

3. Energy & Power in Hydraulic Systems

- **Hydraulic Energy & Power**
- **Efficiency**
- **Pascal's Law** ►
- **Hydraulic Jack**
- **Air-to-Hydraulic Pressure Booster**
- **Conservation of Energy Law** ►
- **Continuity Equation**
- **Hydraulic Power** ►
- **Potential Energy & Kinetic Energy**
- **Bernoulli's Equation** ►
- **Torricelli's Theorem**
- **The Siphon**



Pressure & Flow

3.1 Hydraulic Energy & Power

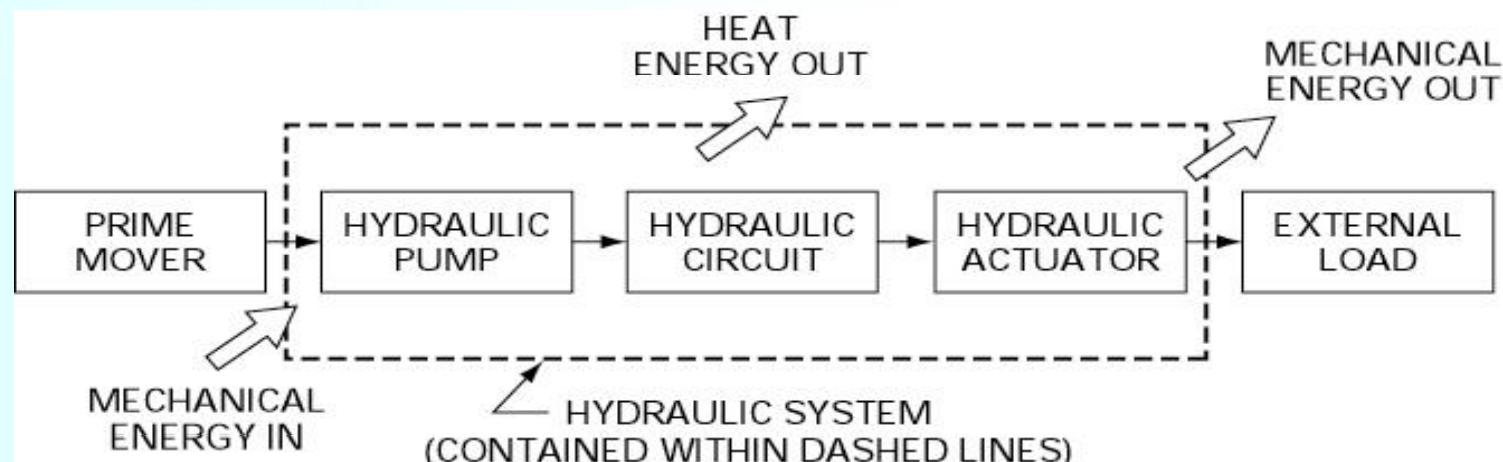
- Energy: the ability to perform work

$$\text{Input } ME - \text{Lost } HE = \text{Output } ME$$

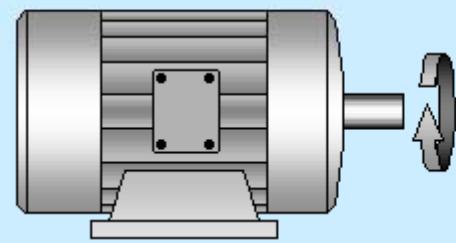
- Power: the rate of doing work or expending energy

- Hydraulic System

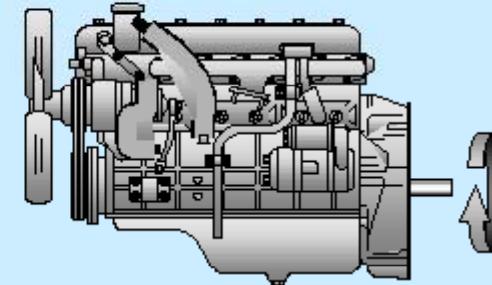
- Is not a source of energy
- Energy transfer system
- Is much more versatile to transmit power
 - variable speed, reversibility, overload protection, high-power-to-ratio, immunity to damage under stalled conditions



