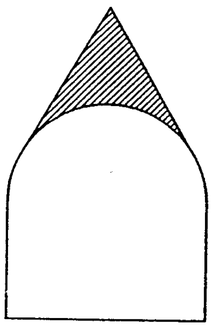
8. Block theory for tunnels and shafts

1) Introduction

- Advantages of tunnels over slopes
  - Make the routes more straight and shorter
  - Avoid extensive surface cut: environmentally friendly
  - Avoid the weak rock of weathered zone
  - Smaller key blocks are involved: tunnel span is far smaller than slope length
- This chapter shows
  - How to choose an optimum tunnel direction and find out removable blocks

2) Geometric properties of tunnels

- Shape and size of removable blocks depend on tunnel shape and size



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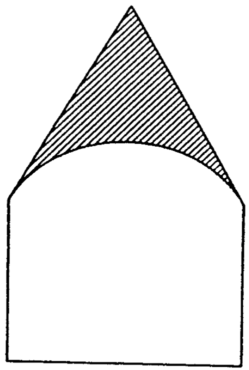

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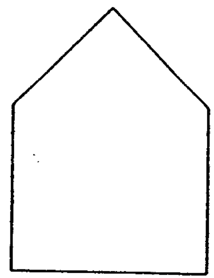

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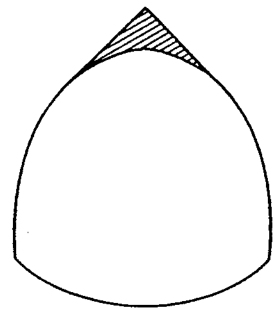

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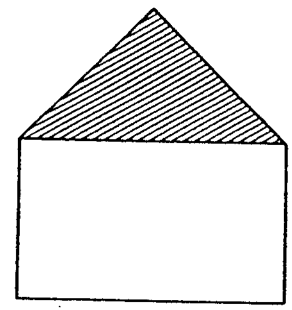

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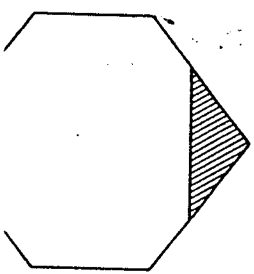

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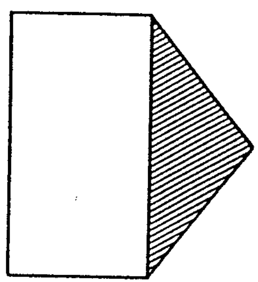

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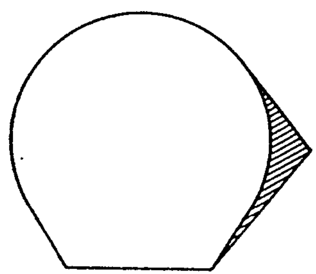

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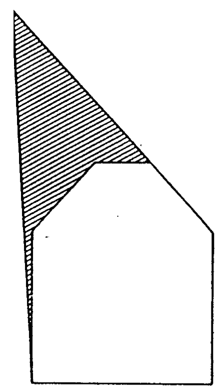

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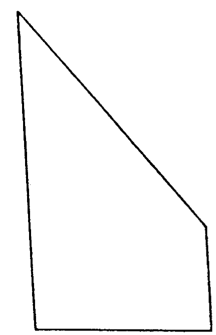

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3) Blocks with curved faces

- Local (tunnel) coordinate systems
  - Select an upward direction in the tunnel face perpendicular to tunnel axis as $\boldsymbol{y_0}$.
  - Select a tunnel axis direction pointing observers in the tunnel from the face as $\boldsymbol{z_0}$.
  - Select one of the tunnel face strike lines pointing $\boldsymbol{y_0} \times \boldsymbol{z_0}$ as $\boldsymbol{x_0}$ (right-hand rule).
  - Conversion between global and local coordinate systems is possible by using transformation matrix (law):

