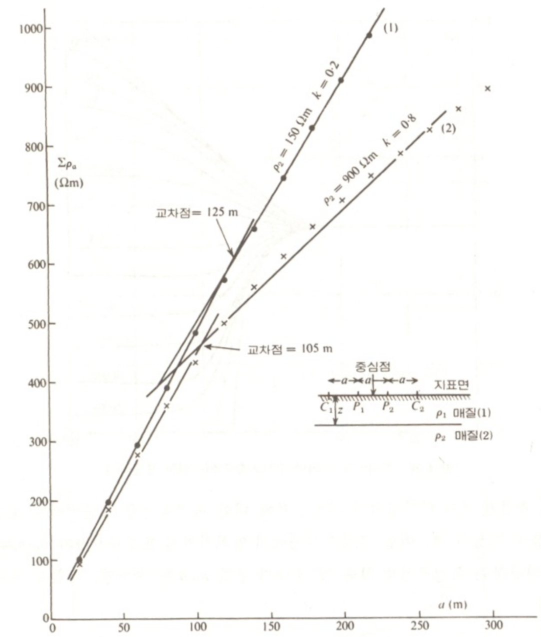


그림 4·37 웬너배열에서 ρ_a의 누적그래프

ρ_1 = 100 Ωm, z = 100 m, (1) ρ_1/ρ_2 = 0.67, k = 0.2; (2) ρ_1/ρ_2 = 0.11, k = 0.8

- A better way to find z is to draw the accumulated resistivities with respect to the electrode spacing.
- For example, the resistivity is 100, 200, 300 Ωm for the electrode spacing of 10 m, 20 m, and 30 m. Then, the accumulated resistivity becomes 100, 300, and 600 Ωm. The accumulated resistivities are plotted against the electrode spacing.
- Draw a straight line connecting the points, and the intersection point indicates the depth of the upper layer (Figure 4.37).
- This method is not accurate, but it can provide a little bit better results than the previous method.
- When the electrode spacing is very small compared to the depth of the first layer, the

method yields good results.

✓ Curve matching

– A much more accurate and dependable method of interpretation in electric sounding involves the comparison of field profiles with characteristic curves.

– The master curves are prepared with dimensionless coordinates. Equations (8.38) to (8.40) can be put in this form by dividing ρ_a by ρ_1. The ratios ρ_a/ρ_1 are then plotted against a/z, L/z, or r/z, that is, the electrode spacing divided by the depth of the upper bed. The curves are plotted on logarithmic paper.

– The sets of curves are constructed either for various values of k between ± 1 or for various ratios of ρ_2/ρ_1 between 0 and $+\infty$.

– (Fig. 8.21)

– The characteristic curves are generally drawn on a transparency. Match a field result sliding the master sheet around on the field profiles until the latter coincides more or less with one of the master curves (or can be interpolated between adjacent master curves). The respective coordinate axes must be kept parallel.

– The point where ρ_a/ρ_1 = a/z =1 on the master sheet then determines the values of ρ_1 and z on the field curve axes, while the actual curve fit gives the value of k and ρ_2.

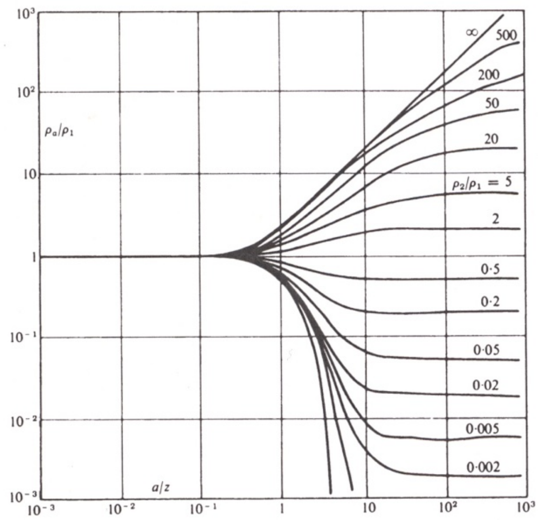


Figure 8.21. Wenner spread – master curves for two horizontal beds. (From Keller and Frischknecht, 1966.)

When the lower bed is an insulator, $\rho_2 = \infty$ and $k=1$.
Then the apparent resistivity increases indefinitely with electrode spacing.

Because all the current will flow in the upper bed, it is possible to determine the value of ρ_a when $\rho_2 = \infty$ by calculating the electric field at the midpoint of the current electrodes

Because their separation is much larger than the thickness of the upper bed, it is reasonable to assume a uniform current density from top to bottom.

Then the current from either electrode is found by integrating over a cylindrical equipotential surface of radius r and height z.

$$I = \int_0^{2\pi}\int_0^{z} J_r \, d\theta \, dz = 2\pi z r J$$

The current is doubled because there are two current electrodes.

$$E = 2\rho_1 J = \rho_1 I / \pi r z$$

For the Wenner array

$$\rho_a = \frac{2\pi a \Delta V}{I} = \frac{2\pi a}{I}\int_a^{2a} E \, dr = \left(\frac{2a\rho_1}{z}\right)\ln 2$$

$$= 1.39\left(\frac{a\rho_1}{z}\right)$$

For the Schlumberger layout $L = r$

$$\rho_a = \frac{\pi L^2}{I}\frac{\partial V}{\partial r} = \frac{L\rho_1}{z}$$

For the dipole - dipole system

$$\rho_a = -\left(\frac{\pi r^3}{I}\right)\frac{\partial^2 V}{\partial r^2} = \frac{r\rho_1}{2z}$$

✓ Interpretation by asymptotes

- When the lower bed has very large resistivity, the characteristic two-layer curve becomes a straight line for large electrode spacing.
- In the logarithmic plots, this line has a slope of 45° for all of the arrays considered.
- The intersection of this straight edge with the ρ_a axis gives ρ_1. The sloping part of the curve on the right-hand side of the profile. The interface depth can then be found on the horizontal axis from the intersection of the triangle and the horizontal straight edge.

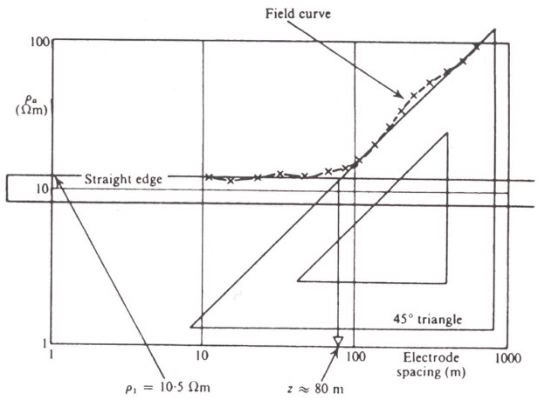
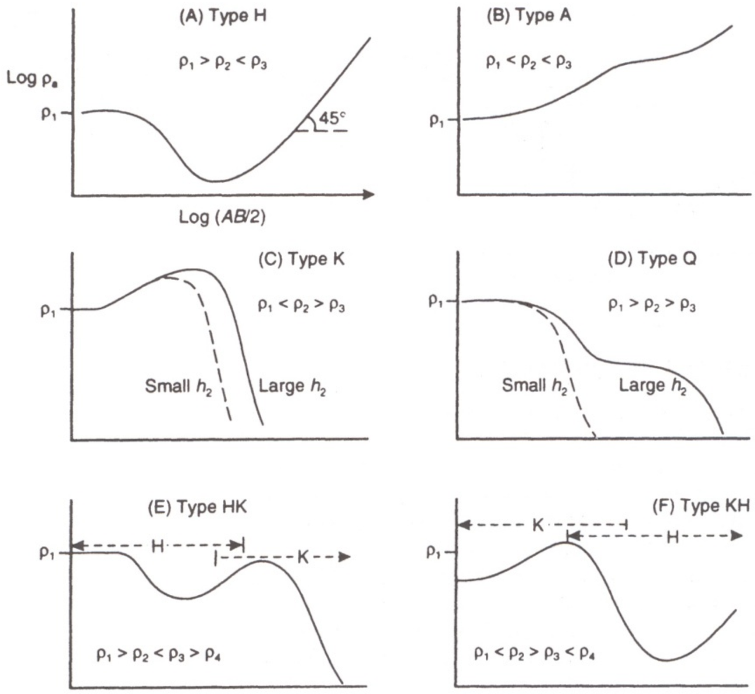


Figure 8.22. Estimate of ρ_1 and z from the 45° asymptote. (After Keller and Frischknecht, 1966.)

➢ Vertical sounding for multiple horizontal layers



- Numerical modeling (수치모델링) and Inversion (역산)

  - ✓ These methods are used for all of the fields of Exploration Geophysics such as gravity, magnetic, seismic, electric, and electromagnetic method.
  - ✓ Developing numerical modeling and inversion algorithms has been a main research topic in Exploration Geophysics.
  - ✓ Since we can know the main expression for apparent resistivity for a model, we can compute the model response using computer programs such as Fortran, C, C++, Basic, Metlab, etc. We usually use finite-difference method (유한차분법) and finite-element method (유한요소법) which are used to solve the main equation approximately.
  - ✓ If a model is given, we can compute the model response with a numerical method. → Definition of numerical modeling.
  - ✓ Numerical modeling should be differentiated from physical modeling. In a physical modeling (축소모형실험), we set up a physical model, and simulate field survey.

  - ✓ Interpretation of data using numerical modeling is carried out by computing the model response for a model, comparing the model response with field data, and modifying the model. In this case, we manually change the model.

- ✓ Inversion is an effective method for interpretation for geophysical survey data. In an inversion algorithm, the above processes of computing the model response, comparing the model response with field data, and modifying the model are all included, but they are carried out automatically rather than manually.

- ✓ In an inversion algorithm, we can set up an objective function, which can be the sum of the squares of errors between field data and modeled data. By computing the gradient of the objective function, the direction to modify the model is determined.

- ✓ In an inversion algorithm, we also use a numerical modeling algorithm.

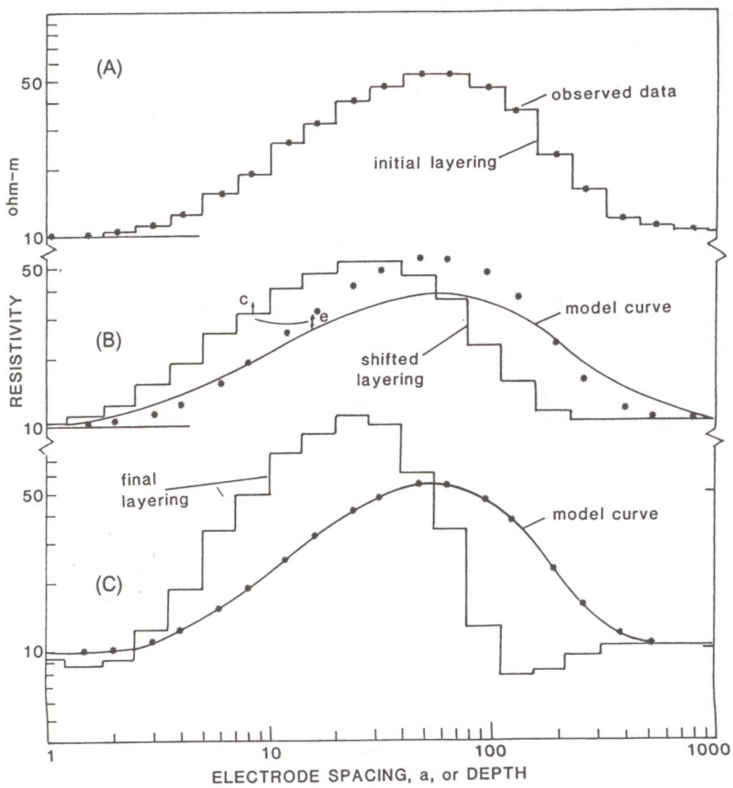
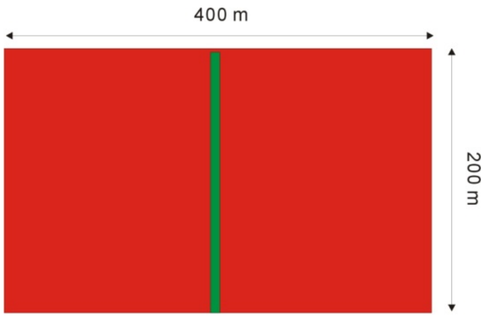


Figure 7.33 Automatic sounding inversion technique. (A) Observed data and initial layering. (B) Shifted layering and resulting model sounding curve. The difference (*e*) between the model and observed curves is used to apply a correction (*c*) to the layering. (C) The final layering and resulting model curve that is closely similar to the observed data. From Barker (1992), by permission

- ✓ A major advantage of the computer approach is that it provides an opportunity to investigate the problems of equivalence and suppression quickly and efficiently.

- ✓ When we use computer software for interpretation of field data, we should consider the advantages and disadvantages of the computer software.

- ✓ Numerical modeling can be used to design field survey. We can determine the electrode spacing through numerical modeling.

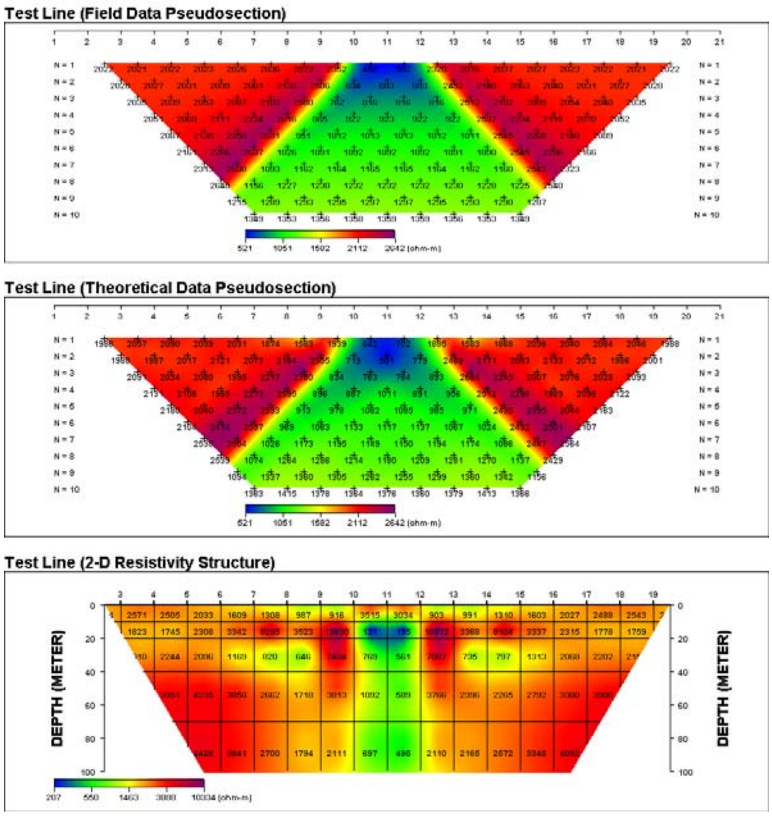


The width of the dyke is 1 m

Electrode spacing is 20 m

Dipole-dipole method is used

The resistivities of dyke and the surrounding rock are 20 and 2000



- ➢ Equivalence and suppression
  - ✓ Equivalence (등가성)
    - – Longitudinal conductance

      $S_L = h/\rho = h\sigma$ for a given layer

      $$S_L = \sum_{i=1}^{n}(h_i/\rho_i) = \frac{h_1}{\rho_1} + \frac{h_2}{\rho_2} + \ldots \frac{h_n}{\rho_n}$$

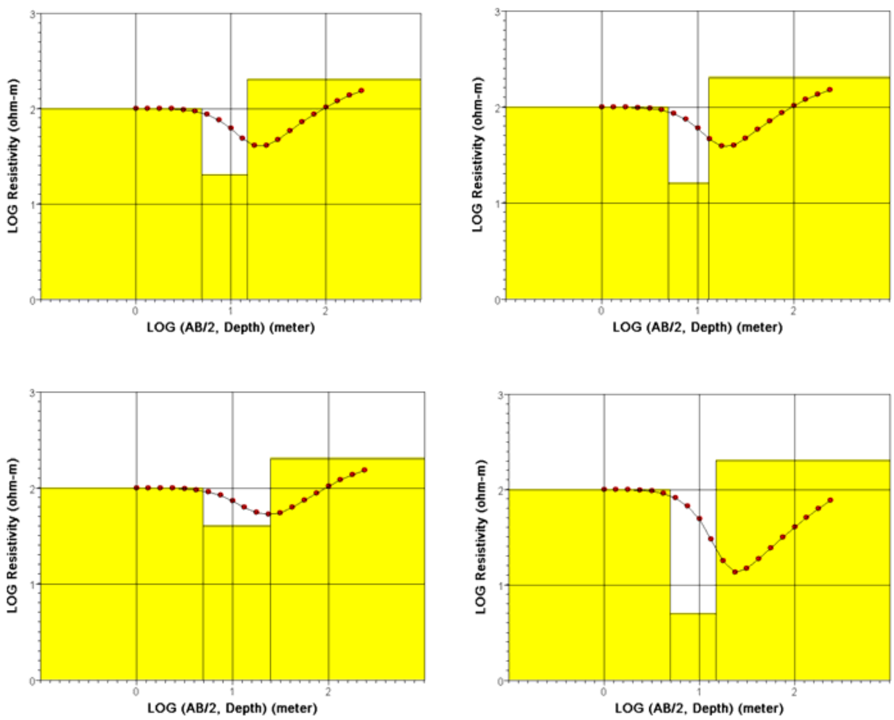
    - – Transverse resistance

      $T = h\rho$

      $$T = \sum_{i=1}^{n}(h_i\rho_i) = h_1\rho_1 + h_2\rho_2 + \cdots + h_n\rho_n$$

- The importance of the longitudinal conductance for a particular layer is that it demonstrates that it is not possible to know both the true layer conductivity (or resistivity) and the layer thickness, so giving rise to layer equivalence.
- The longitudinal conductance of a layer whose resistivity is 100 Ω m and thickness is 5 m is the same as that of a layer whose resistivity is 80 Ωm and 4 m.
- Equivalence needs to be considered during interpretation of sounding curves.

