

3. Graphical methods: Stereographic projection

1) Types of projection

- Projection: Mapping 3D images into 2D ones on a planar surface
- Parallel projection: parallel rays are projected to a planar surface from an object. It is useful to convey measurements of distances or angles.
ex.) Orthographic projection (Fig.3.1, Fig 3.4),
oblique projection (Fig.3.2)
- Perspective projection: nonparallel rays connecting one or more foci and a object are projected to a surface.
It is useful to convey perspective views of objects.
ex.) Equal area projection,
equal angle (stereographic) projection

1) Equal-Area projection ()

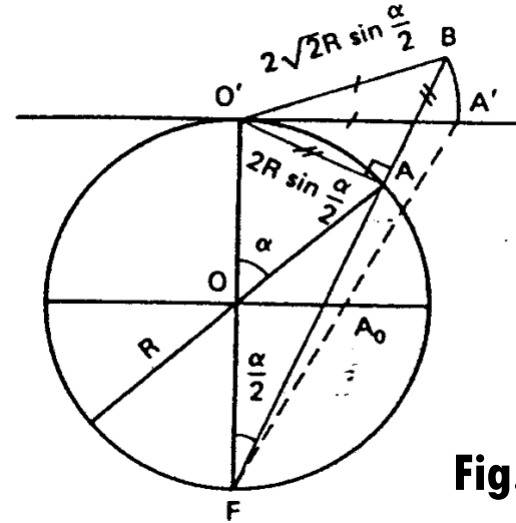
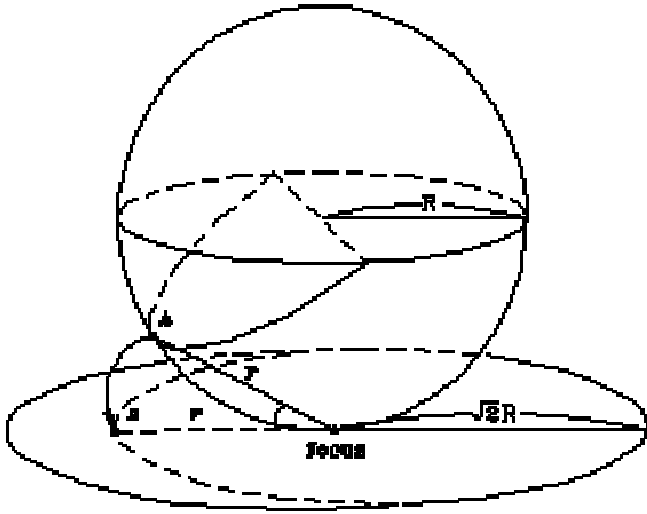
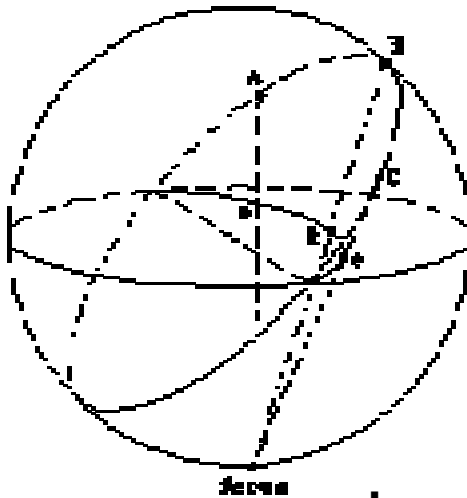


Fig. 3.5

Advantage: Area of a small circle is preserved.

Disadvantage: Shape of a small circle is distorted according to its location on the sphere.