	$\hat{x}$	$\hat{y}$	$\hat{z}$
$\hat{x}_0$	$\cos\theta_{xx_0}$	$\cos\theta_{yx_0}$	$\cos\theta_{zx_0}$
$\hat{y}_0$	$\cos\theta_{xy_0}$	$\cos\theta_{yy_0}$	$\cos\theta_{zy_0}$
$\hat{z}_0$	$\cos\theta_{xz_0}$	$\cos\theta_{yz_0}$	$\cos\theta_{zz_0}$

$$\begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = \begin{bmatrix} \cos\theta_{xx_0} & \cos\theta_{yx_0} & \cos\theta_{zx_0} \\ \cos\theta_{xy_0} & \cos\theta_{yy_0} & \cos\theta_{zy_0} \\ \cos\theta_{xz_0} & \cos\theta_{yz_0} & \cos\theta_{zz_0} \end{bmatrix} \begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix}$$

$\hat{x}$, $\hat{y}$, $\hat{z}$: global coordinate axes

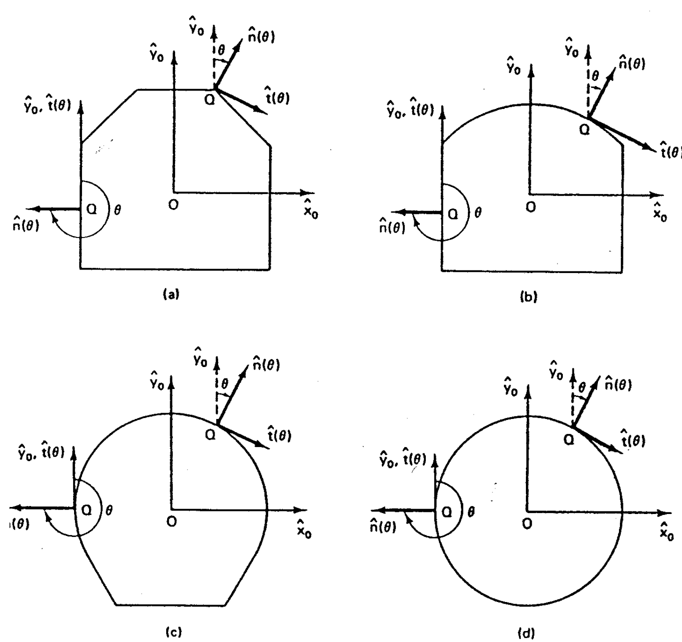
$\hat{x}_0$, $\hat{y}_0$, $\hat{z}_0$: local (tunnel) coordinate axes

3) Blocks with curved faces

- Local coordinate systems for points on the tunnel boundary
  - Use 2D transformation matrix for coordinate conversion:

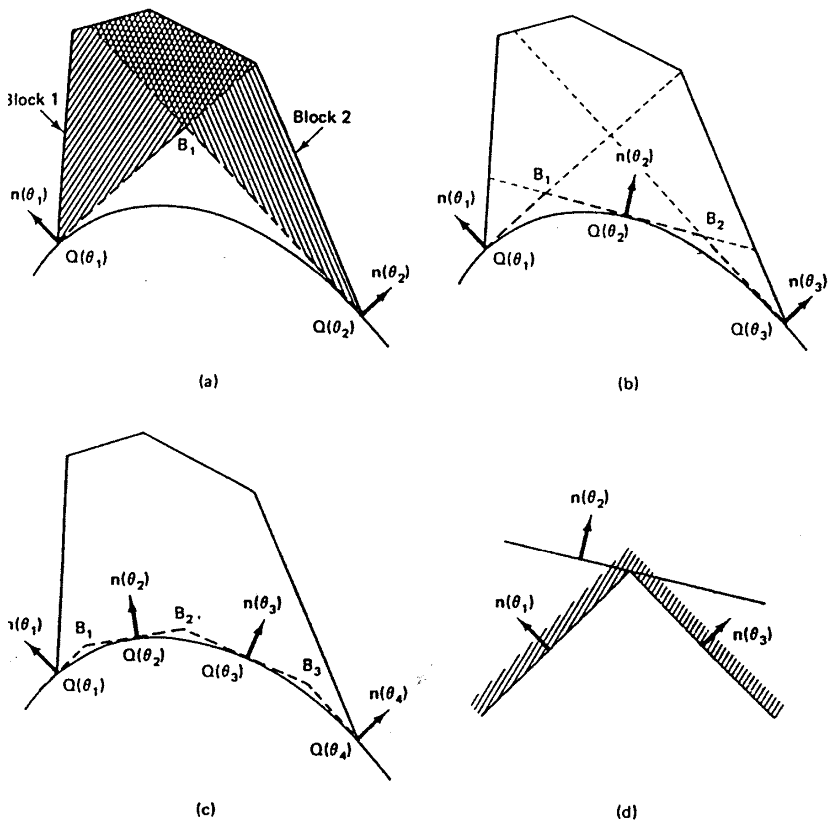
$$\begin{array}{ccc} & \hat{x}_0 & \hat{y}_0 \\ \hat{t}_\theta & \cos\theta_{x_0 t_\theta} & \cos\theta_{y_0 t_\theta} \\ \hat{n}_\theta & \cos\theta_{x_0 n_\theta} & \cos\theta_{y_0 n_\theta} \end{array}$$