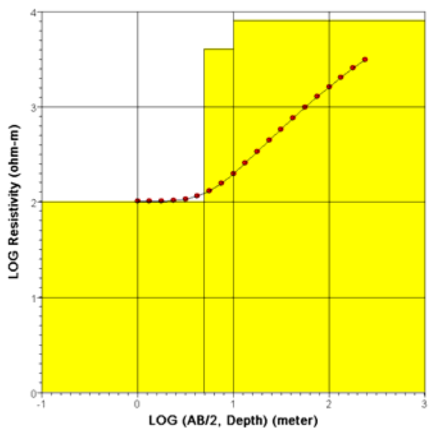
Layer	Model 1	Model 2	Model 3	Model 4
1	5 m / 100Ωm	5 m /100Ωm	5 m /100Ωm	5 m /100Ωm
2	10 m / 20Ωm	8 m/ 16Ωm	20 m/ 40 Ωm	10 m / 5 Ωm
3	200Ωm	200Ωm	200Ωm	200Ωm



- The longitudinal conductance S_L for this layer is h_2/ρ_2=constant, and as long as the thickness and resistivity are changed within limits so as to maintain that ratio, there will be no appreciable change in the resulting apparent resistivity curve.
- All the pairs of h_2/ρ_2 are electrically equivalent.

- ✓ Suppression (은폐성)

  - This is particularly a problem when three (or more) layers are present and their resistivities are ascending with depth or descending with depth.
  - The middle intermediate layer may not be evident on the field curve and so its expression on the apparent resistivity graph is suppressed.



  - For a layer to be suppressed, its resistivity should approach that of the one below so that the resistivity contrast between the top and the suppressed layer is comparable to that between the top and lowermost layers is.
  - The effects of missing such a suppressed layer can have major effects on the estimation of the depth to bedrock.

  - If the intermediate layers are thin with respect to those overlying, and if either equivalence or suppression is suspected, master curves will provide no solution. However, equivalent or suppressed layers can be modeled very effectively using computer methods in conjunction with knowledge of what is geologically reasonable for the field area in question.

(8) Applications and Case Histories

- ➢ Subsurface collapse features

- ✓ In a small village in east Devon, a 5 m diameter hole appeared overnight in the middle of the road. The local water main had been ruptured and had discharged for over 12 hours and all the water had disappeared down a fissure into underlying limestone. Several of the local buildings started to crack badly, and on investigation it was found that the rafted foundations of several houses had broken and the houses were literally cracking open at the seams, resulting in the emergency evacuation of the residents.
- ✓ A resistivity survey was initiated in order to determine the subsurface extent of the problem prior to drilling. The Wenner array was applied with electrode separations of 10, 15 and 20 m.
- ✓ The resulting apparent resistivity values were plotted as a contour map. It is clear that where the hole had appeared, there was a deep infill of clay.

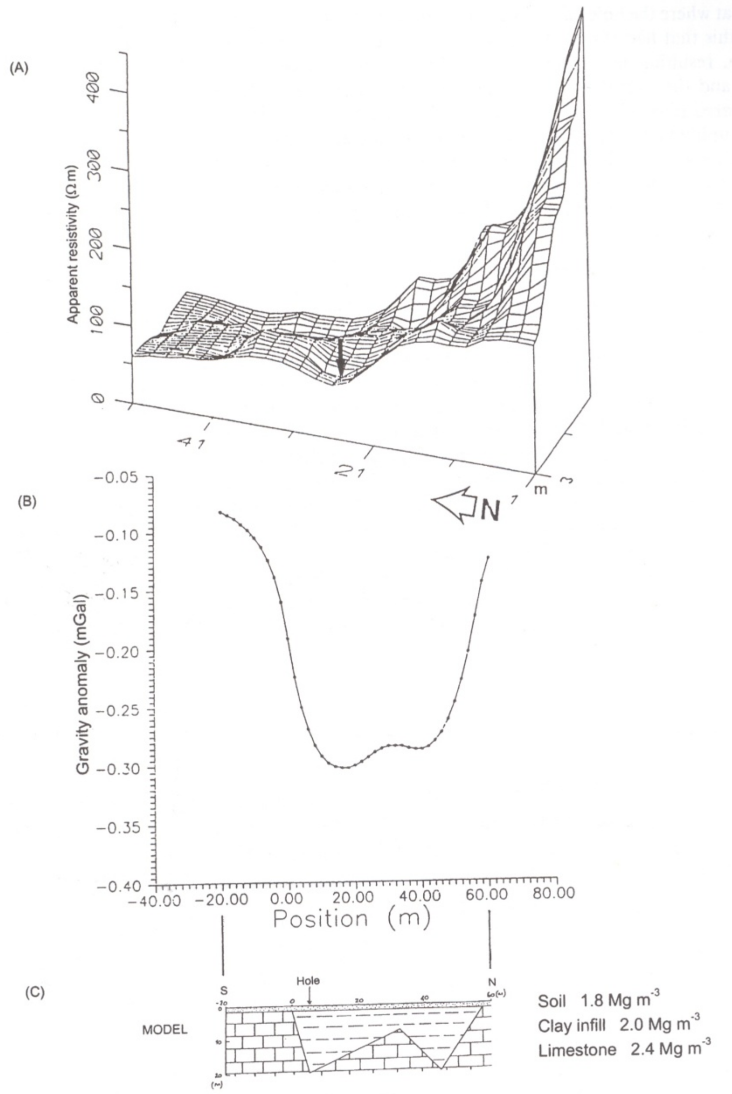


Figure 7.39 (*opposite*) (A) Apparent resistivity isometric projection obtained using constant-separation traverses with an electrode separation of 10m. (B) Modelled microgravity profile that would be expected for the geological model shown in (C): interpreted depth to limestone constrained by drilling. A north–south profile is shown in Figure 7.20. The position of a clay-filled solution feature is arrowed

- Location of buried foundations
  - ✓ Electrical resistivity subsurface imaging was used at a disused railway yard in order to locate old foundations concealed beneath railway ballast.
  - ✓ The electrical resistivity subsurface imaging survey was carried out adjacent to a metal chain-link fence, and an old diesel tank, and about 3 m from an existing building.
  - ✓ Despite extremely high electrode contant resistances, a 25 m long array was surveyed with an inter-electrode separation of 1 m.
  - ✓ The final pseudo-section of true resistivities against depth shows a general increase in resistivity with depth. It revealed two areas of extremely high resistivity (> 125000 Ωm).

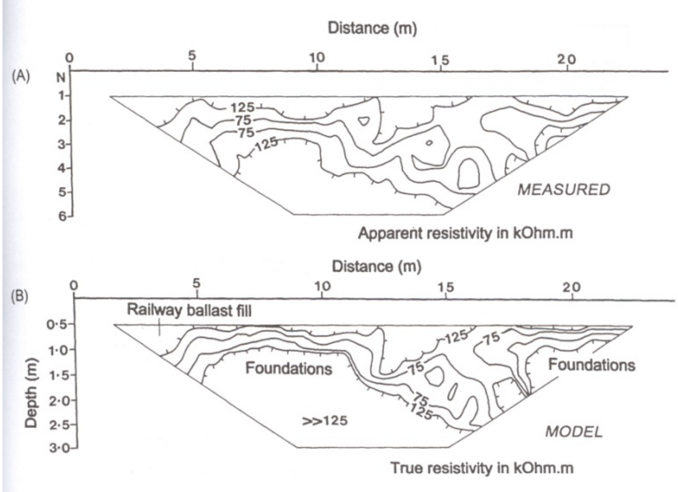


Figure 7.42 Electrical resistivity subsurface imaging pseudo-sections: (A) apparent resistivity profile, and (B) true resistivity–depth profile, over buried concrete slabs at 1 m depth. From Reynolds and Taylor (1995), by permission

- Goundwater and landfill surveys
  - ✓ Detection of saline groundwater
    - – In the mid-1950s, a comprehensive electrical resistivity survey programmed was initiated in order to map out saline groundwater in areas of the Netherlands below or at mean sea level. Figure 7.43 shows schematically the nature of the hydrogeology in the western part of Netherlands.
    - – The vertical electrical soundings provided a means of obtaining information about the vertical distribution of fresh, brackish and saline water bodies and their areal extent (Figure 7.44)

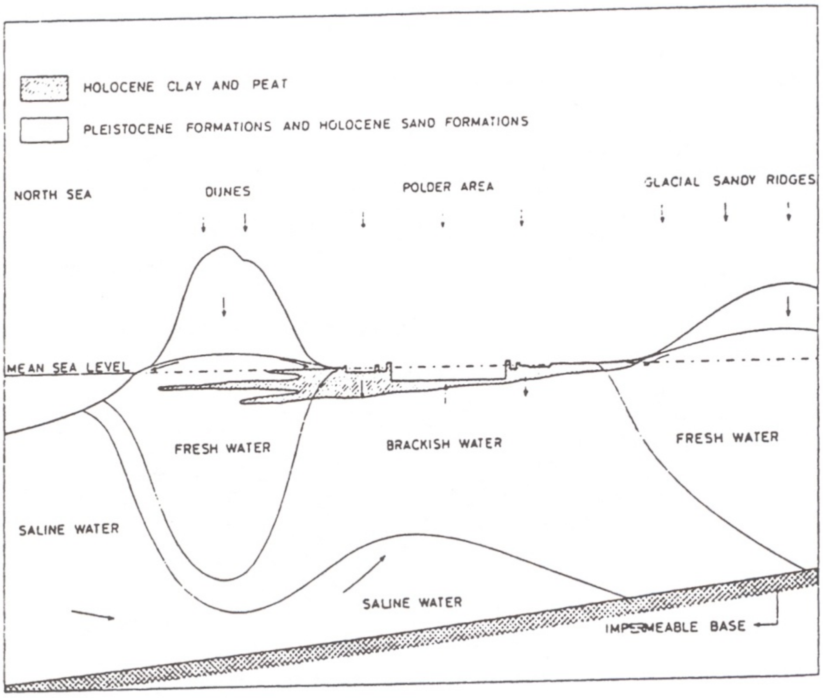


Figure 7.43 Schematic hydrogeological cross-section for the western part of the Netherlands. From van Dam and Meulenkamp (1967), by permission

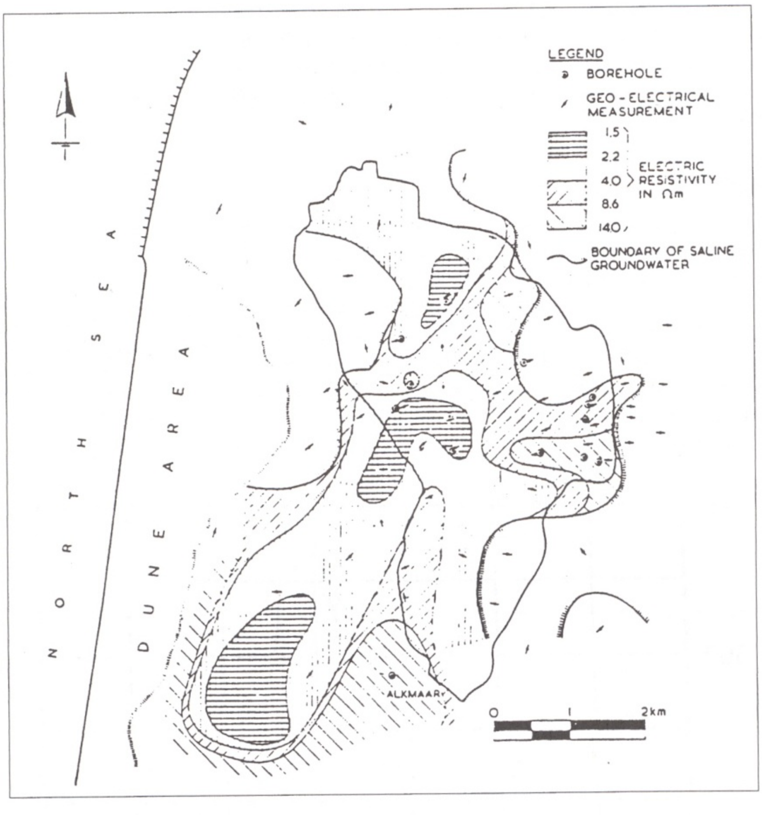


Figure 7.44 Distribution of resistivities of a sand layer with a saline groundwater boundary in Noord Holland as determined by many vertical electrical soundings. From van Dam and Meulenkamp (1967), by permission

- ✓ Groundwater potential

  - In Kano state, northern Nigeria, an internationally funded aid program was established in the 1980s to provide tudewells with handpumps for 1000 villages in rural areas. Village populations ranged from several hundred to no larger than 2000 people, but all were in very remote locations. Failure to obtain a reliable supply of water would have resulted in many of the villages being abandoned and the populations moving to the larger towns, thereby compounding the local problems of sanitation and health, education and employment, and the demise of rural culture and skills.

  - It was first recommended that geophysics was necessary to locate groundwater; boreholes drilled anywhere would succeed. In practice, borehole failures were in excess of 82 % of holes drilled, particularly in the southern areas.

  - Geophysical methods were then called upon to improve the failure rate.
  - Predominantly, vertical electrical soundings were used on sites selected following initial hydrogeological and photogeological inspection.

  - Careful analysis of the vertical electric sounding data with the subsequent borehole information, led to the compilation of a database of typical formation resistivities and their likely hydrogeological potential.

- ✓ Landfills

  - There is no such thing as a typical landfill – Some are extremely conductive, others are resistive relative to the surrounding media.
  - There are many geophysical variables and care must be taken no to presume a particular geophysical response for any given site.

Spontaneous (Self) Potential Methods

1. Introduction

- The self-potential or spontaneous polarization (SP) method was devised in 1830 by Robert Fox who used copper-plate electrodes connected to a galvanometer to detect underground copper sulfide deposits in Cornwall, England.
- The method has been used since 1920 as a secondary tool in base metal exploration, characteristically to detect the presence of massive ore bodies, in contrast to the induced polarization method which is used predominantly to investigate disseminated ore bodies.
- In recent years, the SP method has been extended to groundwater and geothermal investigations, and can also be used as an aid to geological mapping, for example, to delineate shear zones and near-surface faults.
- The SP method is the cheapest in terms of equipment and the simplest to operate in the field.
- The phenomenon of self-potentials is utilized more extensively in borehole well logging than in surface applications

2. Occurrence of self-potentials

- The SP method is passive, i.e. differences in natural ground potentials are measured between any two points on the ground surface.
- The potentials measured can range from less than a millivolt (mV) to over one volt, and the sign (positive or negative) of the potential is an important diagnostic factor in the interpretation of SP anomalies.
- Self-potentials are generated by a number of natural sources, although the exact physical processes are still unclear.
- A summary of the common types of SP anomaly are listed in Table 8.1. The geometry of geological structures as well as compositional variations create SP anomalies.
- Natural ground potentials consist of two components, one of which is constant and unidirectional and the other fluctuates with time. The constant component is due primarily to electrochemical processes, and the variable

component is caused by a variety of different processes ranging from alternating currents induced by thunderstorms and by variations in the Earth's magnetic field, to the effects of heavy rainfall.

- In mineral exploration, components of the SP are called the mineral potential and the background potential, respectively. The 'background' potentials can be used in geothermal and hydrogeological investigations as the main measured anomaly.

Table 8.1 Types of SP anomalies and their geological sources

Source	Type of anomaly
Mineral potentials	
Sulphide ore bodies (pyrite, chalcopyrite, pyrrhotite, sphalerite, galena) Graphite ore bodies Magnetite + other electronically conducting minerals Coal Manganese	Negative ≈ hundreds of mV
Quartz veins Pegmatites	Positive ≈ tens of mV
Background potentials	
Fluid streaming, geochemical reactions, etc.	Positive +/− negative ⩽ 100 mV
Bioelectric (plants, trees)	Negative, ⩽ 300 mV or so
Groundwater movement	Positive or negative, up to hundreds of mV
Topography	Negative, up to 2 V

3. Origin of self-potentials

- The common factor among the various processes for self-potentials is groundwater.
- The potentials are generated by the flow of water, by water acting as an electrolyte and as a solvent of different minerals, and so on.
- There are three ways of conducting electricity through rocks: by dielectric, electrolytic and electronic conduction. The electrical conductivity of porous rocks depends on their porosity and on the mobility of water to pass through the pore spaces (hence dependent upon ionic mobility and solution concentrations, viscosity, temperature and pressure).

(1) Electrokinetic potentials (전기역학 전위)

- ✓ An electrokinetic potential (E_k) forms as a result of an electrolyte flowing through a capillary or a porous medium, the potential being measured across the ends of the capillary.

