

4. Frictional Losses in Hydraulic Pipelines

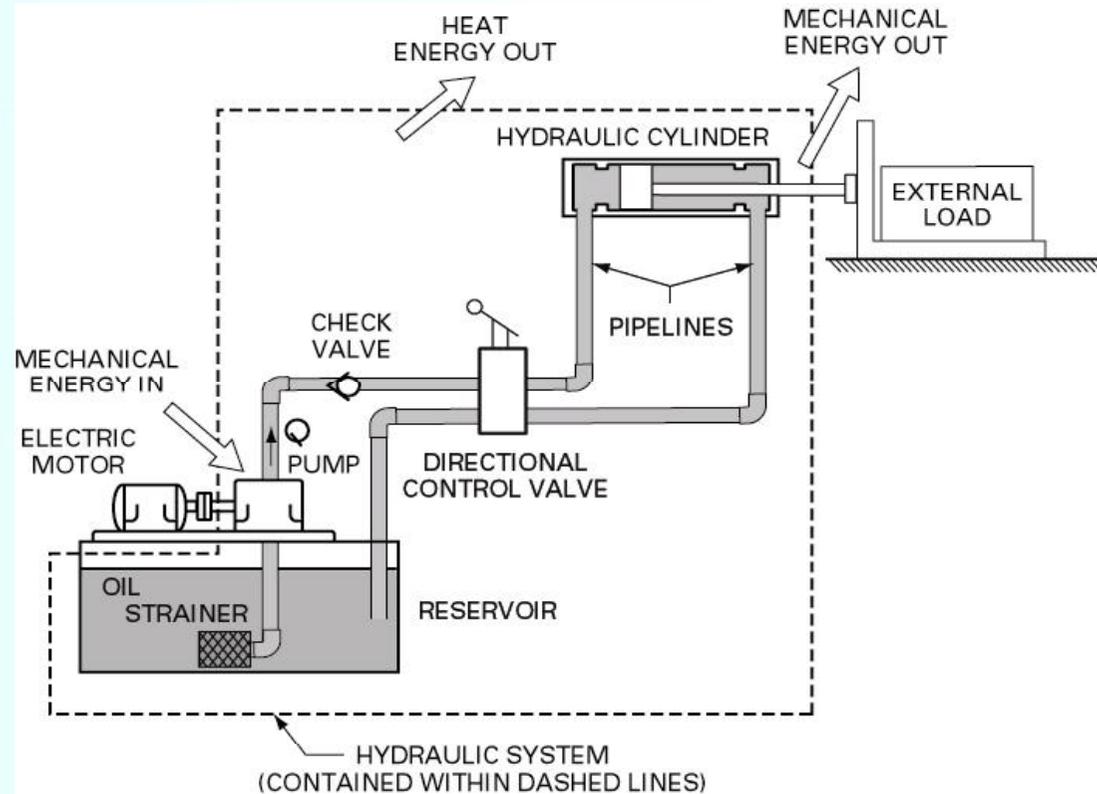
- Laminar & Turbulent Flow
- Reynolds Number ▶
- Darcy's Equation
- Moody Diagram ▶

- Frictional Losses & Friction Factor
- Losses in Valves & Fittings: K Factor
- Equivalent-Length Technique ▶
- Hydraulic Circuit Analysis

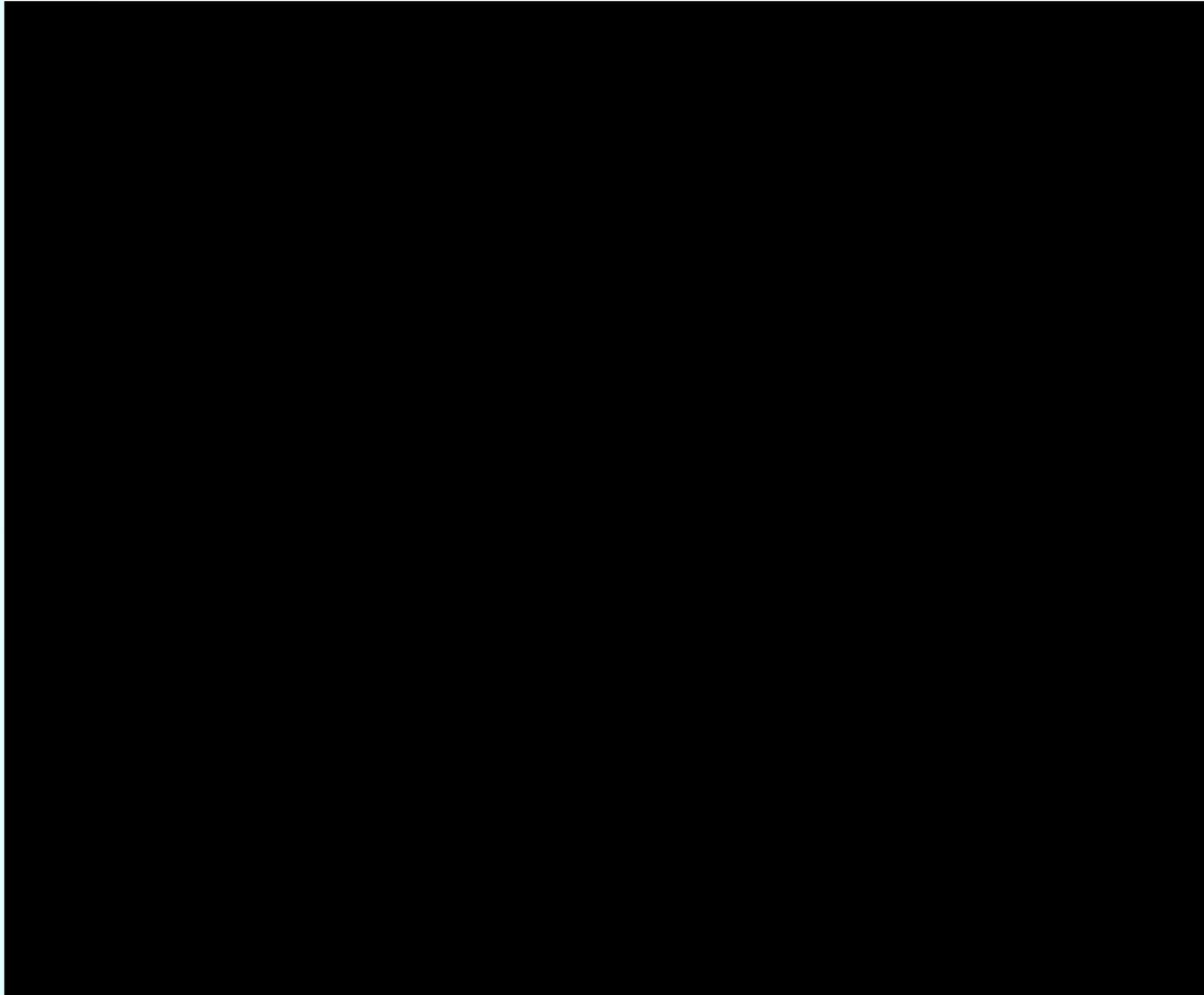
- 4.A Bulk Modulus ▶
- 4.7.A Orifice Flow 오리피스 ▶

4.1 Energy Transfer including Energy Losses

- Energy Losses
 - Frictional Fluid Flow
 - Valves & Fittings: Bends, couplings, tees, elbows, filters, strainers
- Selection of the proper sizes
 - Pipes, Valves, Fittings

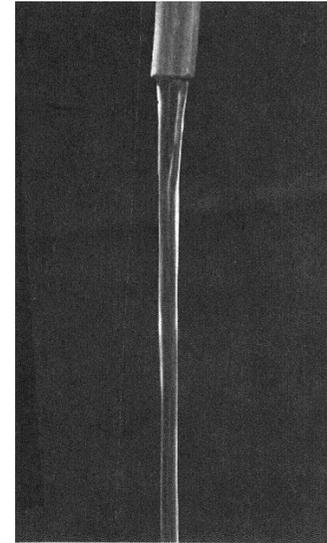
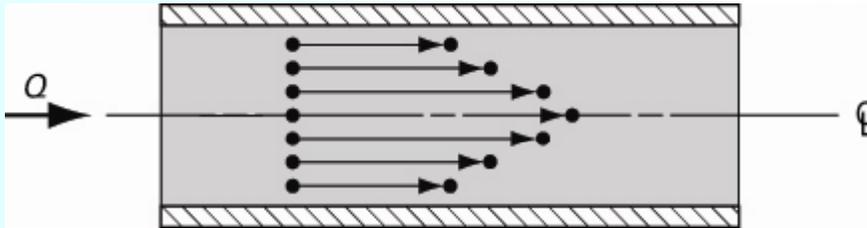


유체의 흐름

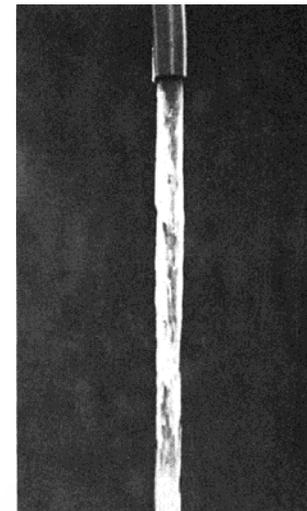
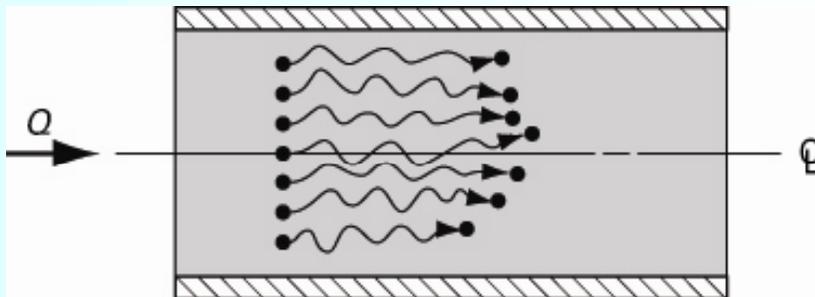