$$\begin{bmatrix} t_\theta \\ n_\theta \end{bmatrix} = \begin{bmatrix} \cos\theta_{x_0 t_\theta} & \cos\theta_{y_0 t_\theta} \\ \cos\theta_{x_0 n_\theta} & \cos\theta_{y_0 n_\theta} \end{bmatrix} \begin{bmatrix} x_l \\ y_l \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_l \\ y_l \end{bmatrix}$$



3) Blocks with curved faces

- Space pyramids of curved blocks (faces)
  - Space pyramid is defined as an intersection of space pyramids of two tangential planes at both ends



4) Tunnel axis theorem

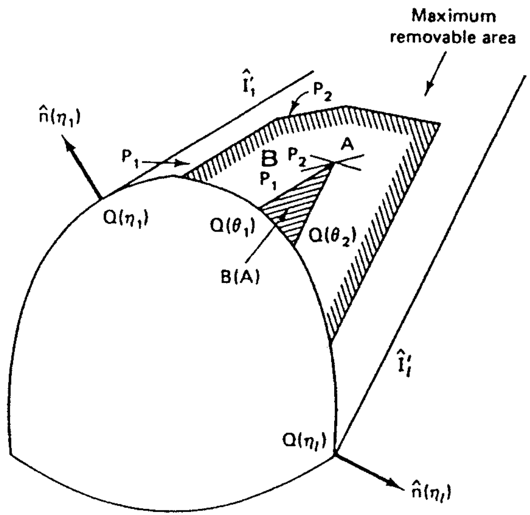
- Theorem: JP is a removable block of a tunnel if and only if $\pm\hat{a} \notin JP$

- No. of the infinite blocks is always 2 and 0 for non-repeated and repeated sets, respectively (Table 8.1).

- No. of the removable blocks is n^2-n.

5) Maximum key block (removable area)

• Maximum removable blocks are triangular in a tunnel section
- There is a maximum size for a given tunnel geometry beyond which blocks are no longer removable.
- All the joints of a maximum removable (key) block always have a common intersection point in a rock mass.
- JP of a removable block is not empty and it has a triangular shape.
→ The edges of the maximum removable blocks are among the JP's edges.



5) Maximum key block (removable area)

- Edge matrix

- Testing matrix is [T]=[I]·[D] (Direction-ordering index·Block code)

  $T = sign[(n_i \times n_j)\cdot n_k]\cdot[d_{ll}]$

- Testing matrix shows which edge is included in an intersection space of half spaces defined by joint planes (the edge is an intersection vector satisfying all the inequalities of joint planes).

- The edges satisfying all the inequalities lacks – (minus) or + (plus) in the corresponding row of the testing matrix.