- ✓ Electrofiltration, electromechanical or streaming potentials.
- ✓ According to Helmholtz's law, the flow of electric current is related to the hydraulic gradient and a quantity known as the electrofiltration coupling coefficient, C_E.

$$E_k = \frac{\varepsilon \mu C_E \delta p}{4\pi\eta}$$

ε : dielectric constant

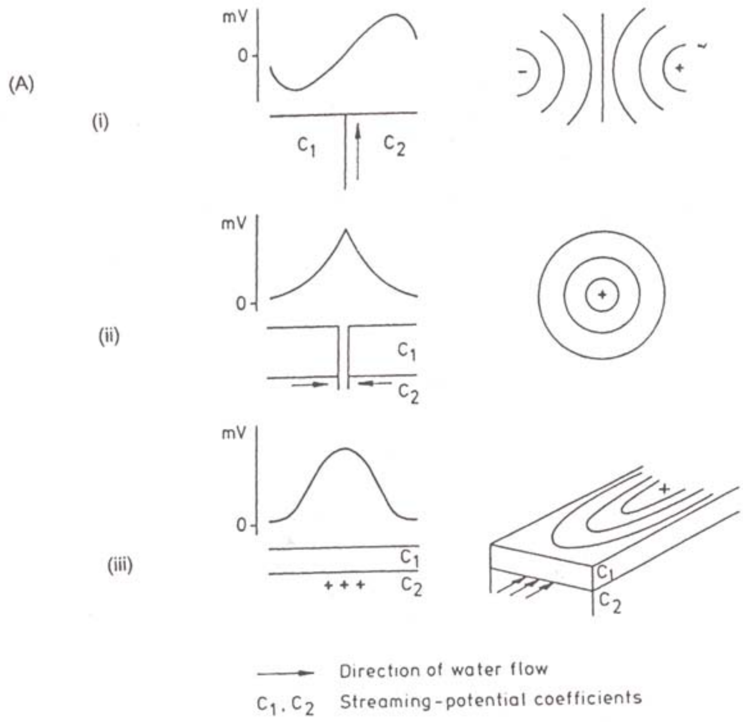
μ : resistivity

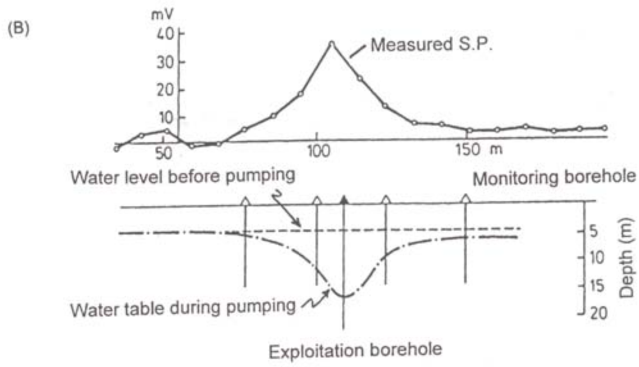
η : dynamic viscosity of the electrolyte

δp : pressure difference

C_E : electrofiltration coupling coefficient

- ✓ Figure 8.1: Graphs of electrokinetic potentials obtained for different geological situations.





- The potentials tend to increase in positive with the direction of water flow as the electric charge flows in the opposite direction. Consequently, negative charge flows uphill and can result in spectacular SP anomalies on topographic highs.
- For examples, Gay (1967) reported a potential of -1842 mV measured on a mountain top near Hualgayoc, Peru (minerialization: alunite); Nayak (1981) measured a value of -1940 mV on a hill of unmineralized quartzites in Shillong, India; Corwin and Hoover Volcano, Adak Island, Alaska.
- Superimposed on the topographic flow generation of self-potentials, potentials of 5 mV are caused by the rapid percolation of water such

as from heavy rainfall.

- Variation in soil moisture content may also produce locally variable SP signals. When the electrode is in the wetter soil, the SP signals become increasingly positive.
- The small but measurable SP anomaly lasts only as long as the water flow.
- Potentials of the order of tens of millivolts can be induced artificially through pumping groundwater (Figure 8.1B).

✓ Another factor that needs to be taken into account is thermoelectric coupling, which is the production of a potential difference across a rock sample throughout which a temperature gradient is maintained. The effect may be caused by differential thermal diffusion of ions in the pore fluid and of electrons and donor ions in the rock matrix. The process is called the Soret Effect.

(2) Electrochemical potentials (전기화학 전위)

✓ Transient background diffusion (or liquid-junction) potentials (확산전위 또는 액간접촉전위) up to tens of millivolts may be due to the differences in the mobilities of electrolytes having different concentrations within groundwater.

✓ For this mechanism to explain the continued occurrence of back ground potentials, a source capable of maintaining imbalances in the electrolytic concentrations is needed, otherwise the concentrations differences will disappear with time by diffusion.

$$E_d = -\frac{RT(I_a - I_c)}{nF(I_a + I_c)}\ln\left(C_1 / C_2\right) \qquad E_N = -\frac{RT}{nF}\ln\left(C_1 / C_2\right)$$

I_a and I_c are the mobilities of the anions and cations

C_1 and C_1 : the solution concentration

n : ionic valence

F : Faraday's constant(96487$Cmol^{-1}$)

R : is the universal Gas constant(8.314JK^{-1})

T : absolute temperature

✓ The Nernst (shale) potential (E_N) occurs when there is a potential difference between two electrodes immersed in a homogeneous solution and at which the concentrations of the solutions are locally different.

- ✓ The form of the equation for the Nernst potential is a special case of that for the diffusion potential and can easily be combined to form the electrochemical potential.
- ✓ The Nernst potential is of particular importance in well logging, in which case it is referred to as the shale potential.
- ✓ The higher the temperature and the greater the concentration differences, the larger the electrochemical potential will be.
- ✓ For this reason, the measurement of self-potentials is important in the exploration for geothermal resources where the temperatures are obviously elevated and the concentrations of salts within the groundwater are also likely to be high.
- ✓ Further electrochemical potentials are attributable to adsorption of anions (음이온의 흡착) on to the surface of veins of quartz (석영) and pegmatite (페그마타이트). It is known as adsorption (or zeta) potentials.
- ✓ For example, an anomaly of up to + 100 mV has been measured over vertical pegmatite dykes within gneiss (Figure 8.2). In addition, adsorption potentials may account for the observed anomalies over clays where the solid-liquid double layer may generate a potential.

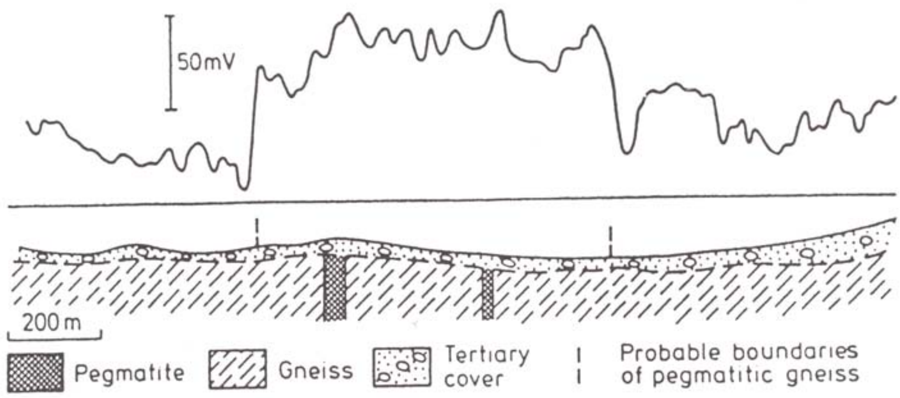


Figure 8.2 An SP profile across pegmatite dykes in gneiss. From Semenov (1980), by permission

(3) Mineral potentials (광화 전위)

- ✓ Mineral potential is of most importance in the use of SP in mineral exploration. It is associated with massive sulfide ore bodies.
- ✓ Large negative anomalies can be observed particularly over pyrite (황철석) and chalcopyrite (황동석) and other good electronic conductors.
- ✓ Sato and Mooney's hypothesis: Where a sulfide ore body straddles the

water table, they gain electrons above the water table. In contrast, below the water table, oxidation is dominant and ions lose electrons.

- ✓ The role of the massive ore body is to permit the flow of electrons from the lower half of the ore body to the upper half. The net result of this process is that the upper surface becomes negatively charged and the lower half becomes positively charged, which causes the negative SP anomalies.
- ✓ This hypothesis does not explain all the occurrences of self-potentials, which indicates that the actual physical processes are more complicated and not yet fully understood.
- ✓ Where clay overlies a massive sulfide ore body, the mineral potential may be suppressed to the extent that no anomaly is observed. This may be a result of the adsorption potential (which tends to be positive) having the same amplitude as the mineral potential (negative polarity) and thus cancelling each other out.

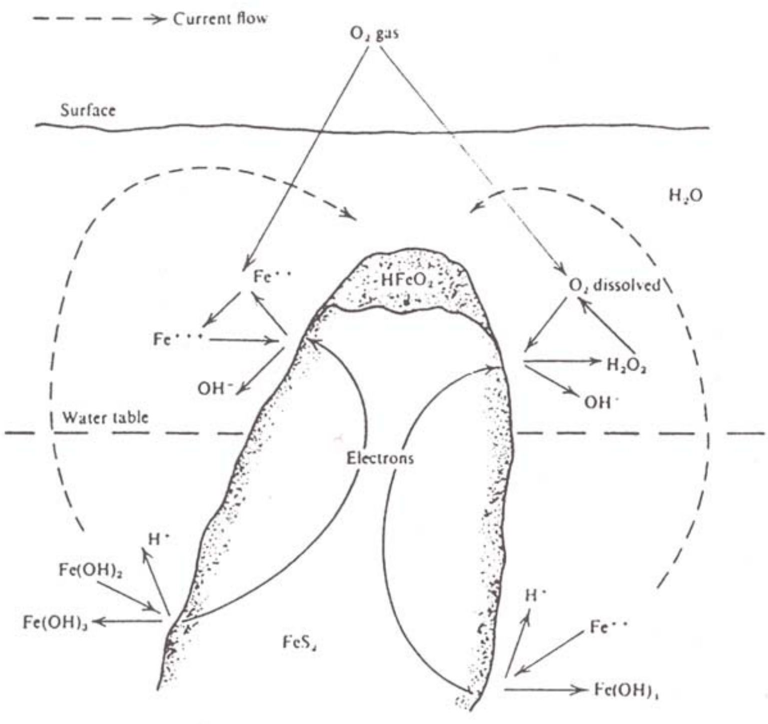


Figure 8.3 Physicochemical model proposed by Sato and Mooney (1960) to account for the self-potential process in a massive sulphide orebody. Reproduced by permission

4. Measurement of self-potentials

- The measurement of self-potentials is very simple.
- Two non-polarized porous-pot electrodes (비분극 전극) are connected to a precision multimeter with input impedance greater than 10^8 ohms and capable of measuring to at least 1 mV.
- Each electrode is made up of a copper electrode dipped in a saturated solution of copper sulfate which can percolate through the porous base to the pot in order to make electrical contact with the ground (Figure 8.4). Alternatively, a zinc electrode in saturated zinc sulfate solution or silver in silver chloride can be used.

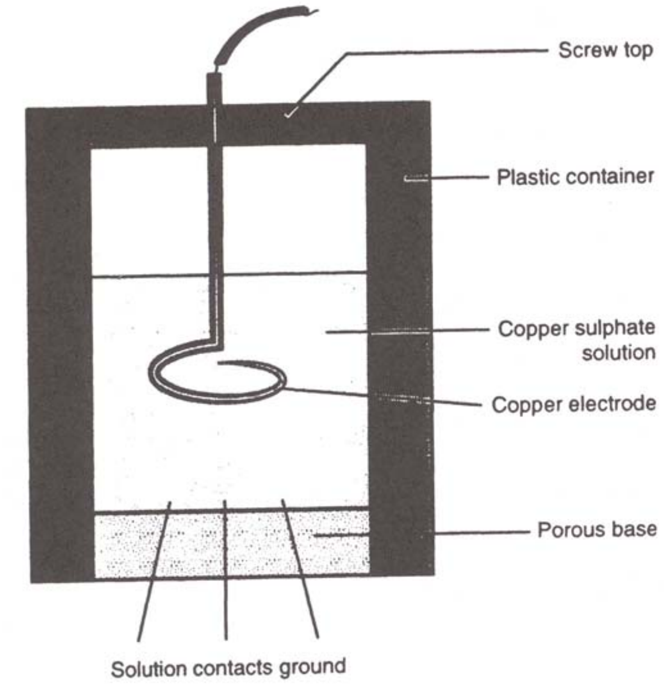


Figure 8.4 Cross-section through a porous pot electrode

- There are two field techniques, both of which are carried out at right-angles to the suspected strike of the geological target.

- The potential gradient method (전위변화율법) uses two electrodes, at a fixed separation, typically 5 m or 10 m, between which the potential difference measured is divided by the electrode separation to give a potential gradient (mV/m). The point to which this observation applies is the midpoint between the two electrodes. Check if the correct polarity of the potential is recorded.

- The potential amplitude method (전위진폭법) is to keep one electrode fixed at a base station on unmineralized ground and to move the second one along the traverse. This method removes the problems of confusing polarity and accumulating errors. Check if the temperature of the electrolyte in the mobile pot does not differ significantly from that in the reference electrode. Potential difference will be produced due to temperature difference. The temperature coefficient for copper-copper sulfate is about 0.5 mV/°C (about 0.25 mV/°C for silver-silver chloride electrodes).

- The self-potential consists of a static and a variable alternating component. The latter, which can have frequencies typically in the range of 5-10 Hz, is caused by atmospheric effects and its long period component may have amplitudes that are of the same order as the static mineral potential.

- Electrical noise can also result if measurements are made too soon after heavy rain or too close to running surface water, as streaming potentials may then swamp any mineral potential.

- The maximum depth of sensitivity of the SP method is around 60–100 m, depending on the depth to the orebody and the nature of the overburden.

5. Corrections to SP data

(1) Removing regional trend
- ✓ Self-potential measured over a large area (of the order of many square kilometers) may have a regional trend due to 'telluric' currents of ≥ 100 mV/km. To interpret the anomaly due to mineralization, its anomaly has to be isolated.

(2) Topographic correction
- ✓ Telluric currents are also affected by changes in elevation.
- ✓ If the surface slope of a survey area is large (> 20˚), the SP minimum may be well displace from its cause and subsequent drilling may miss the ore body completely.

(3) The effects of bioelectric potentials caused by vegetation
- ✓ Passing from bare ground into an area of vegetation can cause a negative potential of several hundred millivolts, comparable to a mineral potential

due to a sulfide ore body.

6. Interpretation of self-potential anomalies

- SP anomalies are often interpreted qualitatively by profile shape, amplitude, polarity (positive or negative) and contour pattern.
- The top of an ore body is then assumed to lie directly beneath the position of the minimum potential. If the axis of polarization is inclined from the vertical, the shape of the profile will become asymmetrical with the steepest slope (Figure 8.5).

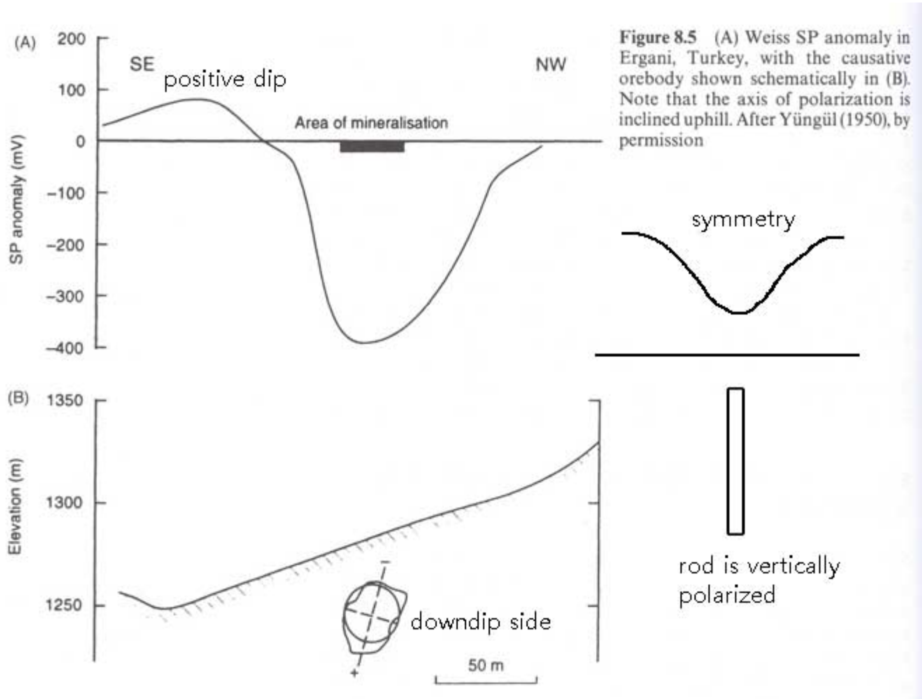


Figure 8.5 (A) Weiss SP anomaly in Ergani, Turkey, with the causative orebody shown schematically in (B). Note that the axis of polarization is inclined uphill. After Yüngül (1950), by permission

- Interpretation is complicated when two or more geological features give rise to superimposed SP anomalies (Figure 8.6). An anomaly over graphitic phyllites is characteristically large (-740 mV) owing to mineral electrochemical potentials. A second anomaly (-650 mV) has been produced by electrokinetic potentials associated with water flow through permeable disintegrated conglomerates.

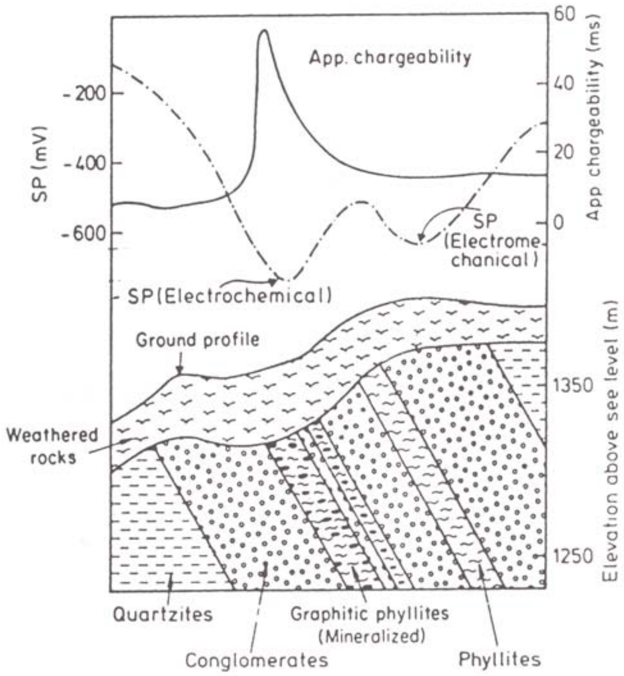


Figure 8.6 Two SP minima with different causes: one produced by electrochemical processes associated with mineralised graphite phyllites, and one caused by electrokinetic processes due to the flow of water in per- meable disintegrated conglomer- ates. From Nayak (1981), by per- mission

- Consider two graphite bodies in gneiss (Figure 8.7) in two different models. The first is where the graphites dip towards each other in a synclinal structure, in which case the negative centers associated with each polarized body are well separated, resulting in an anomaly with two negative minima. The second is where the graphite bodies dip away from each other in an anticlinal structure, in which case the two negative centers are very close together, and may even combine to form one large negative minimum. The separation between the two minima is equal to the separation of the tops of the graphite bodies.