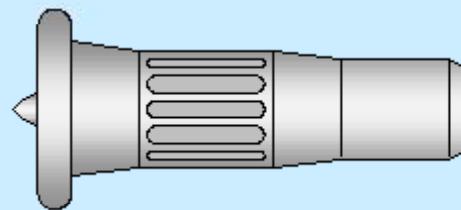
Source of Power



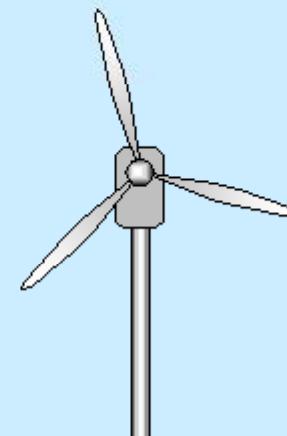
ELECTRIC MOTOR



INTERNAL COMBUSTION
ENGINE

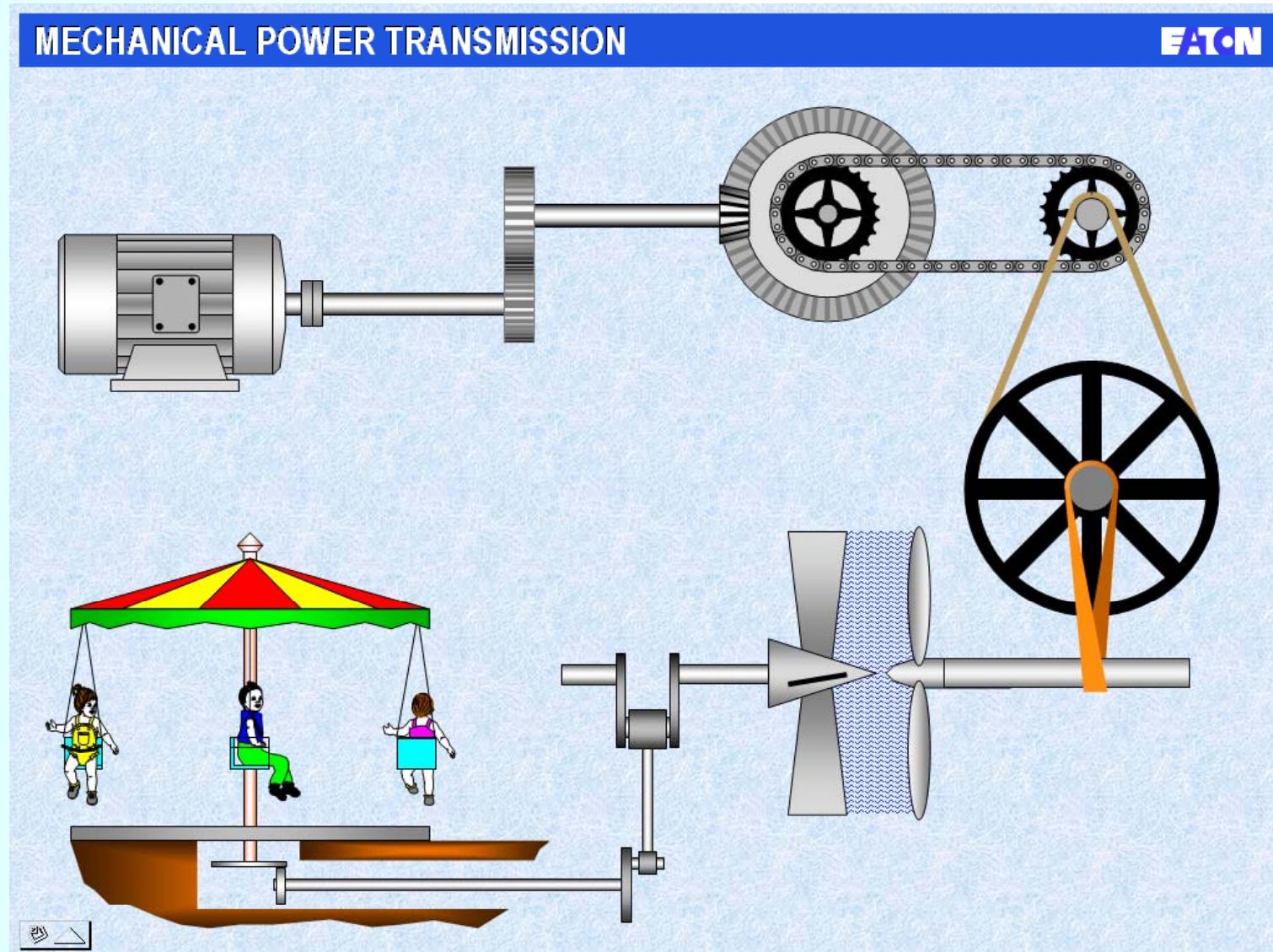


GAS TURBINE

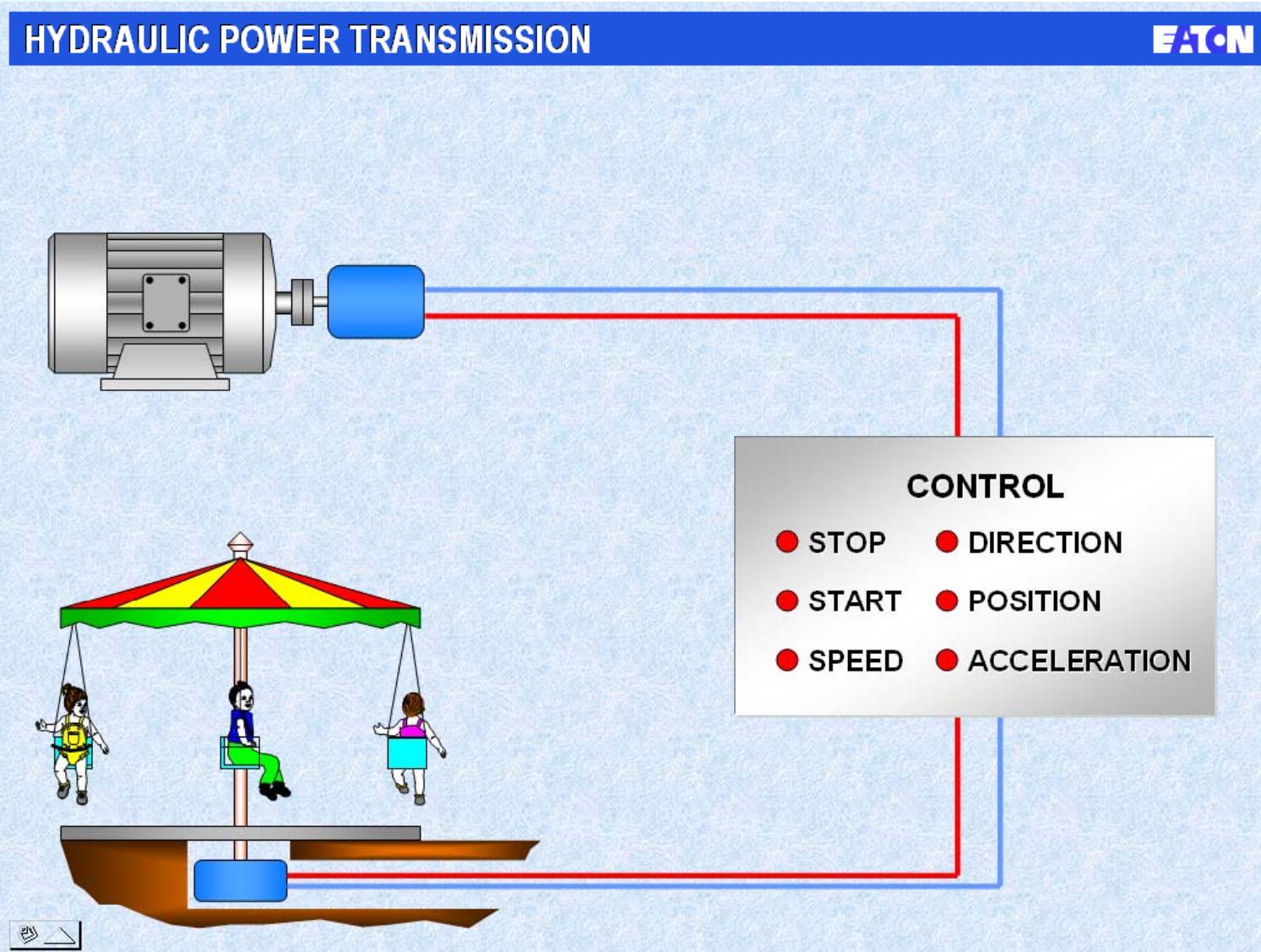


ALTERNATIVE

Mechanical Power Transmission



Hydraulic Power Transmission



3.2 Review of Mechanics

■ Newton's 1st Law

- 관성의 법칙: *An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force.*

■ Newton's 2nd Law

- 가속도의 법칙: *The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object.*

■ Newton's 3rd Law

- 작용 반작용의 법칙: *For every action, there is an equal and opposite reaction.*



Mechanics

■ Linear Motion

- $v = s/t$
- $F = ma$
- $W = Fs$
- $P = W/t = Fs/t = Fv$

■ Rotational Motion

- $\omega = \theta/t$
- $T = Fr$
- $W = T\theta$
- $P = T\omega$
- **Torque horsepower, Brake horsepower(BHP)**

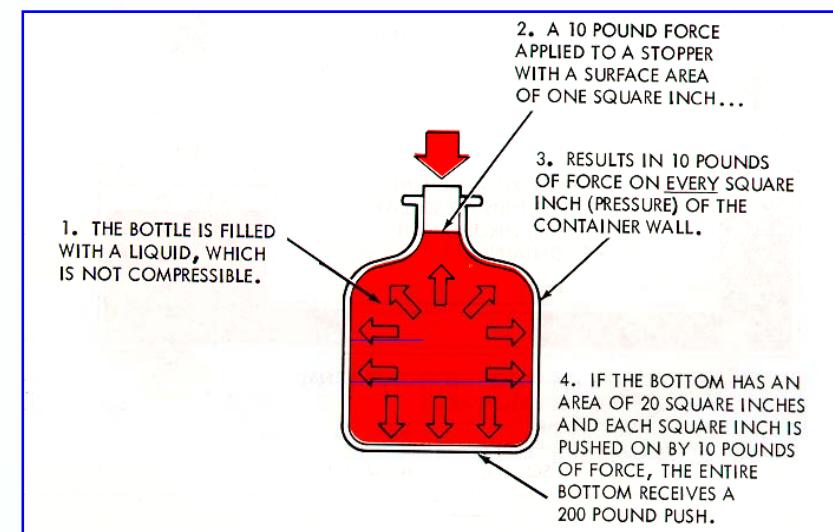
■ Efficiency

- $\eta = \text{output power/input power} \times 100 (\%)$
- 동력손실: 유압유의 누설, 유체내부의 마찰, 기계적 마찰