- The edges above are represented as corners of JPs in a stereographic projection.

$$(I)\cdot(D) = \begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \{E\} = \begin{pmatrix} -1 \\ 0 \\ 1 \\ -1 \\ 0 \\ -1 \end{pmatrix}$$

5) Maximum key block (removable area)

- Limiting edges in the tunnel section

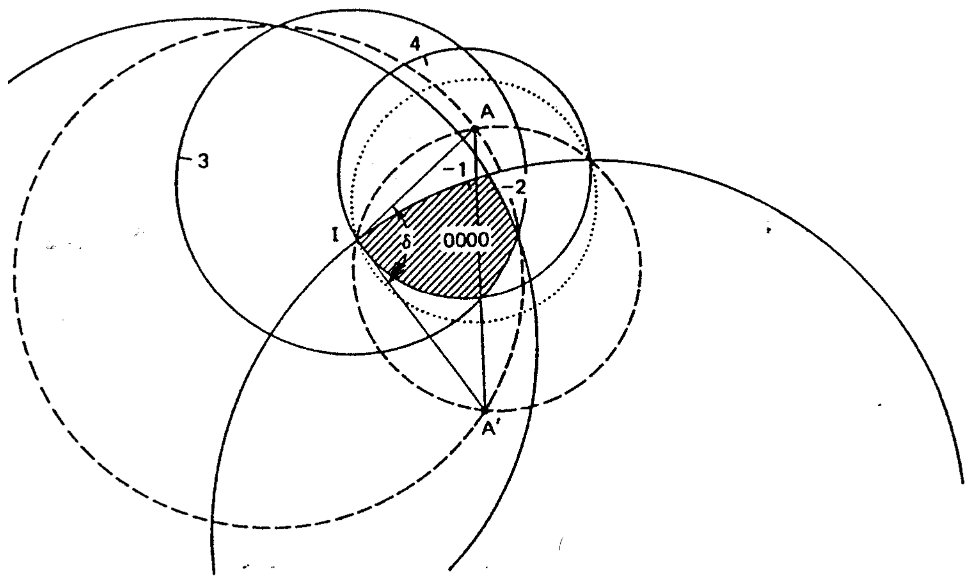
- Let the JP edges I_1'... I_m'.
- Normal vectors of limiting edges defined by $I_i' \times \hat{a}$ make acute or obtuse angles with other inner edges.
- Limiting edges, therefore, can be found by checking out $B_{ik} = sign[(I_i' \times \hat{a}) \cdot I_k']$
- When I_i' is an limiting edge, $-B_i(I_i' \times \hat{a})$ indicates outward normal of the edge. (B_i is 1 when B_{ik}, $k=1, ..., m,$ is 1 or 0 while it is -1 when B_{ik} is -1 or 0).

6) Maximum key blocks analyzed by stereogrphic projection method

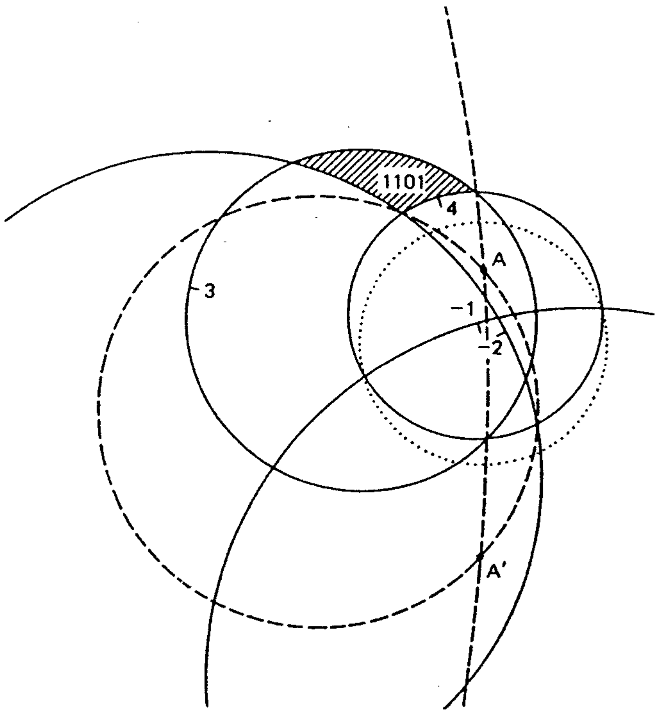
• Constructing the limit planes of a given JP