4.2 Laminar Flow & Turbulent Flow

■ Laminar Flow



■ Turbulent Flow



Fluid Flow in Pipe Line

■ 유체의 흐름에 영향을 미치는 힘의 종류

- 체적력: 중력, 부력
- 관성력
- 내부 마찰력: 점성력
- 표면장력: 전기전자력

■ 대부분의 경우, 체적력과 관성력이 지배적인 영향을 미침

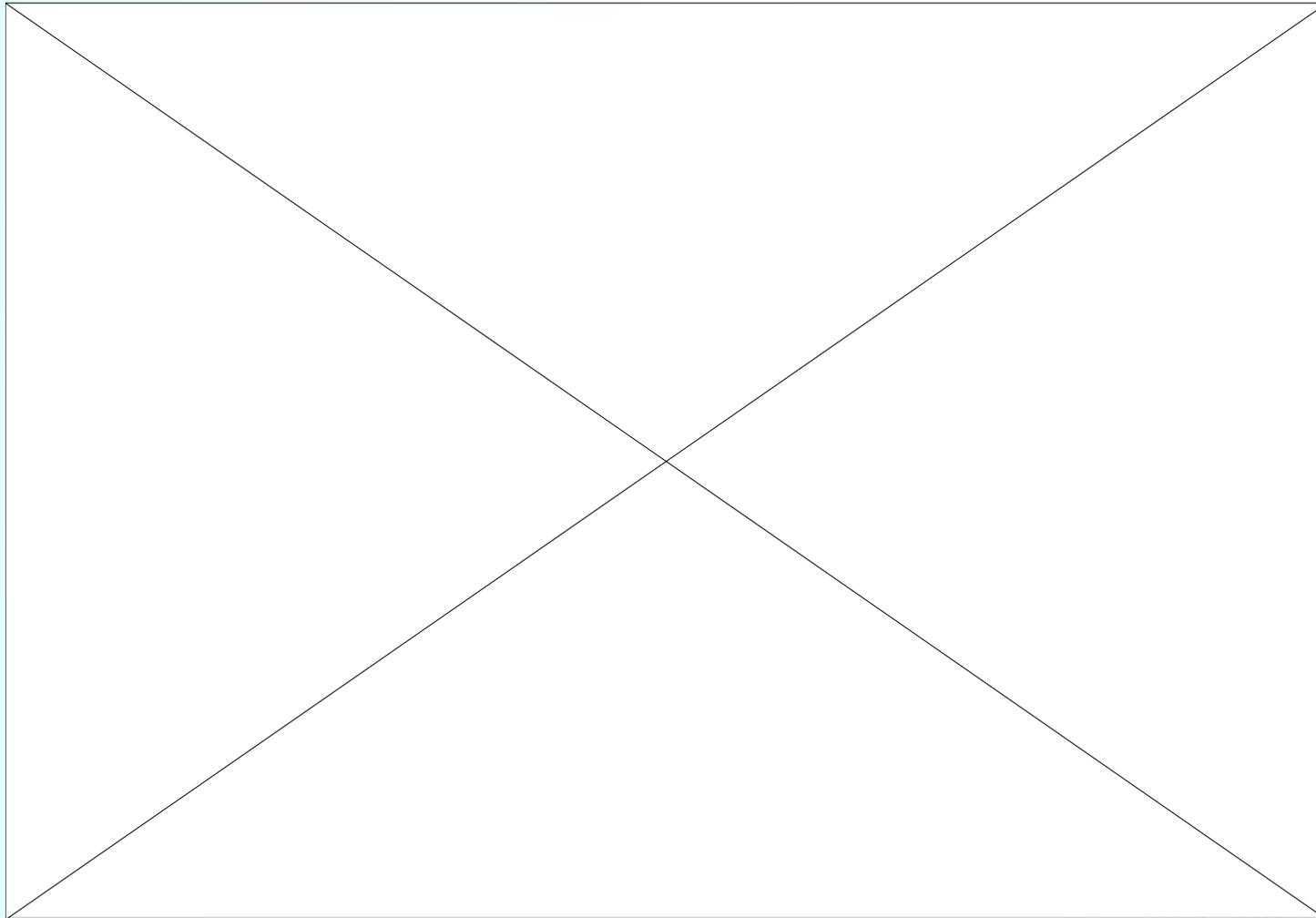
- **Reynold Number**: 점성력에 대한 관성력의 비

$$N_R = \frac{\text{관성력}}{\text{점성력}} = \frac{\rho u a}{\mu}$$

■ Reynolds Number에 의한 유동의 분류(관내 유동의 경우)

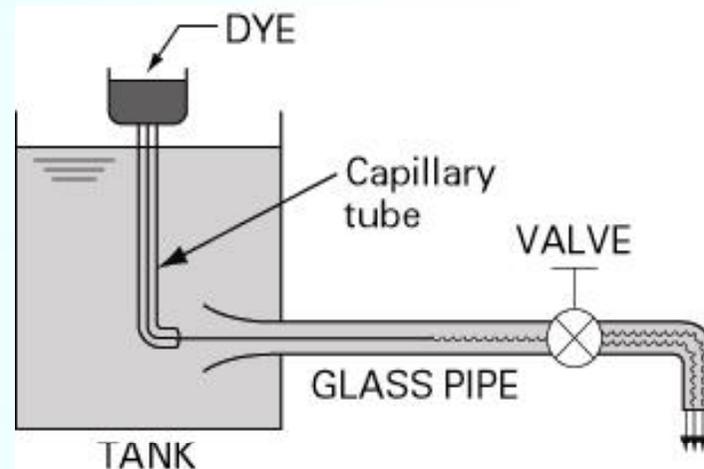
- **Laminar Flow** (점성력에 의해 지배되는 흐름): $N_R < 2000$
- **Transition Flow** (천이 구역): $2000 < N_R < 4000$
- **Turbulent Flow** (관성력에 의해 지배되는 흐름): $4000 < N_R$

Reynolds Experiment



4.3 Reynolds Number

■ Reynolds' Experiment



- The nature of the flow depends on the dimensionless parameter

$$N_R = \frac{vD\rho}{\mu}$$

- If N_R is less than 2000, the flow is laminar.
- If N_R is greater than 4000, the flow is turbulent.
- Reynolds numbers between 2000 and 4000 cover a critical zone between laminar and turbulent flow.



4.4 Darcy's Equation

■ Head Loss (H_L)

- Losses in pipes
- Losses in valves and fittings

■ Head losses in pipes: Darcy's Equation

$$H_L = f \left(\frac{L}{D} \right) \left(\frac{v^2}{2g} \right)$$

- f : friction factor (dimensionless)
- L : length of pipe (m)
- D : pipe inside diameter (m)
- v : average fluid velocity (m/s)
- g : acceleration of gravity (m/s²)

4.5 Frictional Losses

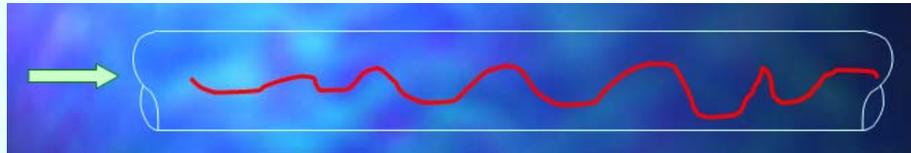
■ Laminar Flow: Hagen-Poiseuille equation



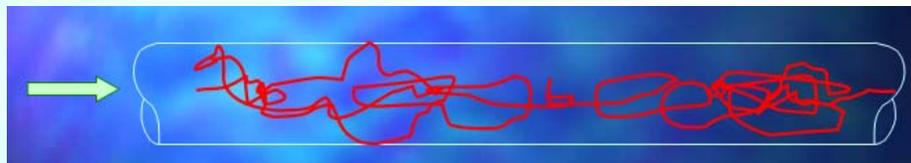
$$f = \frac{64}{N_R}$$

$$H_L = \frac{64}{N_R} \left(\frac{L}{D} \right) \left(\frac{v^2}{2g} \right)$$

■ Transition Flow

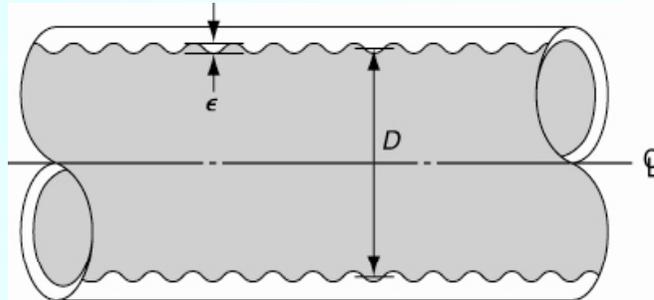


■ Turbulent Flow: The Moody diagram



4.6 Effect of Pipe Roughness

■ Pipe absolute roughness (ϵ)