3.3 Pascal's Law

Pascal's law

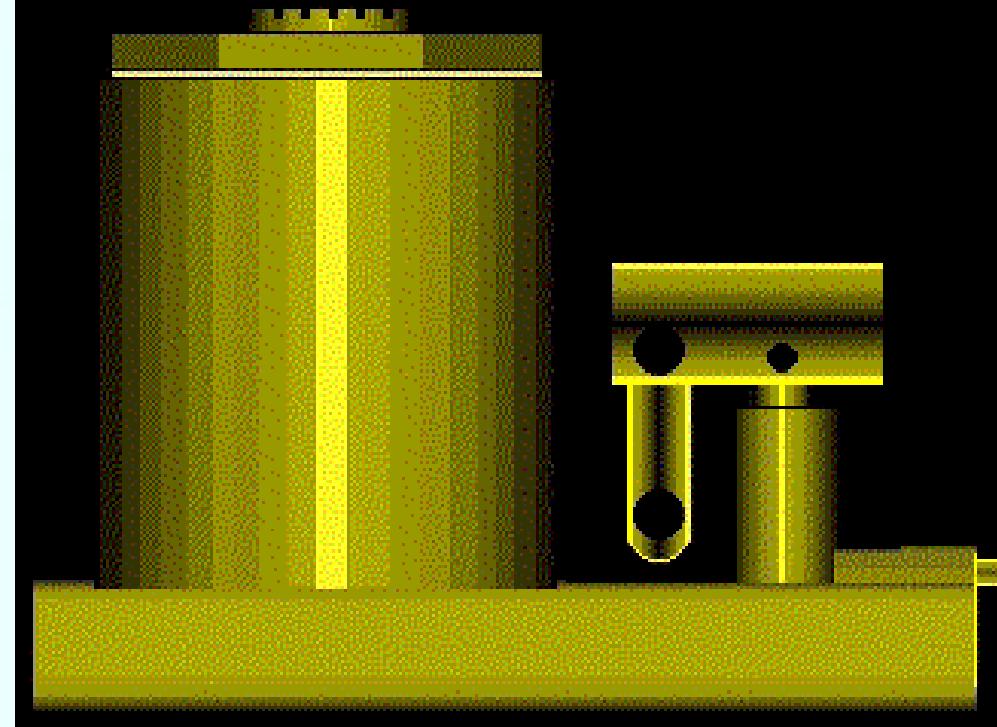
- 유체의 압력은 관에 대하여
직각으로 작용한다.
- 각 점의 압력은 모든 방향으로 같다.
- 밀폐된 용기 속의 유체의 일부분에
가해진 압력은 동시에 각 부에 같은
세기를 가지고 전달 된다.
- 기계시스템의 레버에 의한 힘의
증폭은 유공압시스템에서 파스칼의
법칙에 의한 힘의 증폭과 유사하다.



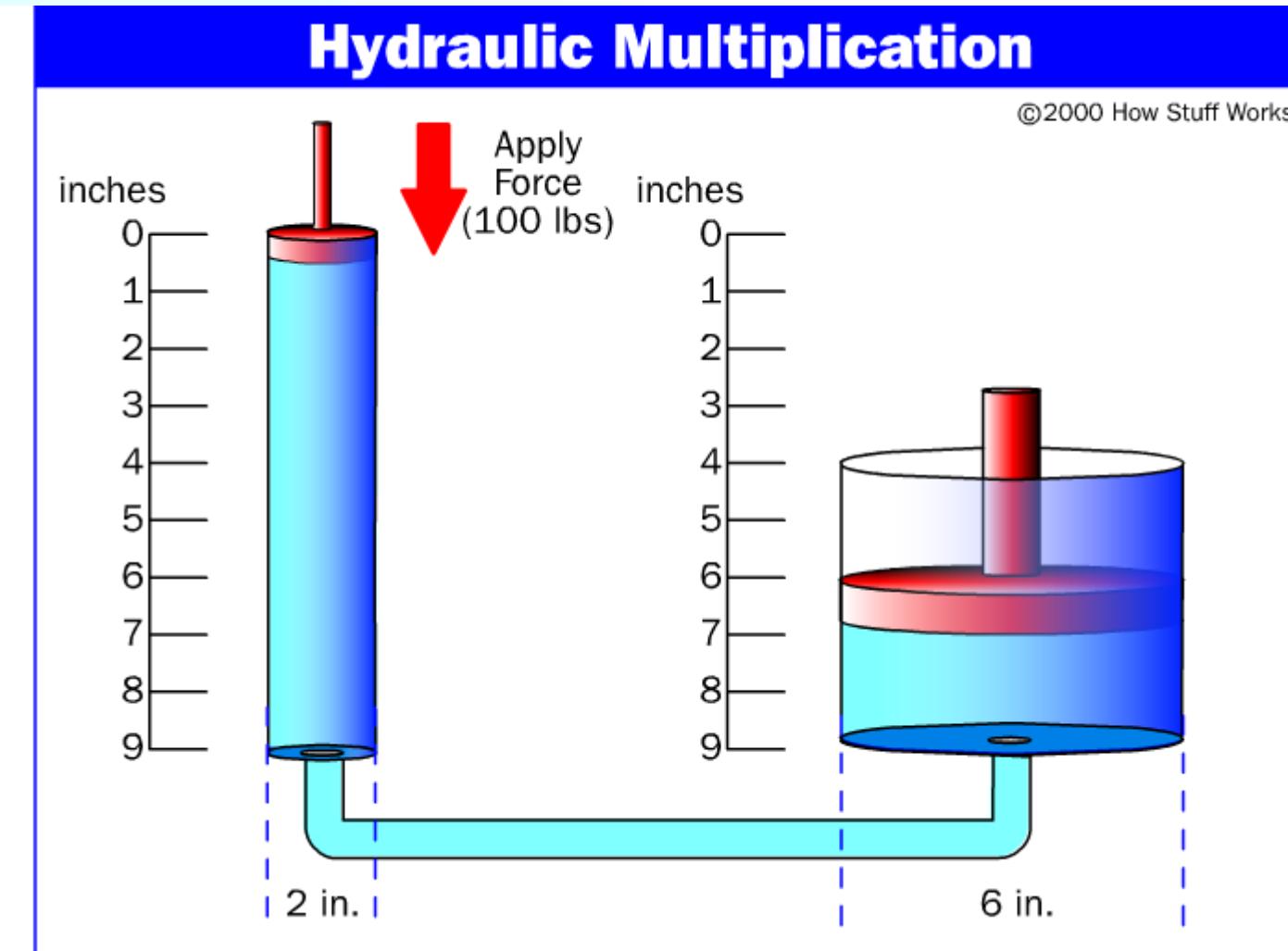
$$P = F_1/A_1 = F_2/A_2$$

3.4 Hydraulic Jack

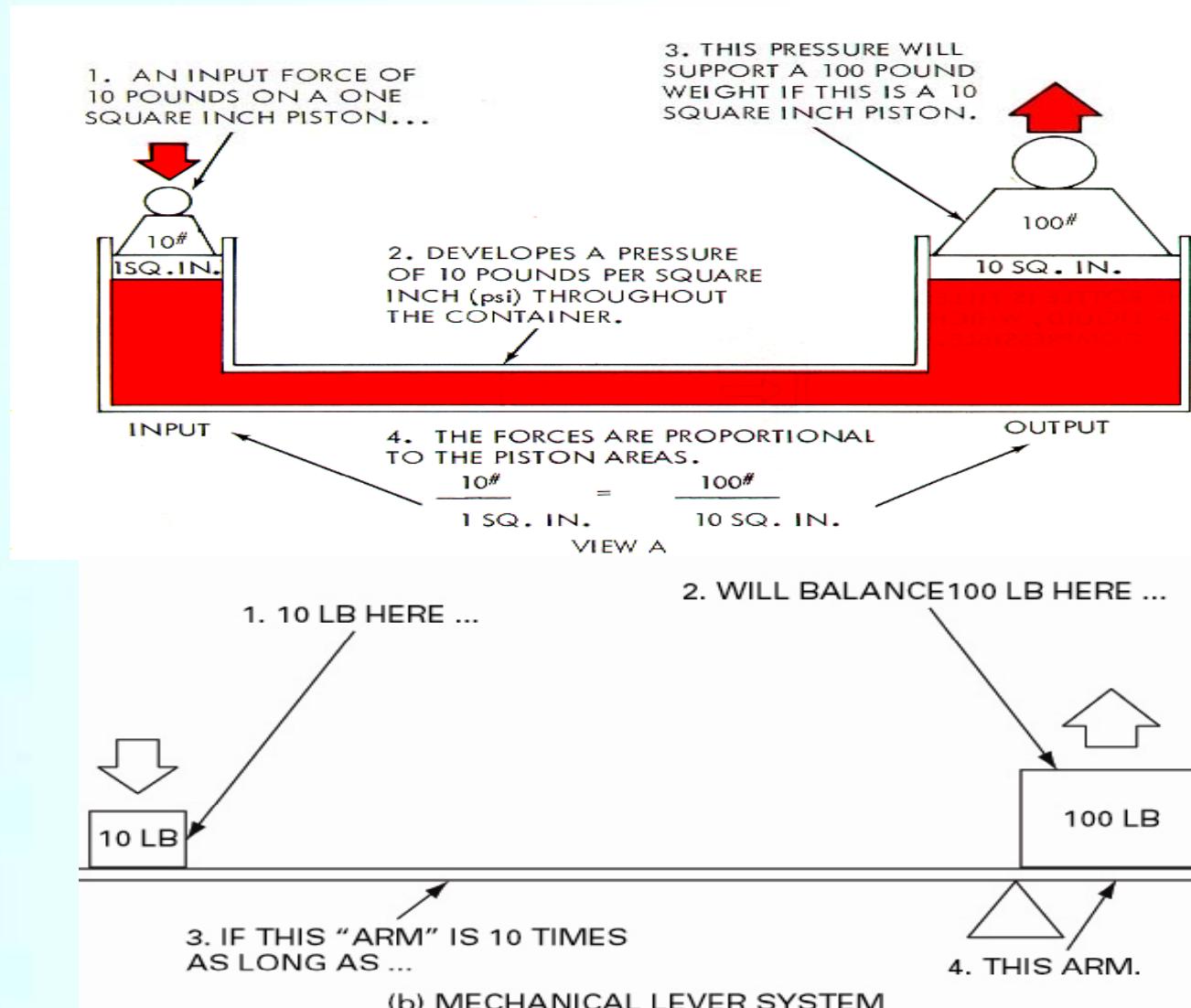
Position Jack &
Close Release...