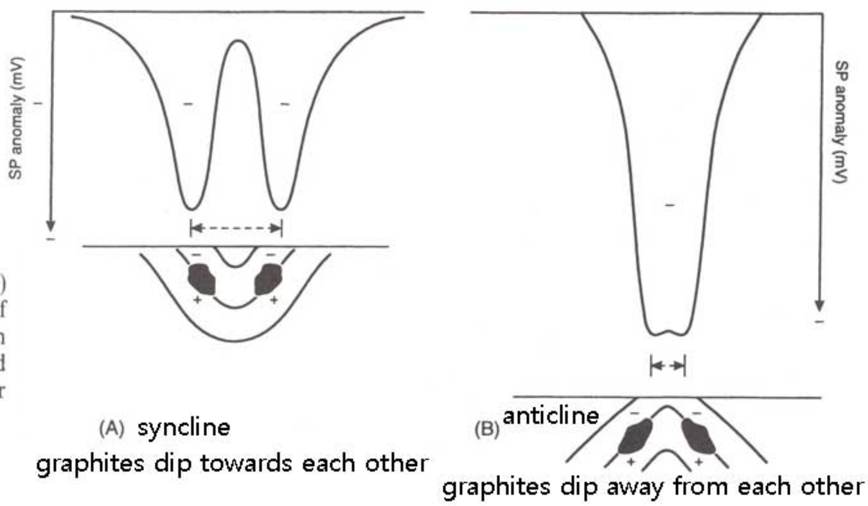


Figure 8.7 SP anomalies due to (A) two graphite bodies with axes of polarisation inclined away from each other (in syncline), and (B) inclined towards each other (in anticline). After Meiser (1962), by permission

7. Applications and case histories

(1) Geothermal

- ✓ The hydrogeological regimes associated with geothermal fields are often complex. Water bodies can have highly differing temperatures and salinities, and be highly mobile. Consequently, streaming potentials may be well developed and hence may be measured using the SP method.

- ✓ Roosevelt Hot Springs, Utah, USA

  Figure 8.10: Alunite (명반석) and pyrite(황철석) occur in the zone, both of which normally produce negative polarity anomalies that may be evident on the profile within 1 km west of the reference electrode position. The area within 1 km to the east of the reference electrode has a positive anomaly of about +80 mV which is thought to be due to the geothermal activity.

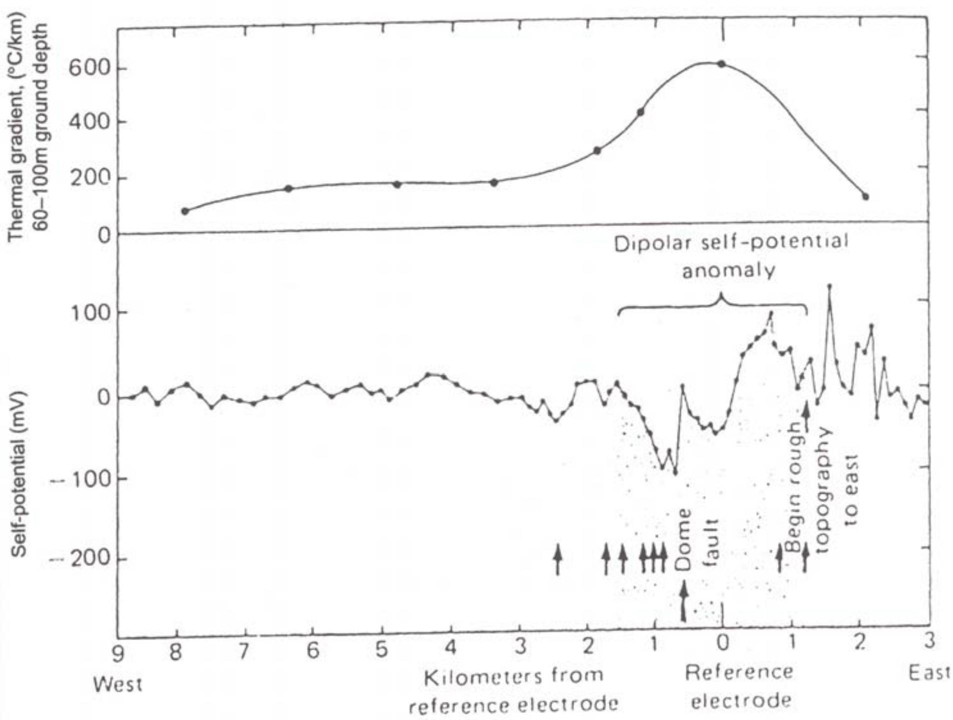
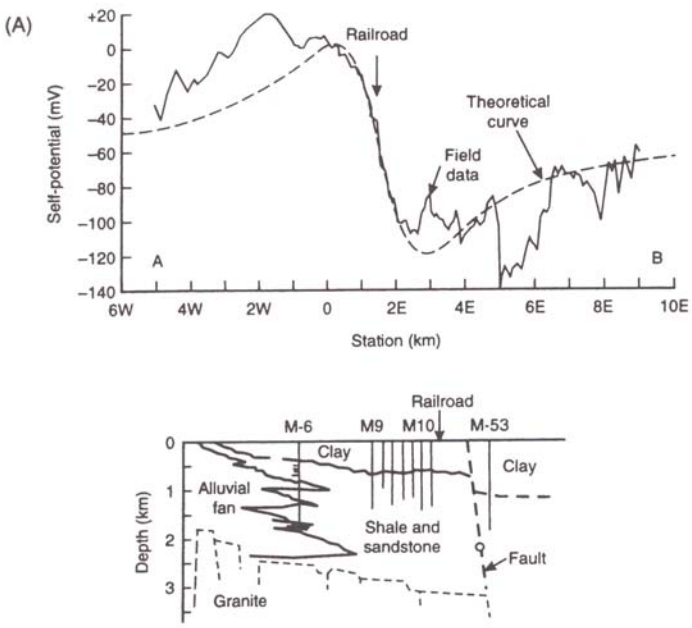


Figure 8.10 Thermal gradient and self-potential profiles over the Dome Fault Zone, Roosevelt Hot Springs, Utah. Arrows denote points at which mapped faults cross the SP survey line. From Corwin and Hoover (1979), by permission

✓ Cerro Prieto geothermal field, Mexico

Significant SP anomaly of some 150 mV peak-to-peak amplitude, associated with the Cerro Prieto geothermal field in Mexico, has been reported. The anomaly is centred over the Hidalgo Fault which is thought to provide a major conduit for geothermal fluids. (Figure 8.11)



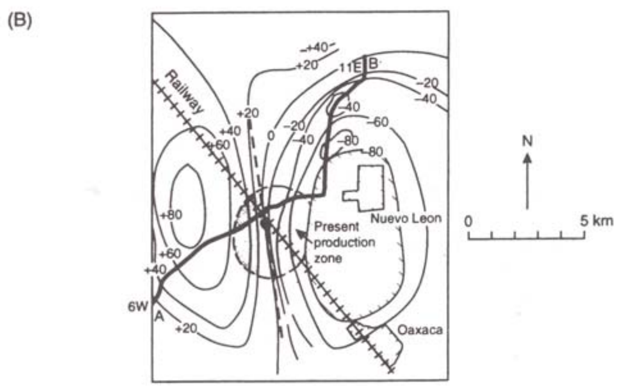


Figure 8.11 (*opposite*) (A) Self-potential anomaly along the profile A–B over the Cerro Prieto Geothermal Field, Mexico, with a simplified geological cross-section. After Corwin and Hoover (1979), by permission. (B) Self-potential map over the same field (profile line A–B marked) showing a distinct positive–negative couplet with the geothermal production area being midway between the two parts of the anomaly. After Fitterman and Corwin (1982), by permission

(2) Location of massive sulfide ore bodies

- ✓ Sulfide ore body at Sariyer, Turkey

  Figure 8.14: Chalcopyrite and pyrite occur in varying concentrations within a massive deposit within andesite and below Devonian schist.

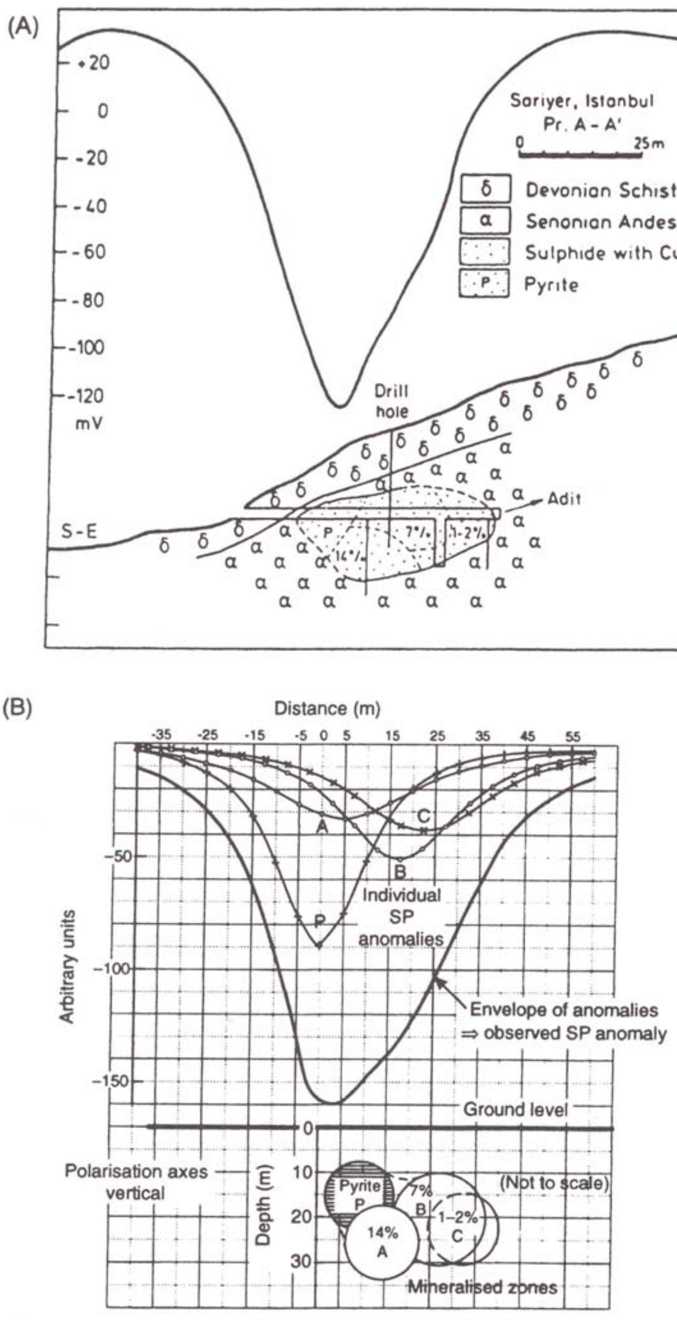


Figure 8.14 (A) Self-potential anomaly across a pyrite orebody at Sariyer, Turkey. The borehole is located at the location of the topographically corrected SP minimum. (B) An equivalent model assuming each segment of the orebosy conforms to a sphere with its axis of polarisation vertical with the corresponding individual SP anomalies and their envelope. After Yüngül, (1950), by permission

(3) Leak detection within embankments

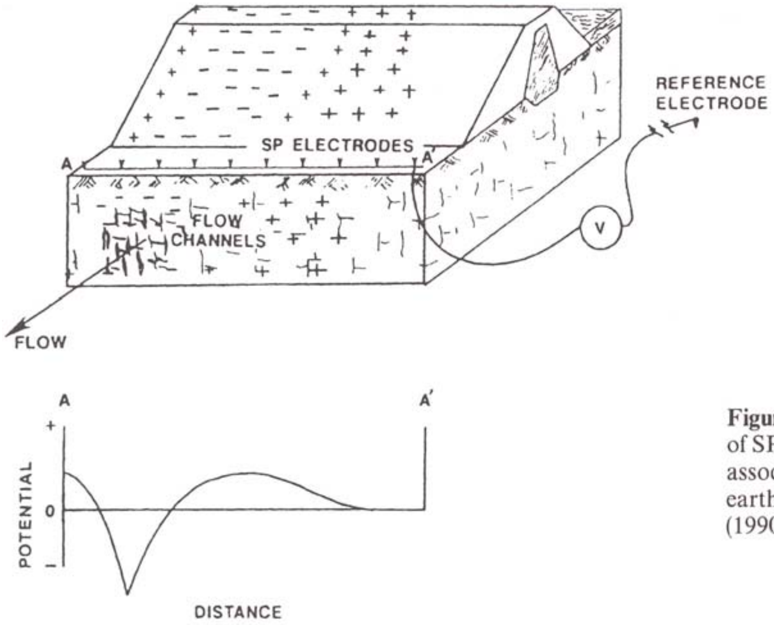


Figure 8.18 Schematic of the concept of SP anomalies generated by features associated with seepages through earth dams. From Butler and Llopis (1990), by permission

Electromagnetic Method

1. Introduction

- In some classifications, the electromagnetic (EM) method is regarded as a kind of the electrical method. In other classifications, the electromagnetic method includes the electrical method.

- Main differences between the electrical method and the electromagnetic method:

  - ✓ Source
    - In the electric method: a direct current (DC) or a very low-frequency alternating current
    - In the electromagnetic method: electromagnetic waves resulting from a relatively high-frequency alternating current

  - ✓ Transmitter and receiver
    - In the electric method: current and potential electrodes are used for transmitters and receivers.
    - In the electromagnetic method: loops or coils, which are not needed to be contacted on the ground, are used.
      ➔ Can be used in aerial survey as well as in ground survey

    - When currents are transmitted directly into the earth, the technique is called "a galvanic method". When currents are generated without the transmitter directly contacting the earth, the technique is called "an inductive method".

    - Galvanic techniques depend upon good electrode contact and thus are not appropriate in areas that have high resistivity at the surface such as dry sand or glaciers. → EM method can be useful in such areas.

    - The electric resistivity methods are (   ), whereas most electromagnetic methods are (   ).

– In galvanic methods, depth penetration for a given resistivity structure is controlled only by the array geometry, in inductive techniques deeper penetration can also be obtained by using lower frequencies.

- Electromagnetic response can be induced by natural earth currents (MT: magnetotelluric method) or by artificial or controlled sources (EM in a narrow sense).

  - ✓ EM: most EM systems employ an active transmitter so that the source geometry and frequency can be controlled.

  - ✓ MT: the ambient electromagnetic radiation from ionospheric oscillations and from lightning discharges is utilized as a source for the telluric, magnetotelluric, and audio-frequency magnetic techniques.

    – Advantages of natural sources are the avoidance of the financial and logistical problems of a transmitter, and the availability of low frequency energy, which is expensive to generate artificially. However, sufficient signal strength is not always available, particularly in the 0.1 – 10 Hz frequency range.

- The electromagnetic techniques have broad range of different instrumental systems.

- Probably the first electromagnetic method to be used for mineral ore exploration was developed by Karl Sundberg in Sweden following the First World War. The sundberg method was developed in 1925 and was also used in structural mapping in hydrocarbon exploration. Other pioneering work was done in the early 1930s by a Russian geophysicist V.R. Bursian. Other electromagnetic methods have been available commercially only since the Second World War and particularly since the mid-1960s.

- EM methods are especially important, not only in mineral and hydrocarbon exploration, but increasingly in environmental geophysics applications. For hydrocarbon exploration, the EM methods have not been widely used. These days, however, marine MT begins to be used in oil exploration. The marine MT data can be useful when it is not easy to acquire seismic exploration data. The MT data can provide regional subsurface information, and thus can be used as a reconnaissance method.

- The range of applications for EM surveying. (Table 10.1)

Table 10.1 The range of applications for EM surveying*

Mineral exploration
Mineral resource evaluation
Groundwater surveys
Mapping contaminant plumes
Geothermal resource investigations
Contaminated land mapping
Landfill surveys
Detection of natural and artificial cavities
Location of geological faults, etc.
Geological mapping
Permafrost mapping, etc.

*Independent of instrument type

2. Elementary Electromagnetic Theory

- To understand the propagation and attenuation of electromagnetic waves, it is necessary to use **Maxwell's equations** in a form relating the electric and magnetic field vectors:

  E: electric field intensity (V/m), **B**: magnetic flux density (tesla T), **J**: current density (A/m^2), **H**: magnetic field intensity (A/m), **D**: the electric displacement (C/m^2)

  ✓ Faraday's law: $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$

  ➔ An electric field exists in the region of a time-varying magnetic field, such that the induced emf (electromotive force) is proportional to the negative rate of change of magnetic flux.

  ✓ Ampere-Maxwell law: $\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$

  ➔ A magnetic field is generated in space by current flow and that the field is proportional to the total current (conduction plus displacement)

  ➔ Maxwell introduced displacement current in Ampere's law.

  ➔ The displacement current (∂D / ∂t) is a quantity that arises in a changing electric field and has the units of electric current.

  ✓ Gauss' law for electric fields

  ➔ This integral (the net electric flux through the surface) is proportional to the net electric charge q_{enc} enclosed by the surface

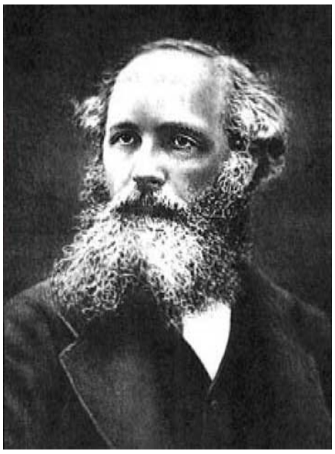
  $$\nabla \cdot \mathbf{D} = \nabla \cdot \left(\varepsilon \mathbf{E}\right) = q_{enc}$$

  ✓ Gauss' law for magnetic fields

  ➔ There can be no net magnetic flux through the surface because there can be no net "magnetic charge" (individual magnetic poles) enclosed by the surface.