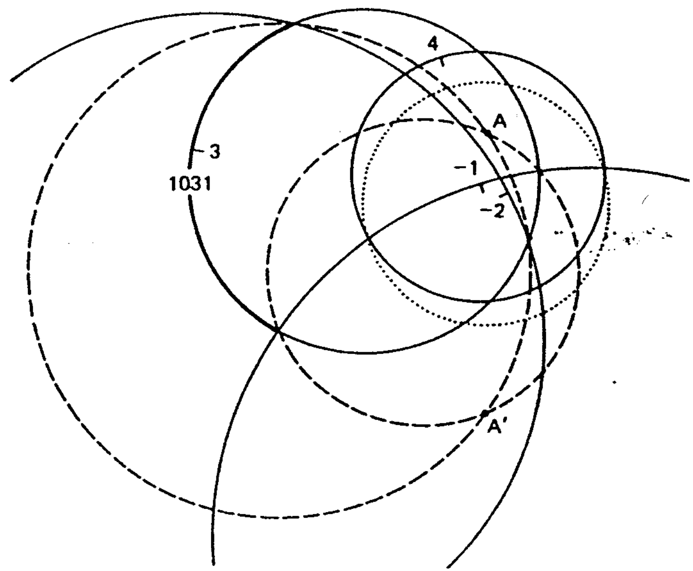
-Draw great circles of the limit planes passing through tunnel axis vectors, $\pm\hat{a}$, and JP edges.

- Select two great circles which do not intersect the inner area of the JP.
- Check out whether the JP is inside or outside of the great circles.