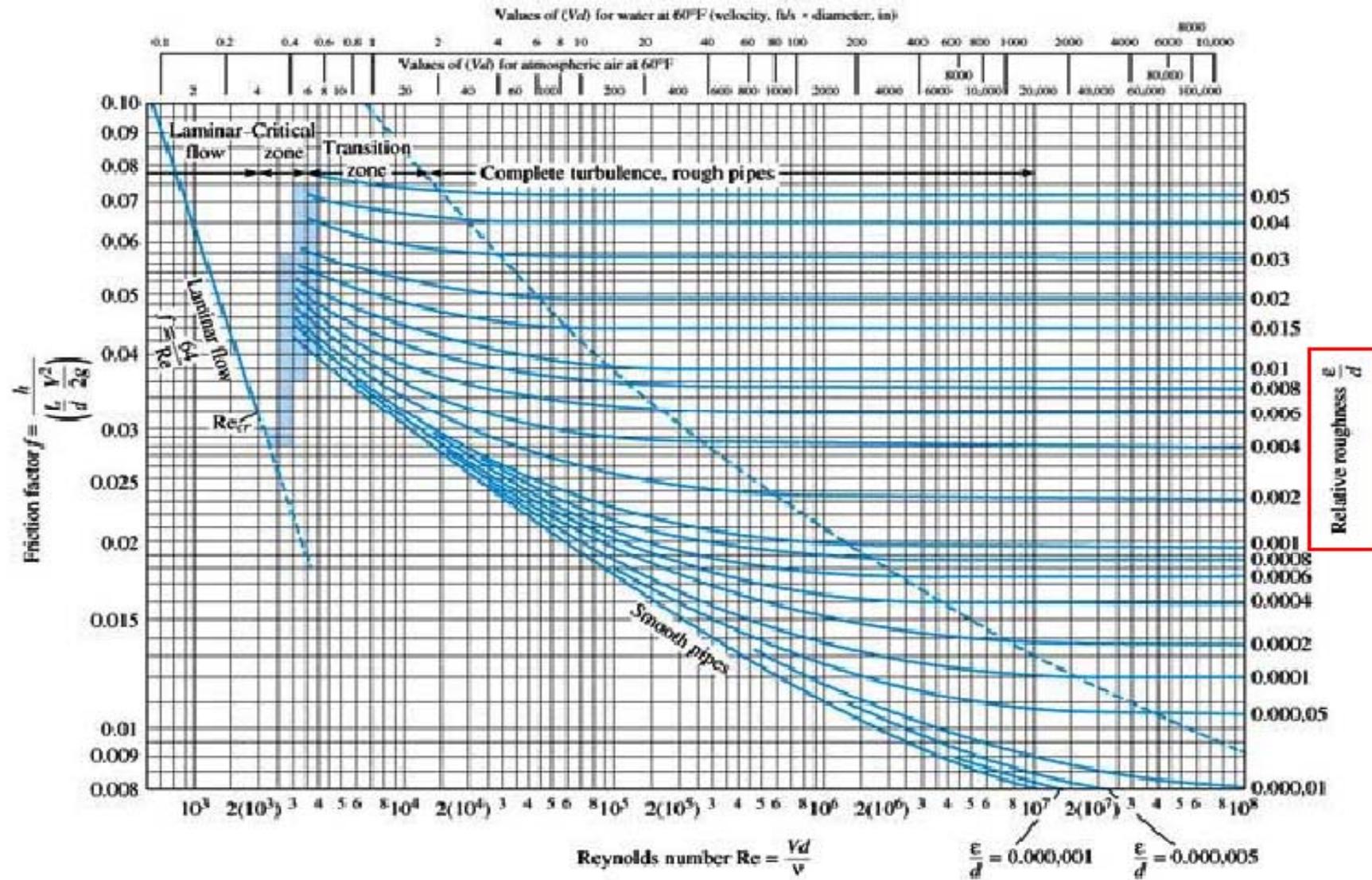


$$\text{Relative roughness} = \frac{\epsilon}{D}$$

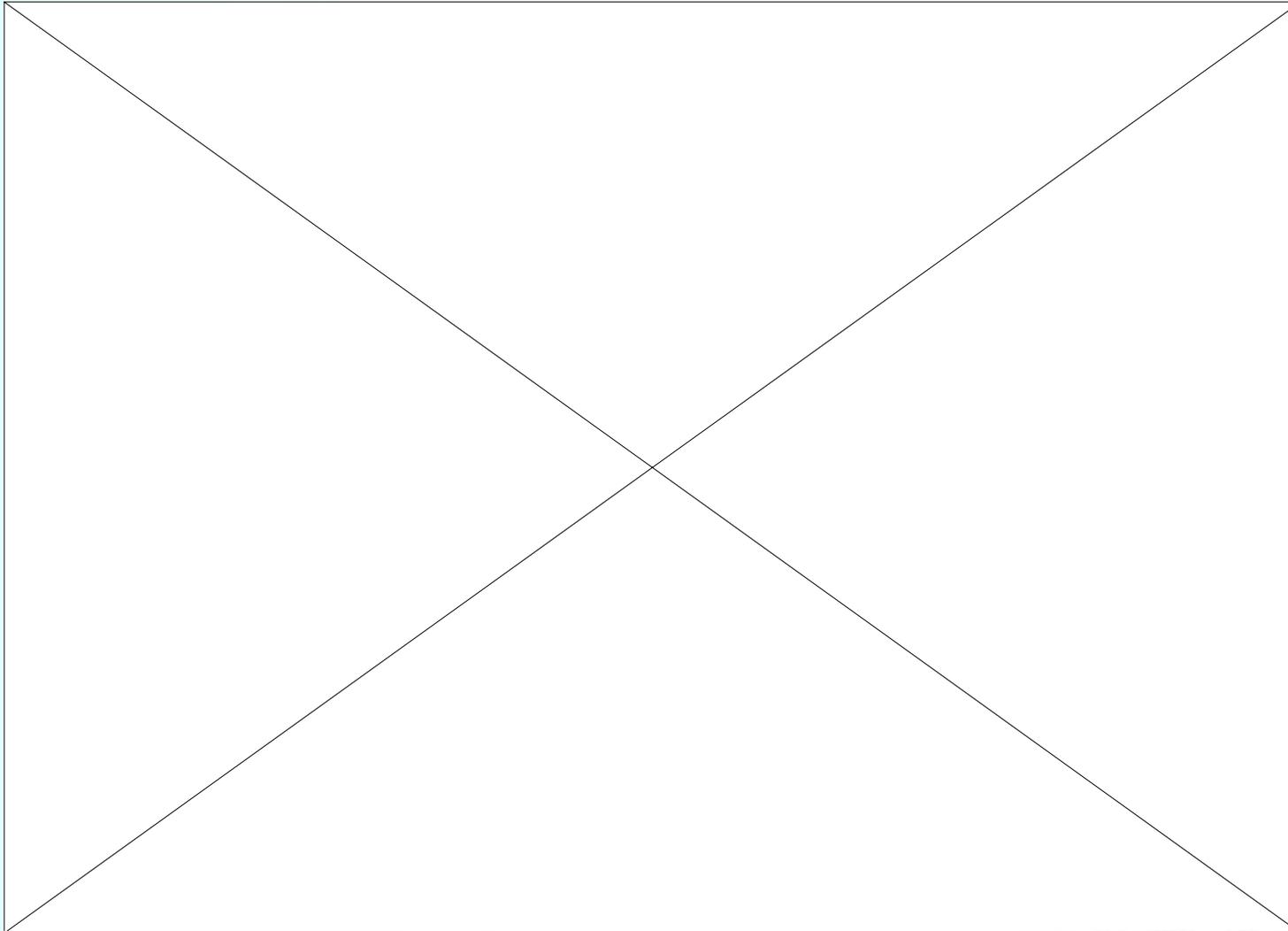
■ Typical values of absolute roughness

TYPE OF PIPE	ABSOLUTE ROUGHNESS	
	ϵ (in)	ϵ (mm)
GLASS OR PLASTIC	SMOOTH	SMOOTH
DRAWN TUBING	0.00006	0.0015
COMMERCIAL STEEL OR WROUGHT IRON	0.0018	0.046
ASPHALTED CAST IRON	0.0048	0.12
GALVANIZED IRON	0.006	0.15
CAST IRON	0.0102	0.26
RIVETED STEEL	0.072	1.8

Moody Diagram



Finding Friction Factor using Moody Diagram



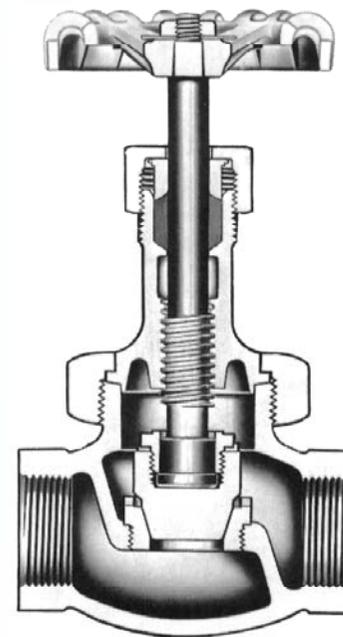
4.7 Losses in Valves & Fittings

■ The K Factor

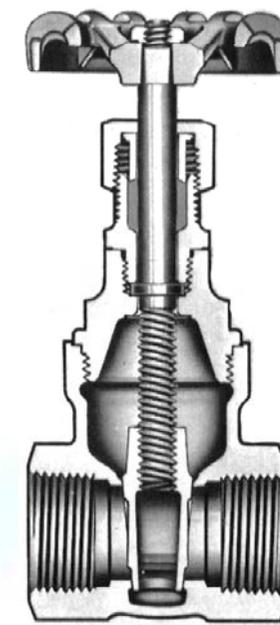
$$H_L = K \left(\frac{v^2}{2g} \right)$$

■ K factors of common valves and fittings

VALVE OR FITTING	K FACTOR
GLOBE VALVE: WIDE OPEN	10.0
1/2 OPEN	12.5
GATE VALVE: WIDE OPEN	0.19
3/4 OPEN	0.90
1/2 OPEN	4.5
1/4 OPEN	24.0
RETURN BEND	2.2
STANDARD TEE	1.8
STANDARD ELBOW	0.9
45° ELBOW	0.42
90° ELBOW	0.75
BALL CHECK VALVE	4.0



Globe Valve
(10.0)



Gate Valve
(0.19)

Bend & Elbows



45° elbow
(0.42)



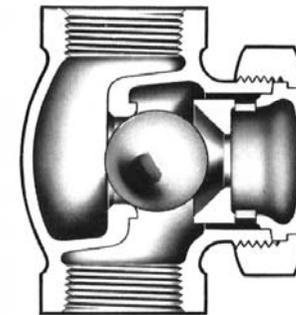
90° elbow
(0.75)



Tee
(1.8)