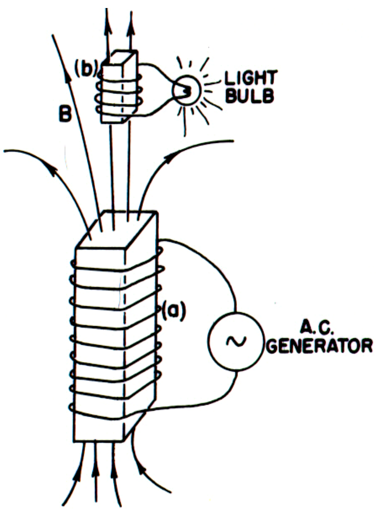
  ➔ There must always be as much magnetic flux into the surface as out of it, and the net magnetic flux must always be zero.

✧ **Maxwell's achievements**



– Electricity and magnetism were originally thought to be unrelated.
– In 1865, James Clerk Maxwell provided a mathematical theory that showed a close relationship between all electric and magnetic phenomena
– Electric field lines originate on positive charges and terminate on negative charges.
– Magnetic field lines always form closed loop – they do not begin or end anywhere
– A varying magnetic field induces an emf and hence an electric field (Faraday's law)
– Magnetic field is also produced by changing electric field
  ➔ Changing electric field produces magnetic field, which in turn produces changing electric field, which in turn produces changing magnetic field.
– Maxwell concluded that visible light and all other electromagnetic waves consists of fluctuating electric and magnetic fields, with each varying field inducing the other.

✧ **An example (transformer)**



– Current flowing in conductive wire produces **H** field via Ampere's law assuming ωε≪σ: $\boxed{\nabla \times \mathbf{H} = \mathbf{I}}$

– High μ value of core material enhances **B** field: $\boxed{\mathbf{B} = \mu \mathbf{H}}$

– An E field is produced which curls around the magnetic field via Faraday's law: $\boxed{\nabla \times \mathbf{E} = -i\omega \mathbf{B}}$

– These processes result in voltage in the upper coil
$\boxed{\int (\nabla \times \mathbf{E}) \cdot \hat{\mathbf{n}} ds = \oint \mathbf{E} \cdot \mathbf{dl} = V}$ Stoke's theorem
– From V=IR, current will flow in the upper coil, which lights bulb

➢ Electromagnetic equations in an isotropic and homogeneous media conductivity σ, relative permeability μ and relative dielectric permittivity (유전율) ε

* Prove $\nabla \cdot \mathbf{E} = 0$ $\nabla \cdot \mathbf{B} = 0$

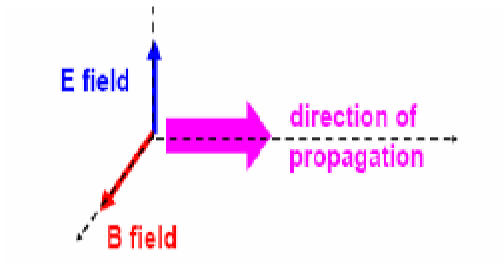
$$\nabla \cdot (\nabla \times \mathbf{E}) = -\nabla \cdot \left(\frac{\partial \mathbf{B}}{\partial t} \right) = -\frac{\partial}{\partial t} (\nabla \cdot \mathbf{B}) = 0$$

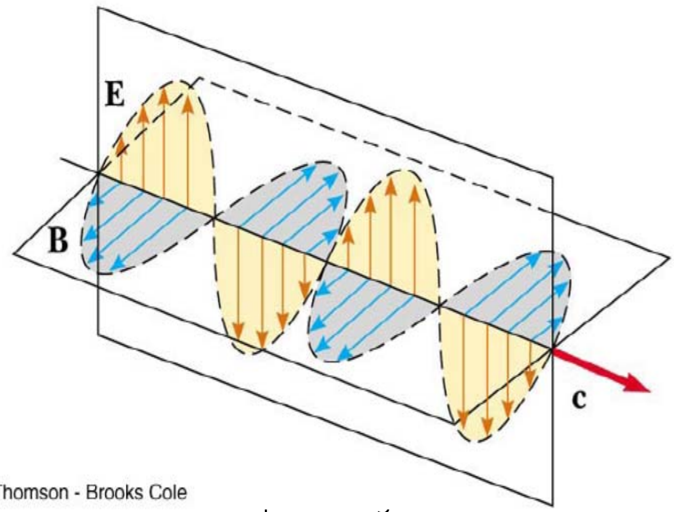
✓ Features of electromagnetic waves

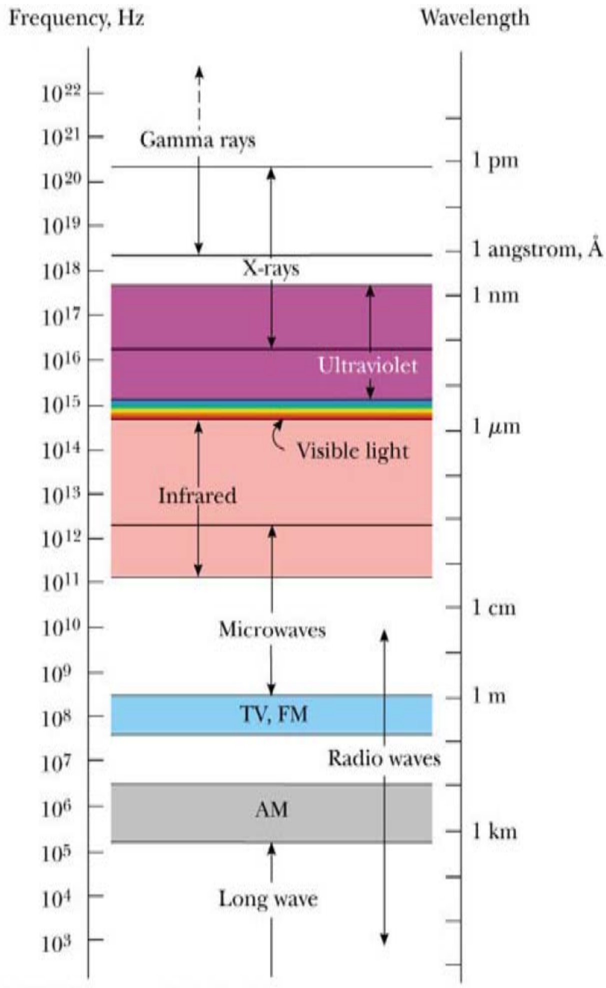
- The electric and magnetic fields **E** and **B** are always perpendicular to the direction in which the wave is traveling. Thus, the wave is a transverse wave
- The electric field is always perpendicular to the magnetic field.
- The cross product E × B always gives the direction in which the wave travels.
- The fields always vary sinusoidally, just like the transverse waves
- Electromagnetic waves travel at the speed of light

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99792 \times 10^8 \, {}^{m}\!/_{s} \quad \text{in the free space}$$

- Light is an electromagnetic wave.







$$\nabla\cdot(\nabla\times\mathbf{H})=\nabla\cdot\mathbf{J}+\nabla\cdot\left(\frac{\partial\mathbf{D}}{\partial t}\right)$$

$$=\nabla\cdot\mathbf{J}+\frac{\partial}{\partial t}(\nabla\cdot\mathbf{D})$$

Assuming that no current source exist within the earth, $\nabla\cdot\mathbf{J}=0$ or the divergence of current density is equivalent to the rate of accumulation of charge density q $\qquad \nabla\cdot\mathbf{J}=\frac{\partial q}{\partial t}=0 \qquad \nabla\cdot\mathbf{D}=\nabla\cdot\varepsilon\mathbf{E}=0$

In regions of finite conductivity, charge does not accumulate to any extent.

$$\left(\begin{aligned}&\nabla\cdot\mathbf{J}=\nabla\cdot\sigma\mathbf{E}=0\\&\qquad\nabla\cdot(\sigma\mathbf{E})\\&\quad=\sigma\nabla\cdot\mathbf{E}+\cancel{\mathbf{E}\nabla\sigma}^{0}\quad(\because\nabla\sigma=0)\\&\qquad\boxed{\nabla\cdot\mathbf{E}=0}\end{aligned}\right)$$

* In a homogeneous and isotropic medium,

$\mathbf{B}=\mu\mathbf{H} \qquad \mathbf{D}=\varepsilon\mathbf{E} \qquad \mathbf{J}=\sigma\mathbf{E}$

$$\nabla\times\mathbf{E}=-\mu\frac{\partial\mathbf{H}}{\partial t}$$

$$\nabla\times\mathbf{H}=\sigma\mathbf{E}+\varepsilon\frac{\partial\mathbf{E}}{\partial t}$$

$$\nabla\times(\nabla\times\mathbf{E})=-\mu\frac{\partial}{\partial t}(\nabla\times\mathbf{H})$$

$$\left(\begin{aligned}\nabla\times\nabla\times\mathbf{A}&=\nabla(\nabla\cdot\mathbf{A})-\nabla\nabla\cdot\mathbf{A}\\&=\nabla(\nabla\cdot\mathbf{A})-\nabla^2\cdot\mathbf{A}\end{aligned}\right)$$

$$\cancel{\nabla(\nabla\cdot\mathbf{E})}^{0}-\nabla^2\mathbf{E}=-\mu\frac{\partial}{\partial t}\left(\sigma\mathbf{E}+\varepsilon\frac{\partial\mathbf{E}}{\partial t}\right)$$

$$\boxed{\nabla^2\mathbf{E}=\mu\sigma\frac{\partial\mathbf{E}}{\partial t}+\mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2}}$$

$$\nabla\times(\nabla\times\mathbf{H})=\sigma(\nabla\times\mathbf{E})+\varepsilon\frac{\partial}{\partial t}(\nabla\times\mathbf{E})$$

$$-\nabla^2\mathbf{H}=-\sigma\mu\frac{\partial\mathbf{H}}{\partial t}-\varepsilon\mu\frac{\partial^2\mathbf{H}}{\partial t^2}$$

$$\boxed{\nabla^2\mathbf{H}=\mu\sigma\frac{\partial\mathbf{H}}{\partial t}+\mu\varepsilon\frac{\partial^2\mathbf{H}}{\partial t^2}}$$

* If we choose sinusoidal time variations for $\mathbf{E}$ and $\mathbf{H}$

$$\mathbf{E}(t)=\mathbf{E}_0 e^{i\omega t}$$

$$\nabla^2\mathbf{E}=\mu\sigma i\omega\mathbf{E}_0 e^{i\omega t}-\mu\varepsilon\omega^2\mathbf{E}_0 e^{i\omega t}$$

$$\boxed{\nabla^2\mathbf{E}=i\omega\mu\sigma\mathbf{E}-\omega^2\mu\varepsilon\mathbf{E}}$$

$$\mathbf{H}(t)=\mathbf{H}_0 e^{i\omega t}$$

$$\boxed{\nabla^2\mathbf{H}=i\omega\mu\sigma\mathbf{H}-\omega^2\mu\varepsilon\mathbf{H}}$$

Helmholtz equation

$$\left(\nabla^2+k^2\right)\mathbf{A}=0$$

- Attenuation of EM fields
  - ✓ The wave is attenuated in traveling through some media but not in free space.
  - ✓ Corresponding wavelength in the free space:
    - if the periodic frequencies employed in the field data are about 3000 Hz, $\omega \le 2\times10^4$
    - $\lambda = 2\pi c/\omega = 2\pi\times3\times10^8/\omega$ m = $9.42\times10^4 \ge 90$ km
    - Because the distances involved in field layouts are usually less than 1 or 2 km, the phase variation resulting from propagation is negligible.
    - ✧ $\varepsilon = \varepsilon_r\varepsilon_0$, $\varepsilon_0 = 8.85\times10^{-12}$ F/m (dielectric permittivity of the free space),
      ε_r= relative dielectric permittivity: 80 for water, 10 for rocks, and 1 for air. $\varepsilon = 10\varepsilon_0 = 9\times10^{-11}\ \mathrm{F}/\mathrm{m}$
    - ✧ $\mu = \mu_r\mu_0$, $\mu_0 = 1.3\times10^{-6}$ H/m (magnetic permeability of the free space.
      μ_r= relative magnetic permeability : less than 3 even in ferromagnetic minerals
      normaly $\mu_r = 1$, $\mu = \mu_0 = 1.3\times10^{-6}\ \mathrm{H}/\mathrm{m}$
  - ✓ In the air, $\sigma=0$, $\varepsilon=\varepsilon_0$, and $\mu=\mu_0$

$$\nabla^2\mathbf{E}=i\omega\mu\sigma\mathbf{E}-\omega^2\mu\varepsilon\mathbf{E}=[-(2\times10^4)^2\cdot1.3\times10^{-6}\cdot9\times10^{-11}]\mathbf{E}$$

$$\approx 46.02\times10^{-10}\approx5\times10^{-9}\mathbf{E}$$

  - ✓ In rocks of low conductivity, $\varepsilon=10\varepsilon_0$, $\mu=\mu_0$, and $\sigma=10^{-3}$ S/m

$$\nabla^2\mathbf{E} = [i(2\times10^4\times1.3\times10^{-6}\times10^{-3}) - 4\times10^8\times1.3\times10^{-6}\times9\times10^{-11}]$$
$$= [i\times2.6\times10^{-5} - 4.6\times10^{-8}]\mathbf{E} \approx 0$$

- ✓ In rocks of high-conductivity, $\varepsilon=10\varepsilon_0$, $\mu=\mu_0$, and $\sigma=10^3$ S/m

$$\nabla^2\mathbf{E} = [25i - 4.6\times10^2 - 8]\mathbf{E} \approx 25i\mathbf{E}$$

- ✓ Identical relations hold for H.
- ➔ In air and in poorly conducting rocks

$\nabla^2\mathbf{E} \approx 0$, $\nabla^2\mathbf{H} \approx 0$ $\Rightarrow$ Laplace equation

- ➔ Within a good conductor

$\nabla^2\mathbf{E} \approx i\omega\mu\sigma E \approx \mu\sigma\dfrac{\partial\mathbf{E}}{\partial t}$, $\nabla^2\mathbf{H} \approx i\omega\mu\sigma\mathbf{H} \approx \mu\sigma\dfrac{\partial\mathbf{H}}{\partial t}$ $\Rightarrow$ Diffusion equation

- ➢ Solutions for the diffusion equation
  - ✓ The diffusion equations are generally difficult to solve. However, there is one important case in which a solution is readily obtained → the wave is plane polarized.
  - ✓ Assume the wave is propagating along the z axis so that the xy plane is the plane of polarization.
  - ✓ Solutions for H

$$\nabla^2\mathbf{H} \approx i\omega\mu\sigma\mathbf{H} = \mu\sigma\frac{\partial\mathbf{H}}{\partial t}$$
$$H = H_y(z,t) = H_0e^{i\omega t+mz}$$
$$\frac{\partial H_y}{\partial t} = i\omega H \ , \ \nabla^2 H = (\frac{\partial^2 H_y}{\partial z^2}) = m^2H$$
$$m^2H = i\omega\mu\sigma H$$
$$m = \pm(1+i)\sqrt{\frac{\omega\mu\sigma}{2}} \quad (\because \frac{(1+i)^2}{2} = i)$$

H must be finite when $z=+\infty$, $m=-(1+i)a$ $\quad (a=\sqrt{\dfrac{\omega\mu\sigma}{2}})$

$$H_y = H_0e^{i\omega t-(1+i)az} = H_0e^{-az+i(\omega t-az)}$$

* real part $\rightarrow$ $H_y = H_0e^{-az}\cos(\omega t - az)$

  - ✓ Skin depth is defined as a criterion for the penetration of electromagnetic waves, and indicates the depth in which the signal is reduced by 1/e.

$$\frac{H_y}{H_0} \approx e^{-az} = e^{-z\sqrt{\frac{\omega\mu\sigma}{2}}} = e^{\sqrt{\frac{2\pi f\mu}{2}\frac{1}{\rho}}} = e^{-2\times10^{-3}z\sqrt{\frac{f}{\rho}}} \quad (\because \sigma = \frac{1}{\rho})$$

Skin depth $\rightarrow \dfrac{H_y}{H_0} = \dfrac{1}{e}$

$$2\times10^{-3} z\sqrt{\frac{f}{\rho}} = 1$$

Skin depth $z = \dfrac{1}{2\times10^{-3}}\sqrt{\dfrac{\rho}{f}} = 500\sqrt{\dfrac{\rho}{f}}$

Table 6.1. and 6.2.

As a crude rule of thumb,

If $z\left(\dfrac{f}{\rho}\right)^2 > 10^3$: attenuation will be large and vice versa

Table 6.1. *Attenuation of EM waves.*

f (Hz)	ρ (Ωm)	$\lvert H_y/H_0\rvert$ for $z = 30$ m	z for $\lvert H_y/H_0\rvert = 0.1$ (m)
10^3	10^{-4}	0.00	0
10^3	10^{-2}	0.00	0
10^3	1	0.15	37
10^3	10^2	0.83	370
10^3	10^4	0.98	3,700
10	10	0.94	1,160
10^2	10	0.83	370
10^4	10	0.15	37
10^6	10	0.00	3.7

Table 6.2. *Skin depth variation with frequency and resistivity.*

f (Hz)	$\rho = 10^{-4}$ Ωm z_s (m)	10^{-2} Ωm z_s (m)	10^0 Ωm z_s (m)	10^2 Ωm z_s (m)	10^4 Ωm z_s (m)
10^{-3}	160	1,600	1.6×10^4	1.6×10^5	1.6×10^6
10^{-2}	50	500	5,000	5×10^4	5×10^5
10^{-1}	16	160	1,600	1.6×10^4	1.6×10^5
1	5	50	500	5,000	5×10^4
10	1.6	16	160	1,600	1.6×10^4
10^2	0.5	5	50	500	5,000
10^3	0.16	1.6	16	160	1,600
10^4	0.05	0.5	5	50	500
10^6	0.005	0.05	0.5	5	50
10^8	—	0.005	0.05	0.5	5

- Solutions for the current

$\nabla\times\mathbf{H} = \sigma\mathbf{E} + \varepsilon\dfrac{\partial\mathbf{E}}{\partial t}$ (In EM, $\omega\varepsilon \ll \sigma$, $\dfrac{\partial\mathbf{E}}{\partial t} \leftarrow$ neglected)

$$\nabla\times\mathbf{H} = \sigma\mathbf{E} = \mathbf{J}$$

$$J_y = J_z = 0$$

$$J_x = -\frac{\partial H_y}{\partial z} = -\frac{\partial}{\partial z}\{H_0 e^{-az}\cos(\omega t - az)\} = aH_0 e^{-az}\{\cos(\omega t - az) - \sin(\omega t - az)\}$$

$$= \sqrt{2}aH_0 e^{-az}\cos(\omega t - az + \frac{\pi}{4}) = \sqrt{\omega\mu\sigma}H_0 e^{-z\sqrt{\frac{\omega\mu\sigma}{2}}}\cos(\omega t - z(\frac{\omega\mu\sigma}{2})^{\frac{1}{2}} + \frac{\pi}{4})$$

- ✓ The amplitude of the current is ($\sqrt{\omega\mu\sigma}$) times that of the magnetic field at all points.
- ✓ Because Jx is proportional to Hy, the current flux exhibits the same skin effect as the magnetic field
- ✓ In a good conductor, it is concentrated near the surface

- ✓ When $(\omega\mu\sigma/2)^{1/2}$ is small, the magnetic field will propagate through the medium without much attenuation and in the process will fail to induce any appreciable current flow in it. → little secondary magnetic field generated

- ✓ When $(\omega\mu\sigma/2)^{1/2}$ is large, the large surface current creates a large secondary magnetic field, out of phase with the original, which partially or completely cancels the primary field

- ✓ When the medium has intermediate conductivity, there will be some secondary magnetic field developed.