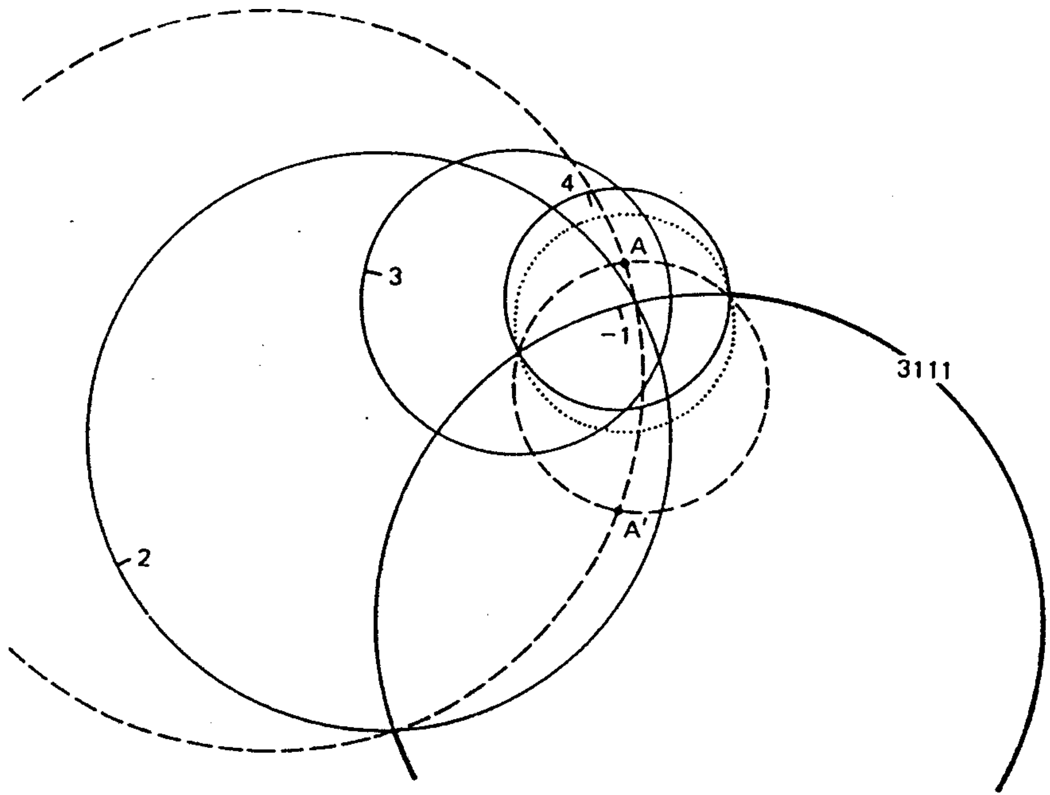


6) Maximum key blocks analyzed by stereogrphic projection method





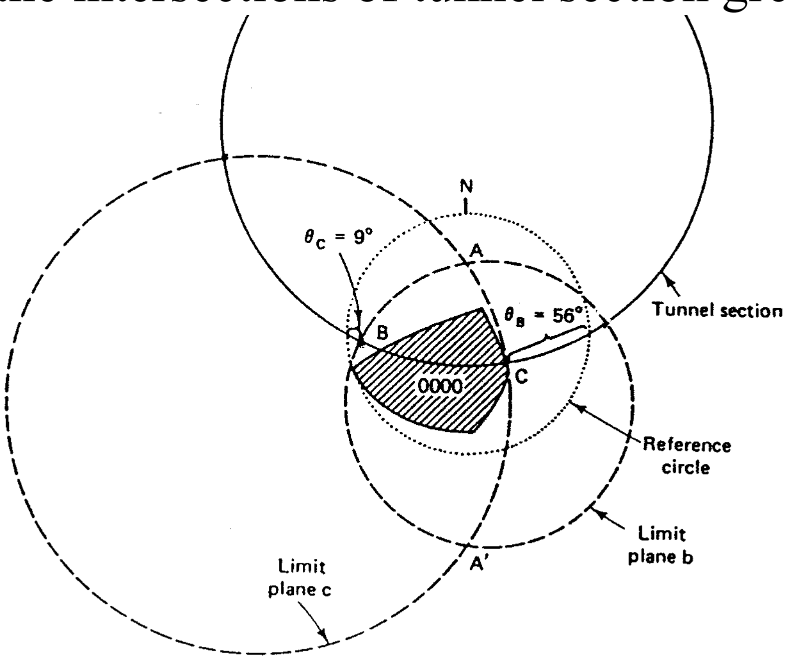
6) Maximum key blocks analyzed by stereogrphic projection method

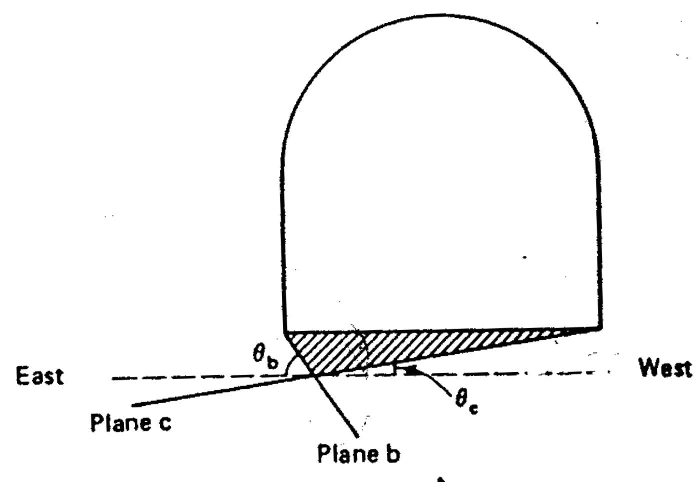


6) Maximum key blocks analyzed by stereogrphic projection method

• Calculating angular intervals of the limit planes

- Draw a great circle of tunnel section perpendicular to tunnel axis vectors.
- Find out intersection points between the tunnel section and reference circle: They represent a horizontal line in the tunnel section used for measuring angles.
- Measure the angle from the intersection points to the limit edges represented by the intersections of tunnel section great circles and limit planes.





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6) Maximum key blocks analyzed by stereogrphic projection method