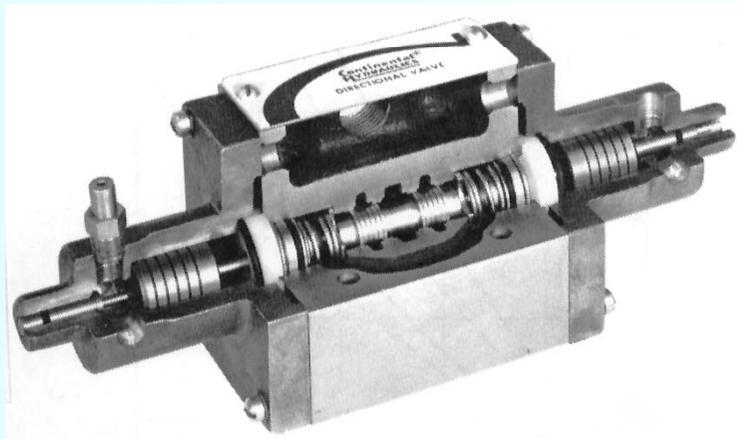
Return bend
(2.2)



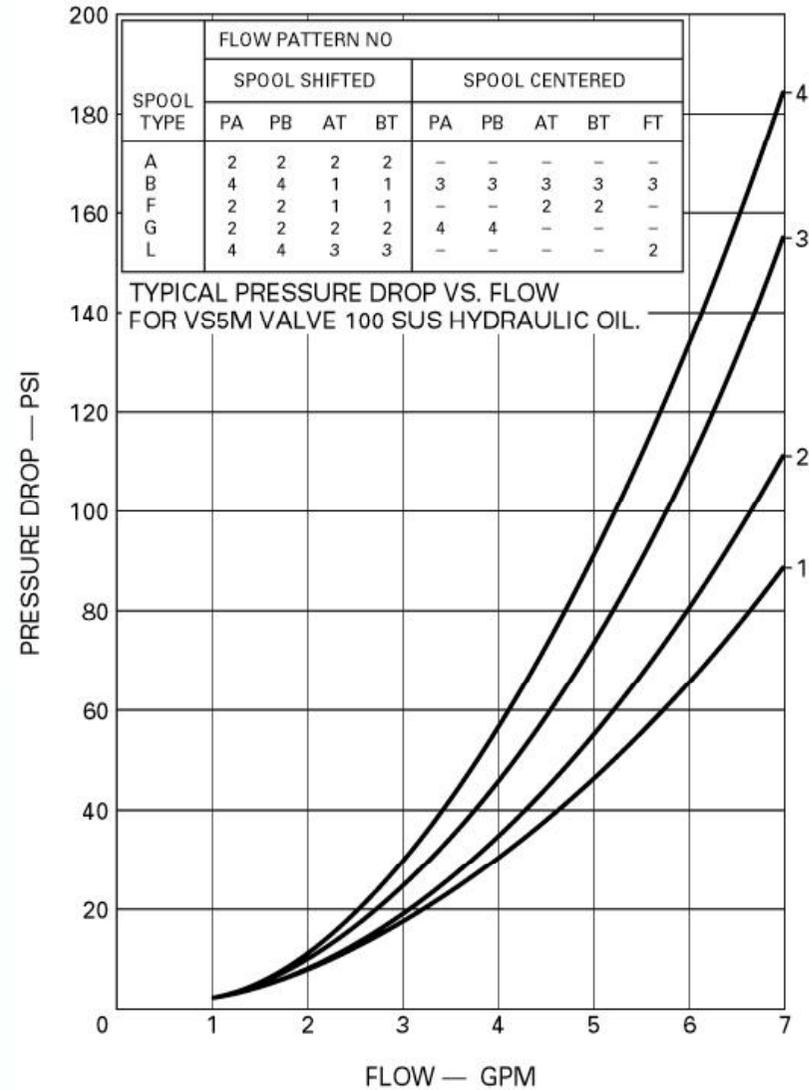
Ball check valve
(4.0)

Pressure Drop vs. Flow Rate Curves

Directional Control Valve



Orifice Flow :



4.8 Equivalent-Length Technique

- The equivalent length of a valve or fitting

$$H_{L(\text{valve or fitting})} = H_{L(\text{pipe})}$$

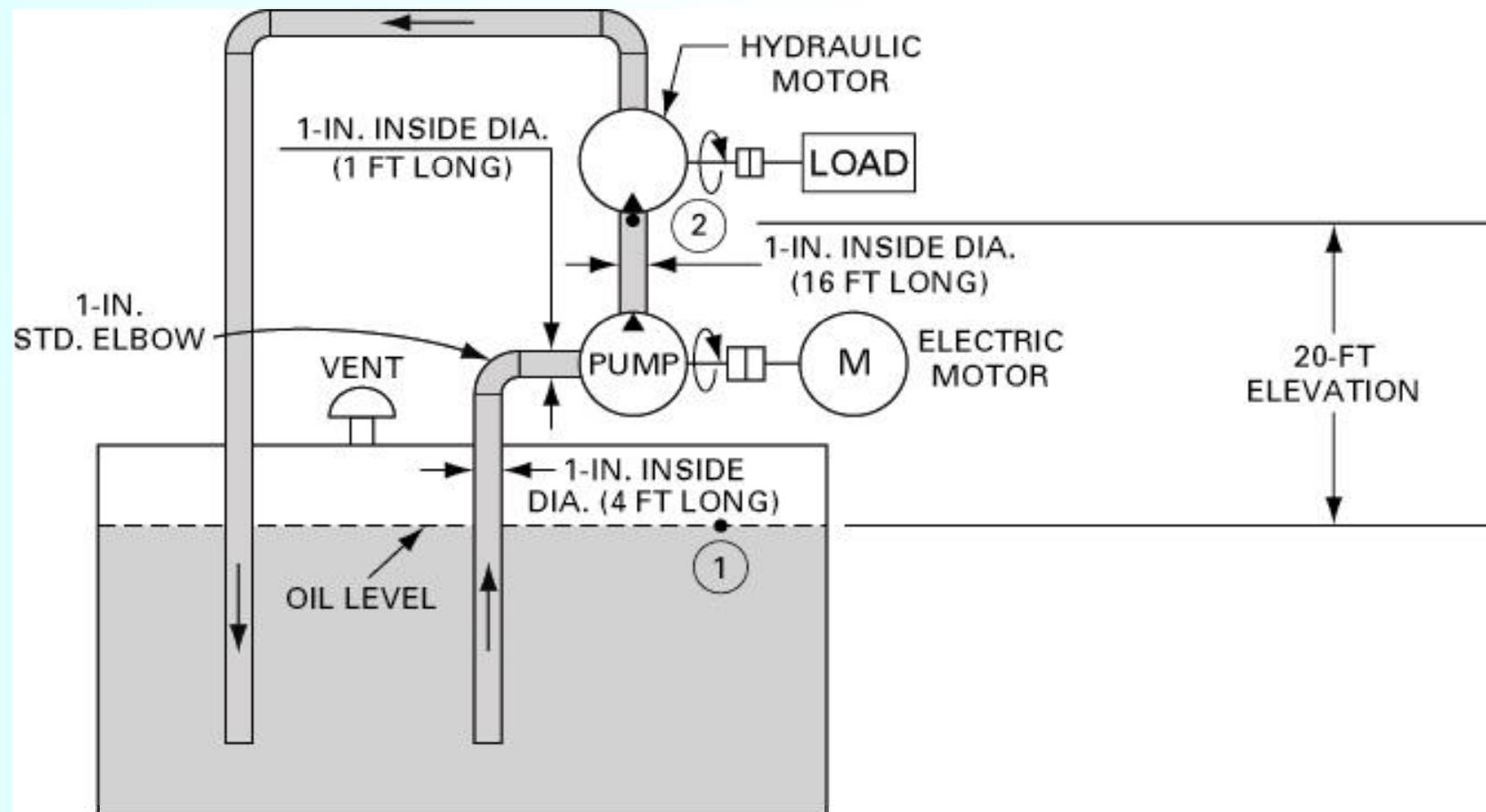
$$K \left(\frac{v^2}{2g} \right) = f \left(\frac{L_e}{D} \right) \left(\frac{v^2}{2g} \right)$$

$$L_e = \frac{KD}{f}$$



4.9 Hydraulic Circuit Analysis

■ Energy losses due to friction



4.A Bulk Modulus

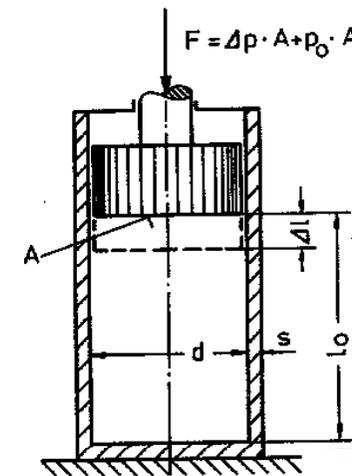
■ 체적 팽창 계수(β)의 정의

- 압력의 변화량을 체적의 변화비로 나눈 값

$$(\beta = -\frac{\Delta P}{\Delta V / V} = -\frac{\Delta P}{\Delta V} V)$$

- 유체시스템의 스프링(감쇠) 효과를 나타내는 변수
- 유체 압축율 (compressibility)의 역수를 나타낸다. 즉, 유체의 압축율이 크면 체적 팽창 계수가 작고 유체의 압축율이 작으면 체적 팽창 계수가 크다

- 그림에서 초기에 $P_o \cdot A$ 의 힘이 작용하는 피스톤에 $\Delta P \cdot A$ 만큼의 힘을 추가로 가하면 밀폐된 실린더 내의 체적 변화량은 체적 팽창계수 β 를 사용하여 다음과 같이 나타낼 수 있다.



$$\Delta V = A \cdot \Delta l = V_o \cdot \frac{\Delta P}{\beta}$$

Effective Bulk Modulus

- 유효체적팽창계수 (β_e : Effective bulk modulus)
 - 실제 체적 팽창 계수는 유체에 포함된 공기의 양 또는 용기 자체의 부피 변화등에 따라 민감하게 영향을 받는다
 - 이러한 여러 가지 변수의 영향을 고려한 체적 팽창 계수를 유효 체적 팽창 계수라 한다.