➢ Boundary conditions

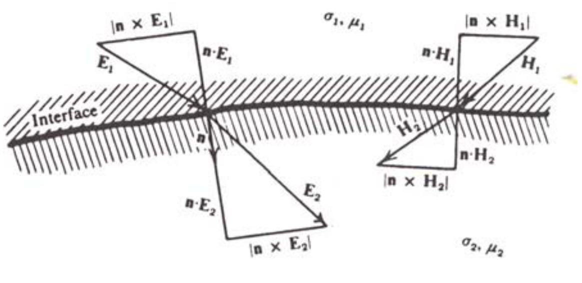


Figure 6.14. Boundary conditions on EM fields at an interface.

① The tangentioal component of electric field

$$\mathbf{n}\times(\mathbf{E}_1 - \mathbf{E}_2) = 0$$

② The tangential component of magnetic field

$\mathbf{n}\times(\mathbf{H}_1-\mathbf{H}_2)=0$

③ The normal component of current density

$\mathbf{n}\bullet(\sigma_1\mathbf{E}_1-\sigma_2\mathbf{E}_2)=0$

④ The normal component of magnetic flux

$\mathbf{n}\bullet(\mu_1\mathbf{H}_1-\mu_2\mathbf{H}_2)=0$

3. Principles of EM surveying

- Electromagnetic methods use the response of the ground to the propagation of incident alternating electromagnetic waves which are made up of two orthogonal vector components, an electric intensity (E) and a magnetic force (H) in a plane perpendicular to the direction of travel.
- For geophysical applications, frequencies of the primary alternating field are usually less than a few thousand hertz. The wavelength of the primary wave is of the order of 10-100 km while the typical source-receiver separation is much smaller (4 – 100 m). → The propagation of the primary wave and associated wave attenuation can be disregarded (Figure 10.8).

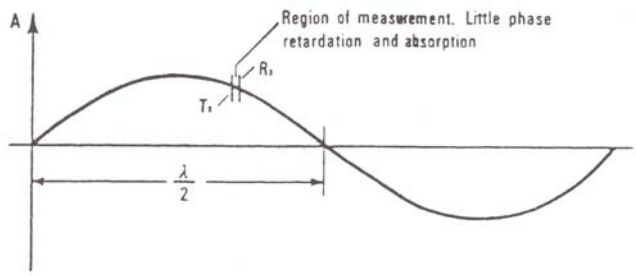


Figure 10.8 The physical separation of a transmitter (Tx) and receiver (Rx) is very small in relation to the wavelength of EM waves with frequencies greater than 3 kHz. Consequently, attenuation due to wave propagation can be ignored. Reproduced with permission from Beck (1991)

- The primary electromagnetic field, generated in a transmitter coil, propagates above and below the ground. When the EM radiation travels through subsurface media, it is modified slightly relative to that which travels through air.
- If a conductive medium is present under the ground, the magnetic component of the incident EM wave induces eddy currents (alternating currents) within the conductor. These eddy currents then generate their own, secondary, EM field (Figure 10.9).

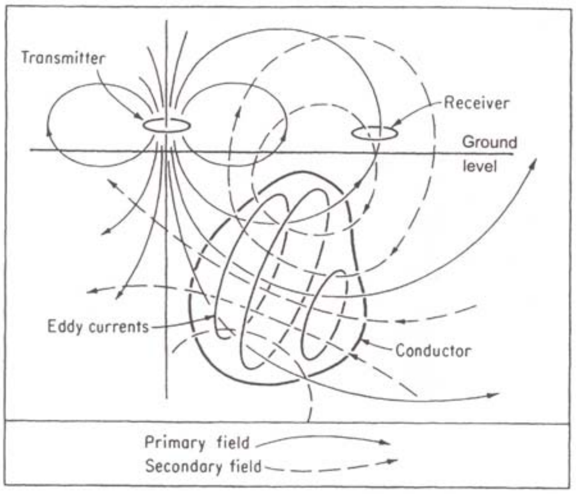


Figure 10.9 Generalized schematic of the EM surveying method. Reproduced with permission from Grant and West (1965)

- A receiver detects both the primary field that travels through the air and the secondary field.
- The measured response will differ from the primary field in both phase and amplitude. The degree to which these components differ reveals important information about the geometry, size, and electrical properties of any sub-surface conductor.

A. Phase

- Assume that the ground is composed of three components: inductance (L), resistance (R) and capacitance (C) (Figure 10.10).

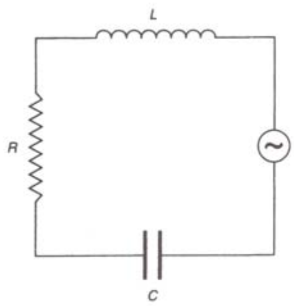


Figure 10.10 Basic electrical circuit containing capacitance (C), inductance (L) and resistance (R), the three electrical components that describe the equivalent behaviour of the ground.

- The relationship between the applied alternating voltage and the current (Refer to the textbook on Physics)
  - ✓ The amplitude (E) of an alternating voltage changes sinusoidally
    $E = E_0 \sin \omega t$
  - ✓ The current (I) within the equivalent circuit is described by

$$I = E_0[\{\omega L - (\frac{1}{\omega L})\}^2 + R^2]^{-\frac{1}{2}} \sin(\omega t - \alpha)$$

➔ The current (I) has the phase lag α

$$\alpha = \tan^{-1}\left\{[\omega L - (1/\omega L)]/R\right\}$$

- In EM exploration, in accordance with the properties of an EM wave, a primary magnetic field is applied (P) which is (in phase) with its orthogonal electric components E: The primary magnetic wave is ($P = H_0 \sin \omega t$) (Fig.10.11A).
- According to Faraday's law, the time-varying primary magnetic field induces voltage to a subsurface conductor.
- The voltage induced into a secondary perfect conductor has phase lag of ($\frac{\pi}{2}$), compared to the incident primary magnetic field. → The induced voltage will be zero when the magnetic field is either at its maximum or minimum (Figure 10. 11B).

- Eddy currents within a conductor take a finite time to be generated, arising from an induced voltage. → this is expressed as the phase lag α (Figure 10.11C), which depends on the electrical properties of the conductor. In good conductors, the phase lag is large, whereas in poor conductors the phase lag is small.
- Figure 10.12 shows the relationship between the primary, secondary, and resultant fields. The primary magnetic field is P. The induced voltage lags ($\frac{\pi}{2}$) behind the primary magnetic field. The secondary current or magnetic field lags α behind the induced voltage. As a result, the secondary magnetic field lags ($\frac{\pi}{2} + \alpha$) behind the primary magnetic field.
- The component of the secondary field, which has the phase lag of π, is called (real) or (phase) component (Hs sinα), whereas the component of the secondary field, which has the phase lag of π/2, is called (imaginary), (out-phase), or (quadrature) component (Hs cosα).

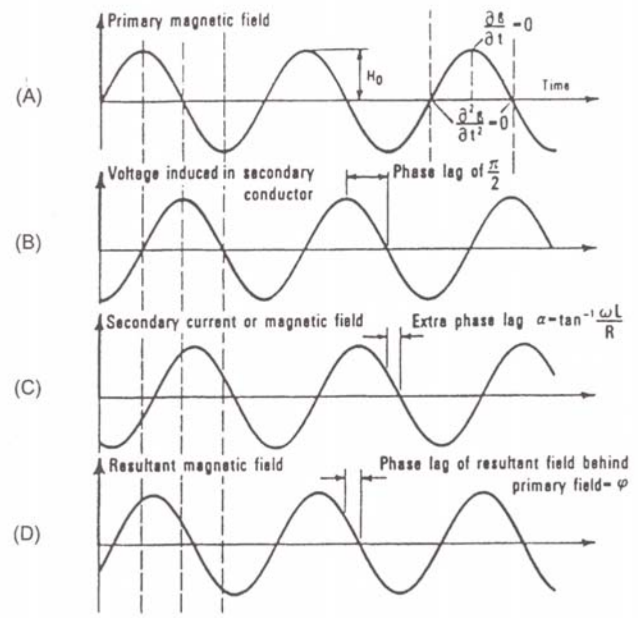


Figure 10.11 Relationships between induced voltages and associated phase lags between primary, secondary and resultant magnetic fields. Reproduced with permission from Beck (1981).

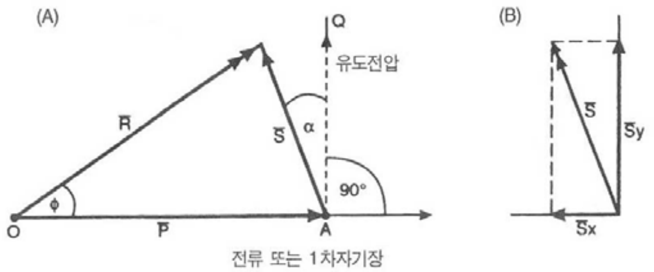


그림 10.12 (A) 1차장, 유도기전력, 2차장, 합성장 벡터 및 위상, (B) 2차장의 동상 및 이상성분. 2차장은 1차장과 평행한 동상성분(S_x)과 수직인 이상성분(S_y)으로 분해된다.

B. Elliptic polarization

- The detector in an EM field system, generally a small coil with many turns of fine wire, measures the secondary field produced by a subsurface conductor, in the presence of the primary field. Consequently, the detected signal is a combination of the primary and one or more secondary fields. In general the combination is a magnetic field that is elliptically polarized.

$$H_P = A\sin\omega t \qquad H_S = B\cos(\omega t - \alpha)$$

$$\sin\omega t = \frac{H_P}{A} \qquad \cos(\omega t - \alpha) = \frac{H_S}{B}$$

$$\sin^2\omega t + \cos^2\omega t = 1 \qquad \therefore \cos\omega t = \sqrt{1-(\frac{H_P}{A})^2}$$

$$\cos(\omega t - \alpha) = \cos\omega t\cos\alpha + \sin\omega t\sin\alpha$$

$$\cos(\omega t - \alpha) = \sqrt{1-(\frac{H_P}{A})^2}\cos\alpha + \frac{H_P}{A}\sin\alpha = \frac{H_S}{B}$$

$$\sqrt{1-(\frac{H_P}{A})^2}\cos\alpha = \frac{H_S}{B} - \frac{H_P}{A}\sin\alpha$$

$$\{1-(\frac{H_P}{A})^2\}\cos^2\alpha = (\frac{H_S}{B})^2 - 2(\frac{H_S}{B})(\frac{H_P}{A})\sin\alpha + (\frac{H_P}{A})^2\sin^2\alpha$$

$$\cos^2\alpha = (\frac{H_S}{B})^2 - 2(\frac{H_S}{B})(\frac{H_P}{A}) + (\frac{H_P}{A})^2$$ → Eq. of ellipse

when $\alpha = \frac{\pi}{2}$,

$0 = (\frac{H_S}{B} - \frac{H_P}{A})^2$, $\therefore H_S = \frac{B}{A}H_P$ → Eq. of a straight line

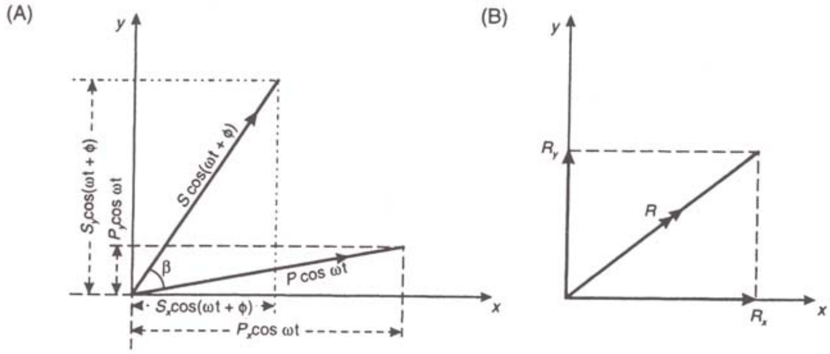
when $\alpha = 0$,

$$1 = (\frac{H_S}{B})^2 + (\frac{H_P}{A})^2$$

when A=B,

$B^2 = {H_S}^2 + {H_P}^2$ → Eq. of a circle

- If a detector coil lies in the plane of polarization, a null signal would be obtained. Some of the early EM methods were based on this fact; the dip and azimuth of the polarization ellipse and its major and minor axes were measured.
- If the detector coil is rotated about its vertical or horizontal diameter, it will not always be possible to find a perfect null position, but we can find the position where the minimum signal is detected.



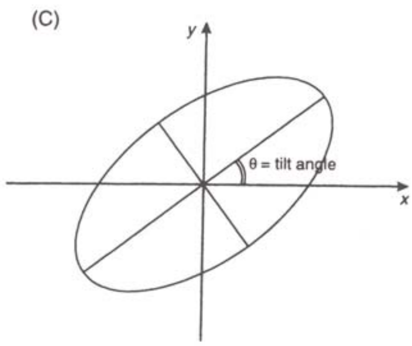
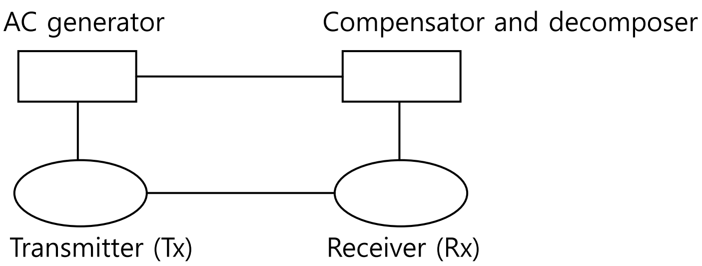


Figure 10.13 (A) Secondary field orientated in space at an angle β to the primary field. (B) The horizontal and vertical components R_x and R_y of the resultant of the summation of primary and secondary fields. (C) The ellipse of polarization inclined at the tilt-angle θ to the horizontal

- Elliptical polarization
  Combined field is elliptically polarized.
  The tilt angle θ can be determined by rotating receiver.

4. EM survey

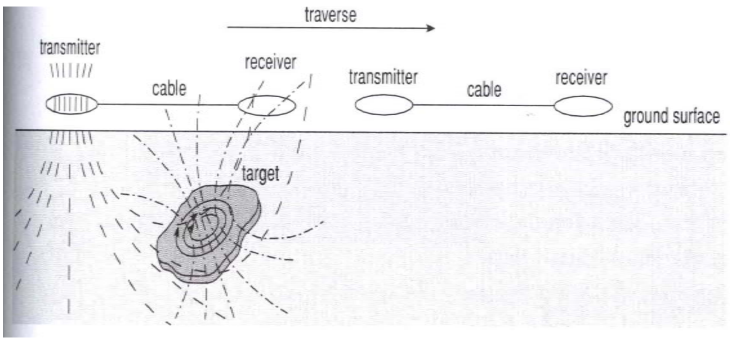
A. EM Systems



- Transmitter is connected to AC generator
- Receiver is connected to compensator and decomposer
- Compensator can be used to remove the primary field, it is possible to measure the secondary field
- Decomposer splits field into real (in-phase) and imaginary (out-of-phase,

quadrature) components

- Penetration depth depends on frequency, ground resistivity, coil separation (L), and is greater for vertical dipole configuration
- As a rule of thumb, penetration is at least L/2



- Current is applied to the loop.
- Current induces magnetic field
- Magnetic fields induce emf, which generates current. The direction of the current is determined so that the primary magnetic field decreases.
- At the receiver, we measure the primary and secondary field.

⇒ magnetic field perpendicular to the receiver loop is measured.

So the receiver is tilted along the magnetic fields, no magnetic fields are measured. The plane where the minimum magnetic field is measured is perpendicular to the ellipse polarization. By doing so, we can note the tilt angle of the ellipse.

B. Types of EM systems

Table 10.2 A classification of electrical and electromagnetic systems

Transmitter type	Receiver type			
	Ground wire	Both wire and small coil	Small coil (ground)	Small coil (air)
Grounded wire				
Galvanic	Resistivity IP		Magnetometric resistivity (MMR) Magnetic IP (MIP)	
Inductive		CSAMT	Some TEM systems	
Small loop			Slingram Horizontal-loop EM Vertical-loop EM Tilt-angle method Ground conductivity meters (GCM) Some TEM systems Coincident loop Borehole systems	Airborne EM Time-domain towed-bird Helicopter rigid-boom
Large loop (long wire)			Large-loop systems Sundberg method Turam Many TEM systems Borehole systems	
Plane wave				
Vertical antenna		VLF-resistivity VLF		VLF
Natural geomagnetic field	Telluric currents			

- Magnetometric resistivity (MMR)
  - ✓ Direct current is injected into the ground through two widely separated electrodes.
    ⇒ similar to electrical resistivity method
  - ✓ We measure the secondary magnetic field arising from the flow of current using an extremely sensitive low-noise magnetometer aligned perpendicular to the line between the electrodes (Figure 10.1).