Hydraulic Multiplication



Simple Hydraulic Jack & Mechanical Lever



Analysis of Simple Hydraulic Jack

Pascal's Law

$$p_1 = p_2$$

$$F_1/A_1 = F_2/A_2$$

$$F_2/F_1 = A_2/A_1$$

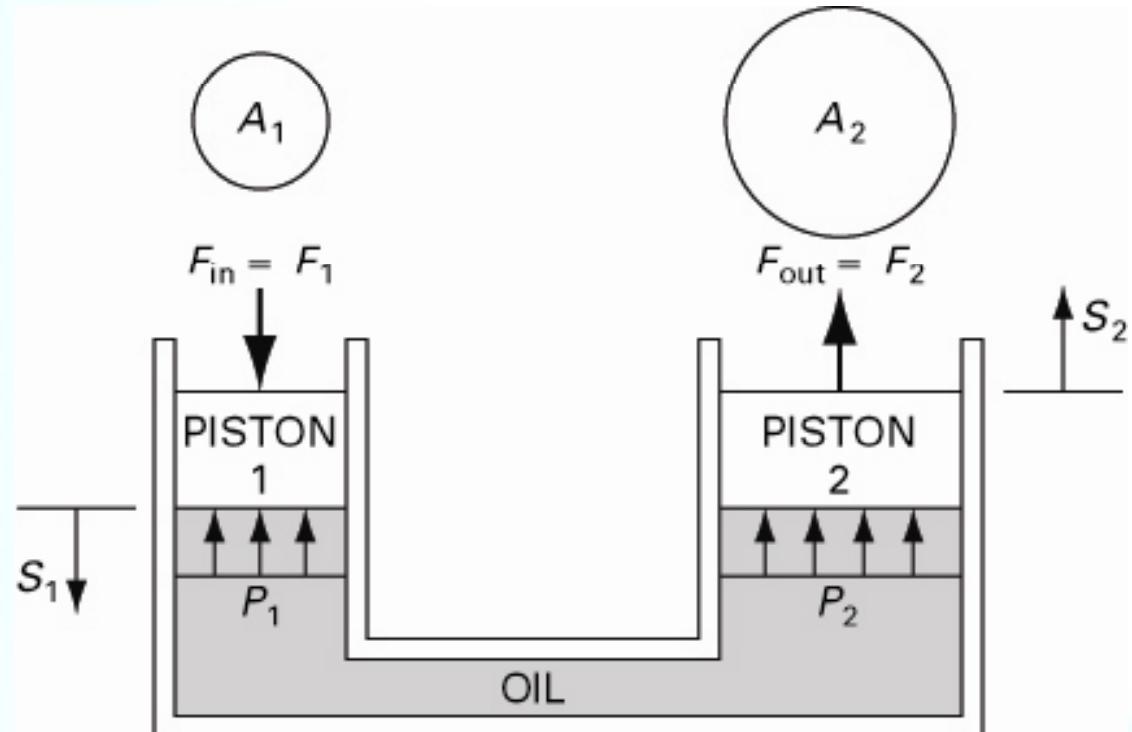
$$V_1 = V_2$$

$$A_1 S_1 = A_2 S_2$$

$$S_2/S_1 = A_1/A_2$$

$$F_2/F_1 = S_1/S_2$$

$$F_1 S_1 = F_2 S_2$$

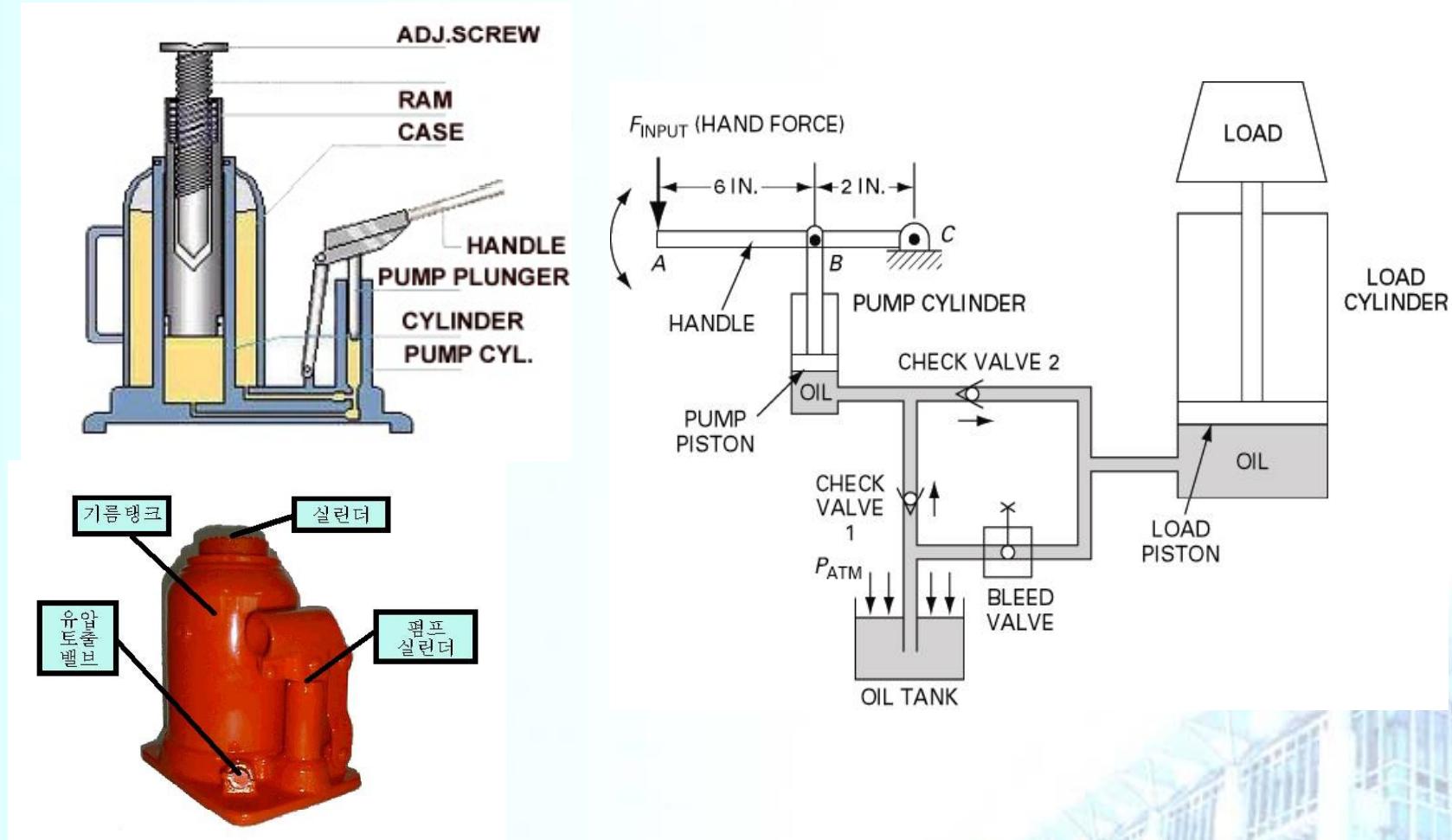


Conservation of energy

- 다른 동력 시스템과 마찬가지로 유압시스템은 동력을 생성할 수 없다.