Example: Effective Bulk Modulus

- 예를 들어 gas와 유체가 혼합된 flexible 용기의 초기 체적은 다음과 같으며,

$$V_t = V_l + V_g$$

피스톤의 움직임으로 압력이 ΔP 만큼 증가하면,

$$\Delta V_t = -\Delta V_l - \Delta V_g + \Delta V_c$$

가 되고 전체 유효 체적 팽창 계수는 다음과 같이 표현 된다.

$$\beta_e = V \cdot \frac{\Delta P}{\Delta V_t}$$

각 부분의 체적팽창계수

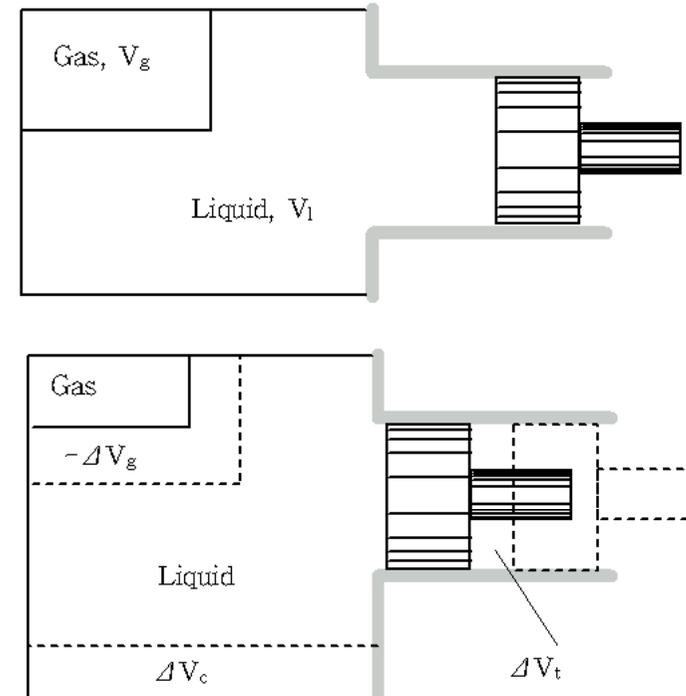
$$\beta_l = -V_l \cdot \frac{\Delta P}{\Delta V_l} \quad \beta_g = -V_g \cdot \frac{\Delta P}{\Delta V_g}$$

$$\beta_c = V_c \cdot \frac{\Delta P}{\Delta V_c}$$

식을 결합하면,

$$\frac{\Delta V_t}{V_t \Delta P} = \frac{1}{\beta_l} + \frac{S}{\beta_g} + \frac{1}{\beta_c} = \frac{1}{\beta_e}$$

$$S = \frac{V_g}{V_l} \approx \frac{V_g}{V_t}$$



Bulk Modulus in Gas & Container

■ Gas

■ Polytropic Process

$$pv^n = k$$

■ isothermal compression: $n=1$

■ adiabatic compression:

$$-V \frac{dP}{dV} = nP = \beta_g$$

$$\beta_g = P$$

$$\beta_g = \frac{C_p}{C_v} P = \gamma P$$

■ Container (for steel pipe)

■ thin-walled cylinder

■ D: inner diameter of pipe

■ t: wall thickness

■ E: modulus of elasticity

■ thick-walled cylinder

$$\beta_c = \frac{tE}{D}$$

$$\beta_c = \frac{E}{2(1+\nu)} \approx \frac{E}{2.5}$$

Bulk Modulus: Effect of air

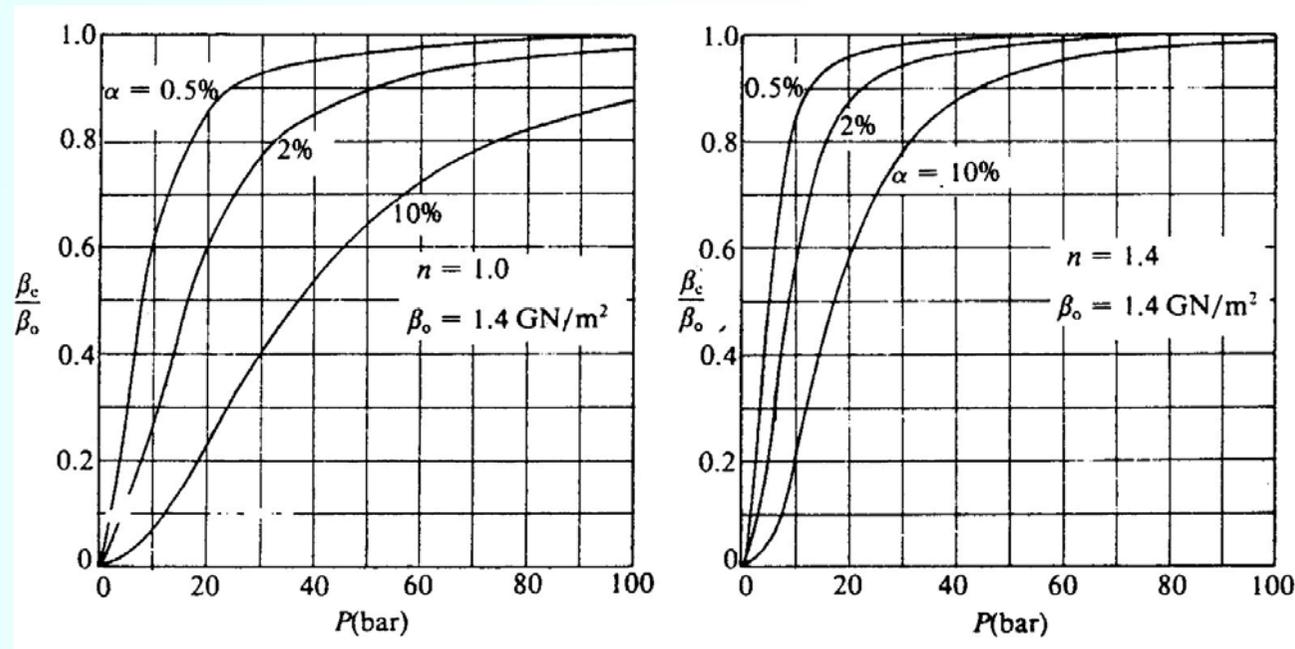
- 유체 내에는 항상 어느 정도의 공기가 포함되어 있으며 일반적으로 이것에 의한 체적 팽창 계수의 변화는 무시할 만하다.
- 그러나 유체 내의 공기가 **bubble** 형태로 존재할 경우에는 체적 팽창 계수에 많은 영향을 주게 된다.
- 유체 내의 공기에 의한 체적 팽창 계수의 변화는 다음 식으로부터 추정할 수 있다.

$$\frac{\beta_e}{\beta_o} = \frac{\left(\frac{P}{P_o}\right)^n + \alpha}{\frac{\alpha \beta_o}{n P} \left(\frac{P}{P_o}\right)^n}$$

- 여기서 α 는 공기와 오일 체적의 비 (V_a/V_o)
- β_o 는 오일의 체적 팽창 계수
- P_o 는 대기압
- P 는 유체의 작동압력
- n 은 압축과정에 따른 계수

Bulk Modulus: Effect of air

- 그림에는 유체 내에 함유된 공기 양에 따른 유효 체적 팽창 계수와 체적 팽창 계수비의 변화를 압력에 따라 나타내었다. 이때 등온(**Isothermal**) 압축의 경우 $n=1$ 이며, 단열(**adiabatic**)의 경우에는 $n=1.4$ 이다.
- 일반적으로 작동 유체의 압력이 높아질 수록 유체내의 공기에 의한 체적 팽창 계수의 변화는 그 영향은 줄어들게 된다.



공기 함유량에 따른 체적팽창 계수의 변화

4.7.A Orifice Flow

■ Orifice Flow

- 유동에 있어 갑작스런 제한을 가하는 부분
- **orifice**를 막 통과한 유체의 속도는 연속법칙을 만족시키기 위하여 상류의 속도보다 더 증가하게 된다.
- **Turbulent Orifice Flow: 대부분의 orifice Flow**
 - 관성력이 지배적
 - 유체 입자들의 가속에 의한 압력 강하 발생
- **Laminar Orifice Flow**
 - 점성력이 지배적
 - 유체 점성에 의한 내부 전단력에 의한 압력 강하 발생



Turbulent Orifice Flow

- orifice 양단의 1과 2에서 Bernoulli's Equation 적용

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

$$V^2 \Big|_{u_1}^{u_2} = -\frac{2}{\rho} P \Big|_{P_1}^{P_2}$$

$$u_2^2 - u_1^2 = \frac{2}{\rho}(P_1 - P_2)$$

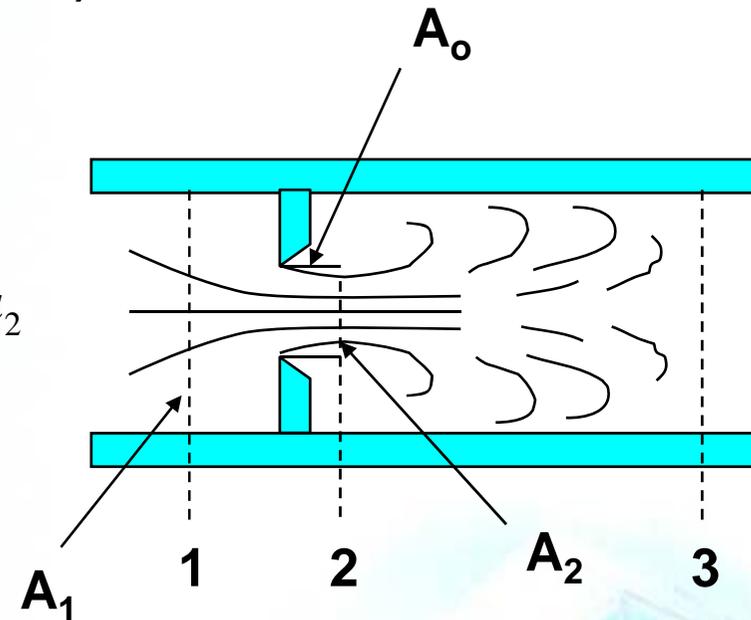
- 비압축성 유체에서의 연속 방정식

$$A_1 u_1 = A_2 u_2 = A_3 u_3 \quad u_1 = \frac{A_2}{A_1} u_2$$

$$u_2 = \frac{1}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{\frac{2}{\rho}(P_1 - P_2)}$$

$$Q = C_v A_2 u_2$$

$$Q = \frac{C_v A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{\frac{2}{\rho}(P_1 - P_2)} = C_d A_o \sqrt{\frac{2}{\rho}(P_1 - P_2)}$$