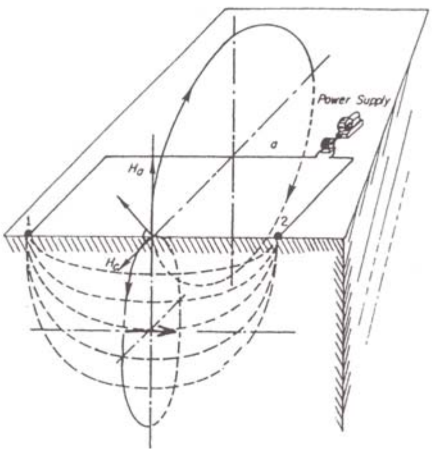


Figure 10.1 Basic concept of a magnetometric resistivity (MMR) field survey layout. Electrodes (1 and 2) are used to inject direct current into the ground. The secondary magnetic field arising from the current flow in the ground is measured at the mid-point by an extremely sensitive magnetometer. Reproduced with permission from Edwards and Nabighian (1991).

- Small-loop systems
  - A frequency-domain EM system consisting of two small coils are moved along a survey line with its separation maintained constant. One is a transmitter, and the other is a receiver.
  - The primary field is nullified so that the in-phase and quadrature components of the secondary field can be measured.

    ⇒ Compensator and decomposer are connected to the receiver.
  - Figure 10.2 shows the various combinations of coil orientation.

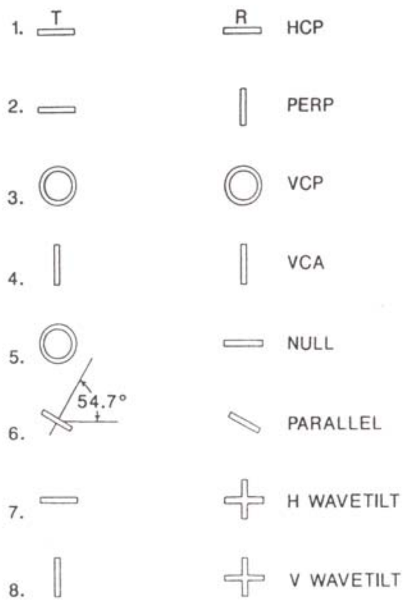


Figure 10.2 Eight common dipolar loop configurations (Tx = transmitter; Rx = receiver). Each rectangle represents the edge-on view of a coil; e.g. to move from configuration 1 to 3, both coils are rotated about a horizontal axis; to move from 3 to 4, each coil is rotated about a vertical axis. HCP = horizontal coplanar; VCP = vertical coplanar; VCA = vertical coaxial; PERP = perpendicular. Reproduced with permission from Frischnecht *et al.* (1991).

- Large-loop systems
  - Sundberg's method and Turam method
  - Sundberg's method uses a long, grounded, insulated wire a few hundred meters to several kilometers long, or a rectangular loop with the long-side laid in the direction of geological strike (Figure 10.3A and B).
  - Typical loop dimensions are 1200 m by 400 m. Measurements are made along profiles at right angles to the cable or long side of the loop. Phase reference is taken by using a feeding coil located close to the source loop/cable using the compensator system shown in Figure 10.3C. Normally only the vertical magnetic field is observed.
  - Turam technique overcomes a significant operational difficulty with the Sundberg method, i.e., the necessity to have a feeding coil close to the source cable/loop.
  - Two separate receiver coils are used which are maintained at a constant

separation, typically 10-20 m (Figure 10.3D). Using the two coils, the ratio of the resultant vertical-field amplitudes and phase difference of the vertical fields are measured.

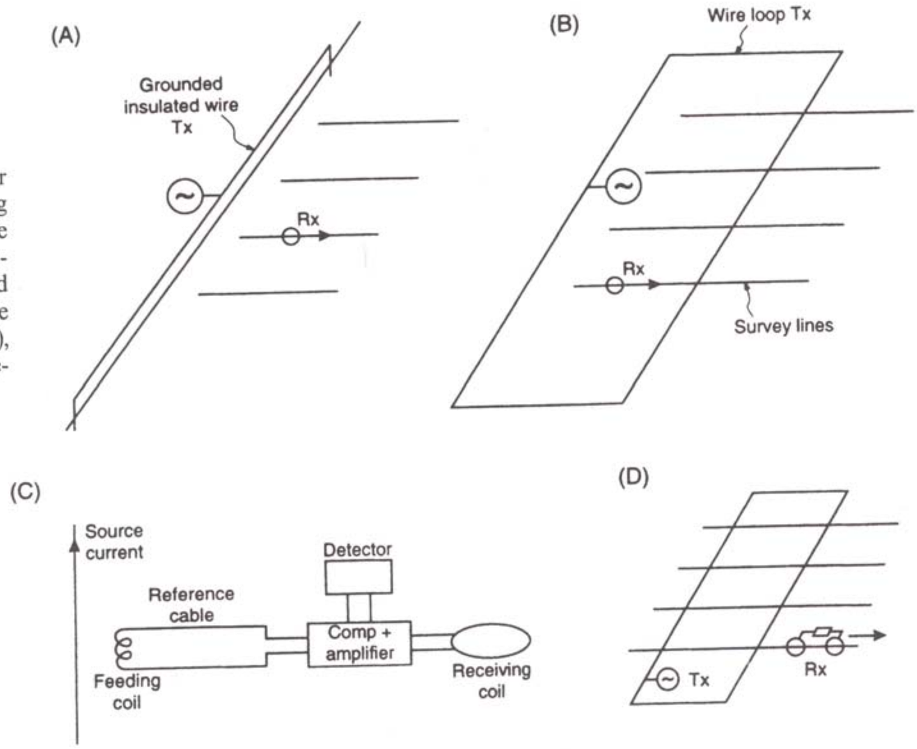


Figure 10.3 (A) Survey layout for the Sundberg method with a long grounded wire, or (B) a grounded wire loop with survey profile lines indicated. Phase reference is determined using a compensator (C) close to the source wire. In the Turam method (D), two separate receiver coils are deployed with a constant separation

- Time-domain system
  - ✓ In frequency-domain system, the secondary field can be measured by nullifying the primary field. In other methods, we measure the combined field of both primary and secondary fields. Because we can know the primary field by design, we can extract the secondary field from the combined field.
  - ✓ In time-domain or transient EM, the primary field is applied in pulses, typically 20-40 ms long which is done by applying current to the transmitter. If we switch off the current, the primary field will disappear. The secondary field is measured after the primary field has been switched off.
  - ✓ The main advantage of the time-domain EM is that the transmitter coil can be used as the receiver.
  - ✓ TEM systems can be used very effectively for depth soundings. Increased depth penetration is achieved by measuring the decay of the secondary

field as a function of time.

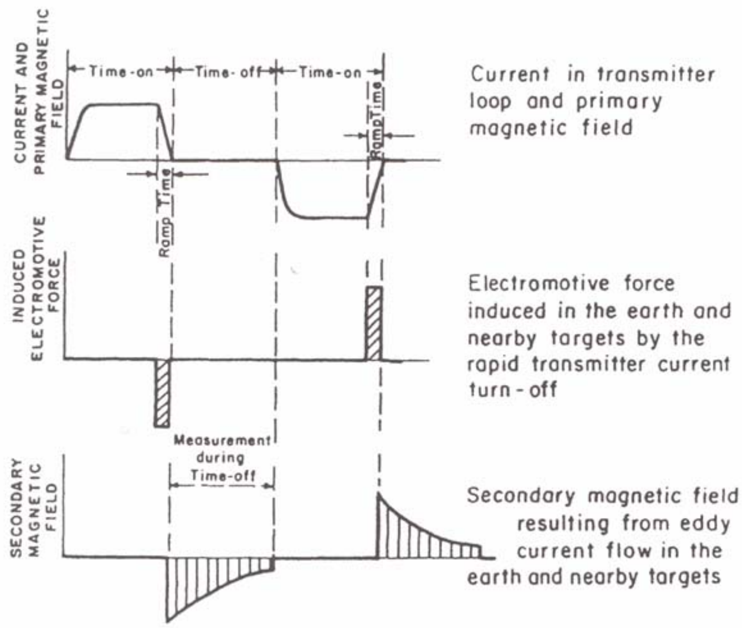


Figure 11.33 Time-domain EM waveforms. From McNeill (1990), by permission

- VLF (very low frequency)

  - ✓ In VLF method, 15-24 kHz radio waves are used. This wave is used to communicate with submarines. At very large distances from the transmitters, the EM field approximates to a plane wave which is used in geophysical exploration (Figure 10.5).

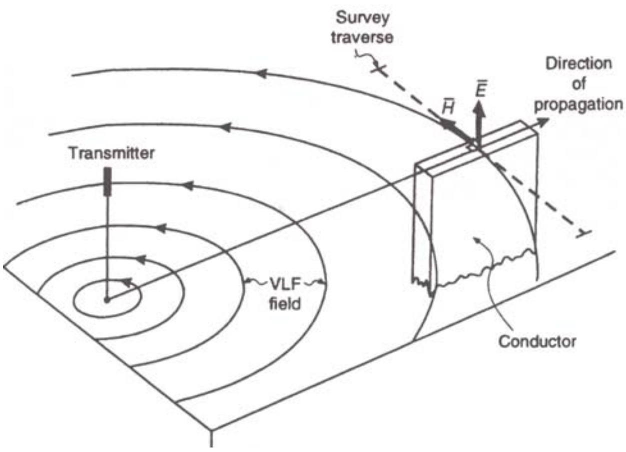
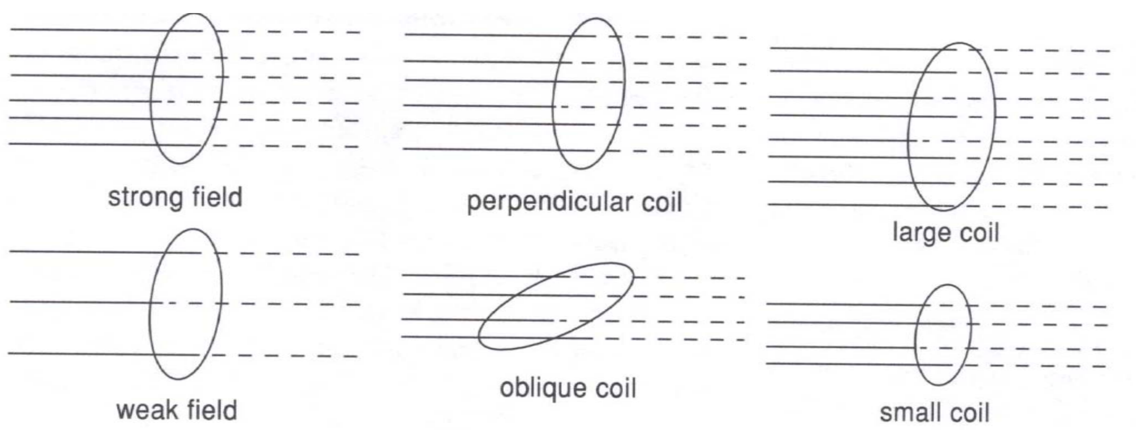


Figure 10.5 Artificial VLF source (e.g. military transmitter) provides a primary EM field which, at a sufficiently large distance, equates to a plane EM wave. Preferred survey directions over a linear conductor are tangential to the VLF field

- ✧ Factors that affect the signal
  - ✓ The signal at receivers depends on the material, shape, and depth of target, and also upon the design and positions of the transmitter and receiver coils
  - ✓ Assume the target to be a loop of wire. The size of the current induced in it by primary field depends on
    - – The number of lines of magnetic field through the loop (magnetic flux)
    - – The rate of change of this number
    - – The material of the loop: The greater conductivity will generate greater signals



C. Airborne EM surveying

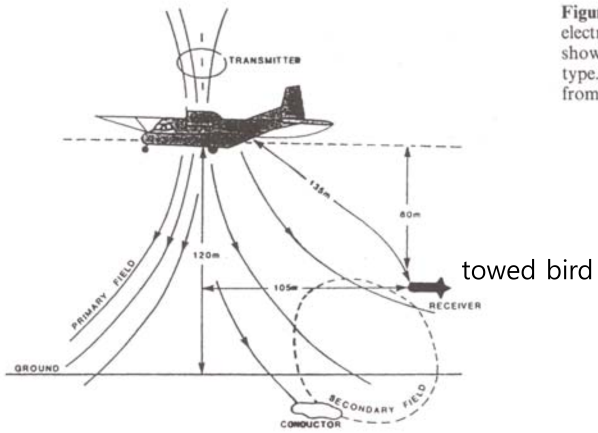


Figure 10.16 Principle of airborne electromagnetic surveying. The system shown deployed is of the towed-bird type. Reproduced with permission from Palacky and West (1991)

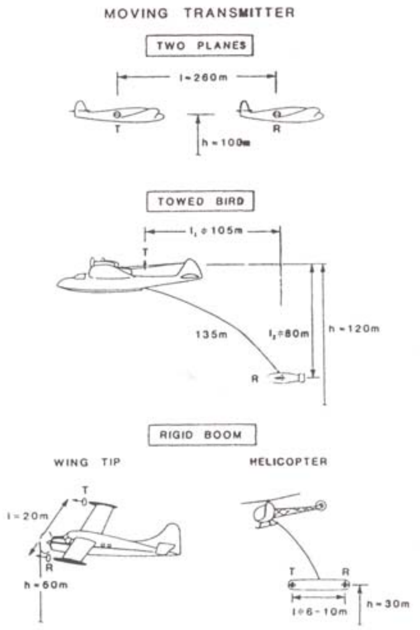


Figure 10.17 Transmitter–receiver geometry of five basic styles of active airborne EM systems. Reproduced with permission from Palacky and West (1991)

D. Borehole EM surveying

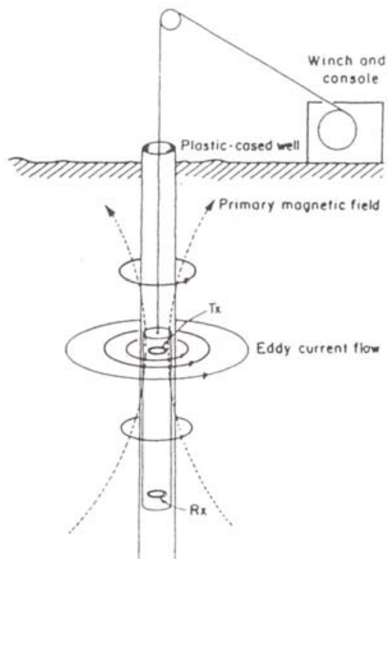


Figure 10.22 The basic principle behind an electromagnetic induction logger for use in boreholes. Reproduced with permission from McNeill (1990)

5. Magnetotelluric (MT) method

- Principles of operation
  - ✓ As a consequence of the presence and fluctuation of the Earth's magnetosphere, natural low-frequency magnetotelluric fields occur which induce alternating currents within the ground.
  - ✓ These currents flow parallel to the ground surface and cover huge areas and are known as 'telluric' currents (지전류) – named after Tellus, the Earth Goddess in Roman mythology.
  - ✓ Two kinds of sources: one is due to distant lightning→ 1-400 Hz, and the other is due to changes in density of conductive plasma (solar wind) impinging on the Earth's magnetic field → 0.0005 and 1 Hz.
  - ✓ The magnitude of the electric field gradient is the order of 10 mV/km.

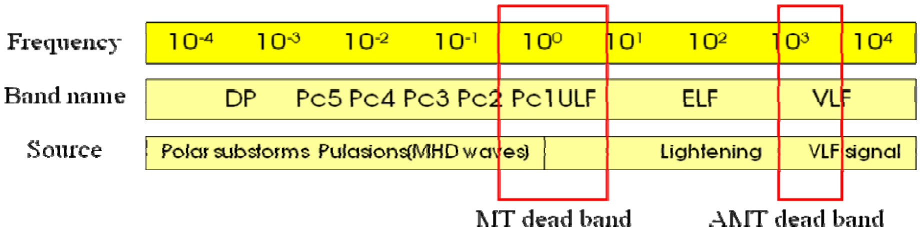
  - ✓ The magnetotelluric method uses measurements of both the electric and magnetic components of the natural time-variant field.

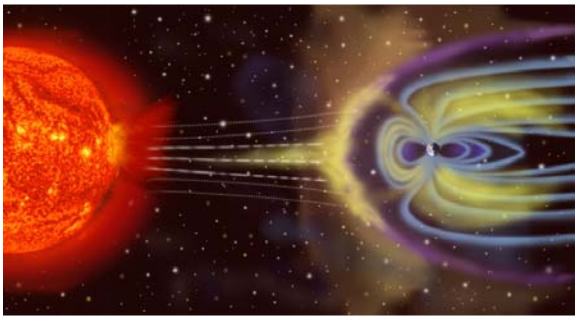
  - ✓ The major advantage is its capability for exploration to very great depth (hundreds of kilometers) as well as in shallow investigations without any artificial power sources (with the exception of 'controlled-source' magnetotelluric method: CSMT, CSAMT)

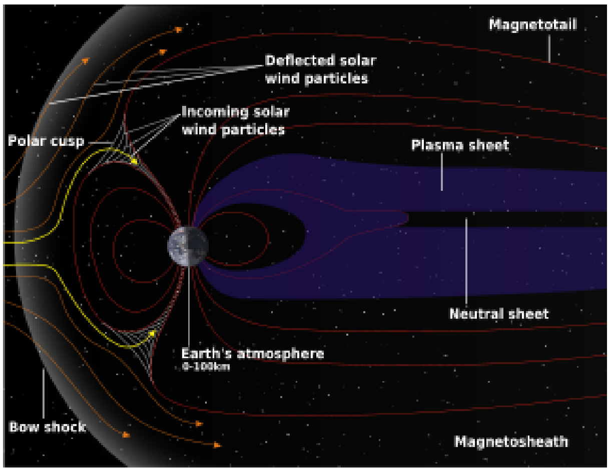
  - ✓ According to frequency range, MT is classified into two methods: audio-frequency MT (AMT or AFMAG, 10-10^4) and MT (0.0001 Hz – 10 Hz).