Hydraulic Jack

■ 19C 영국의 J.Bramah



Air-to-Hydraulic Pressure Booster

- 공간절약, 운영 및 유지관리 비용 절감

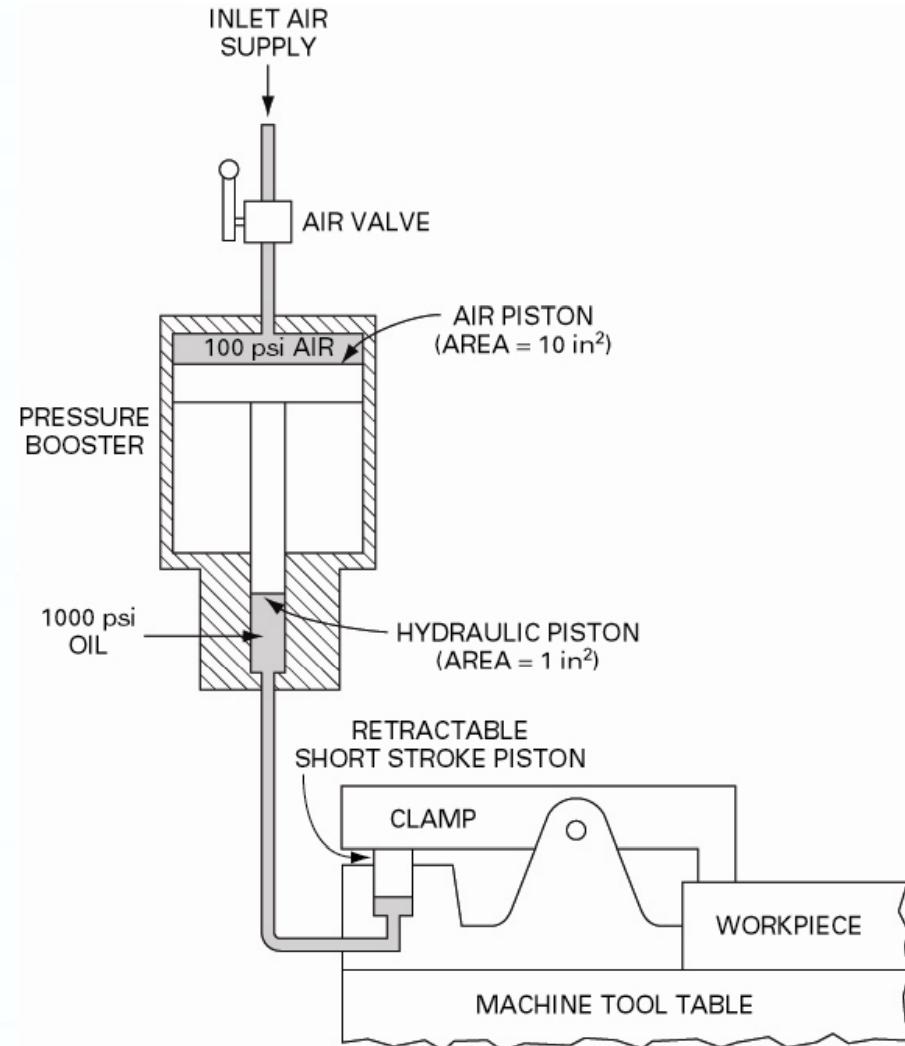
- 압력비

- = 출력오일 압력
/입력공기 압력
- = 공기피스톤 넓이
/유압피스톤 넓이

- 100psi의 공기압으로
30,000psi까지의 유압으로
바꾸어줄 수 있다.

- Pressure ratio

- = $1000\text{psi} / 100\text{psi}$
- = $10\text{in}^2 / 1\text{in}^2$
- = 10



3.5 Conservation of Energy

■ Conservation of Energy

- 에너지 = 힘 X 움직인 거리 : [Nm] [J]
- 입력에너지와 출력에너지는 같다.

$$F_1 s_1 = F_2 s_2$$

시스템의 총 에너지는 항상 같다. 즉, 에너지는 생성되지도 소멸되지도 않는다.

- Energy = Potential Energy + Kinetic Energy

- Elevation Potential Energy (EPE)

$$EPE = WZ$$

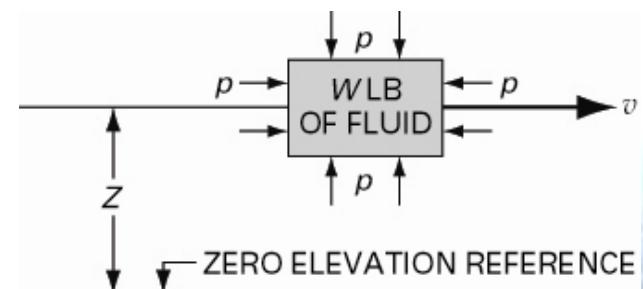
- Pressure Potential Energy (PPE)

$$PPE = WP/\gamma$$

- Kinetic Energy (KE)

$$KE = \frac{1}{2} W/g v^2$$

$$E_T = WZ + WP/\gamma + \frac{1}{2} W/g v^2 = \text{constant}$$



3.6 Continuity Equation

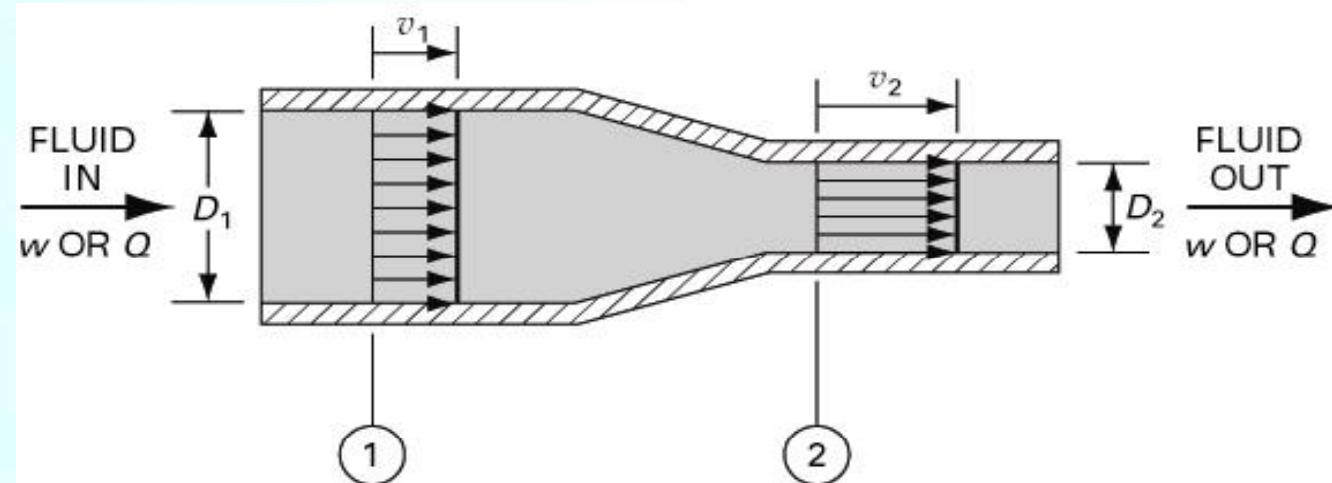
- 정상유동의 관로에서 모든 단면의 무게유량(**weight flow rate**)은 같다.