Orifice Flow Equation

■ Orifice Flow Equation

$$Q = C_d A_o \sqrt{\frac{2}{\rho} (P_1 - P_2)}$$

- C_d : Discharge coefficient
- C_v : Velocity coefficient
- C_c : Contraction coefficient
- A_o : Orifice area
- A_2 : Vena-contracta area
- P_1 : Pressure at 1
- P_2 : Pressure at 2

$$C_d = \frac{C_v C_c}{\sqrt{1 - C_c^2 (A_o / A_1)^2}}$$

$$C_c = \frac{A_2}{A_o}$$

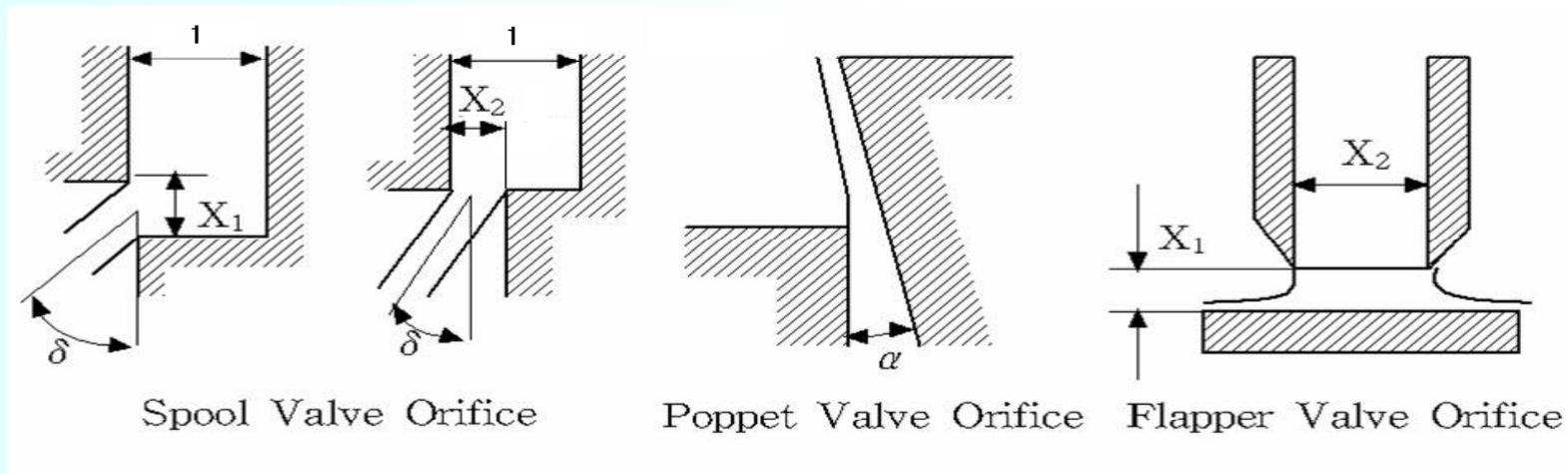


4.7.B Discharge Coefficient

■ Von Mises의 해석

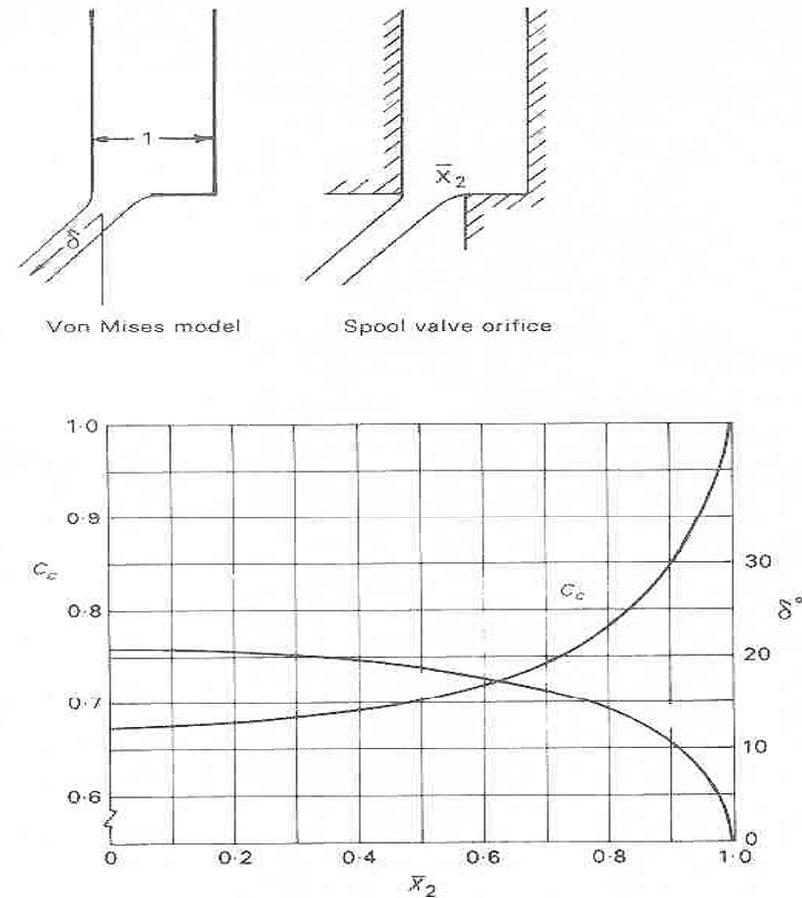
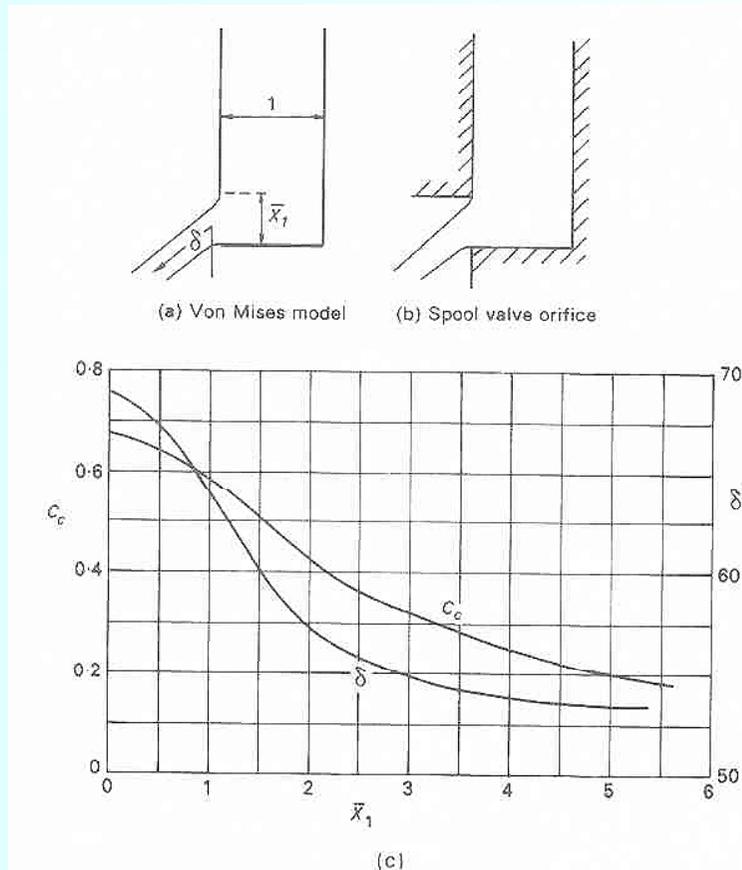
- Steady Flow
- No Friction Loss (Viscous effect are negligible)
- Irrotational Flow
- Two-Dimensional Flow
- Gravity Effect are negligible
- The Fluid is incompressible