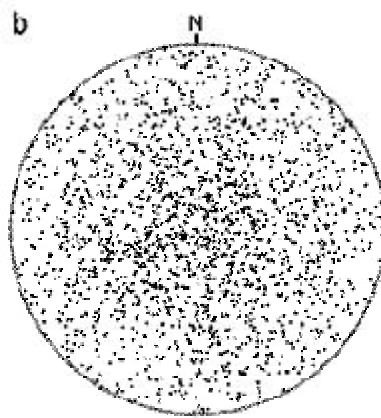
2) Stereographic (equal-angle,) projection



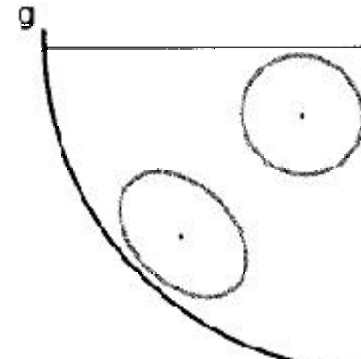
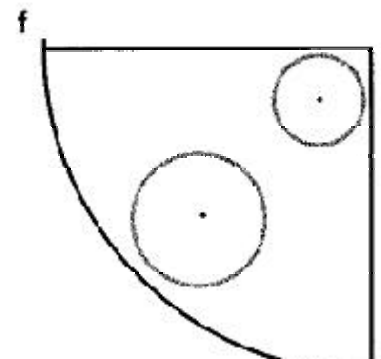
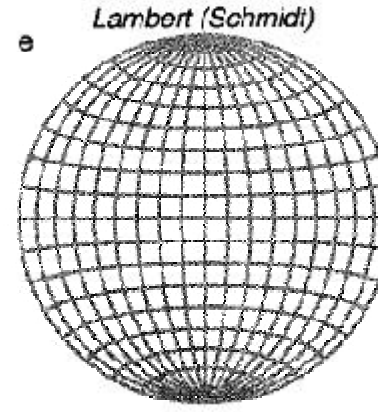
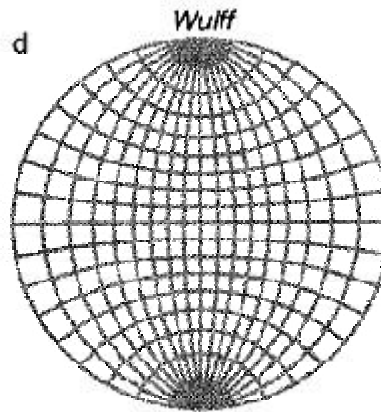
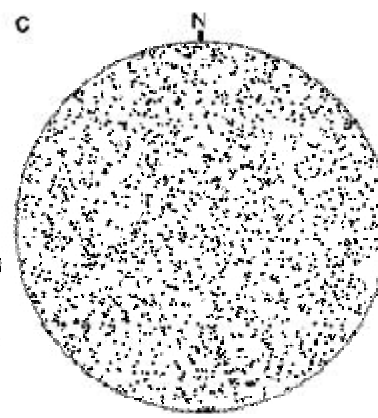
Lower focal point (=upper hemisphere) projection

Advantage: Shape of a small circle (angle) is preserved (conformal).

Disadvantage: Area of a small circle changes according to its location on the sphere.



$N=2000$



2) Stereographic projection of lines and planes

- Projection of a vector
- Projection of a great circle
- Projection of a small circle

3) Stereographic projection of a joint pyramid

- Projection of half spaces

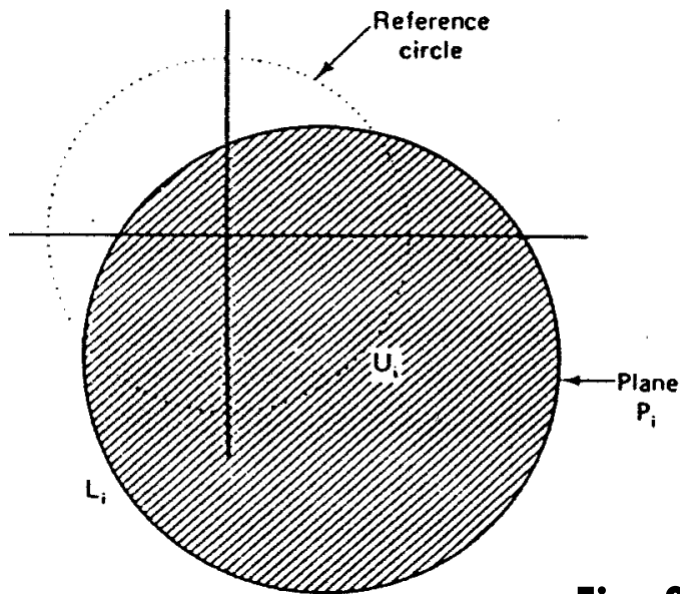


Fig. 3.16

- Joint pyramid: an intersect of joint half spaces shifted to the center of a projection sphere