$$w_1 = w_2$$

$$\gamma_1 A_1 v_1 = \gamma_2 A_2 v_2$$

$$Q_1 = A_1 v_1 = A_2 v_2 = Q_2$$

$$v_1/v_2 = A_2/A_1 = (D_2/D_1)^2$$



Continuity Equation

■ Integral Relation for a Control Volume

■ Conservation of mass

$$\left(\frac{dm}{dt} \right)_{\text{sys}} = 0 = \frac{d}{dt} \left(\iiint_{C.V.} \rho dV \right) + \iint_{C.S.} \rho (\bar{V}_r \cdot \hat{n}) dA$$

■ Derivation of the Continuity Equation for Lumped Fluid Circuit

$$\rho V = m = \text{constant}$$

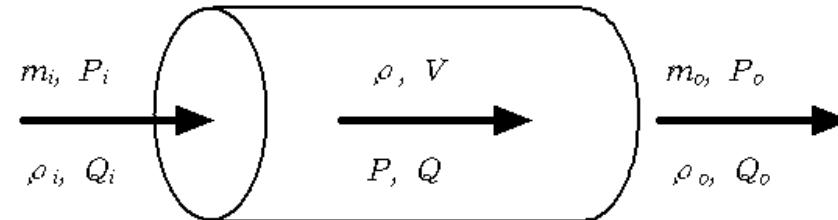
$$V \Delta \rho + \rho \Delta V = 0$$

$$\frac{\Delta V}{V} = -\frac{\Delta \rho}{\rho}$$

$$\beta_e = -\frac{V}{\Delta V} \Delta P = \frac{\rho}{\Delta \rho} \Delta P$$

Generalized Flow-Continuity Equation

■ Generalized Flow-Continuity Equation



■ Mass Flow Rate Continuity

$$\left(\frac{dm}{dt} \right)_{sys} = \frac{d}{dt} \left(\iiint_{C.V.} \rho dV \right)$$

$$\frac{d}{dt}(\rho_i V_i) - \frac{d}{dt}(\rho_o V_o) = \frac{d}{dt}(\rho V) = V \frac{d\rho}{dt} + \rho \frac{dV}{dt}$$

$$\Delta\rho = \frac{\rho}{\beta_e} \Delta P \quad \Delta V \rightarrow 0 \quad \frac{d\rho}{dt} = \frac{\rho}{\beta_e} \frac{dP}{dt}$$

■ Generalized Flow-Continuity Equation

$$Q_i - Q_o = \frac{V}{\beta_e} \frac{dP}{dt} + \frac{dV}{dt} \quad (\text{for Lumped model, } \rho_i = \rho_o = \rho)$$

3.7 Hydraulic Power

■ Hydraulic horsepower (HHP; 유압마력)

- 유체가 액츄에이터에 전달하는 마력

$$\begin{aligned} \text{HHP[kW]} &= p[\text{Pa}]Q[\text{m}^3/\text{s}]/10^3 \\ &= p[\text{kgf/cm}^2]Q[\text{l/min}]/612 \end{aligned}$$

■ Output horsepower (OHP; 출력마력)

- 액츄에이터가 부하에 전달하는 마력
- 출력마력은 마찰손실이나 누설손실에 의하여 항상 유압마력보다 작다.

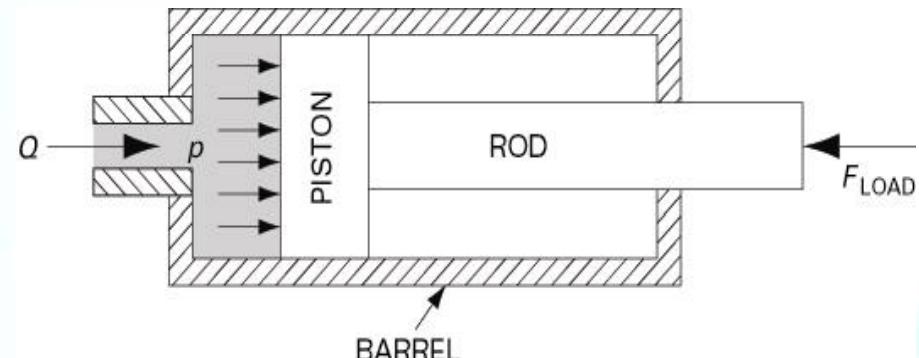
Hydraulic Cylinder Example

■ Questions

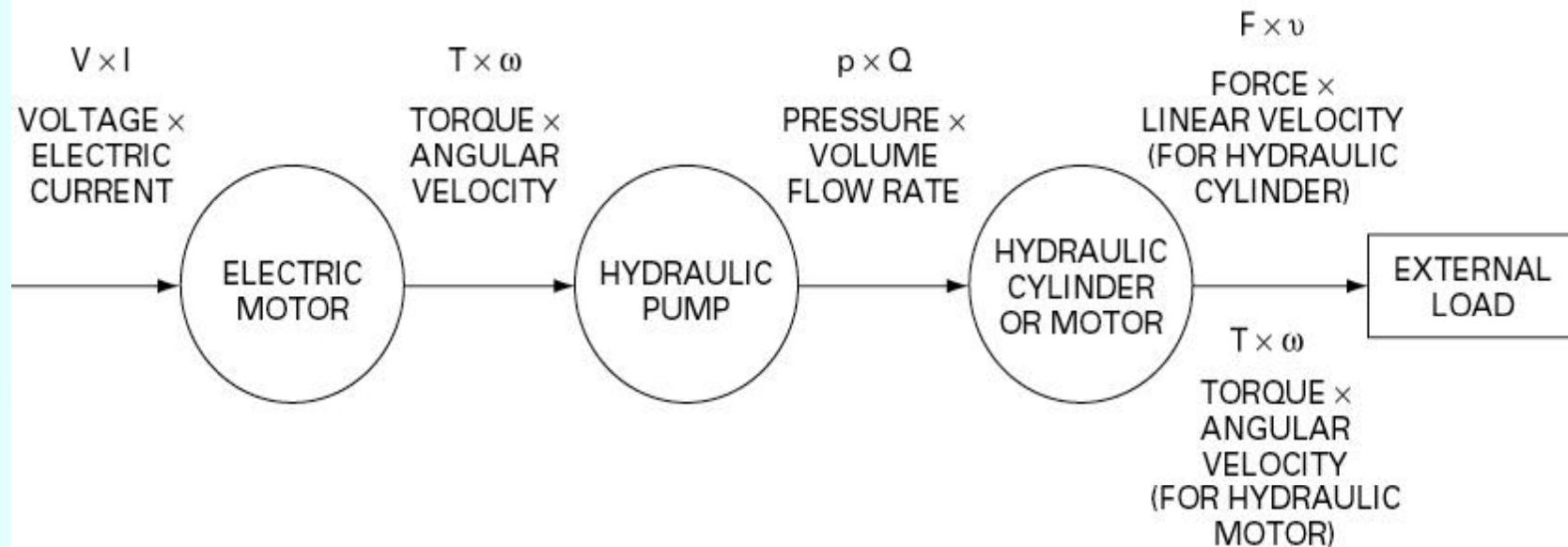
- How do we determine how large a piston diameter is required for the cylinder?
- What is the pump flow rate required to drive the cylinder through its stroke in a specified time?
- How much hydraulic horsepower does the fluid deliver to the cylinder?