■ 일반적인 Orifice 형상



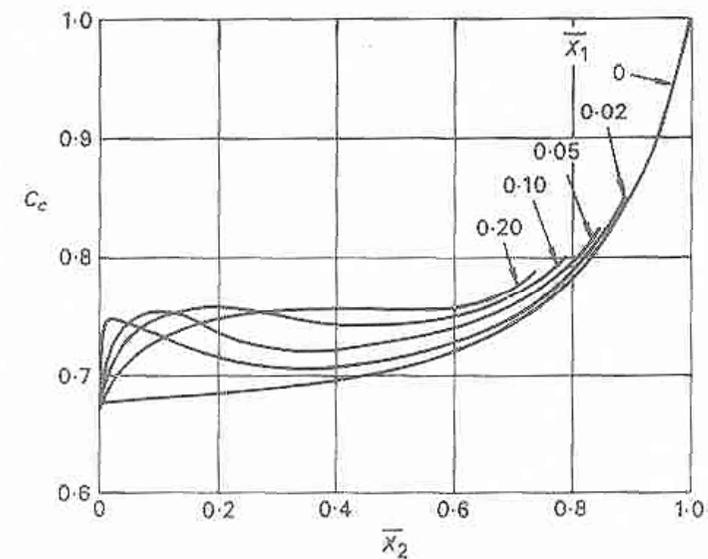
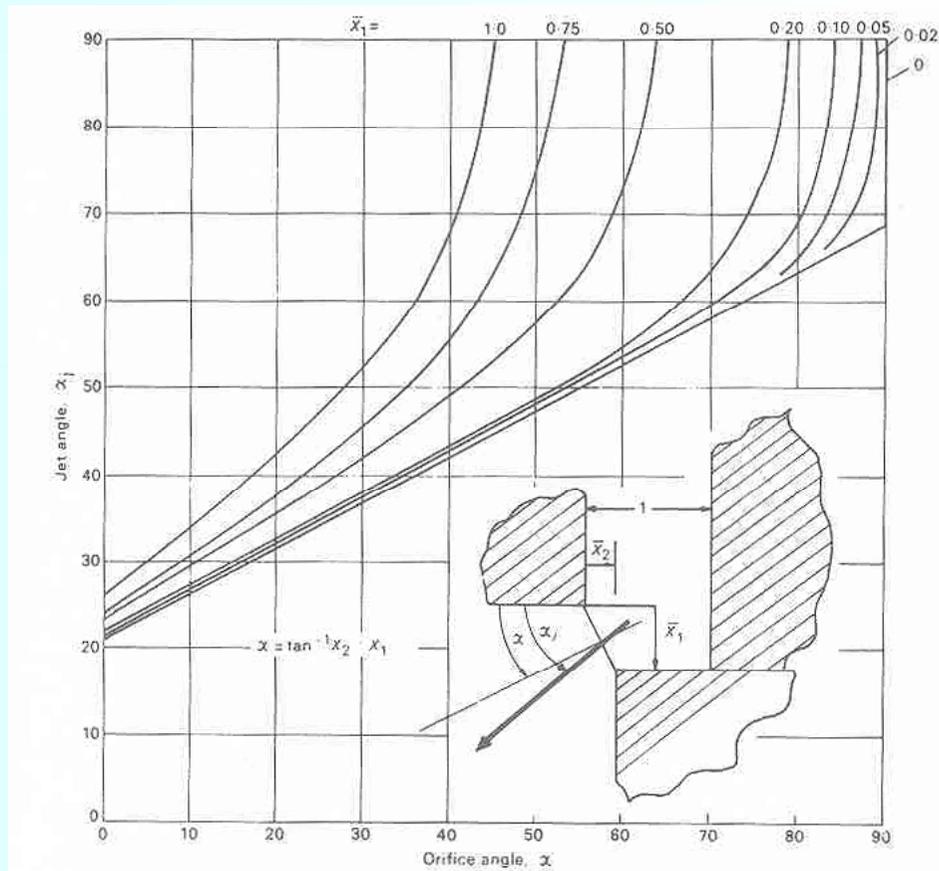
4.7.C Contraction Coefficient

■ Contraction Coefficient and Discharge Angle for Two-Dimensional Spool Valve



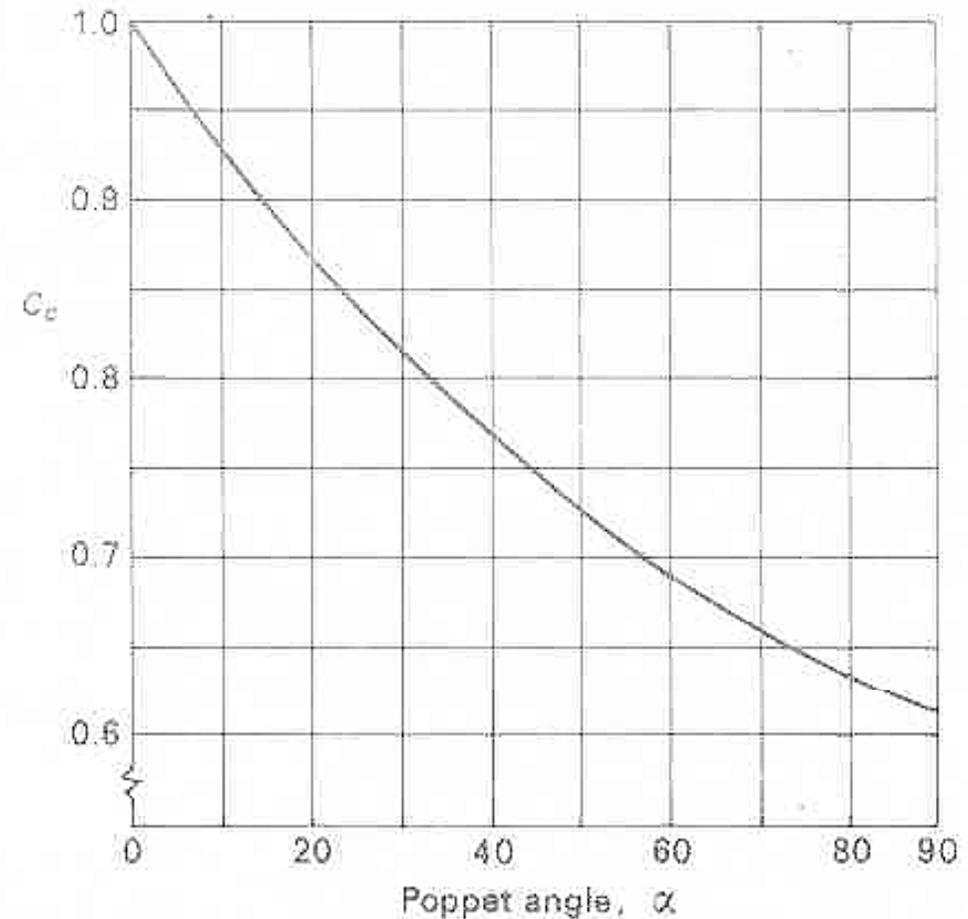
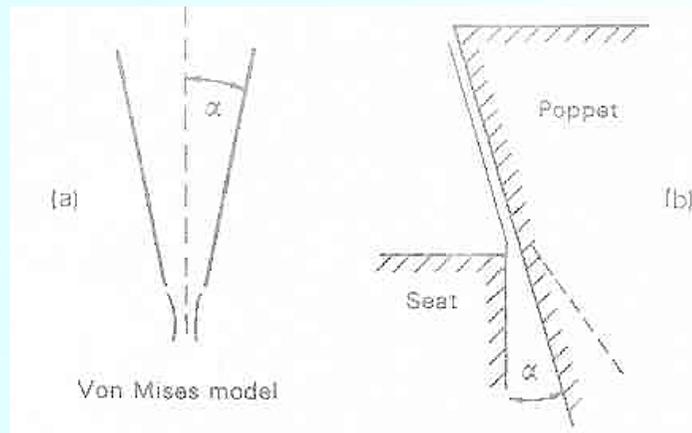
Contraction Coefficient

■ Jet Angle and Contraction Coefficient at Small Clearance for Two-Dimensional Spool Valve