  - ✓ The main disadvantage with the natural-source MT methods is the weak signal strength. The variability in source strength and direction requires substantial amounts of stacking time (5-10h) per site, thus making MT soundings expensive and production rates slow.

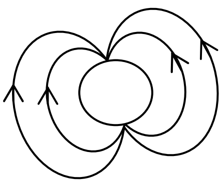
  - ✓ Variability of local thunderstorm sources and signal attenuation around 1 Hz and 2kHz can degrade data quality.



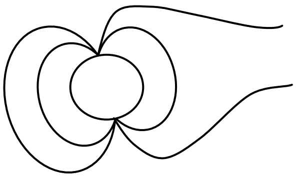




- ✓ Magnetosphere is highly magnetized region around an astronomical object.
- ✓ The shape of magnetosphere of Earth is determined by the extent of Earth's magnetic field, solar wind plasma, and interplanet magnetic field.
- ✓ In magnetosphere, a mix of free ions and electrons from both the solar wind and the Earth's ionosphere is confined by magnetic and electric forces that are much stronger than gravity and collisions.
- ✓ In the early 1970s, David Strangway and Myron Goldstein at the University of Toronto introduced the principle of an artificial signal source which was dependable and strong enough to speed up data acquisition and improve the reliability of results.--> This is called controlled-source MT: CSMT or CSAMT



- ✓ The CSMT methods typically operate within the frequency band 0.1 Hz to 10 kHz.



- ✓ AMT has been used in groundwater/geothermal resource investigations and in the exploration for major base metal deposits over the depth range from 50 -100 m to several kilomiters.
- ✓ The main application of the MT method has been in hydrocarbon exploration, particularly in extreme terrain and to penetrate below volcanic materials, both types of areas where reflection seismology is either extremely expensive or ineffective.

- Since the mid-1970s, CSAMT has been used in an increasing range of applications, and especially since the early 1980s within geotechnical and environmental investigations.
- Nowadays marine CSAMT is popularly used in hydrocarbon exploration as a secondary method.

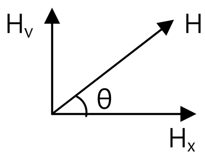
- Impedance and apparent resistivity
  - Assumption
    - The frequencies are so low that displacement currents are negligible.
    - For plane waves of this type, it is clear that horizontal variations in **E** and **H** are small compared with vertical variations.
    - Only periodic frequency variations are considered.
  - If the wave is polarized in the xy plane and traveling in the z direction, and the magnetic vector (whose magnitude is H_0) is at an angle of θ with the x axis, the magnitude of the magnetic components are $H_{x0} = H_0 \cos\theta$ and $H_{y0} = H_0 \sin\theta$.

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla^2 \vec{E} = i\omega\sigma \vec{E} - \mu\varepsilon\omega^2 \vec{E}$$

$$\nabla^2 \vec{H} = i\omega\sigma \vec{H} - \mu\varepsilon\omega^2 \vec{H}$$



$$H = H_0 e^{-az} e^{i(\omega t - az)}$$

$$H_x = H_0 \cos\theta e^{-az} e^{i(\omega t - az)}$$

$$H_y = H_0 \sin\theta e^{-az} e^{i(\omega t - az)}$$

$$H_z = 0$$

$$a = \sqrt{\frac{\omega\mu\sigma}{2}}$$

From Ampere's law

$$\nabla \times \vec{H} = \vec{J} = \sigma \vec{E}$$

$$E_x = \frac{1}{\sigma}(\nabla \times \vec{H})_x = \frac{1}{\sigma}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) = -\frac{1}{\sigma}\frac{\partial H_y}{\partial z}$$

$$= -\frac{1}{\sigma} H_0 \sin\theta(-a) e^{-az} e^{i(\omega t - az)} - \frac{1}{\sigma} H_0 \sin\theta(-ia) e^{-az} e^{i(\omega t - az)}$$

$$= \frac{1}{\sigma} H_0 \sin\theta(1+i) a e^{-az} e^{i(\omega t - az)}$$

$$E_y = \frac{1}{\sigma}(\nabla \times \vec{H})_y = \frac{1}{\sigma}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) = -\frac{1}{\sigma} H_0 \cos\theta(1+i) a e^{-az} e^{i(\omega t - az)}$$

$$\frac{E_x}{H_y} = \frac{1}{\sigma}(1+i)a = (1+i)\sqrt{\frac{\omega\mu}{2\sigma}} = (1+i)\sqrt{\frac{\omega\mu\rho}{2}}$$

$$\frac{E_y}{H_x} = -\frac{1}{\sigma}(1+i)a = -(1+i)\sqrt{\frac{\omega\mu\rho}{2}}$$

$$\left|\frac{E_x}{H_y}\right| = \left|\frac{E_y}{H_x}\right| = \sqrt{\omega\mu\rho}$$

$$\Updownarrow \qquad \Updownarrow$$

$$Z_{xy} \qquad Z_{yx} \quad \Rightarrow \text{ Impedance}$$

$$\rho_{xy} = \frac{1}{\mu\omega}\left|Z_{xy}\right|^2 \qquad \rho_{yx} = \frac{1}{\mu\omega}\left|Z_{yx}\right|^2$$

$$\phi_{xy} = \tan^{-1}\frac{\mathrm{Im}(Z_{xy})}{\mathrm{Re}(Z_{xy})} \qquad \phi_{yx} = \tan^{-1}\frac{\mathrm{Im}(Z_{yx})}{\mathrm{Re}(Z_{yx})}$$

- ρ_{xy}, ρ_{yx} have the same values for shallow parts. ⇒ interpret subsurface by 1-D
- For deeper parts, they different values. ⇒ interpret subsurface by 2-D

➢ Field Survey

- ✓ The general field layout is shown in Figure 11.59. It comprises two orthogonal electric dipoles to measure the two horizontal electric components and two magnetic sensors parallel to the electric dipoles to measure the corresponding magnetic components.
- ✓ A third sensor measures the vertical magnetic component.
- ✓ The magnetic sectors are made up of coils with several tens of thousands of turns around highly permeable iron cores with a total sensor length typically 2 m long.
- ✓ Five fields are measured simultaneously as a function of frequency: two horizontal electric components, and three magnetic components (E_x, E_y, H_x, H_y, H_z)
- ✓ By measuring the changes in the magnetic (H) and electric (E) fields over a range of frequencies, an apparent resistivity sounding curve can be produced.

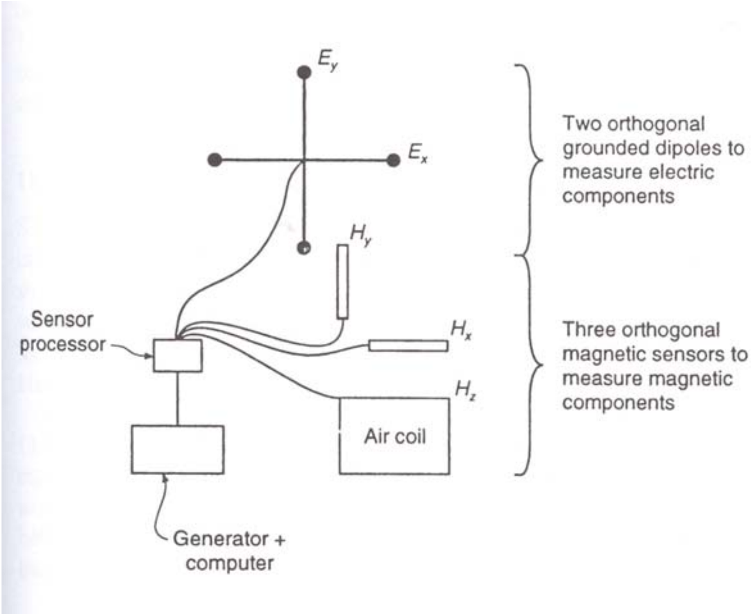


Figure 11.59 Generalised field lay-out for a magneto-telluric survey

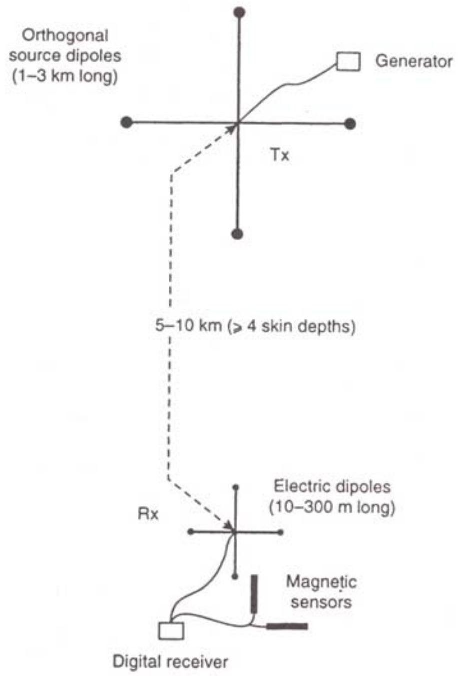
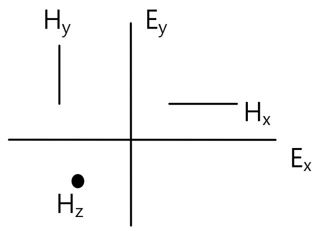


Figure 11.60 Generalised field lay-out for a controlled-source magneto-telluric survey

H_x, H_y, H_z should have some distances.
When they are closely deployed, they can cause some noises.



- ✓ For controlled-source MT surveys, either a loop or grounded dipole is used as a transmitter with the same measurement configuration.
- ✓ The grounded dipole is typically between 1 km and 3 km long, commonly two orthogonal grounded dipoles are used to provide two different source polarizations.
- ✓ The source may be several kilometers away from the receiver sensors.

- Interpretation
  - Figure 11.62 shows two examples of magneto-telluric soundings

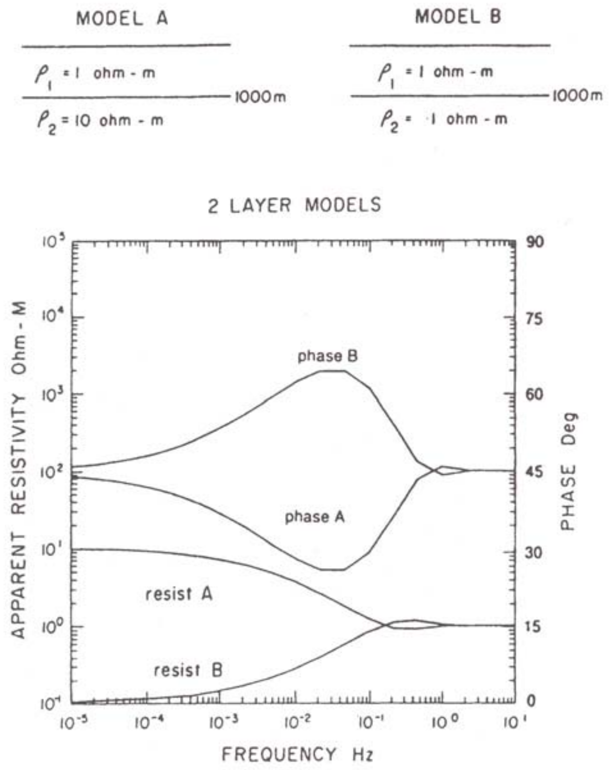


Figure 11.62 Magneto-telluric apparent resistivity and phase responses of 2-layer models. Model A = resistive basement; Model B = conductive basement. From Vozoff (1991), by permission

  - Model A for a resistive basement (ρ=10Ωm)
    - With decreasing frequency, the phase difference increases to achieve a peak at mid frequencies. The apparent resistivity increases to reflect the basement value
  - Model B for a conductive basement (ρ=0.1Ωm)
    - The apparent resistivity is asymptotic to a constant value (0.1) at low frequency, and phase difference should revert to 45°.
  - The apparent resistivity is sensitive to near-surface inhomogeneities. At middle and higher frequencies, the phase difference is more sensitive to deeper structures than apparent resistivity.
  - Very shallow features may not be evident on the phase difference sounding while they may be seen on the apparent resistivity data.

  - In CSAMT surveys, considerable amounts of data processing may be necessary prior to inversion
    - Preprocessing is for the removal of errors and noise
    - For data enhancement, normalizing, static correction, filtering and

derivative calculations may be applied. Regional effects can be removed by deconvolution.--> enhance subtle lateral effects in survey areas with complex layering.

- Applications and case histories
  - ✓ Mineral exploration
    - – Gold mineralization due to hydrothermal alteration in central Washington: The silicified zone is a major gold producer.
      When granite rises, the surrounding rocks can be hydrothermally altered.

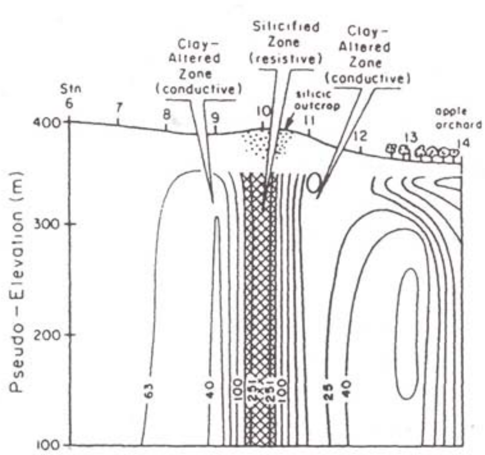


Figure 11.64 CSAMT apparent resistivity pseudosection over a silicified reef several kilometres from a gold mine which exploits this same structure. From Zonge and Hudges (1991), by permission

    - – In gold exploration in north-central Nevada

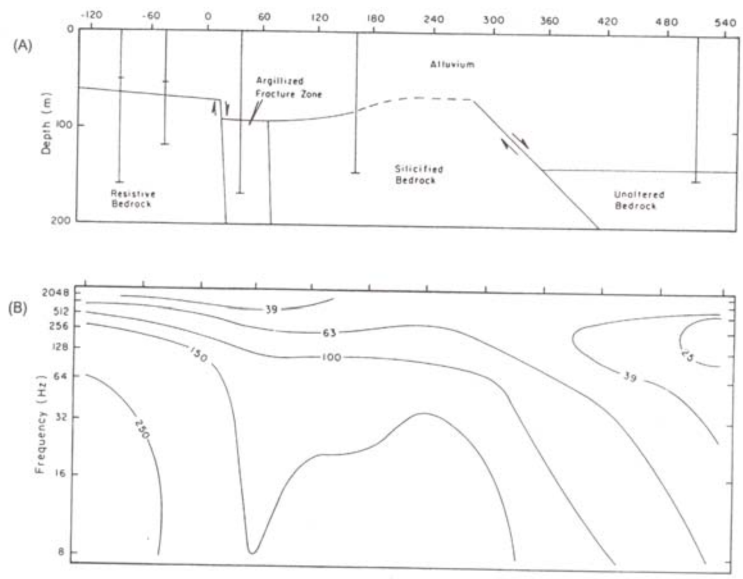


Figure 11.65 CSAMT results of a survey over a buried basement structure associated with a hydrothermal gold deposit: (A) geology and CSAMT results, and (B) CSAMT Cagniard resistivity data, courtesy of Phoenix Geophysics. From Zonge and Hughes (1991), by permission

  - ✓ Environmental applic
    - – Water plume

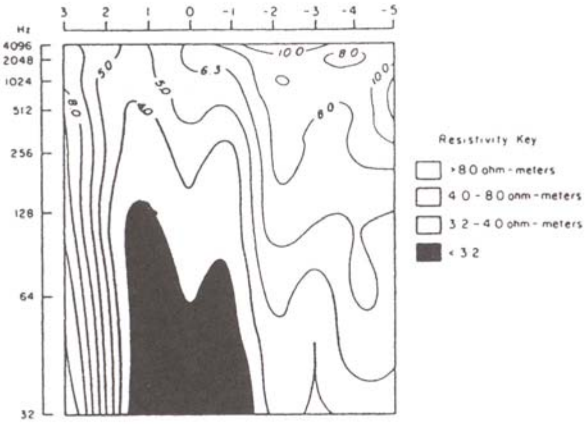


Figure 11.69 Apparent resistivity pseudosection near an abondoned injection well, showing the conductive plume (shaded) associated with upwardly migrating brine. From Zonge and Hughes (1991), by permission

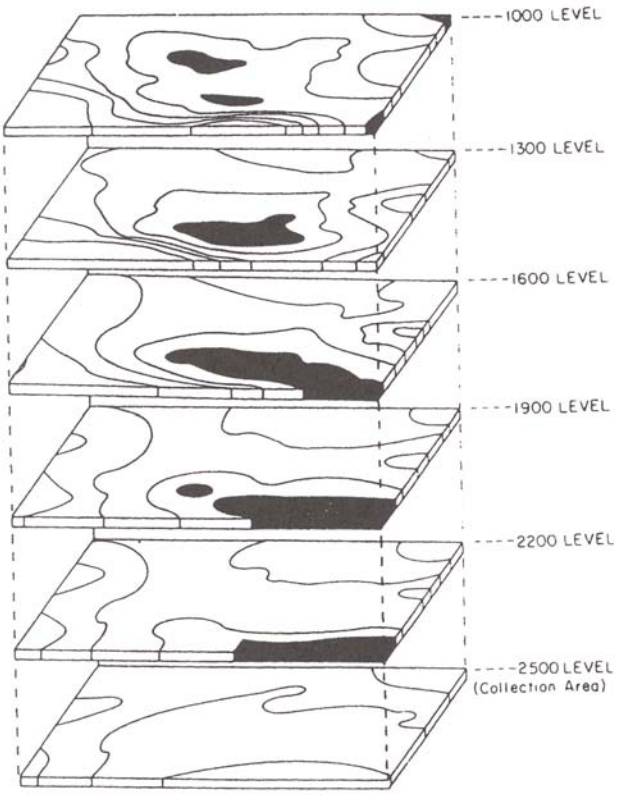


Figure 11.70 CSAMT depth-level resistivity slices; conductive zones are shown shaded. The largest areal extent of contamination can be seen at the 1900 level. From Zonge and Hughes (1991), by permission