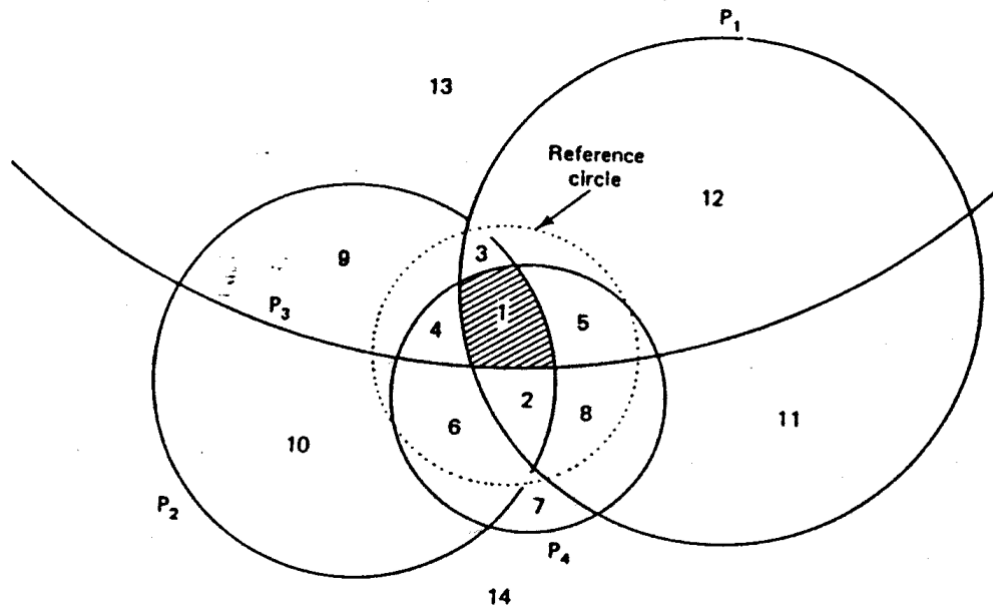


Fig. 3.17

- Intersection of two joints

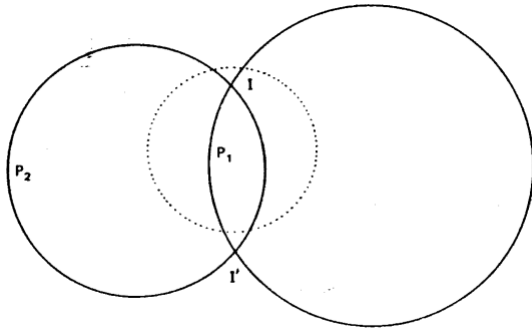


Fig. 3.12

- Arc of joint pyramid

4) Additions

- Normal to a given plane
- Plane normal to a given vector (line)
- $(X_0, Y_0) \rightarrow (X, Y, Z)$

- Center of a great circle passing two points
 - Vector analysis
 - Graphical method

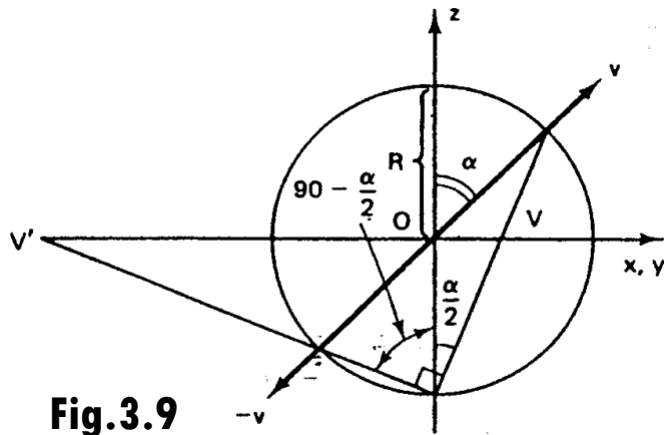


Fig.3.9

- Orthographic projection of a vector on a plane
- Vector analysis
- Graphical method