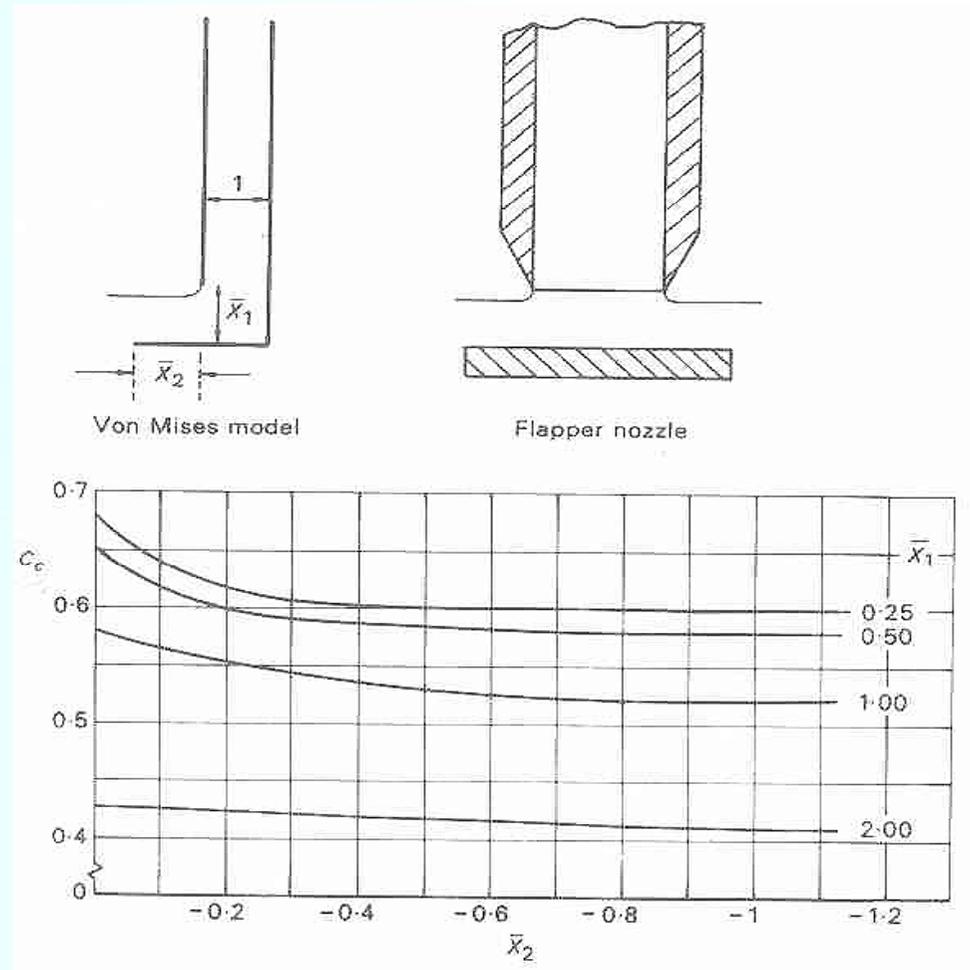
Contraction Coefficient

■ Contraction Coefficient Variation with Poppet Angle



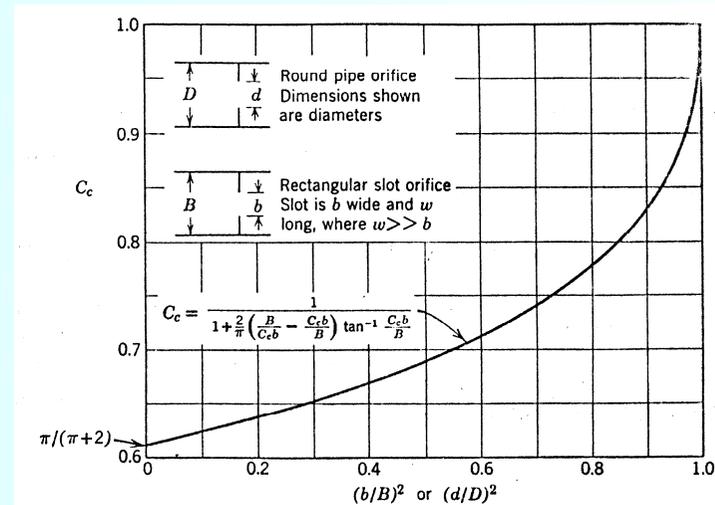
Contraction Coefficient

■ Contraction Coefficient for Flapper Nozzle

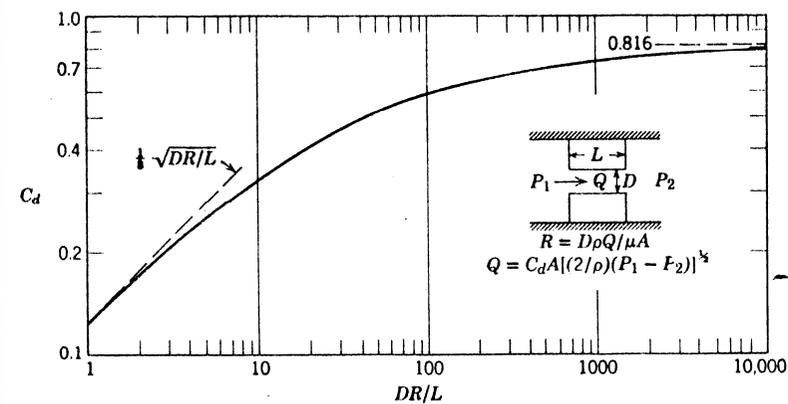


4.7.D Discharge Coefficient의 실험적 고찰

Slot Type Orifice

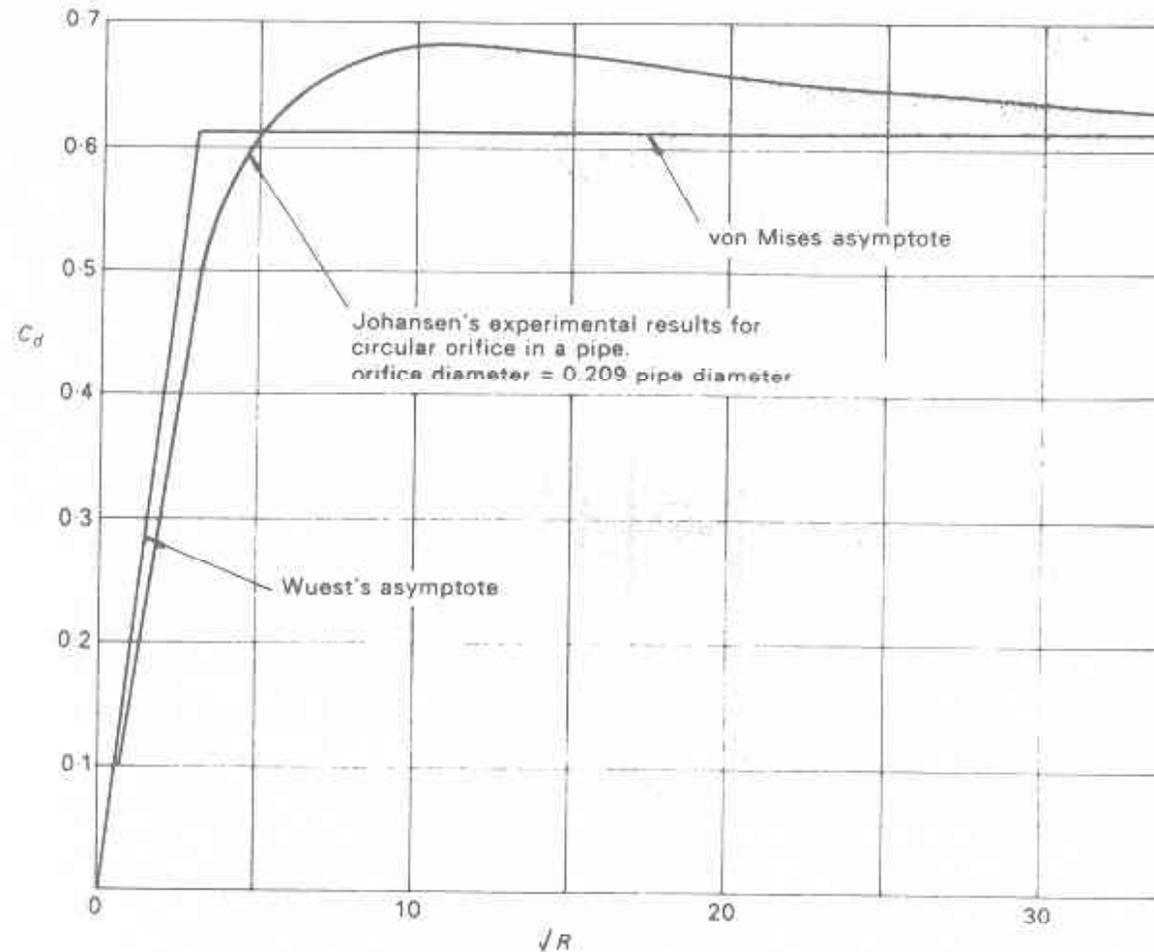


Short Tube Orifice



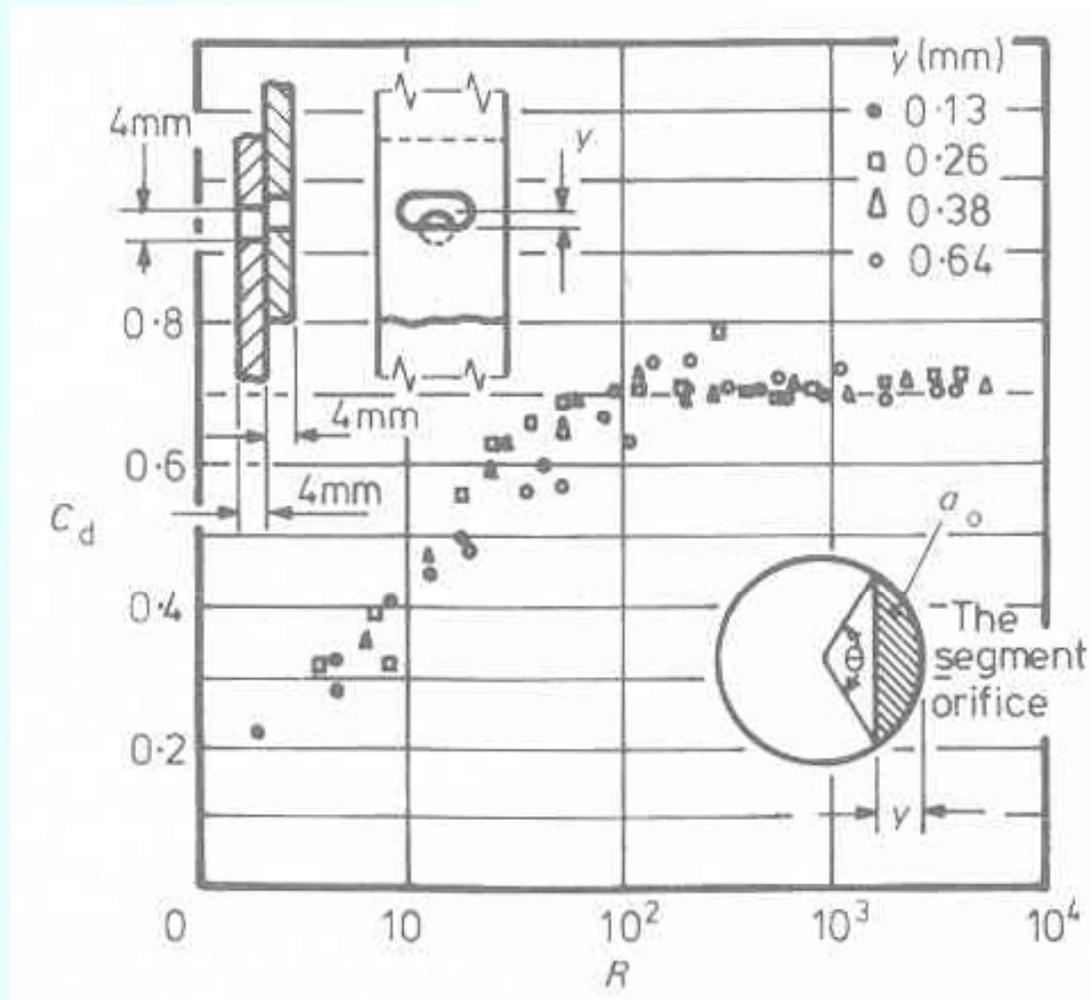
Discharge Coefficient vs Reynolds No

- for a circular sharp edged orifice in a pipe



Experimental Variation of Discharge Coefficient

■ for segmental orifice



Report

■ Text Problems

■ 4-22

■ 4-28

■ 4-36

■ 4-38

■ Due date: 2주 후