■ Answers

- $pA = F_{\text{load}}$
 $A = F_{\text{load}}/p$
- $V_D = AS$
 $Q = V_D/t = AS/t$
 $= Av$
- $E = FS = pAS$
 $P = E/t = pAS/t$
 $= pAv = pQ$
 $\text{HHP[kW]} = p[\text{Pa}]Q[\text{m}^3/\text{s}]/10^3$
 $= p[\text{kgf/cm}^2]Q[\text{l/min}]/612$

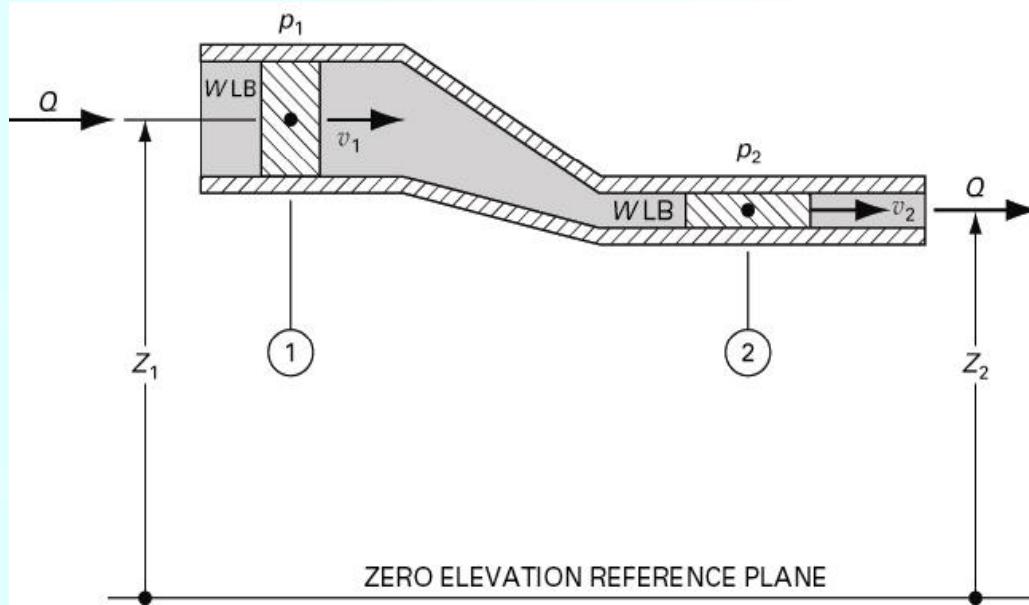


Conversion of Power



- **Mechanical power = force \times linear velocity**
 $=$ torque \times angular velocity
- **Electrical power = voltage \times electric current**
- **Hydraulic power = pressure \times volume flow rate**

3.8 Bernoulli's Equation



Z : **elevation head**
 p/γ : **pressure head**
 $v^2/2g$: **velocity head**

- 관로의 마찰손실을 무시할 수 있다면 1지점과 2지점에서 **W lb**의 유체가 갖는 총 에너지는 같다.

$$WZ_1 + Wp_1/\gamma + Wv_1^2/2g = WZ_2 + Wp_2/\gamma + Wv_2^2/2g \quad : [Nm]$$

$$Z_1 + p_1/\gamma + v_1^2/2g = Z_2 + p_2/\gamma + v_2^2/2g \quad : [m]$$

Bernoulli's Equation

■ Bernoulli's Equation

- Incompressible, steady state에서 유선을 따라 Euler's equation을 적용

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant along streamline}$$

■ Basic differential momentum equation for an infinitesimal element

$$\rho g - \nabla p + \mu \nabla^2 V = \rho \frac{dV}{dt}$$

■ Navier-Stokes Equation

- Newtonian Fluid의 경우 전단력이 속도 구배에 비례

$$\rho g - \nabla p + \nabla \cdot \tau_{ij} = \rho \frac{dV}{dt} \quad \tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

■ Euler's Equation

- Inviscid Flow의 경우 점성력을 무시

$$\rho g - \nabla p = \rho \frac{dV}{dt}$$

Energy Equation

- 유체 1lb의 덩어리가 1지점에서 갖고 있는 총 에너지에서 펌프에 의해 가해지는 에너지를 더하고, 유압모터에 의해 제거되는 에너지를 빼고, 마찰손실에 의한 에너지를 빼면, 이 유체가 2지점에 도달했을 때의 총 에너지와 같다.

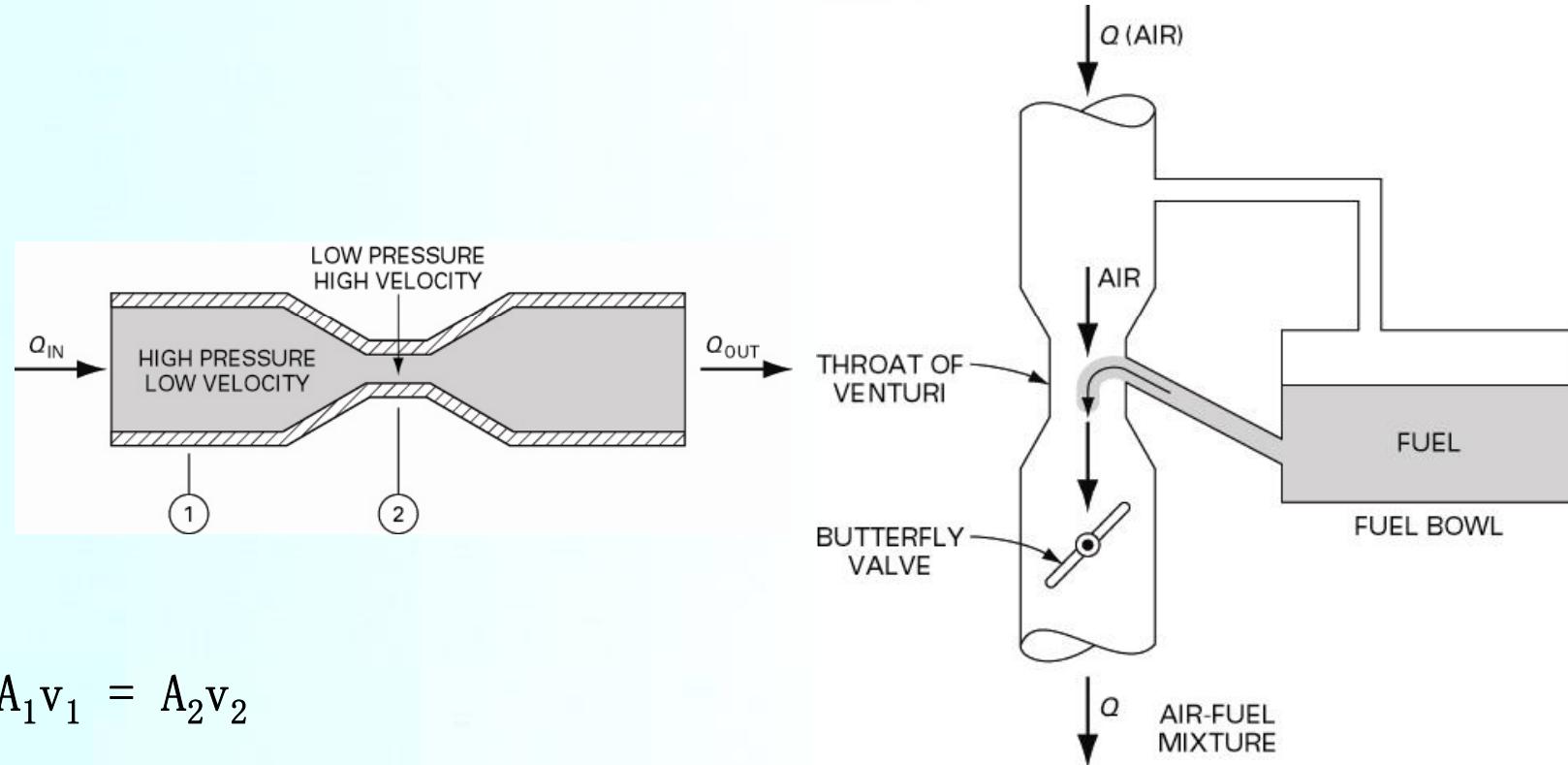
$$Z_1 + p_1/\gamma + v_1^2/2g + H_p - H_m - H_L = Z_2 + p_2/\gamma + v_2^2/2g \quad : [m]$$

- H_L : head loss
- H_p : pump head
- H_m : motor head

$p = \gamma H$, $HHP[kW] = p[Pa]Q[m^3/s]/10^3$ 로부터

- $H_p[m] = 1000 HHP[kW]/Q[m^3/s]$

Venturi Application



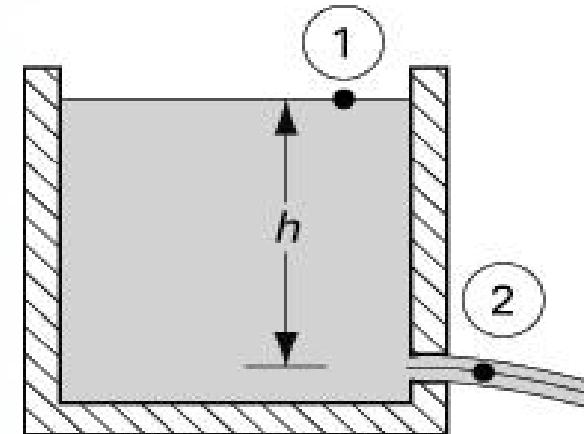
$$A_1 v_1 = A_2 v_2$$

$$p_1/\gamma + v_1^2/2g = p_2/\gamma + v_2^2/2g$$

$$p_1 - p_2 = \gamma/2g(v_2^2 - v_1^2) > 0$$

3.9 Torricelli's Theorem

- 이상적인 유체의 자유분사(free jet) 속도는 중력가속도와 수두의 곱의 2배에 제곱근을 취한 것과 같다.



$$\begin{aligned}
 Z_1 + p_1/\gamma + v_1^2/2g + H_p - H_m - H_L &= Z_2 + p_2/\gamma + v_2^2/2g \\
 h + 0 + 0 + 0 - 0 - 0 &= 0 + 0 + v_2^2/2g \\
 v_2 &= \sqrt{2gh}
 \end{aligned}$$

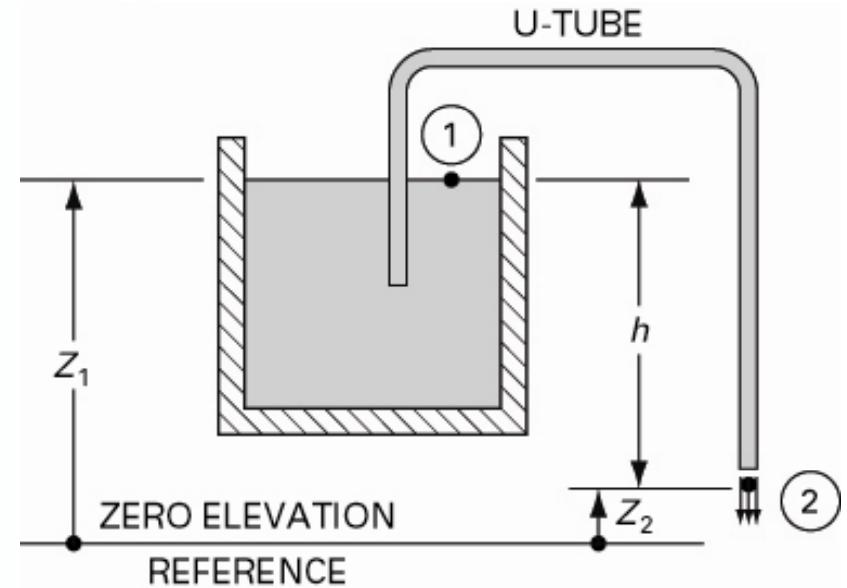
- 이상유체가 아니면 구멍으로 밀어내는 순수한 압력수두가 작아져서 분사속도가 줄어들며, 또한 실제의 분사속도는 유체의 점도에 영향을 받는다.

$$v_2 = \sqrt{2g(h-H_L)}$$

Operating Principle of Siphon

Conditions

- The elevation of the free end must be lower than the elevation of the liquid surface inside the container
- The fluid must initially be forced to flow up from the container into the center portion of the U-tube. This is normally done by temporarily providing a suction pressure at the free end of the siphon.

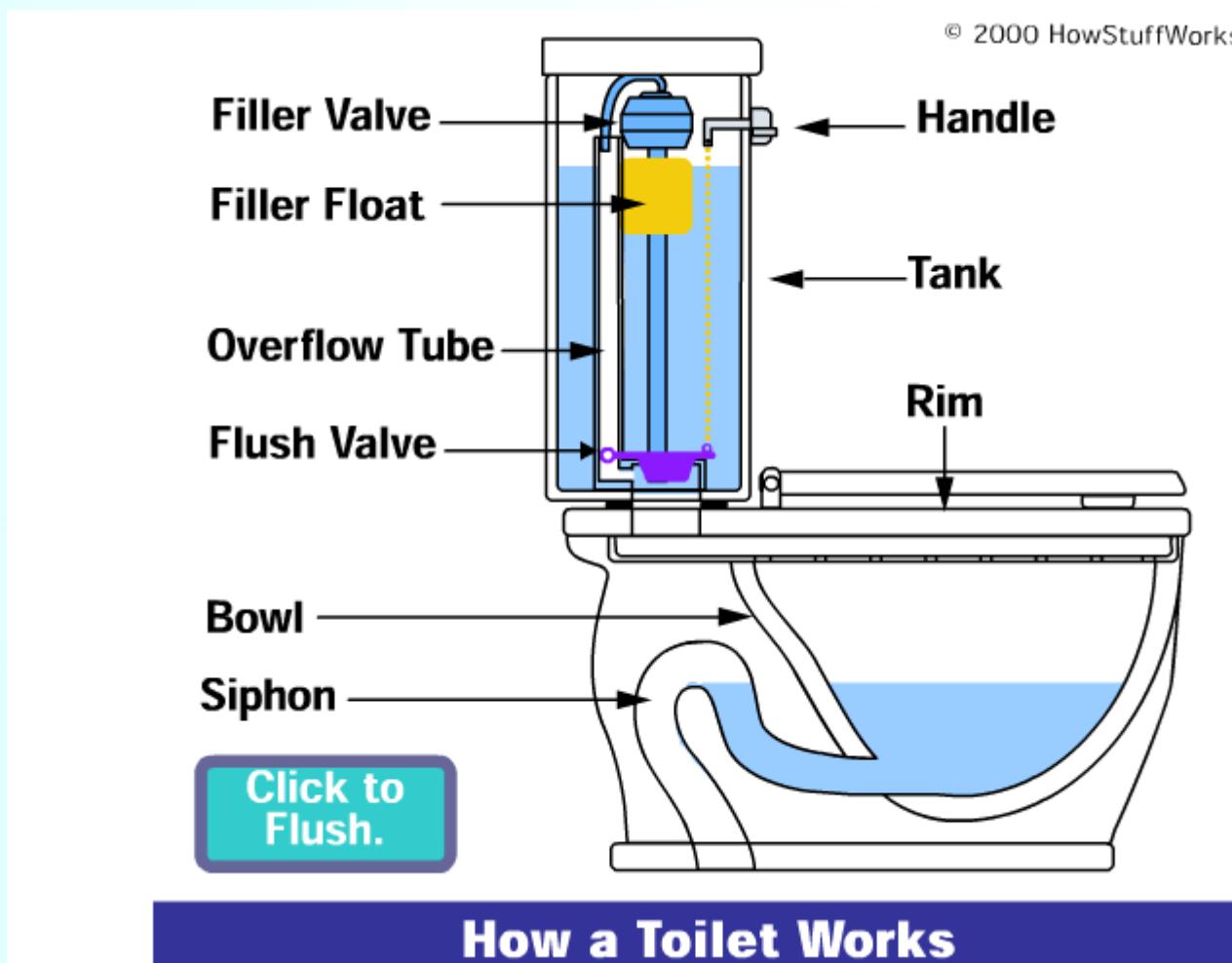


$$Z_1 + p_1/\gamma + v_1^2/2g + H_p - H_m - H_L = Z_2 + p_2/\gamma + v_2^2/2g$$

$$Z_1 + 0 + 0 + 0 - 0 - H_L = Z_2 + 0 + v_2^2/2g$$

$$v_2 = \sqrt{2g(Z_1 - Z_2 - H_L)} = \sqrt{2g(h - H_L)}$$

The Siphon: Toilet



Report

■ Text Problems

- 3-3
- 3-25
- 3-40
- 3-60

■ Due date: 2주후